Wind Energy

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Lecture 06: Fluid Mechanics - External flows

Welcome back so, now what we're going to do we'll continue our discussion on the fundamental of the fluid mechanics. So, so far as you recall that what we have discussed so far basic of fluid statics where we looked at how one can take a fluid element and then find out the pressure and the forces then We have talked about different cell forms, so all the governing equations that we have discussed. And then we looked at how the non-dimensionalization is done for the equation system, so that we kind of come up with some of the non-dimensional parameters. And from there, then obviously, we talked about geometric similarity. kinematic similarity, then dynamic similarity so all these. So obviously this dynamic uh i mean the dimensional analysis this always helps to avoid conducting too many experiments, if somebody is talking about multiple computations so what you can have you can have some non-dimensional number which can provide you some kind of a resume map from where you can actually pick your design data.

So that is the advantage of having dimensional analysis that if you can group them in some kind of an this non-dimensional parameters and then find out the design data then that would make the process faster. So today we continue with this dimensional analysis a little bit more onto this and then we can move. So, here we start with this fluid flows over an object and this experiences some kind of a resistance which is known as the drag force. So, that means we, so what we are talking about here that we take in particular object and if there are fluid flows over it, so there would be some kind of separation or whatever.

So, essentially the drag force, that is what we would like to find out. And this is an incompressible flow with smooth object. So, the drag force could be function of the characteristic length, velocity, viscosity, density. One can conduct the dimensional analysis and find out those numbers so that that can actually allow so here is your basic mlt system and this is your all the variables which are going to give you the drag force. So, What we are interested to conduct the model study.

In a wind tunnel to know the drag. So our variables here is 5. The fundamental dimensions are 3. So there would be m equal 2. Then our repeating variables would be 3.

So we will end up getting 2 dimensional or pi group. one is pi one and the second is the in terms of pi two which is the viscosity and, finally once we do the dimensional analysis obviously the details we are not working out here we are always kind of advised to look at the textbook if you really want to go into the details how things are done because important thing here is that we are just trying to refresh your memory about these different things so that we can move for aerodynamic calculations of them. Now having said that, what you can see the drag on a car which will be a function of like that and this is the matrix that you get so this is your one pi variable, this is pi two, so this is your drag coefficient which is a function of pi two. So, one can put in terms of two non-dimensional number, the drag coefficient should be. Now, you can do some testing.

Drag on a car $F = f\left(L, u, \rho, \mu\right)$

$$\pi_1 = \frac{F}{\rho u^2 L^2}$$

$$\pi_2 = \frac{\mu}{\rho u L}$$

	M	L	T
u	0	1	-1
L	0	1	0
ρ	1	-3	0
F	1	1	-2
μ	1	-1	-1

$$\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right)$$

Drag coefficient

$$C_D = \psi\left(\frac{1}{\mathrm{Re}}\right)$$

And the testing a model car in a wind tunnel. So this is geometric similarity. So this is the prototype, which will have the actual length scale, then the flow conditions, the viscosity, density. And this is the model car. So one can always try to find out what could be the reasonable scale ratio.

So, and then how to achieve the kinematic similarity. So, the kinematic similarity, then you put the model inside the test section, you get the other parameters like this drag balance moving belt. So, kinematic similarity to achieve that you need some essentially the moving platform so this platform has to move. So, this needs to move so that you can find out the kinematic similarity. So, what we need to match when you try to match the actually the prototype and the model parameter we need to match the Reynolds number.

So we need to conduct the test, where we need to match that Reynolds number. So the Reynolds number of model and prototype that would go as per the definition of the Reynolds number but obviously this velocity length and these are used in the I mean they are going to be different. If you take the same properties of the fluid finally you get the CD of this model and prototype and you find out. So, the analysis shows that the model that you are testing, that ratio that will get you the scale ratio. So seems some unrealistic for automobile applications.

$$\mathrm{Re}_m = \mathrm{Re}_p \quad \Rightarrow \left(\frac{\rho u L}{\mu}\right)_m = \left(\frac{\rho u L}{\mu}\right)_p$$

Taking same properties of fluid $\Rightarrow (uL)_m = (uL)_p$

Now let's match the $\mathbf{C_D}$ $\Rightarrow \left(\dfrac{F}{\rho u^2 L^2}\right)_p = \left(\dfrac{F}{\rho u^2 L^2}\right)_m$

$$\Rightarrow F_p = F_m \frac{\left(u^2 L^2\right)_p}{\left(u^2 L^2\right)_m} \qquad \Rightarrow F_p = F_m$$

So that kind of an, so that means if I do that, then my city of the model and prototype would have some kind of an ratio of 10, that means the model velocity should be 10 times the prototype velocity which is quite unrealistic. Typically, if you let us say at the peak condition the car may be running at 100 kph. So, what are the probable solutions? You go to a different kind of tunnel. Or you use some bigger model. You use different fluid.

