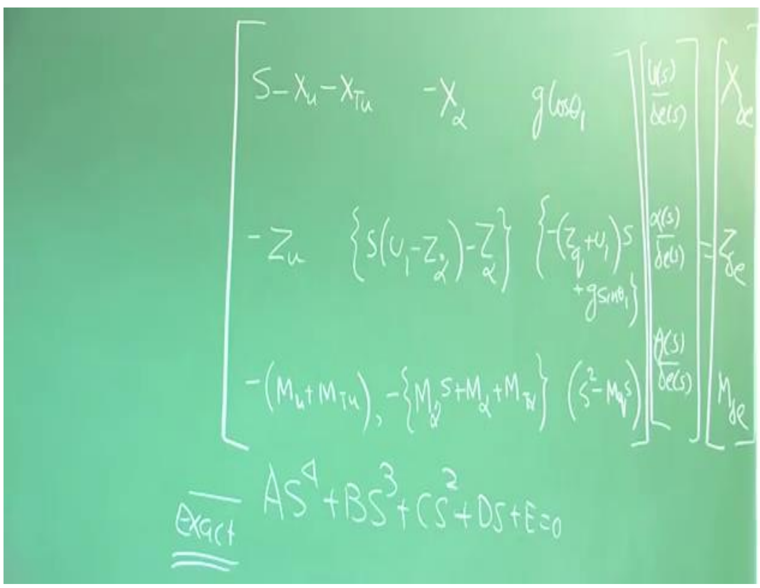
Aircraft Dynamic Stability & Design of Stability Augmentation System

Professor A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology Kanpur

Module 4
Lecture No 24
Long Period Mode (Phugoid) Approximation

Dear friends, we are continuing our discussion on short period mode. And you know what exactly we are talking now is about approximation. Why we are doing this? We want to get a simplified equation so that some numbers, some parameters we can identify which will help us in analysing the dynamic stability of an airplane.

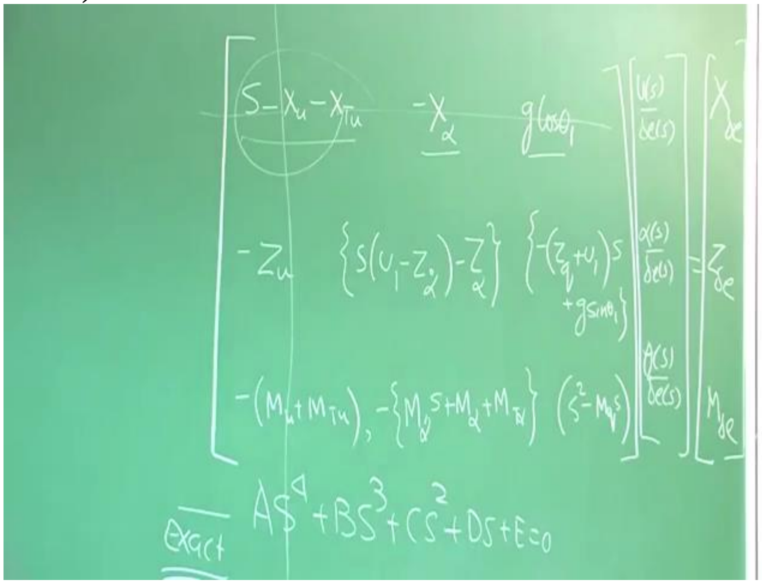
(Refer Slide Time: 01:06)



In this short period approximation, this is important, this is an approximation because we know that the exact is here. This is the exact. We have to find the roots of this equation. And generally for business type aircrafts, conventional aircrafts, you will find that it has 2 complex pair of complex conjugates. And in 1 complex conjugate pair, we find that the real part is large negative as compared to the other conjugate pair.

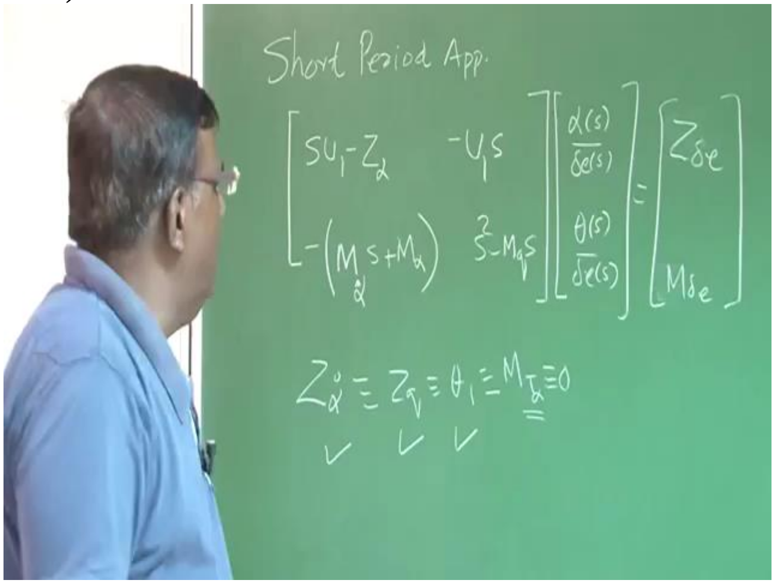
And we try to understand the physics behind it. And we know now that the airplane in longitudinal mode can get excited primarily in short period. Like, it just oscillates like this and comes back to equilibrium in a very short time. And what are the assumptions? That during this short time, total velocity remains constant. That means, the perturbed U is 0. So one approximation is, perturbed U is 0. And this understanding will help us in making this matrix in a reduced order form.

