Fundamentals of Combustion (Part 2)
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Lecture – 61
Theoretical Analysis of a Two-Dimensional Diffusion Flame

Let us start this lecture with the thought process from Albert Einstein who says, look deep into nature, and then you will understand everything better. But unfortunately we do not have time to look deep into nature. And nor we understand anything in life or in otherwise. So, in the last lecture we basically looked at a very simplified analysis based on the phenomenology, what is happening then trying to the relate that those things.

And we have seen that we can derived a relationship for laminar flame height or flame length sometimes people call it flame length also for a z diffusion flame and we have extended that analysis for turbulent z diffusion flame. And we have seen it is roughly matching at least the trend of the experimental data. And, now will be looking at laminar flames and we look at theoretical analysis of this and we are considering here a two dimensional z diffusion flame.

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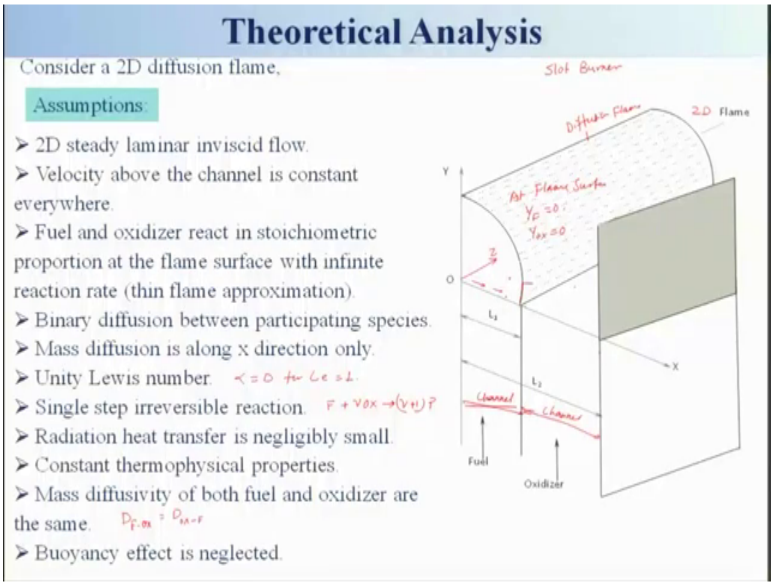

Here that is a two slots are there right these are the two slots and these are two passages with two dimensional having L l is a length and this L 2 minus L 1 this portion right which is in which oxidizer passing through keep in mind that this is the X direction and

Y direction and Z direction in this way right and this flame is basically two dimensional in nature ok. You can call it as a slot burner because, this is like a slot right. And generally people do not use that much except maybe these thing people always be happy with the tube burner or the jet kind of thing burner.

So, where you know you use a circular pipe or some other pipe and then you use it and we will be using this analysis for this two dimensional diffusion flame because, this is basically 2 D diffusion flame right this is a diffusion flame and 2 D in nature. That means what it is basically changing with respect to X and Y and the Z direction, it is not changing at all right that is why it is 2 D and people will be always be happy with the axis symmetric burner also right.

Let us consider is simple one and we will make some assumption and some of the assumption will be repeated or whatever we have done for finalize will be valid here that is first is this is a two dimensional steady laminar in viscid flow; that means, it is not viscous if it is not there. And it is the study and velocity above the channel right these are the channels right these are all channel, the channel through which fuel is passing and this is another channel right where, the velocity above the channel that means, here it will be remaining constant right kind of things you can say that right and it will not be changing and we will be using basically thin flame approximation.

I have already discuss in which fuel oxidizer react in stoichiometric proportion at the flame surface with finite, infinite reaction rate that means, reaction rate will be very very fast right. And that means, what is the meaning of this that means, on this flame surface right if it is oxidizer is coming passing through it will not cross this flame surface right. It will not enter into that means, no oxygen will be found in this region where right in this region no fuel will be found, no fuel will be passing through the flame surface and reaching the oxidizer stream or oxidizer side right that is the thing.

That means at the flame surface in this flame surface what will be Y F will be 0 and what else Y oxidizer also will be 0 at flame surface. Keep in mind this is your flame surface right and that we call it as a thin film approximation. And a binary diffusion between participating species right we are saying only fuel and oxidizer is there two species which will be playing important role, but in real situation it will not be, why? Because, a chemical reaction will be taking place when chemical reaction taking place there will be several number of species will be participating right. And, that too if it is a z diffusion flame soot will be coming there will be patch and there will be some other things which will be quite complex right.

