Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 3
Lecture No 18
q and Delta dot derivatives

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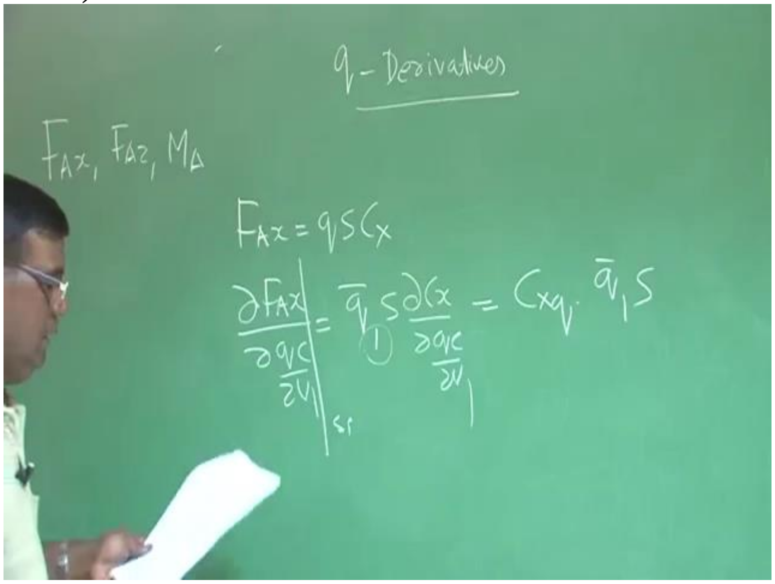

Now let us come to Q derivatives. We have done U derivatives, alpha derivatives, alpha dot derivatives. Now Q derivatives. Okay, again we start like, FAX, FAZ and MA. Why we are taking these 3 things? Please understand, we are trying to make longitudinal perturbed equation of motion. And these things we are doing to find out the perturbed forces when the aircraft is perturbed in the longitudinal mode.

That is either this, this or pitching motion. That is why, FAX, FAZ and MA are being considered. So I can easily write, FAX is equal to QS CX. To DFAX by DQC by 2U1 will be equal to Q1 bar SD CS by DQ C by 2U1. This 1 again because we have to evaluate this as steady-state. Okay. So this I can write as CXQ into Q bar S. It goes without saying, CX Q means DCX by DQC by 2U1.

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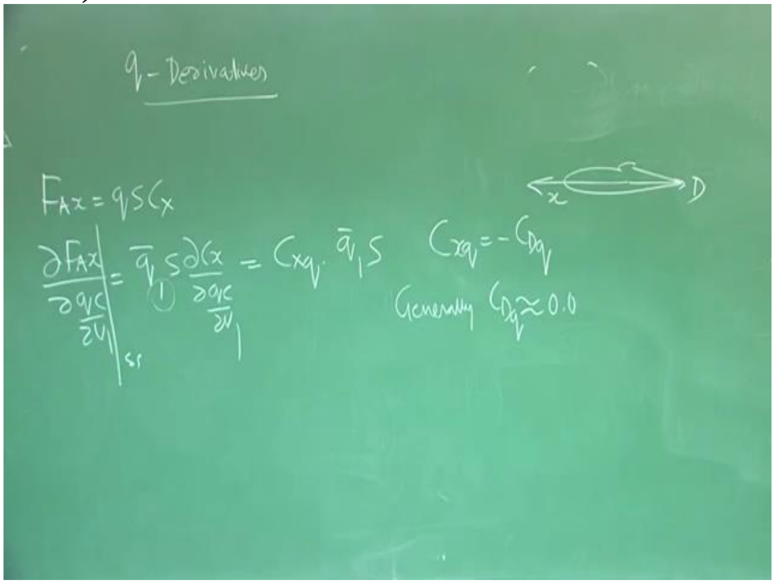

And for small angle, we can assume that this is - the DQ because you know the axiss system what we are using, X is in this direction. However, drag will be in the opposite direction. So there will be a - sign. And then, another relief for us, generally see DQ is very negligible. So we will take it as 0. Please understand, these assumptions are in no way restricting you to use finite value and go for complete analysis.

