Fundamentals of Combustion (Part 2)
Dr. D. P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 70
Spray Combustion Model

Let us start this lecture with a thought process from Warren Buffet, who says there seems to be a some perverse human characteristics that lies to make the easy things difficult and that is true for most of us we try to make the life difficult for us and others. In the last lecture, we basically initiated discussion on the Spray Combustion Model and we have jotted down all the assumptions, which will be using for this analysis. And earlier, we have looked at the fuel air ratio relationship and let us look at that and see how we can simplify it further and express it.

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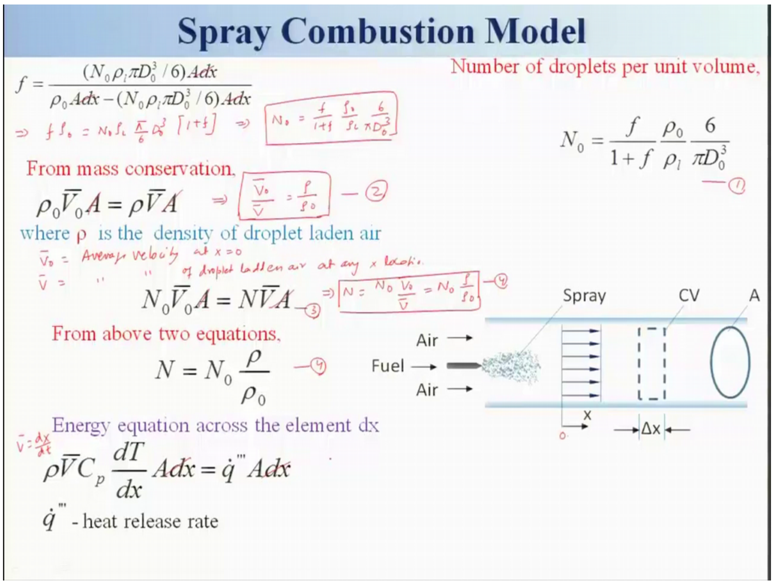

So, if you look at I can cancel this out right, I can write down here as the f rho 0 and is equal to N 0 rho l pi by 6 D 0 cube, I can take out is equal to 1 plus f right and this we can write it down basically as the N 0 is equal to f 1 plus f rho 0 by rho l right into 6 by D 0 cube pi should be there right. So, this is the expression we can get and that basically says, that number of droplets per unit volume at in the beginning will be dependent on the density of the mixture, density of the liquid and also the initial diameter of the droplet mono dispersed droplet and f is the fuel oxidizer ratio.

And we will be basically carrying out mass conservation equation and at this location any location you can say, we are talking about initially at this is X is equal to 0 right rho 0 V 0 A area cross sectional area and this is the density at any location apart from the initial state and average velocity apart from the initial state and of course, cross sectional area, you can cancel it out and keep it mind that this rho is the density of droplet laden air right. And V 0 dash is basically the average velocity at X is equal to 0 and V average is equal to average velocity of droplet laden air right at any location at any X location right.

So, from this I can get that one that is basically V average divided by V is nothing, but your rho by rho 0 right, this I can say this is basically, I can say this is equation 1 and this I can say equation 2 right. And keep in mind that this average velocity at any location right will be changing it will be basically, increasing or decreasing? It will be increasing because of fact that your density is decreasing due to the heat addition. So, therefore, the average velocity at any location apart from the initial state it will be increasing along with the X.

So, now we will have to do also the number balanced like kind of things. So, we can write down this is basically the N 0, N 0 is the number density into this is the V average velocity at the initial state and V is equal to N is the number density of the droplet at any location and V is the average. So, you can cancel it out and you can say, this is as basically N is equal to N 0, V 0 divided by V and is equal to if you look at N 0 rho by rho 0 right, this I can say basically equation kind of 4, I can say and this I can say 3 right.

Now, let us consider combining this above 2 equation of course, I have already derived this thing right, this is equation 4 right and combining this above 2 equations, equation 2 and 3 and energy equation across the element d x we can write down as basically rho V, because at any location this is the average velocity Cp dT by dx into A into dx, this is the volume right is equal to q dot triple dash the heat released into A by dx, you can basically cancel it out and keep in mind that this V is what we have assumed, this V is a will be changing with respect to time and then whenever it will be marching with the x right.