(Refer Slide Time: 02:38)



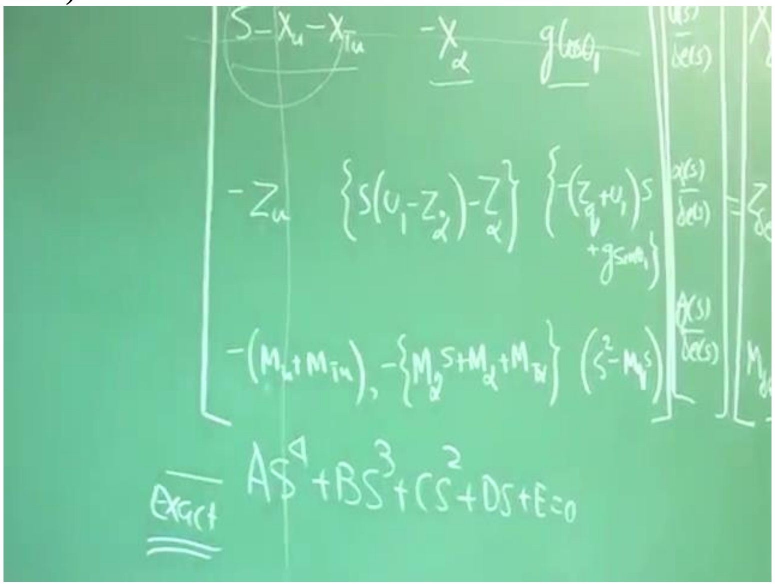
What we did was, since the perturbed U is 0, that means this equation, X - XU, this into U, this into this, this into this equal to X Delta E. That is a superfluous equation because there is no change. So we are dropping this equation. second thing we realise, since U of S, perturbed U is 0, so this multiplied by this becomes 0, this multiplied by this becomes 0. So we also delete this. Okay, clear? This into this, this into this makes no sense. So we also delete this part of the matrix.

(Refer Slide Time: 2:56)



And finally we have a 2 by 2 matrix of this form. But be careful, we have assumed Z alpha dot, ZQ, Theta 1 that is airplane Theta, pitch angle is 0. And we have also put MT alpha equal to 0. But generally, you can understand that MT alpha is not 0. But no problem.

(Refer Slide Time: 3:17)

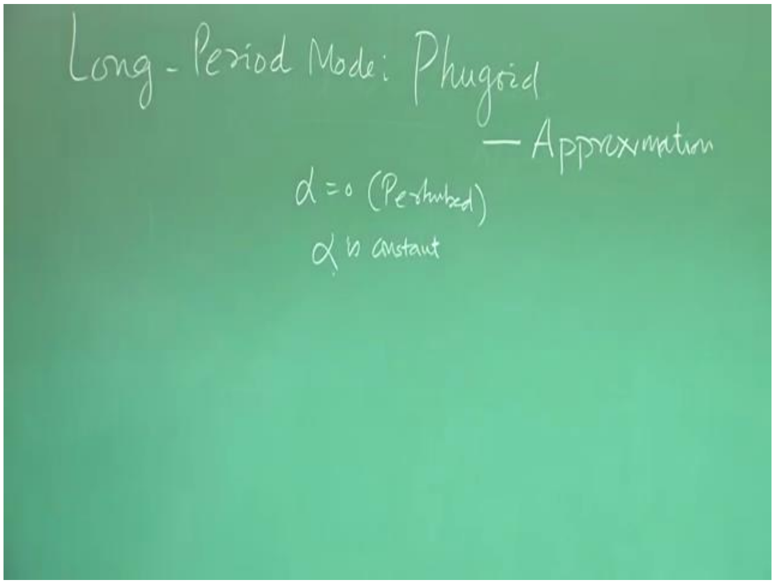


We know here, M alpha and MT alpha. So this MT alpha, the value if you know, it can be clubbed into M alpha. So for us, and the petition could be, M alpha has both components because of aerodynamic moment as well as because of moment due to thrust as a function of alpha or as a variant of alpha. In a sense, how does it change with alpha?

So this becomes a very very simple matrix to handle. And from there, we found out what is Omega N short period which we got through approximation. The damping ratio, through this approximation we got. Then we compared with the exact solution and we found for this airplane, the values are fairly close and we say generally, short period approximation is a good approximation.

Now coming back to the other mode in which the airplane generally gets excited in longitudinal plain is, one is short period like this. Another is it can go like this and then damped out. This is long period and that is called Phugoid mode. So we will be now talking about Phugoid mode.

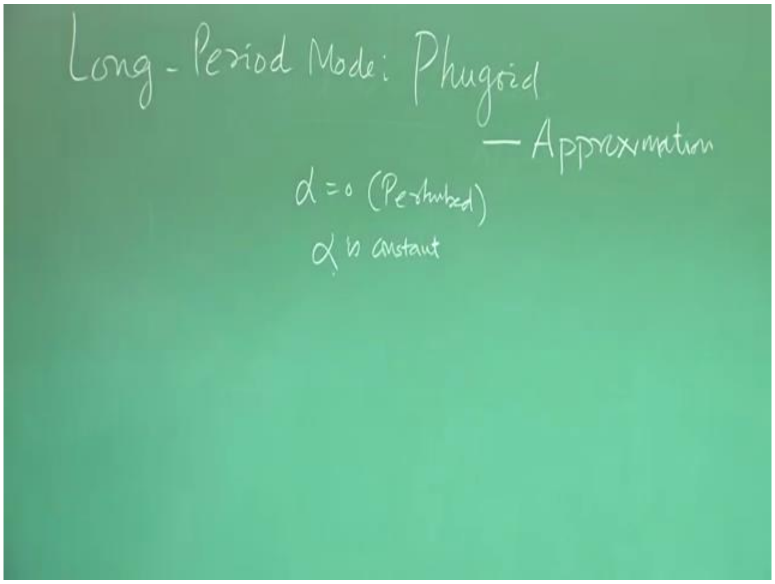
(Refer Slide Time: 4:30)