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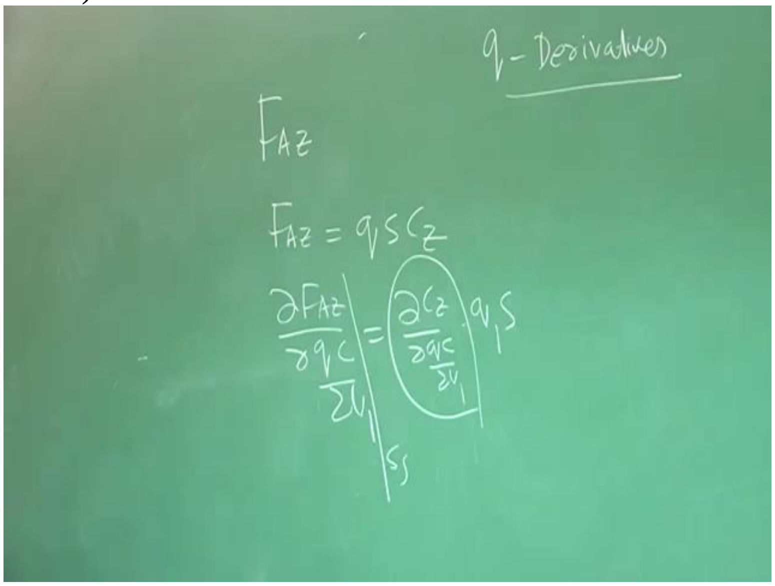

Now we will handle FAZ and we again write FAZ QSCZ and DFAZ by DQ C by 2U1 evaluated as steady-state is equal to DCZ by DQ C by 2U1 into Q1S. The catch point is, how to evaluate DCZ by DQ C by 2U1. Now let us see what is actually happening.

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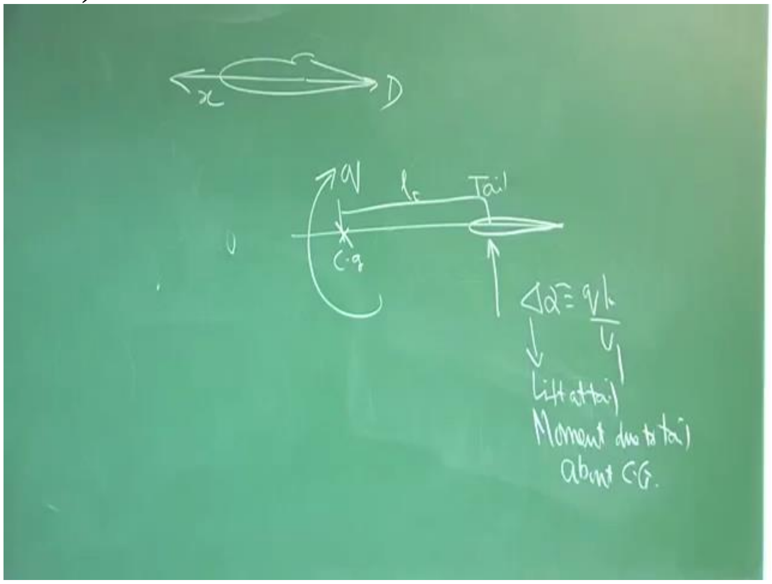

Suppose this is the tail and this is the CG of the airplane and if it is going for a Q, pitch up like this. What will happen? This man will go down. Train will go down. As it goes down, the relative at speed, it will see. And roughly I can say, Delta Alpha equal to QLT by U1. I am saying, this is from AC. This is LT. I am assuming, net force will be acting, represented at the AC of the tail. And this Delta Alpha will give lift at tail.

Also it will give moment due to tail about CG. Because for us, moment about CG is extremely important for analysing its response. With this understanding, we have a clear cut indication that this is one of the important derivatives. Let us evaluate this.

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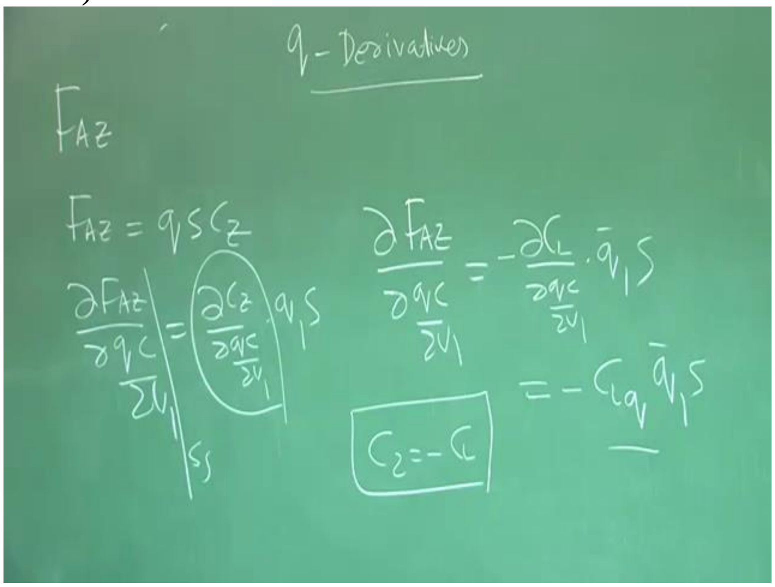