So, however, we will be doing for simplicity and mass diffusion is along the X direction that means, the mass has to diffuse in this direction only in the X direction. There will not be any mass diffusion in the Y direction however; both the diffusion should be there we had seen. But here the mass diffusion along the Y direction is predominant as compare to the mass diffusion along the Y because, along the Y what will be predominant, the momentum will be predominant along the y direction vertical direction right.

So, therefore, as compared to the mass diffusion along the X the Y direction mass diffusion is negligibly small that is the meaning of this. And unity Lewis number, unity Lewis number means basically alpha that is thermal diffusivity is equal to mass diffusivity right and that means, that is the things what it will be unity Lewis number for Le is equal to 1. And single step chemistry which will be there single step chemistry means is a 1 mole of fuel is reacting with nu moles of oxidizer going to the product of nu 1 per nu plus 1 kg of the product and radioactive heat transfer is negligibly small.

But flame surface always will be radiation will be you do not know, but we are not considering like in the pre misplace also you did not consider that. And constant themophysical properties as usual we have taken this assumption. And mass diffusivity for both fuel and oxidizer are same that means, D F of oxidizer between is equal to D oxidizer fuel basically that you know between this binary both are same; need not to be same, but we are considering to be same and buoyancy effect is neglected.

That means, there is no effect of buoyancy in this case what we are considering. So, with these things we will have to now look at the various mass conservation equations. Keep in mind that what we are saying that velocity above the channel you know is basically remaining constant everywhere that means, the V x if you look at is basically along with the X direction what will be the V x will be 0, only the domain end velocity is V y that is a thing what we will be considering.

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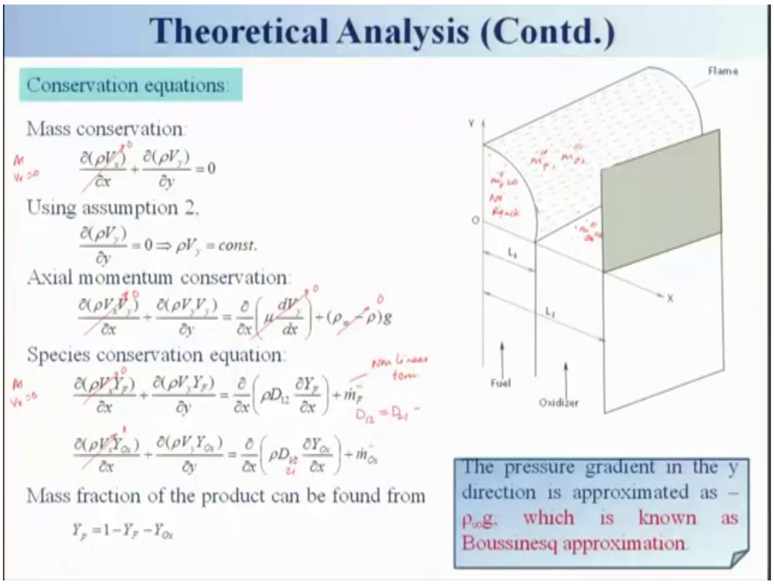

So, let us look at conservation equation mass conservation for the steady flow. We know that this is dou rho V x divided by dou x plus dou rho V y divided by dou y is equal to 0 and in this case there is a V x is not changing in this flame region. So, therefore, this term will be 0, as V x is 0 that we are considering. Means what is saying is basically that is with assumption 2 what you have done, that is with this what we are saying is that rho V x is equal to 0 to integrate is rho V y is constant right. That means, V y and this density you know which will be remaining constant throughout this flame domain right in the along the this region that is the meaning of this and we will look at y momentum because x momentum already is 0. So, we are not considering.

So, axial momentum what we are considering along the Y direction is rho V x dou rho V x v y divide by dou x plus dou V y V y divide by dou y is equal to dou by dou x mu dou v y by dou x and rho infinity minus rho is into g right. And this is basically done you know because of fact that you might be knowing this row g is the gravitational effect right and rho infinity is the outside ambient thing what will be talking about and that is due to the Boussinesq approximation, because the pressure gradient in the Y direction is approximated as rho infinity g right which is an approximation which is known as Boussinesq approximation that will be doing. And keep in mind that this is we are not considering the buoyancy effect therefore, this will be 0 and this V x as it is 0. So, this term will be 0 right and this is a inviscid flow.