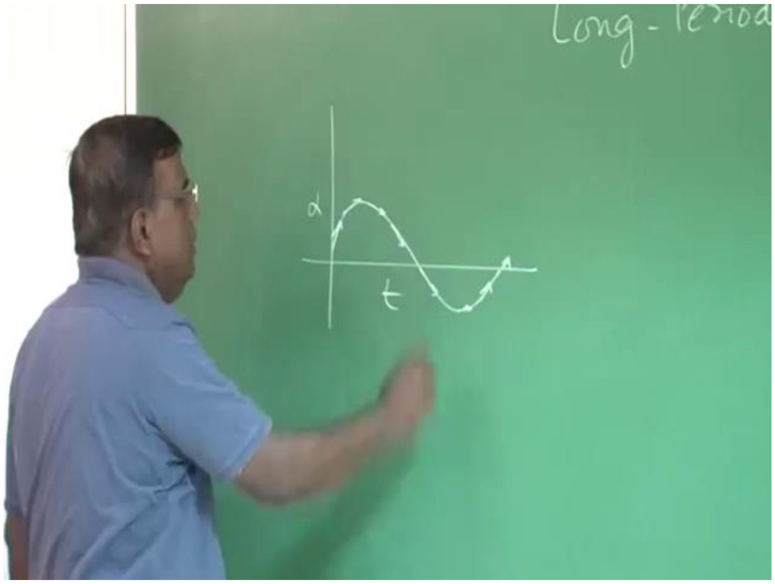


So we are talking about long period mode or Phugoid, again the keyword is approximation. In the long period mode, it is something like this. It gets excited like this and then comes back to equilibrium. It is not a very good but okay okay approximation if we assume alpha remains constant. That means perturbed alpha is 0.

This is not a good approximation for your information but okay, it is a classical approach. And you will see that this also will give us some very relevant insights to understand the longitudinal dynamics of an airplane.

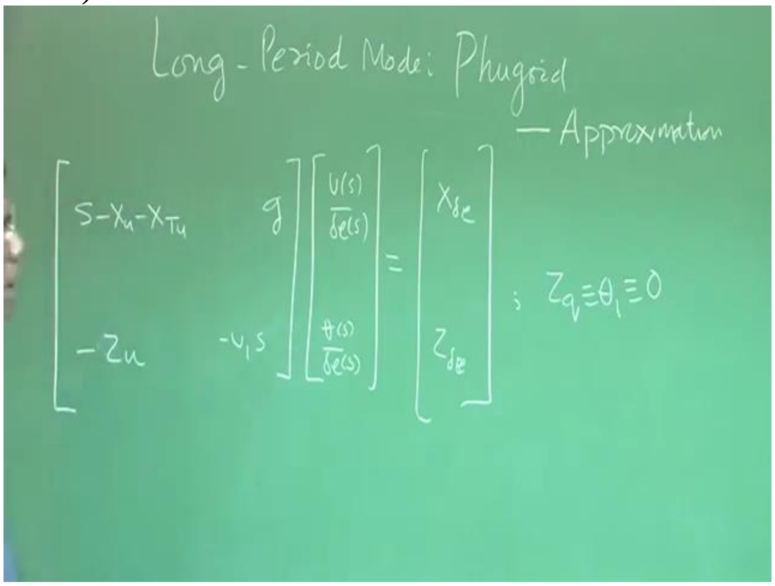
(Refer Slide Time: 05:45)





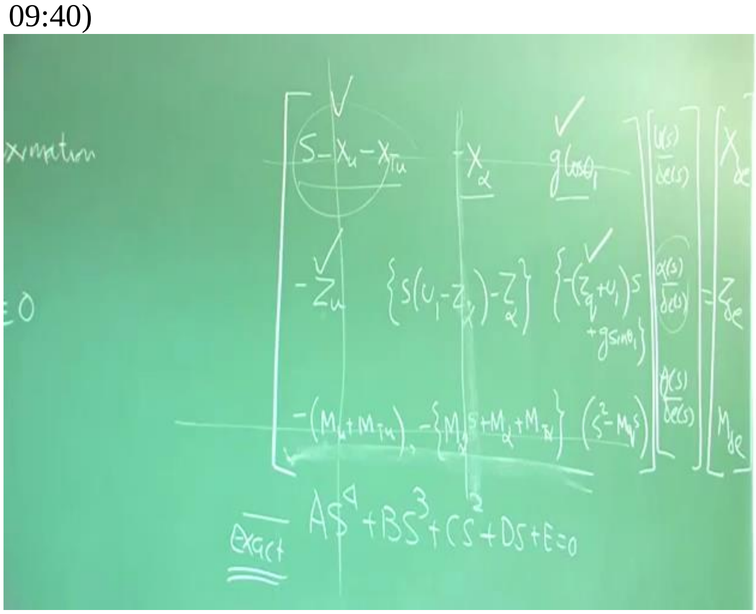
So the moment I talk about perturbed alpha is 0, this is perturbed. Or I say, alpha total is constant and that is why when I draw it like this, although very approximate, if I draw it like this, as if the airplane is going like this. So there is no change in angle of attack. I repeat, this is not a very good approximation. But okay. If I assume that alpha perturbed is 0 because alpha total is constant then we can again see how best we can simplify this matrix. And that is exactly what we will be doing now. Let us watch out.

(Refer Slide Time: 07:25)



So let me first write this matrix. Then I will explain. This is X - XU - XTU. Then G. Then - ZU and - U1S. And here is U of S by Delta E of S and here Theta of S by Delta E of S. This is equal to X Delta E and this is Z Delta E. And you could see that I can always assume ZQ and equal to Theta 1 equal to 0. I put another condition also. Let us see how we have got this.