You can see yourself, I can write DFAZ by D QC by 2 U1 is equal to - DCL by DQ C by 2U1 into Q1S. How do I write this? Because I know, this is a fair enough assumption, CZ is equal to - CL when we are talking about small perturbation. So DCZ by DQC by 2U1 is - DCL by DQ C by 2U1. Or this is by an audition, it is written as - CL Q into Q1 S. The question comes, can you derive the expression for CL Q?

Answer is very obvious. Because you know the mechanism how this CL comes because of Q. It comes through Delta Alpha. So it should be very straightforward for us and let us find out the expression for CLQ.

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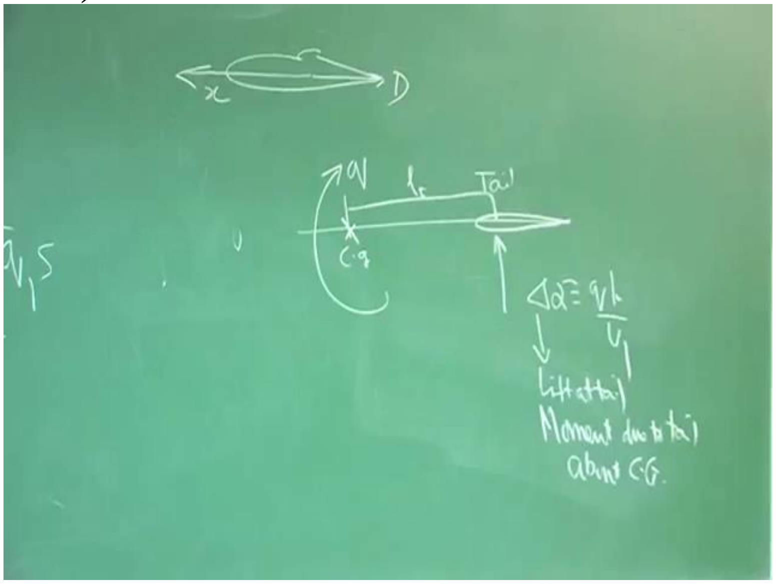

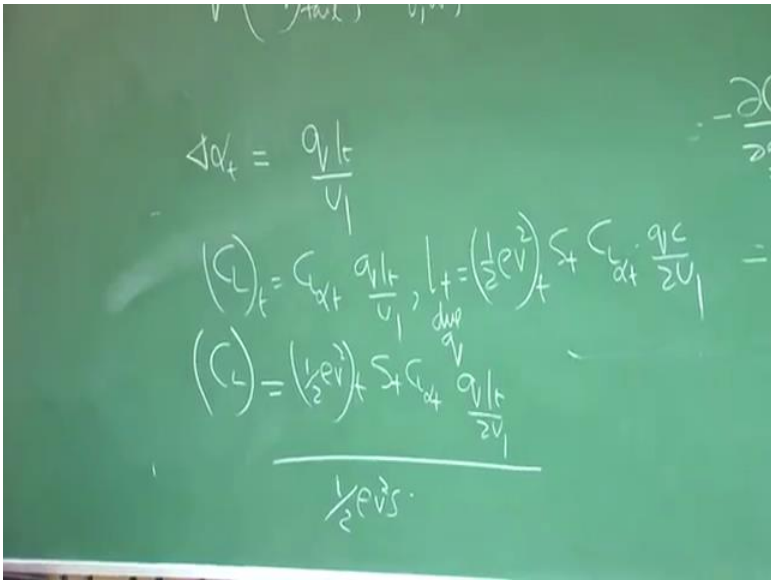

Please understand, this CLQ could be because of CLQ tail. Then because of CLQ and also fuse Lodge and other components but we are mostly convinced that CLQ tail will be predominant. So we will try to derive the expression for CLQ tail. So we know that Delta Alpha tail because of Q is Q LT by U1 which we have demonstrated here. So what is the CL at the tail? It will be CL alpha tail into LT by U1.