So, therefore, this term will be 0 right and then rho V right is basically this is rho V y is equal to 0 this is because the remaining constant. So, therefore, this also will be. So, there is no need to solve any momentum equation as such right now we will be looking at, looking at these thing this is the species conservation equation as it right. That is rho V x dou rho V x Y F dou x plus dou rho V y Y F dou y is equal to dou dou x rho D 1 2 that is basically filled oxidizer dou F x into mass generation of the fuel per unit volume right.

And similarly for the oxidizer we can have in the similar way and keep in mind that this is both are same I could have put that the D 1 2 is equal to D 2 1 right it could have been basically 2 1 right that is why I have putting that 1 2. So, this is again that there is a source term for the oxidizer and product of course, I need to write if mass because if I solve this equation and get the mass product you can get very easily Y P is equal to 1 minus Y F minus oxidizer right if I get this both the thing you can get very easily. So, you simplify this equation what you will get this V x is 0 right, this term will be 0 as V x is 0 right and similarly this also will be 0 right. So, you will get basically right these equations whichever is there. Now, how to solve this is the very important one has to look at it.

Because you are having non-linear terms these terms are non-linear terms right and this is of course, the second derivative terms right which is the has to be looked at. So, question arises how you will be solving this is a very important one. What will be doing for that we will be using a thin flame approximation right. That is: what is that thin flame approximation? At this surface right what will happen that is basically m dot at the flame surface right this will be all this will be occurring that means, the fuel will be getting consumed right because, this will be mass will be fuel will be getting consumed and then oxidizer will be getting consumed right, isn’t it fuel and oxidizer will be reacting and then encoding the product will be formed.

So, at this flame surface this all this terms will be there. But if I look at this point this place is it like a mass of the fuel will be consumed, will be any reaction term here no reaction right where in these domain this region along with the z nothing no reaction will be occurring. Similarly in this region right except the flame surface there will not be any reaction that means, these term will be 0 if you look at m dot F will be 0 here right and F cannot get into you know oxidizer cannot get into. So, therefore, that does not arise similarly in this region what will happen? m dot triple dash oxidizer will be 0 are you getting at the flame surface this term will be there which one?

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Source term of the fuel and oxidizer will be active where?

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On the flame surface apart from the flame surface this will be 0 on the fuel side m dot triple dash F will be 0 on the oxidize side m dot triple dash oxidizer will be 0. That you should keep in mind are you getting, this is the thing which is will be used to look at that ok.

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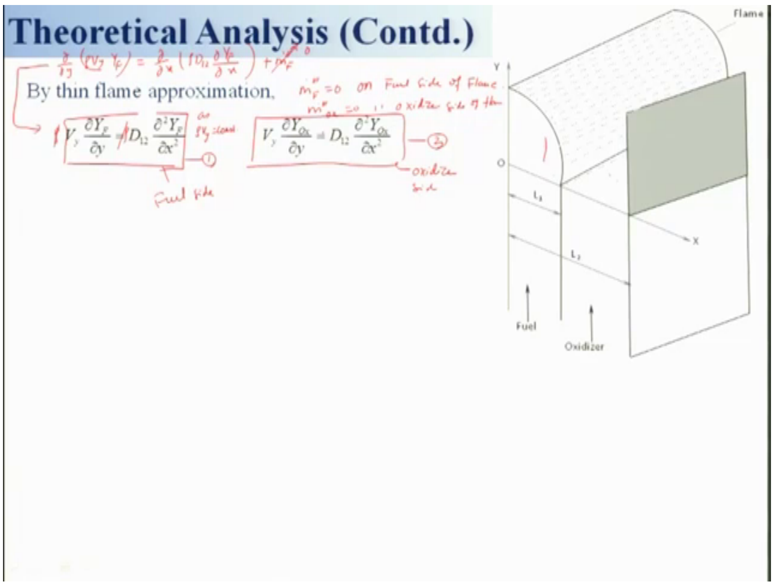

I will do again tell you that thing because that will be getting into that because at the thin flame approximation. So, what will be getting is right if you look at that for this thin flame approximation right m dot F will be 0 on flame side sorry on fuel side of flame right m dot triple dash triple dash oxidizer will be 0 on oxidizer side of flame. I think flame I should not let me write alright basically from the flame onwards that is the thing that is the surface.