(Refer Slide Time: 09:40)



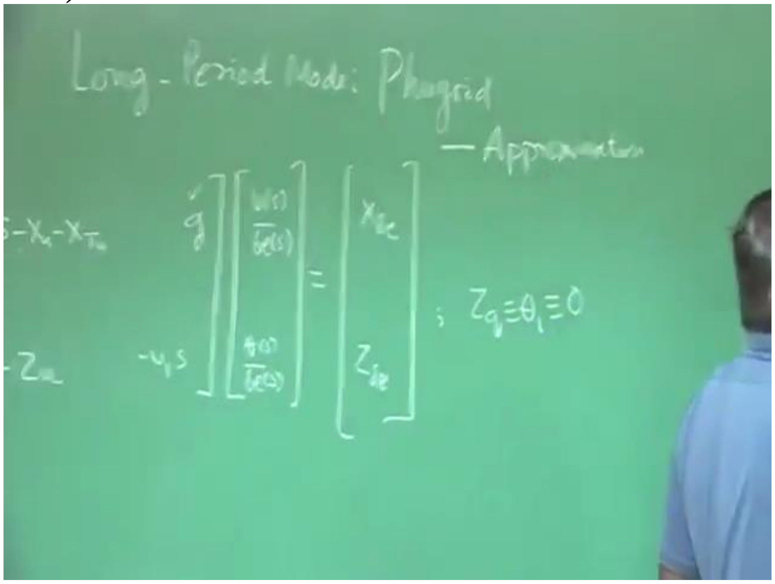
Since alpha perturbed, this is alpha perturbed, this is 0 so I have got this one, this into alpha when I do this, this into alpha, this will come, this into alpha, this will also come. So then, this column, this one I will delete. Do you see what I am saying? If I open this matrix, this will be this term into this term + this term into this term + this term into this term. Right?

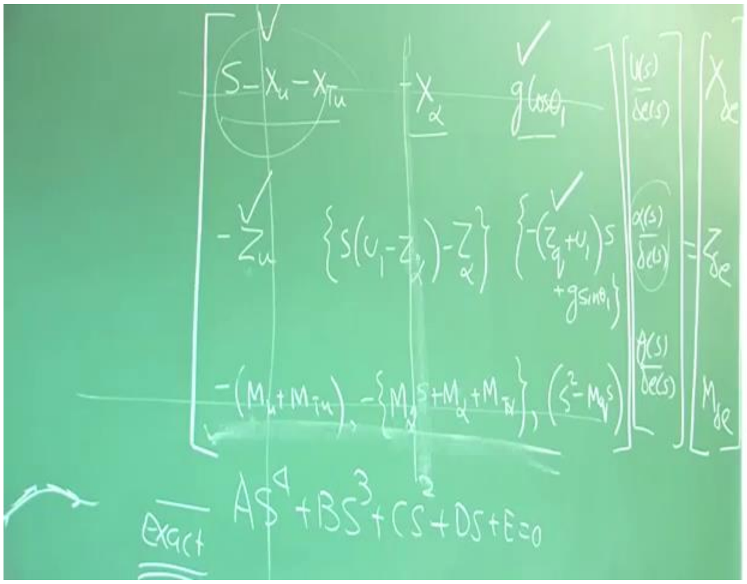
Now since alpha perturbed is 0, X alpha into alpha is (inaudible 8:15) Z alpha dot - Z alpha into alpha of S, this one into alpha of S. Again I repeat, X alpha into alpha of S, SU1 - Z alpha dot - Z alpha into alpha of S. Then again, M alpha dot into S + M alpha into alpha of S. This will vanish. So I am dropping this.

Also please understand, this third equation, this is related to moment coming about the Y axis, M alpha. But here, we are assuming, the airplane is going like this. So this oscillation is not that dominating. Moment about Y axis is not that dominating. It is going like this.

So as a first approximation, I am also saying this man is out. We have it is also apparently superfluous. So then I am left with, this goes, this goes. So what I am left with? I am left with this term, this term, this term and this term. Right? That is exactly what you find here.

(Refer Slide Time: 9:35)





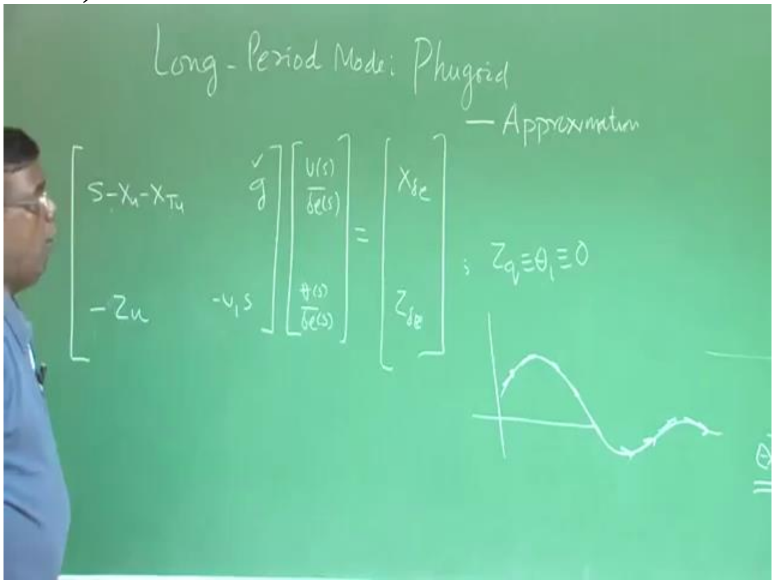
X - XU - XTU. And for G Cos Theta 1 we are assuming Theta1 equal to 0. So this G is here and for here - ZU, - ZU is here and if you see here, ZQ, we have assumed to be 0, Theta 1 is 0. So only - U1S. And that is exactly here.