So what is lift at the tail because of Q due to Q will be equal to half Rho V square at tail S tail into CL which is CL alpha tail into Q C by 2U1. If you want to find out CL based on free stream dynamic pressure, then this will be half Rho V square tail into S tail CL alpha tail Q LT by 2U1 divided by half Rho V square S free stream. That is all. And you know, half Rho V square tail by half Rho V square free stream is Neta.

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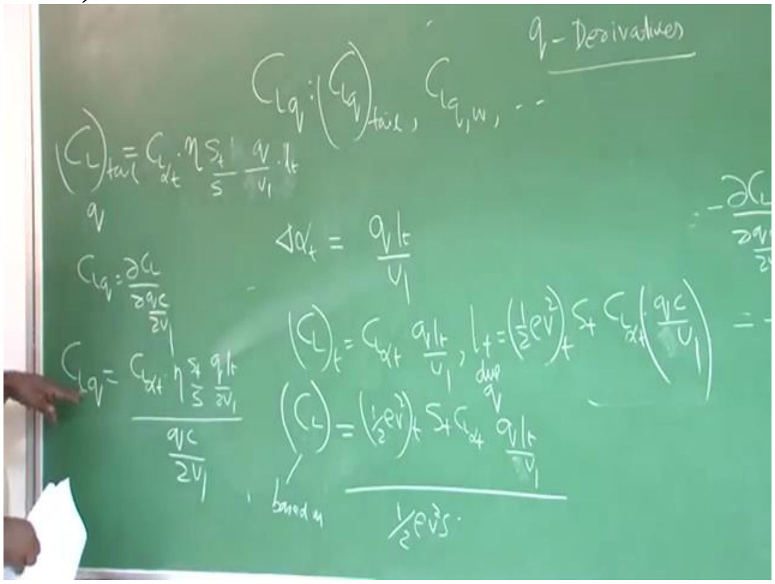

So I can write CL tail because of Q as CL alpha tail into Neta. This is Neta. Then ST LT by S. And Q by 2U1. CL alpha. Half Rho V square by this is Neta. Then L will be LT by C. No, there is no C. So this will be LT. So let me check again.

Half Rho V square tail by half Rho V square free stream is Neta. Neta is here. CL alpha tail. Yes it is here. Q LT by 2U1 is here. Everything is here. S is also here. But S is nothing but S is the wing. Generally, we use that. And if I want to find out CLQ vs DCL vs DQ C by 2U1, the best way to find CLQ is to simply divided by QC by 2U1 because they are linear. QLT by 2U1 divided by QC by 2U1.

This will not be 2. Please understand, this will not be 2. This will be Q LT by U1. That is a mistake from my side. CL alpha tail into QC by U1. I repeat, lift due to Q is half Rho V square tail ST CL alpha tail into alpha. This alpha is nothing but Q LT by U1. So there was a mistake here. So you correct it.

So CL will be, this CL is this an allograft reference, based on free stream dynamic pressure. So I divide by half Rho V square free stream into S. So this to this is Neta. ST CL alpha tail Q LT by U1. And once you want to find out CLQ, this 2 is not here. So I divide by this and then I get an expression which is very well-known which I have done in my last course also. You can refer to that.

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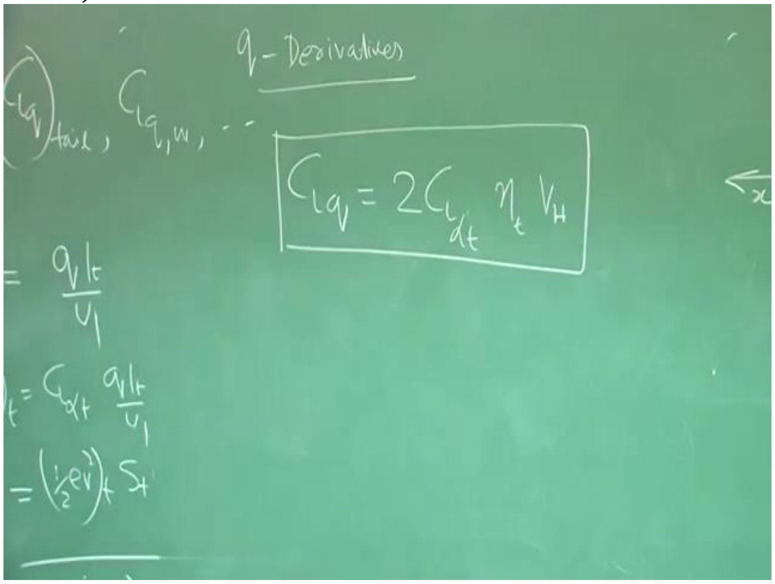