So that, I mean, you try to still match the dimensional parameter. But yes, none of these options are quite easy. But yes, that is some challenge that one should have. So, summary is that for fluid flows over an object, you get drag force. And, then you get the drag coefficient, which is a function of Reynolds number.

So, Cd is essentially ratio of drag force by inertia force and Reynolds number is discussed by inertia. So here, the analysis indicates three different forces. One is drag. One is inertia. And one is discussed.

So, in both the case, the common forces is inertia. So, that's why the analysis scales other forces with respect to inertia. Obviously, this has to do to some extent with the choice of our repeating variables. So, what we have kind of obtained is that, this running curve experiences the drag force, which are the function and from there we get the drag coefficients is. So as we have already talked about that to match these things we need to match the re which may need model testing at the high speed okay so one can indicate i mean run some crude experiment where cd could be constant then we can explain this result from the dimensional analysis that f by rho will square become this, so, again the forces are kind of matched with respect to the inertia, so, somehow one has to manage this.

So, what you have the drag forces, the source is one is the viscous source another is that pressure which is known as the form drag. okay! obviously drag force becomes quite important or very very essential at high velocity and at high Reynolds number your viscous force would be very very small compared to inertia force which is going to be inertia dominated and, if drag dominates that is this is a picture of the cylinder at 0.1 Reynolds number 10,000 Reynolds number So you get different kind of. When we have high Reynolds number case change then we can drop mu. The reason is that this term is essentially arising due to viscous effect.

Using dimensional analysis:

$$\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right) \qquad C_D = \psi\left(\frac{1}{\mathrm{Re}}\right)$$

$$C_D = \frac{\text{drag}}{\text{inertia}} \qquad \frac{1}{\mathrm{Re}} = \frac{\text{viscous}}{\text{inertia}}$$

We are interested to measure the drag on a similar to estimate the drag on the prototype

$$\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right) \qquad C_D = \psi\left(\frac{1}{\mathrm{Re}}\right)$$

So, when we have high Reynolds number that time your viscous force is less dominating rather your inertia force is going to dominate the thing. is the inertia force is going to dominate. So, we can drop that term then we have dimensional analysis if you carry out then you get one pi term which is essentially the coefficient of drag obviously this particular relationship that we get here this is validities for higher Reynolds number which is order of 10 to the power 3 or more if your Reynolds number are less then your viscous dominations or the effect of the viscous that's going to be there. And then you cannot really drop out the term mu.

We can explain this result from the dimensional analysis

$$\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right) \qquad C_D = \psi\left(\frac{1}{\mathrm{Re}}\right)$$

$$C_D = \frac{\text{drag}}{\text{inertia}} \qquad \frac{1}{\mathrm{Re}} = \frac{\text{viscous}}{\text{inertia}}$$

Okay. So, we can conduct the model study up to a point where CD reaches the Re independent value. Here is an example of model testing of a track. where again these are connected with this all moving belt in the tunnel section so you have the model and prototype, this is the length scale ratio, this is the speed ratio, obviously for Re matching you match this so you get the model speed. So which shows if your sound speed is in normal atmospheric pressure temperature is roughly 330 meter per second so this is in the compressible region. If you put them in a tunnel, then it is difficult to get it.

So, you need to use supersonic tunnel. So, depends on the condition, one has to choose, that what kind of tunnel or testing facility has to be used and things like that. Now, if one carry out that model testing inside the tunnel then these are the different velocity speed you get different force so, you get cd versus Reynold number curve so, once you cross certain point here probably Cd becomes independent of Re. okay! now what application to prototype is that here the application is that there are curve that we have got that can be used to obtain data at the for the prototype where again we can use this condition so if the prototype Reynolds number is this at this point Cd is independent then we can find out the Cd and the drag forces. okay! now again coming back to the drag force which has become a function of I mean essentially cd become Re where we will keeping everything CD is ratio of drag by inertia.