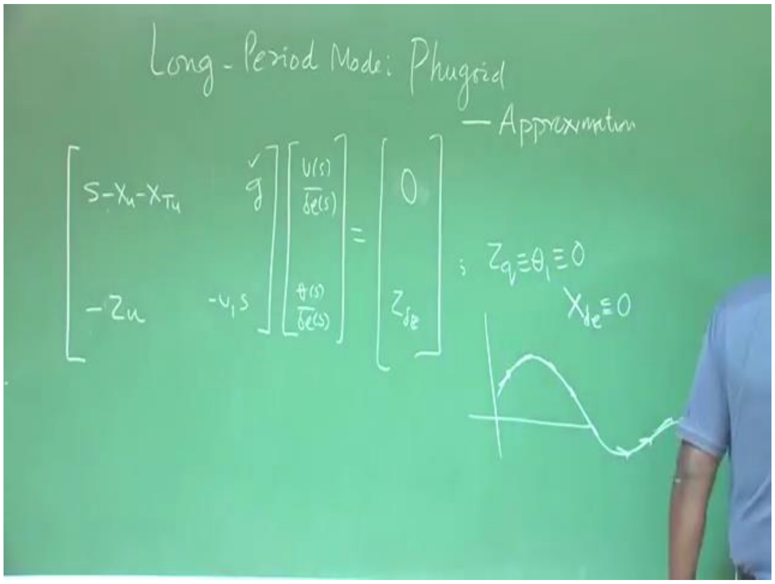
Let me repeat again. When I am talking about Phugoid, what I am assuming? I am assuming, and this is the Phugoid, the airplane is actually through approximation, right? This may not be very accurate. At each point, alpha remaining constant or the perturbed Alpha is 0, the consequence of that, when I multiply these 2 matrices, the second term, X alpha will be multiplied by alpha of S.

Here, the second term, this one multiplied by alpha of S. Here, the second term, this multiplied with alpha of S. Right? All will vanish because perturbed alpha is 0. second thing, the third equation which was here, M Delta E, this is basically representing the pitching moment about the Y axis of the airplane. So this sort of a motion we are talking about.

But actually we are approximating the Phugoid mode. It is not, this approximation is not dominating. This is dominating. So not much of this oscillation, not much of moment about Y axis. So we are saying this is also superfluous. So we are deleting that. This is an approximation.

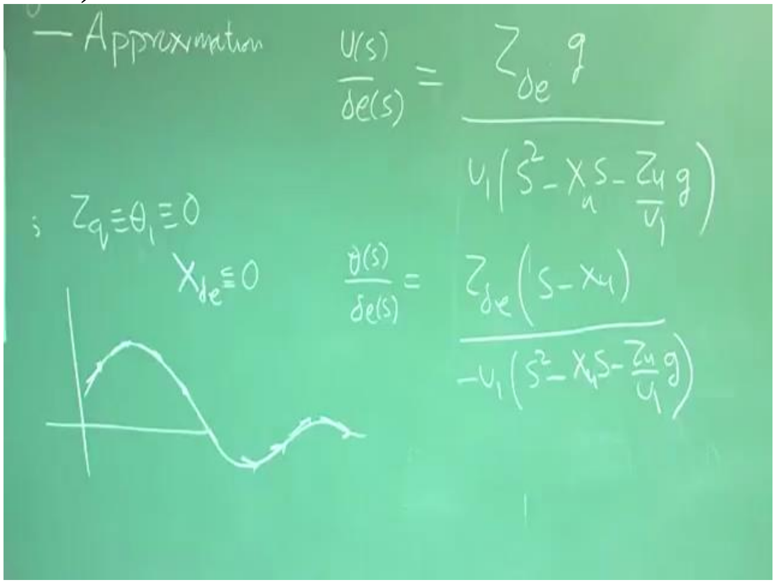
(Refer Slide Time: 11:32)





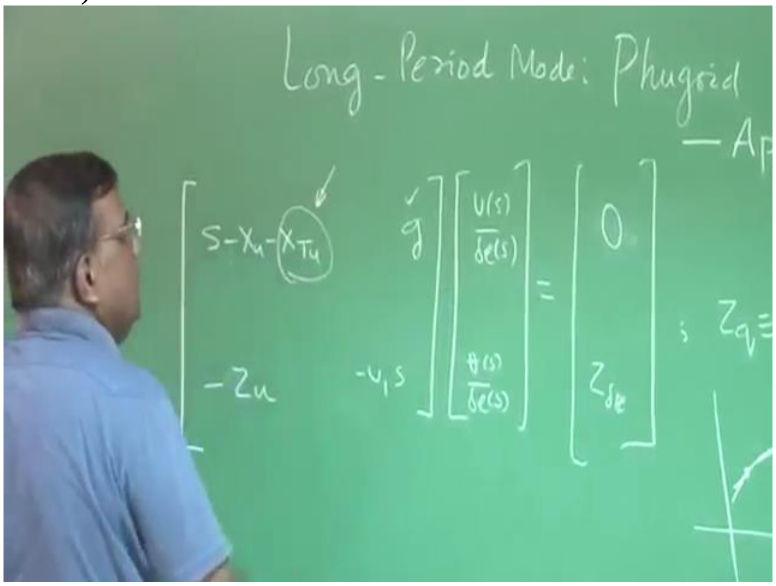
By doing this and putting additional condition, ZQ equal to Theta 1 and equal to 0, we get this simplified version. Also we can do one thing. X Delta E we can assume, it is also negligible. So X Delta E equal to 0. So I put it here. Please understand those approximations what we are doing.