So I will get a CLQ expression as 2 CL alpha tail into Neta tail VH. That is all. See here. If I do this, this and this get cancelled, U1, U1 get cancelled. This 2 goes up. So 2 CL alpha tail. Neta, this Neta T is here. ST LT by S, this C bar, makes it VR. So VH is here. So this is the expression for CLQ.

Note that sign of CLQ is positive which is true because as it goes pitch up it pushes the air down so that the lift is upwards. But then the question is that CLQ will give a nose down moment which is negative. And how do I find that? Very simple. I multiply this with tail moment arm.

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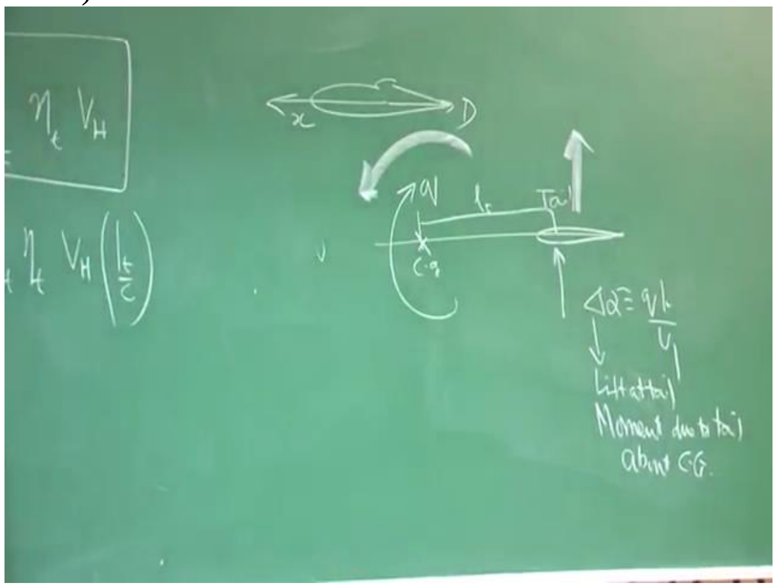

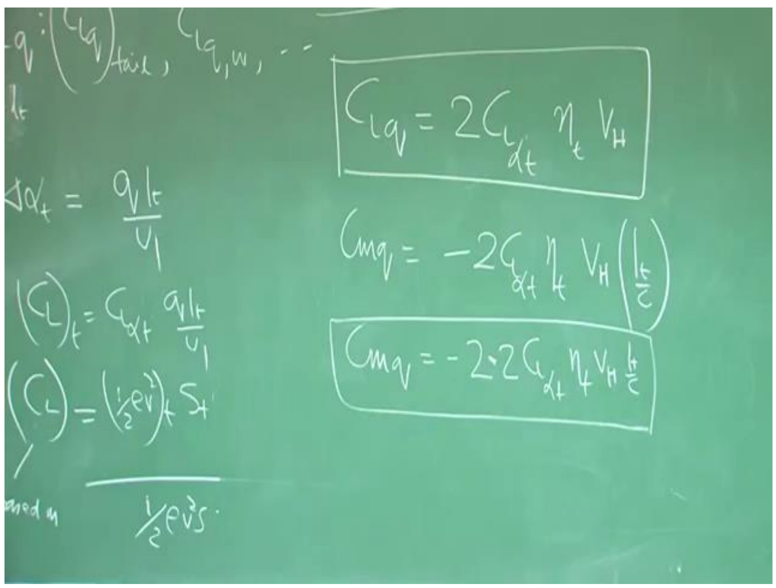

So from here, I write CM Q will be 2 CL alpha tail Neta T VH into LT by C. But I will be wrong if I do not take care of the sign. Because I know because of Q there will be a force upward which will give me a nose down moment. So I have to put a - sign but as I told you this is the expression for a contribution by tail. But generally, wing also will contribute, fuse Lodge will also contribute.