So, basically this V dash will be nothing, but dx by dT right, that is the velocity with along the x direction. So, keep in mind that q dot triple dash heat release rate and right.

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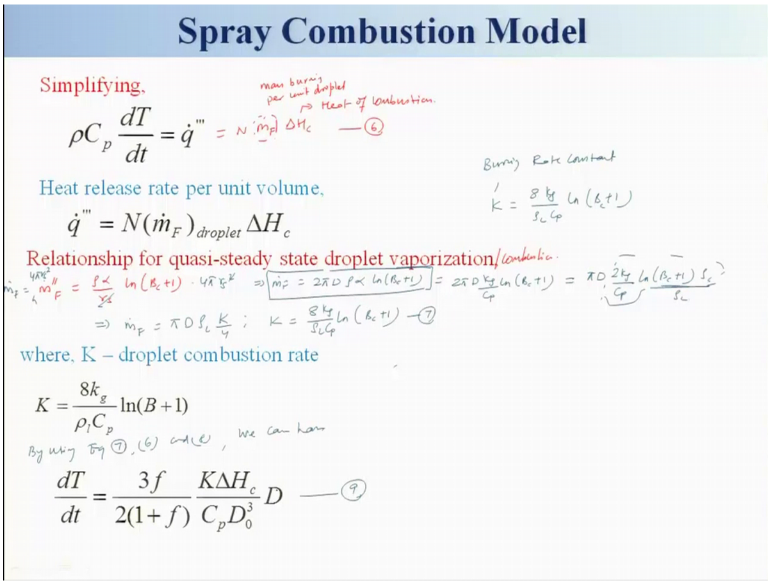

So, you can also write it down simplifying this, I can write down rho Cp dT by dt is equal to q dot triple dash and this is equal to N m dot f into delta HC number density that number of droplets and this is this is basically mass consumption right, mass burning per unit droplet right into delta HC is the heat of combustion, this is basically heat of combustion right.

So, heat is a rate, I have already written there right that equation I can really get and this will be equation, I can say 6 right because q dash is nothing, but that. So, what we need to do is basically, that we will have to relate this quasi steady state droplet vaporizations and you know kind of things into here right and are the combustion right vaporization or combustion right and do that.

So, if we look at we can get basically, we know that f is nothing, but your rho alpha by rs ln Bc plus 1, I can write down right and that is nothing, but your, like you can see what I need, I need basically m dot f; that means, the area has to be multiplied. So, if I want to write down this m dot f is equal to basically, I will have to multiplied by 4 pi rs square. So, I will multiplied by 4 pi rs square. So, this will cancel it out right, I will also look at this 2 rs is nothing, but d. So, i can write down this as basically m dot f is nothing, but two pi D right rho alpha Ln Bc plus one right this I can write down.

Now, this is if you look at I can also take the Louis number equal to 1 and then write down this alpha K alpha is nothing, but your Kg by Cp right. So, I can write down this is

2 pi D Kg by Cp Ln Bc plus 1 right and this I can write down this as pi D right 2 Kg by Cp Ln Bc plus 1 rho L, I can divide rho L right if you recognize this term right this term particularly right, this is coming from the burning rate right.

So, we know the burning rate is basically 8 Kg by rho L Cp Ln V plus C B plus 1 right, this is your burning rate constant right, already we have derived for the quiescent, you know atmosphere conditions right. So, therefore, I can write down here, very simply that is m dot f is nothing, but your pi D by rho L K by 4 whereas K, I have already written let me write down again Kg by rho L p Ln B c plus 1.