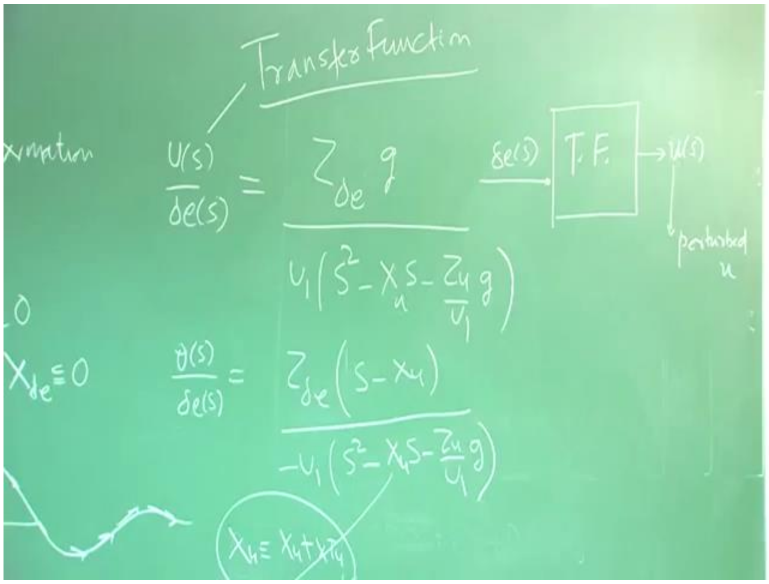
(Refer Slide Time: 11:58)



Now let us handle this 2 by 2 matrix and I can write U of S by Delta U of S by using Kramer's rule, we get this as Z Delta E into G divided by U1 S square - XUS - ZU1 into G. Similarly Theta of S by Delta E of S is given by Z Delta E into S - XU divided by - U1S square - XUS - ZU by U1.

(Refer Slide Time: 13:10)





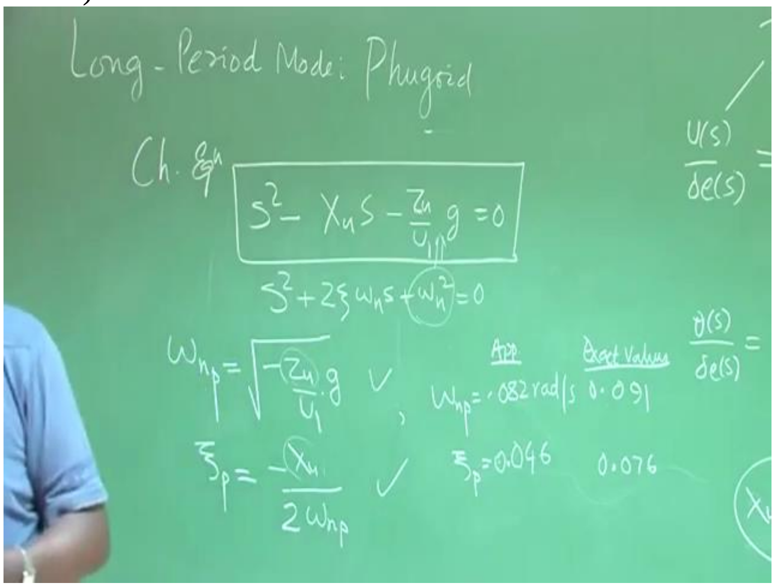
Please note here. This XTU is sitting here and now I am writing only XU here. So you can interpret, this XU if you are taking thrust into account, this XU is nothing but XU because of aerodynamic + XTU diet means, the thrust if it has been absorbed in XU. That is why I was telling you, if you know how to handle the aerodynamic part, the thrust part gets added. So its nothing extra you do.

So that should be very very clear to your mind. Now if I see this, what do you call this U of S by Delta E of S or Theta of S by Delta E of S? These are called transfer functions. That is if I draw a block diagram, I will say I am giving input Delta E of S, I am getting output U of S. What is this U? U is the perturbed U. Do not lose the inside. So ideally, this should be small. And what is fixed inside this box? It is this transfer function.

That is if I want to know what is U of S, once I know the transfer function this which is primarily decided by the aerodynamic dimensional derivative and the flight condition, if I know this transfer function, I have to simply multiply by Delta E of S to get U of S in frequency domain. And there are numerical techniques to convert from frequency domain to time domain. So we say inverse Fourier transform.

Fine. Once we understand this, what is over him? Our aim is to ask a question, whether we Phugoid is dynamically stable or not. How do I check? I again follow the approach which we followed for short period approximation. Our aim is to find, we have to answer the question, whether the Phugoid mode is dynamically stable or not.

(Refer Slide Time: 18:30)



And we know the characteristic equation is S square - XUS - ZU by U1 into G equal to 0. Remember, the beauty of understanding initially the second order system. So again this is second order system and we know, standard notation for second order system using natural frequency and damping ratio. So I compare these 2 and find Omega N Phugoid as - ZU by U1 into G. Right?

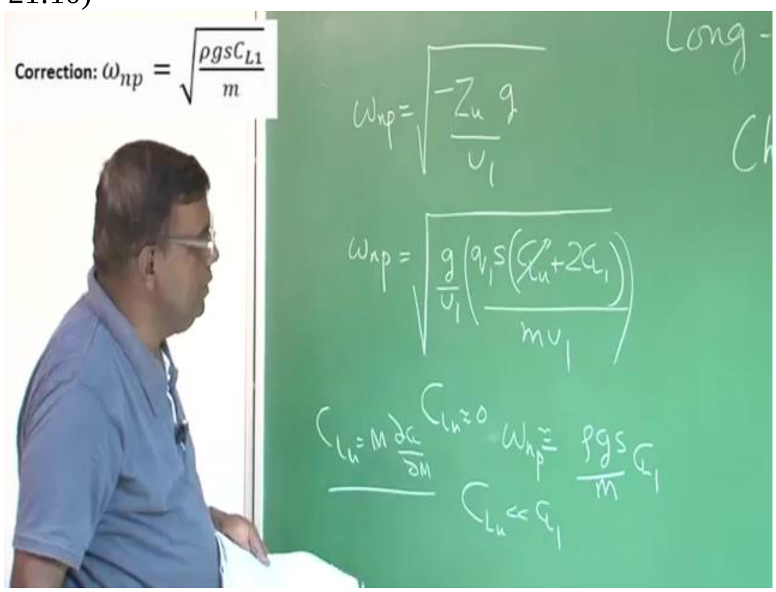
Similarly because this is Omega N square, this is equal to this. 2 Zeta Omega N is - XU. So Zeta T, I find it as - XU by 2 Omega Phugoid. As simple as that. Omega N will be under root of this and 2 Zeta Omega N is equal to mimes XU. So from that I find the expression for natural frequency Phugoid and damping ratio Phugoid.