So you can, roughly at the design stage , you can take CM Q as - 2.2 CL alpha tail Neta T VH LT by C. This is also a very recommended value or expression at design stage. So we know the expression of CLQ and CMQ. We derived this expression just to convince you that although they look very nasty but very simple to estimate.

I know CL alpha tail. I know the ratio of the dynamic pressure. It may be 0.9 or 0.95. I know tail volume ratio. What is the problem? I can find out this derivative. As a designer, we always associate all these derivatives to numbers. Unless you develop that feel, you are not really for designing an aircraft. Typical value of CLQ you will find it will be between 10 to 15 and CMQ also could be - 10 to - 20. Around these typical values for CLQ and CMQ.

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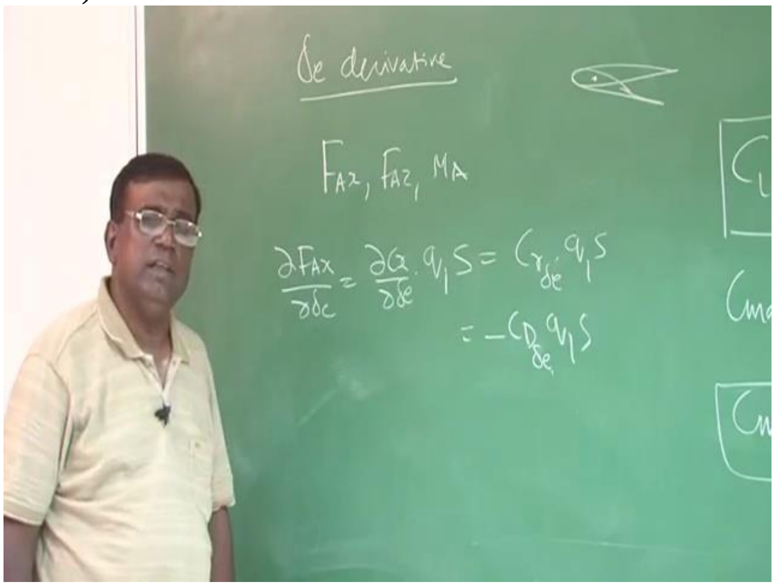

Now what we are left with is the Delta E derivative. So let us see that also out of all these expressions. Because our main aim is to go back these equations and try to understand the short period equations of motions in terms of how to solve them, how to get useful information? These are all meanwhile creating input for solving those equations.

Again we start with because longitudinal is what we are talking about, FAX, FAZ and MA. I will go very straight. DFAX by D Delta E I will write as DCX by D Delta E into Q1S which I can write as CX Delta E into Q1S which is nothing but - CD1 Q1S because - CD1 is - CD Delta E evaluated at steady state into Q1S.

What is the physical meaning of CD Delta E? See, whenever you deflect the elevator, definitely there is going to be a change in lift as well as drag and that is, CD Delta E is reflecting. That is why when you try to trim an aircraft at a most optimal altitude where mostly you will be flying, you ensure that the elevator deflection as almost 0.

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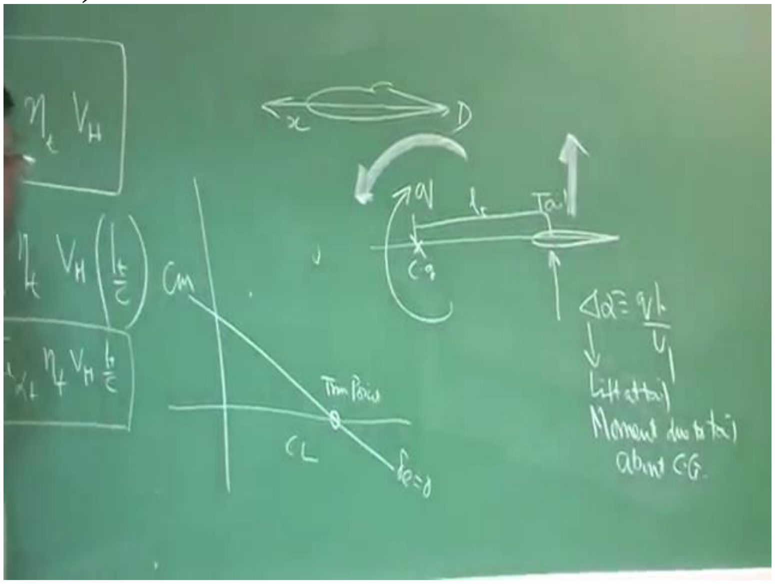

That is you design your aircraft in such a way that your CM vs CL graph looks like this and this is your trim point for Delta E equal to 0. So that your this contribution to drag is 0.