Now, I will let say this is equation 7 right and we have already seen that N is equal to N is equal to N 0 rho by rho 0 from equation, what it was 2 or something 4 from equation 4 I can get this right and this N 0 is nothing, but your f 1 plus f rho 0 by rho L right 6 by pi D 0 q into rho by rho 0. So, rho 0 will cancel it out right and I will get basically f by 1 plus f rho by rho L 6 by pi D 0 q and this is I can say this is 8 right and N already, we know we know that m that f that is 7 and we can put this here and then the equation by using equation 6, 7 and 8 we can get of course, we can get this equation by using equation 7 and this is basically 6 and 8 right, we can have that is K is this constant this droplet combustion rate or constant right and 3 f by this now this equation has to be basically as 9 has to be solve right.

And let us look at that what we need to understand that D is this D which will be varying and this will be following the D square law right.

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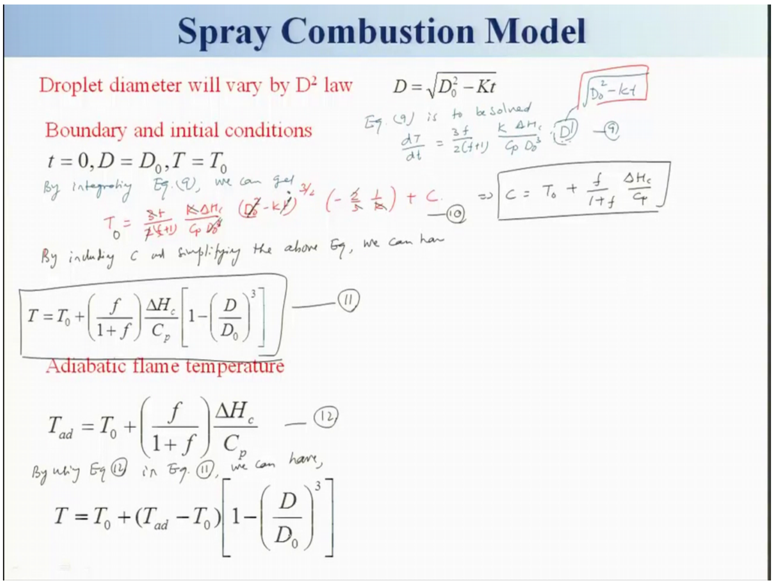

So, therefore, we can write or droplet diameter will vary by D square law that is D is equal to D 0 square minus Kt and the boundary and initial condition will be basically t is equal to 0 D is equal to D 0 and T is equal to T 0 right and like what we will do we will have to equation 9, we need to solve equation 9 is to be solved right, that is dT by dt temperature with respect to time is equal to 3 f 2 f plus 1 K delta HC by Cp D 0 q into D right

So, what will do we will integrate this one right by this is basically 9 by integrating we can get that is T is equal to 3 f 2 1 plus f this is basically, constant and keep in mind that here what we will be using? We will be using this is D 0 square minus Kt root over that I will be using here right this portion right this portion I will be using that.

So, if you do that then what will happen? Then this of course, a constant K I can T I can integrate T is equal to 3 f by 2 f plus 1 then K delta HC by Cp D 0 q and we will have to basically integrate it. So, therefore, D square minus Kt right, 3 by 2 then minus 2 by 3 1 over K and plus C right and when you put this boundary condition T right by applying this boundary condition right, what we will get if T is this thing this will be 0 right, this will be 0 yes or no? This will be 0. So, therefore, D 0 square this one will cancel it out this two will go away right and not do this is 2 because this 3 by 2 right. So, therefore, these will cancel it out whole thing.

So, therefore, you will get basically C what I will be getting and what will be this here? This will be T 0 right C will be T 0 and keep in mind that this also will be cancel it out this K will be cancelling out, this will be cancelling out and this will be cancelling out right, C will be T 0 plus f divided by 1 plus f delta HC by Cp right this is the T we will be getting.

So, then you put that in this equation I can say this is 10 right and then get that here. So, you will get by putting this constant here and by simplifying by including constant and simplifying the above equation, we can have this relation T is equal to T 0 f divided by 1 plus f delta HC divided by Cp and in the bracket 1 minus D divided by D 0 cube. So, this you will get right and keep in mind that this is dependent on D is you know vary right along with the x. So, therefore, temperature also will be vary right and it goes to power and adiabatic flame temperature, we know the T adiabatic is nothing, but T 0 plus f divided by 1 plus f delta HC by Cp this we had done earlier.