And if I want to calculate for an airplane, I know ZU, I know the expression of ZU, I the expression of XU X can easily find out natural frequency, I can easily find out damping ratio. Okay, if we do this for the example airplane that we are doing, this value will come, Omega N Phugoid will come around 0.082 radian per second and Zeta P will come as 0.046. But we recall, the exact values were, exact values were computed by earlier solving AS4 + BS cube + CS square + DS + E equal to 0.

Those values were, this was 0.091 and this was 0.076. Now you could see, natural frequency estimation through approximate method. This is approximate for Phugoid approximation. This is okay. This is a bit close. But as far as damping ratio is concerned, this is not really a value which we would have liked.

And that is why from the beginning I am telling you that Phugoid approximation is not a very very good approximation. But still it has got some additional features which we must know which will help us to design an aircraft or analyse an aircraft dynamics. So with that understanding, now we will do a little bit of, we will play around these 2 expressions and see what is the best information we can get out of this expression?

(Refer Slide Time: 21:10)



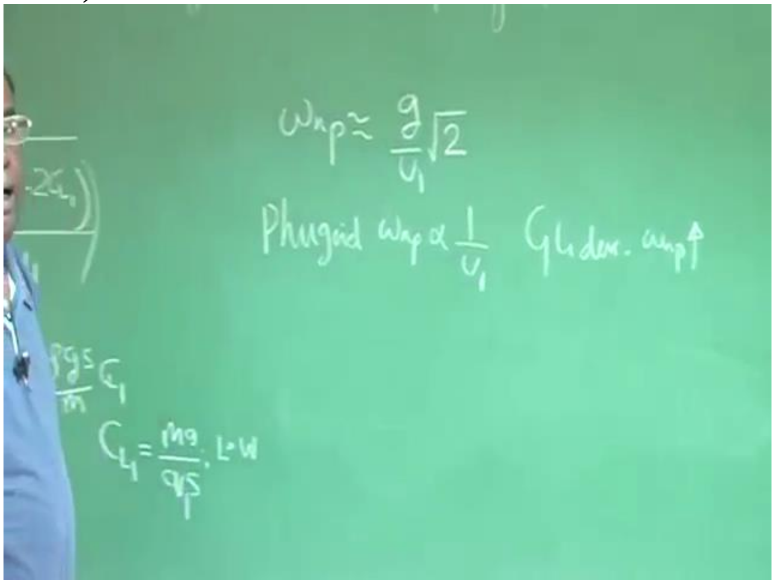
Let us take Omega NP. Omega NP Phugoid is under root - ZUG by U1. We know the expression of ZU so I can substitute that expression here. So we will get something like this. G by U1 into Q1S CLU + 2 CL1 divided by MU1. What I have done? G by U1 is this one. And this is the expression of ZU. There was a - sign. So - and - becomes + so I get an expression like this.

Now I do further approximation. I say CLU is negligible. You remember what is CLU? CLU was M into DCL by DM. Specially for low speed, this value is very very small or 0, up to 0.6 or 0.7 mac number. We are resuming them this value is 0. Then what we have is Omega N Phugoid equal to, this man goes out. So we will have the expression as roughly Rho GS by M into CL1.

Why suddenly it becomes so simple? Because remember, we have assumed that CLU less than less than CL1. Or initially I was talking about, it is 0. The better way of handling this is you can say okay, even if it is supersonic case, high-speed case, CLU will not be 0. So I said my approximation says in any case, CLU will be much less as compared to CL1. So I neglect this. So both way you could see this.

Second statement is better way of handling this situation. Also you know that CL1 equal to MG by Q1S because this comes from lift equal to weight. That is our steady-state. So those 2 information we will use, Q1S by M, G is here. So if I now substitute this here then I get Omega NP has Rho GS by M into CL1.

(Refer Slide Time: 21:43)

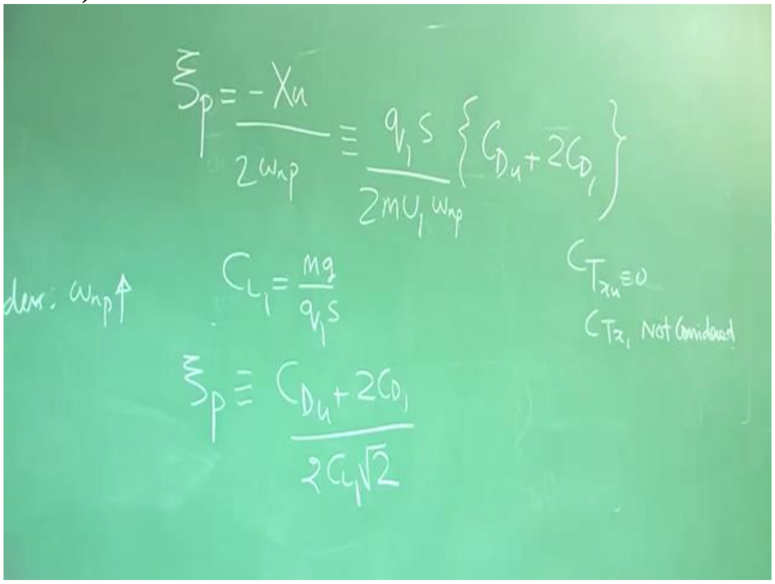


And now if I further see this, if we substitute this CL1 equal to MG by Q1S then we can show that Omega N Phugoid will approximately come like G by U1 root 2. Please do it yourself. You have to just substitute here. All the understanding is, CLU is less compared to CL1 and CL1 is nothing but MG by half Rho V square S which is Q1S at steady-state. If you substitute this, you can easily show that Omega NP will be approximately G by U1 root 2.