This is viscous by inertia. So, again with respect to inertia force all rest of the forces are defined. So, now the earlier example we are talking about high Reynolds number case. Now, we are talking about low Reynolds number case. and lower in my case your inertia is small. So, drag force is equivalent to viscous force so we cannot drop the term of mu okay! but obviously the drag force has two component, one is the pressure drag which is known as formed drag and that is the viscous drag obviously at low Re the viscous part dominates, which one has to take into consideration.

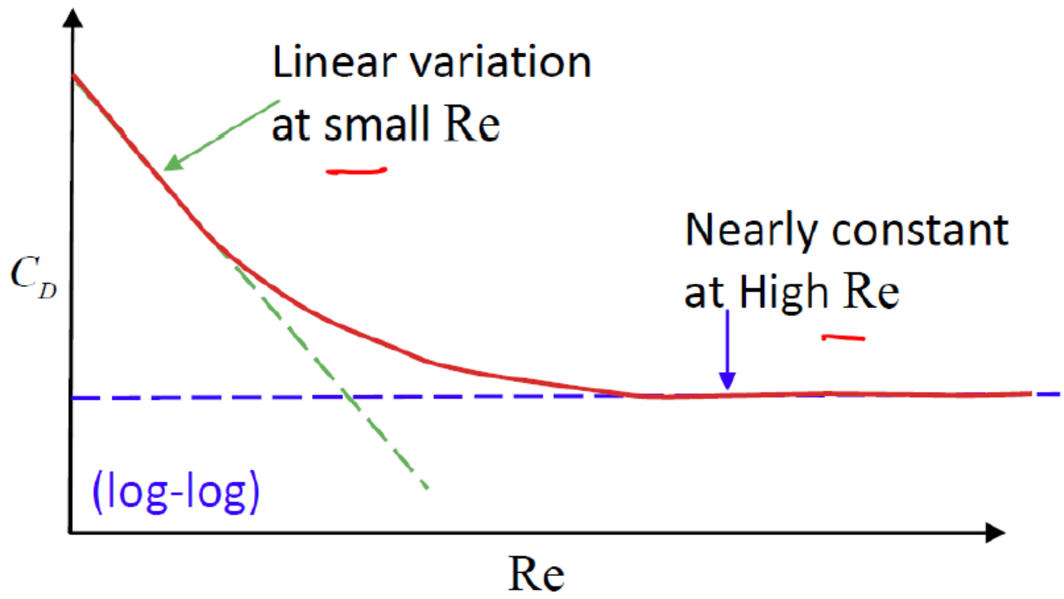
Now, here what we can do for low Re case or that limiting case, we can drop density. And we can conduct the dimensional analysis, then get this parameter, which is also constant. So, basically, CD becomes constant by Re. So, this relationship holds for Re less than 1, which is creeping flow or stokes flow, and quite useful for viscosity measurement, micro flows, biological systems, and things like that. So from the dimensional analysis, what you get, which is a very important aspect of it, I mean, if you look at it, you have two extremes of it.

One is the lower Re condition and there's the higher Re condition. Higher Re condition, inertia is going to dominate. OK, so then drag becomes like this. And the lower Re condition, the viscous is going to be dominating. So, your drag force should retain the component due to the viscous forces.

So, it tells you, I mean, this kind of analysis, the usefulness of this analysis is the following - You can actually, trying to scale the forces and understand the physical process is much better that, what are the forces (obviously the fluid dynamical forces which is going to be dominant or not dominant depending on the situation), for high speed case, if you look at this the drag is proportional to velocity square while as a low speed case it is proportional to u. Obviously, if you are going towards a very low number situation then it would be difficult to start or maintain the motion. So, once you combine these things, then you can always put them in. So, in one particular table or resume map, which can be used for your design data. Here, you have lower Re case, which is obviously you have constant over Re.