So, when I will substitute this in equation 11 and this let say 12, I can use equation 12 in equation 1 and I can get this expression see by using equation 12 in equation 11, we can have like this expression.

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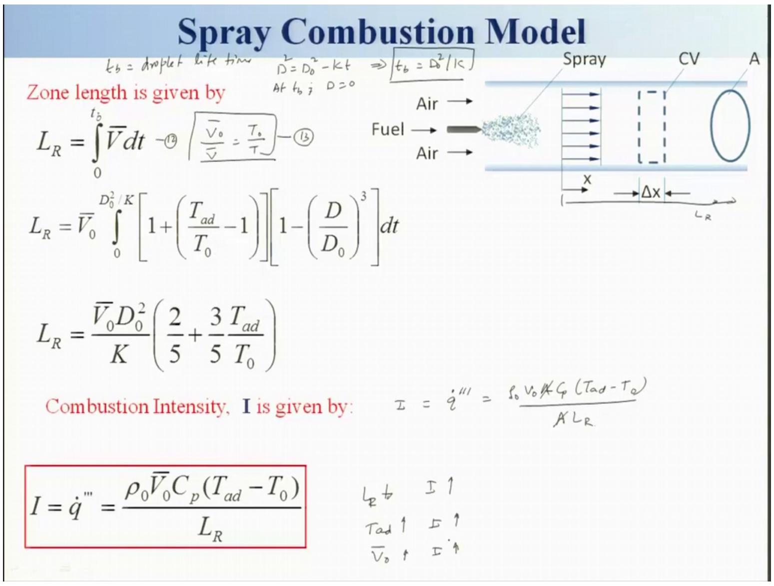

Which is looks to be very simple one and right. So, and what we will be looking at we are getting basically this one.

Now, we will be interested in finding out the zone length; that means, this reaction whatever is taking place in this zone, when this droplet will be consumed completely that is the distance, we can find out the length of the combustors or the we can find out or the reactors. So, this LR I can say this is V average into dt, because this is changing this velocity will be changing and this is from 0 to tb. Tb is your droplet burning time, tb is the droplet lifetime, you can say lifetime; that means, at this is a time in which the droplet diameter become 0 right due to the consumption of the fuel.

So, what will do, you can really put that thing, you know expression and then you will have to integrate because already we have done this V write I can say that, we know that V average I can express in terms of temperature right by the mass conservation right from there looking at densities I can do that this is V 0 average by V average is equal to T 0 by T right.

So, I know this expression right for this thing, I can get that into here in this expression, let say this is the equation I think is it 12 right and this is 13 and then by using this equation and 12 13, you can get this expression I will leave that 2 a derive this thing and keep in mind there is a tb is basically D is equal to D 0 square minus K t. So, therefore, at tb right at tb that what will be D? D will be 0 right at tb D will be 0. So, therefore, this will be t b; t b is nothing, but your D square not divided by K right, which will be integrating you know over 0 to this.

So, when you will do that, you will arrive at a relationship like this and where it is basically dependent on the initial the velocity of the average velocity of the mixture and initial droplet diameter square and also the temperature adiabatic temperature or the flame temperature and lead temperature.

So, the combustion intensity we need to find out it can be determined very easily by considering that I this intensity is basically q dot triple dash is rho 0 and V 0 A Cp and T adiabatic minus T 0 and divided by whatever is happening here is LR right. So, into area so, this area will cancel it out and you can get a very simplified relationship right that is rhoV 0 Cp T adiabatic minus T 0 divided by LR

So, by this you can really find out, what is the intensity, because the intensity basically goes inversely with the length of the, reactor which will ensure the, complete burning of the droplet. So, therefore, if this is smaller right if LR is smaller; that means, I will be higher right intensity will be higher. So, that is the thing, we should keep and if the temperature is higher I will be higher.

So, also V 0 will be higher I will be higher right. So, very simple relationship you can get and then you can derive and this is keep in mind not very accurate, but; however, it can be used for design purposes right. So, what I would suggest, you look at one example and with this I will stop over and.

Thank you very much listening to this lecture.