So, if that is the 0 then what will happen you will get a you will get if I look at the fuel equation that is dou by dou y I can write down rho V y Y F is equal to d by d x rho 1 2 Y

F by x right because, this is 0 right. So, I can write down that as basically if I take this out right inside and rho V y is constant right because rho V y is constant we have already seen from the continuity equation. So, therefore, equal to I can take it out I can write down that as rho V I can say write rho V y d Y F by d y is equal to rho D 1 by 2 dou Y F square by dou x square this is cancel it out right, I am just doing that here because as rho Y is constant already we have seen from the continuity equation.

So that means, I can also write down in the similar way for the oxidizer side right. So, similar way I can write down V y dou Y o x dou Y is equal to dou D 1 2 dou square Y o x divided by dou x square. So, this is the another equation one can think of like to simplified the sum. And, this will be valid where everywhere apart from the flame surface are you getting that means, this is valid for what this side right. And, this equation is valid for oxidizer side. This is for oxidizer side, this is fuel side and these equation will not be valid on the flame surface, these both the equation will not be valid on the flame surface is that clear. And, flame is a very thin one like you can say this thin infinitely thin that is the condition what we are doing.

So, again let me repeat that is the above two equation can only be applied to the outside of the flame surface as both an oxidizer fuel get consumed instantly at the flame surface right as soon as it will come it will be consumed that is all its very thin. So, now, we will have to try to solve these equations 1 and 2 which is a.

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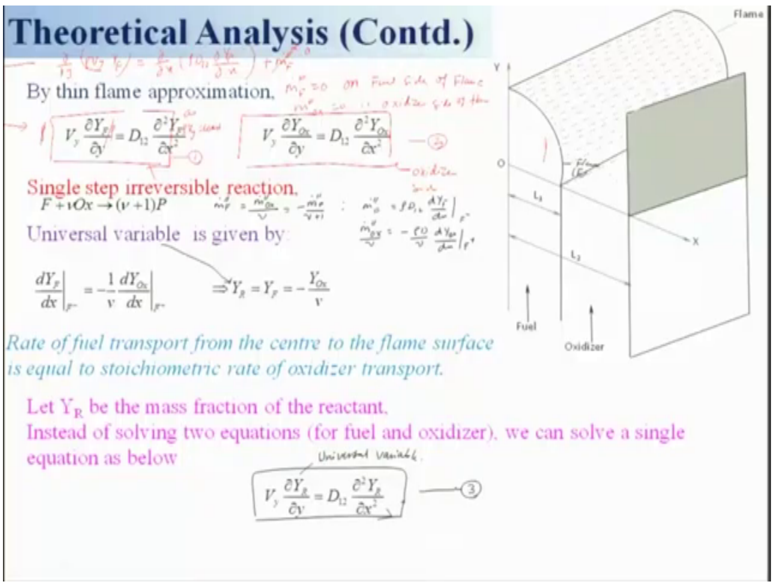

So, let us look at single step irreversible reaction that is fuel 1 mole of fuel is reacting with or one can new moles of oxidizer going to the product of nu plus 1 mole of product. So, if you look at this reaction as m dot triple double mass flux of fuel right is equal to oxidizer by nu is equal to minus product by nu plus 1 because let us say I have taken minus this side or sum.

And from this we can basically look at this m dot flux is equal to what this is equal to this m dot F if you look at is equal to rho D 1 2 d Y F by d x. If you look at this is my flame right flame F you are coming from this side minus flame right, F you can say and similarly I can do for the oxidizer as well right. And similarly I can write down of oxidizer is by nu is nothing, but rho D by nu oxidizer by d x of course, positive side this is.

I can say negative side here like a basically flame surface this is flame F and it is coming from negative side and this is the positive side and same. So, if you look at this is basically this d Y F by dx F is equal to nothing, but 1 by nu d Y by d x at F. So, from this I can say very easily that Y F is equal to Y F is equal to minus Y oxidizer nu and which is nothing, but your Y R. And, this is your universal variable right a one of them if I will get that that means, all this equation 1 and 2 right; I can keep in mind that rate of fuel transport from centre to the flame surface is equal to a stoichiometric rate of this is the proportion right. That is the meaning what is saying and Y F be the mass fraction of the reactant instead of solving two equation of fuel oxide we can solve a single equation. And, that is basically I can say V y dou Y R divided by dou y is equal to D 1 2 dou square Y R by dou x square.

So, I can put that thing and then do that this is my equation we need to solve right and this is the universal variable that is Y R, this is your universal variable right. So, instead of solving two I am solving the one equation. So, let us say this is your equation number 3. So, in the next lecture we will be basically looking at how to solve this equation and get some kind of a flame surface in the other thing.

Thank you very much.