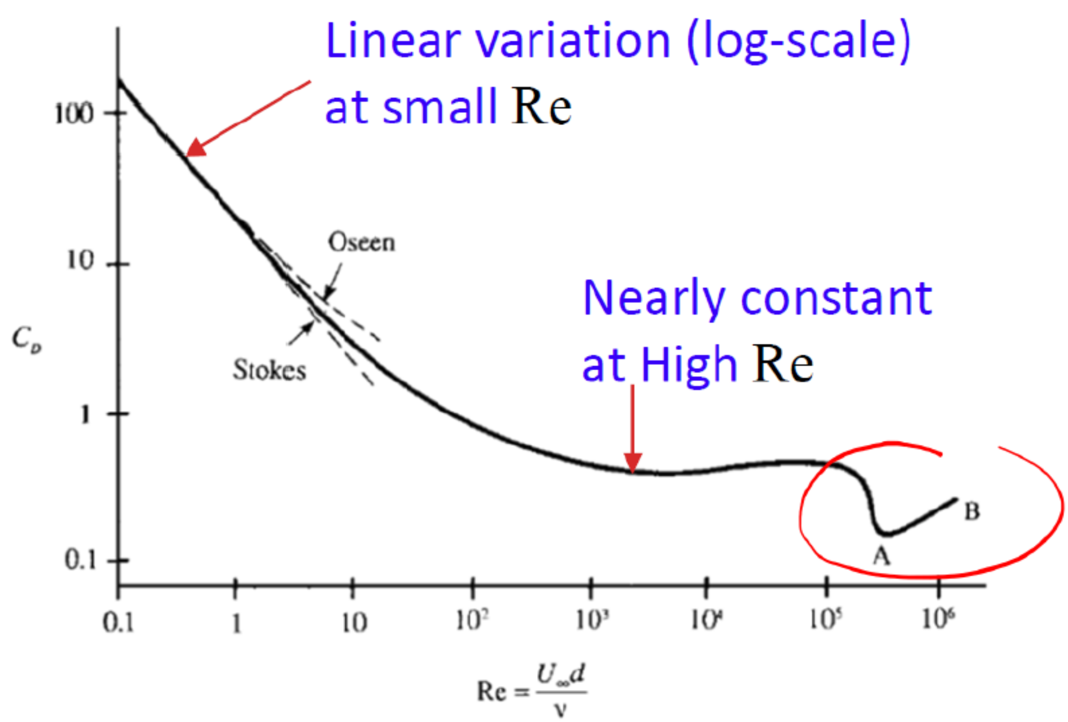
That means Cd is inversely proportional to Reynolds number. In this case, Cd is constant. So, you can see this is a linear variation, then they are nearly constant, which becomes independent of Re. So, this is more like a drag coefficients over a flow over an object, where you get to see this kind of things. Now, here it is an flow over a sphere, here, which is again the data that we are showing here this again, you can find it in any textbook.

So, here you see this linear variation then it is nearly constant then there is a dip. And, this is known as drag crisis. This is a very common curve available in pretty much all the textbook where if you try to in obviously, the viscous pro textbook of fm white or something so what we can try to see that your terminal speed of a falling object which is again the drag is equivalent to the weight then drag becomes L cube. So one case you have low Re case other is high Re case so the drag would become proportional to the u square this is proportional to the u. Obviously the terminal velocity varies differently with length scale for smaller and larger objects.