So what is the message? For Phugoid frequency or natural frequency Omega NP goes inversely with U1. So as the speed is increased, this frequency goes on decreasing. Or reverse way, if the speed is slow, this Phugoid frequency is large. For gliders and all, the speed is not that high. This Phugoid frequency has a tendency to be a little larger.

And that is the fun you have. You can easily excite a Phugoid and really enjoy the Phugoid mode. Now the story does not end here. Now let us see Zeta P.

(Refer Slide Time: 24:25)



Phugoid damping, which by expression is - XU by 2 Omega NP. And again I substitute the value, the expression for XU and all. So I get Q1S by 2MU1 into Omega NP. Here I put CDU + 2 CD 1. I am not taking the thrust effect. Please understand, I am not taking. I am taking CTXU and equal to 0. CTX1 is also not considered. Again we will be leaving this concept, CL1 is MG by Q1S.

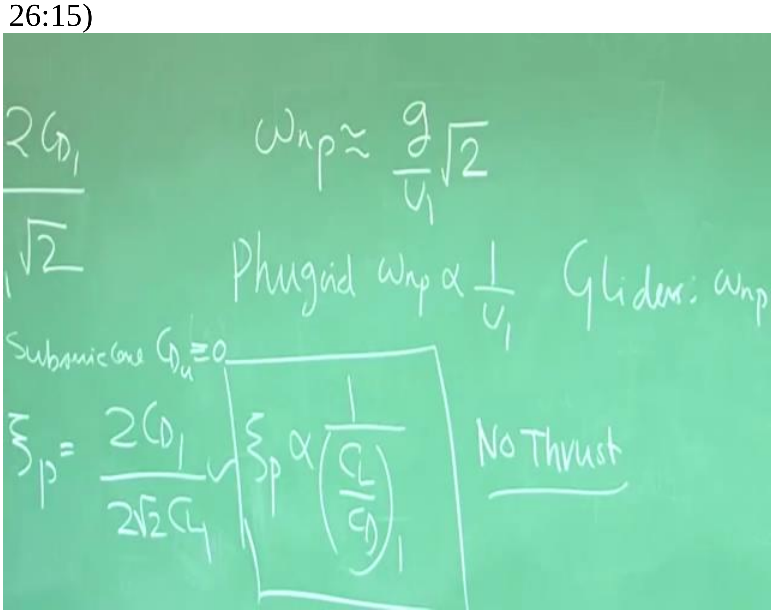
And set it inside the expression of Omega NP also and then we can easily show, if I substitute this, Zeta P will approximately come as CDU + 2CD 1 by 2CL1 root 2. What we have to do to get this expression is, we have to put Omega NP expression which is here, approximate expression. We have to use CL1 is equal to MG by Q1S and then we can modify this expression, Zeta P equal to CDU + 2CD 1 by 2CL1 root 2.

(Refer Slide Time: 24:40)

Zeta P equal to CDU + 2CD 1 by 2CL1 root 2. What next I should do as a designer to get some first-hand feel? I will immediately see CDU will be less as compared to 2 CD 1. And also think, at subsonic speed, CDU will be almost 0. The expression for damping ratio Phugoid is - XU by 2 Omega NP. And we will put, substitute the expression of XU here. Then you get expression for Zeta P as this.

We are aware CL1 is MG by Q1S. It has come from lift equal to wait at steady-state had goes. If I substitute this, I get the expression, Zeta P equal to CDU + 2CD 1 by 2CL1 root 2.

(Refer Slide Time: 26:15)



And for subsonic case, low speed case let us say, CDU can be approximately 0. So we have now Zeta P here, 2 CD 1 by 2 root 2 CL1. And you could see that this implies Zeta P is in Wasilla bottle to serial by CD evaluated at steady-state. That is CL1 by CD 1. This is extremely important. Please understand, this expression is when no thrust. We are mostly talking about the gliding flight.

What is the meaning of this? If CL by CD is very large, specially for gliders, they are very large. They are in the order of 30. That means, the Phugoid damping is very very low. And that is where you enjoy riding. Inside the Phugoid it will go on doing like this, long periods, for longer time. And you enjoy the flight.

So this is very very important as far as gliding is concerned. And you should understand that as you go on increasing CL by CD, the Phugoid damping goes on reducing. And this is a case for no thrust case. And mostly, very very relevant for gliding flight. And we all know by flying also that gliders, we enjoy the gliding flight because we want to enjoy the Phugoid mode and enjoy this motion.

So this was a closer look into Phugoid approximation and short period approximation. We want to avoid using those huge matrix. We found 2 by 2 matrix we will use. And we got some relevant information in short period mode and Phugoid mode through approximation. These are very very handy when you try to feel the initial dimension for designing and aircraft.

And you will see that how will you use it in our design course which will be coming next. At that time, you will appreciate why we are stressing so much time on this. So this is up to the longitudinal mode. Next, now we have to go for lateral directional mode. Thank you very much.