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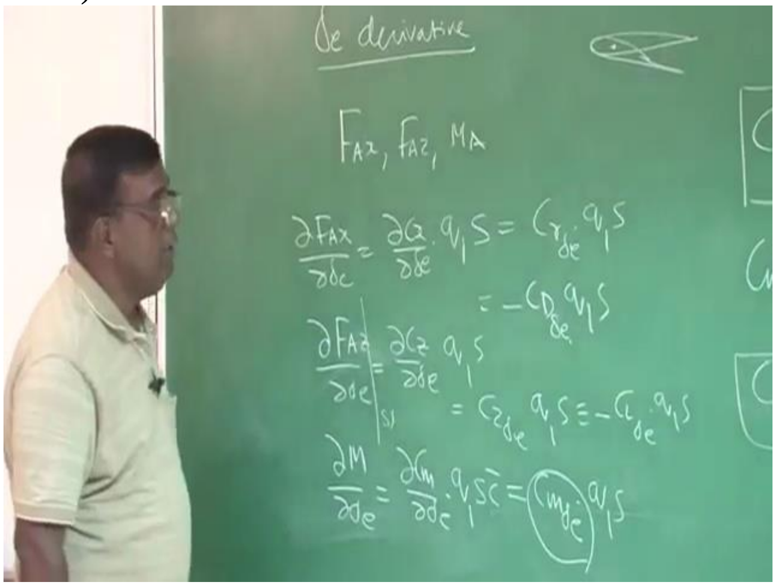

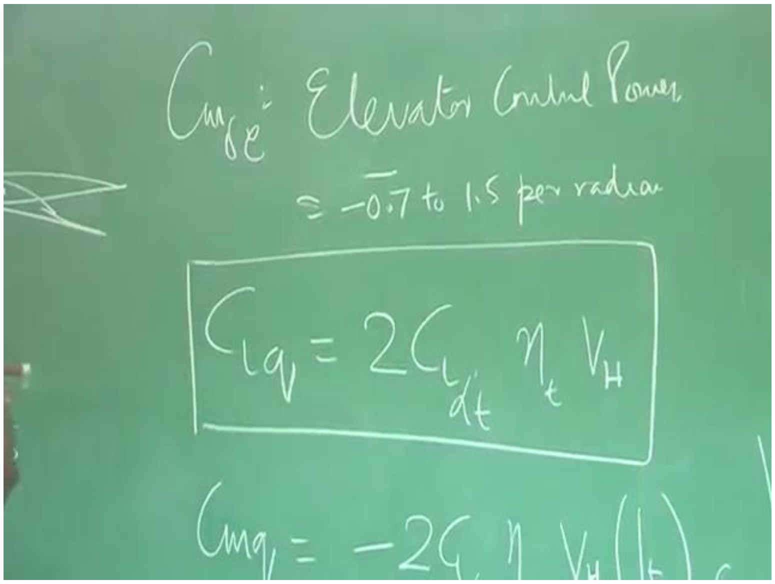

Similarly if I want to find DFAZ by D Delta E this will be DCZ by D Delta E Q1S. It goes without saying that 1 means steady-state because these have to be evaluated at steady state and I am sure you understand this is equal to CZ Delta E into Q1S. And this is - CL D Delta E into Q1S. Because CL and CZ are opposite signs for small perturbation.

And what is CL D Delta E? If deflect the elevator, how much it will generate CL. so these are not very difficult things. Similarly if I write DM by D Delta E, it will be DCM by D Delta E into Q1S C bar and this is very popular, very strong derivative, CM Delta E Q1S. And what is CM Delta E? It is an elevator control power. A typical value is - 0.7 to 1.5 per radian.

So that completes this derivation of all those derivatives which will be required to solve longitudinal perturbed equations of motion. My strong advice will be, please spend some time with pen and pencil and you can also refer my last course were also I have talked about this but in this course, the objective is different. I may be going a little faster because this is not the primary objective.

Primary objective is to build a dynamic stability criteria so that we can design a stability augmentation system. Those who are interested, please follow that module also and derive these expressions yourselves once. And you need to do it once only. That is all. After that, you remember the formula, note down at some worksheet. Whenever you require, you can refer it, calculate it.