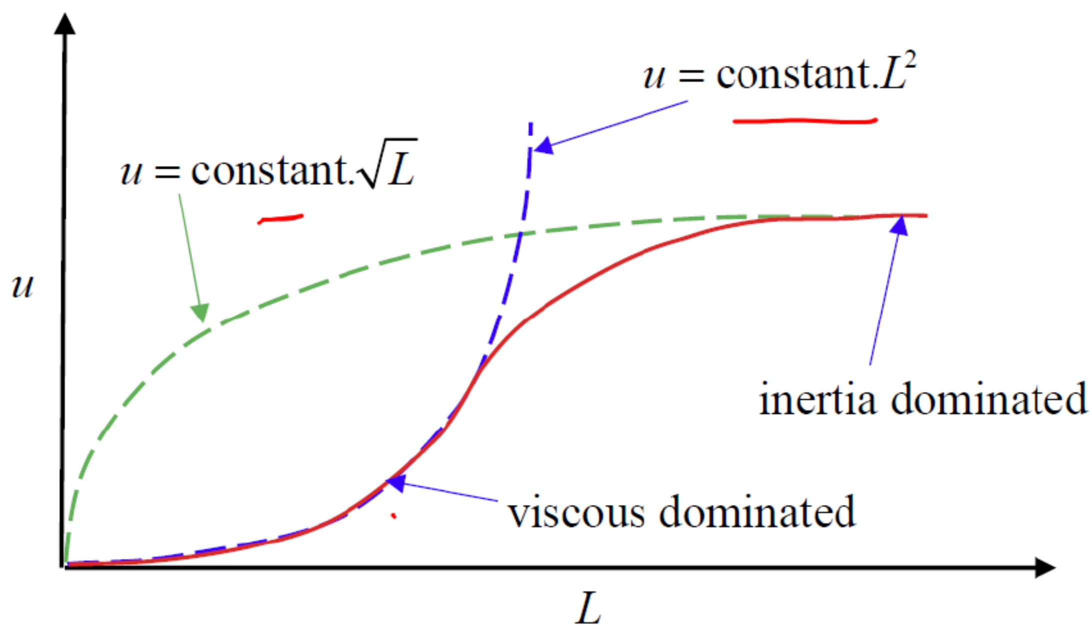


Drag coefficient for flow over an object (conceptual)

Drag coefficient for flow over a sphere (experimental)



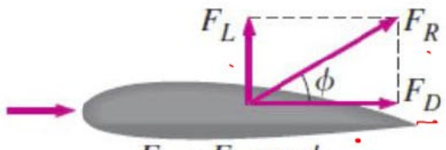
Obviously depending on the size of the object that can vary. So, the point here is that you can so again this is an conceptual plot u versus l, so, this is a situation where you have a discuss dominated so u is constant into l square this is constant to root l this is inertia dominated so you get different kind of situation that means one case is a low re case the other case is a high re case Okay. So, that gives you an idea about now that's the advantage that one has while doing dimensional analysis, that you can capture certain phenomena easily rather obtain the design point from the this kind of analysis. So now, what we are going to do we are going to look at uh the other part of it is just like an flow or external flows and, then we quickly touch upon the turbulence and all these things, so that, so this is what if you see the way we are kind of having our discussion or try to refresh your basics of the fluid mechanics. So I'm reiterating that so that you can refresh your memory and you can go back and look at the textbook.



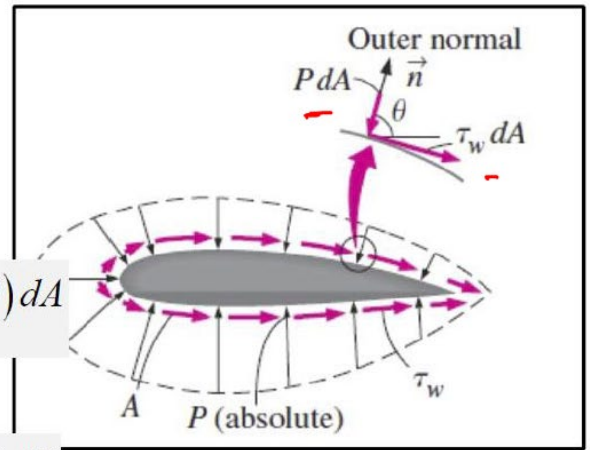
We looked at quickly the fluid statics, then we looked at the basic system, closed systems and all these things. Then we looked at the integral from obviously through RTP and then the basic governing equations of conservation laws. different cell form, dimensional analysis, application of the Navier-Stokes equation, meanwhile we have done some Bernoulli's equations, then we talked about the basic definitions of streamlines, sticklines and all these things, then we moved to this flow over bodies and all these things where we are looking at the dimensional analysis to kind of getting these things. And then, now we are moving towards the external flows, which one can think about when we talk about the wind turbine and things like that, those are external flows. And then also a little bit talk about turbulence.

Obviously, not in depth of turbulence because, as you know turbulence itself is an itself is a topic which can be taught over a complete course. So, what we are going to do the important aspect of the turbulent field and what are the things that we might need. So, we will try to refresh and basic fundamentals of the turbulent flows, their properties and then we move to the discussion on the wind turbine. Okay, so here if you look at an external flows which are again as i've said, these are very very common in practice here is a aircraft which is flying so this is a flow when the aircraft is an external flow, we have talked about wind turbine we have talked about the buildings, So, study of external flows lead to the concept of boundary layer, one of the crucial developments in the fluid mechanics. Obviously, internal flow also experiences boundary layer, boundary layer growth, their separations, everything.

But yes, external flows are. Now, we can talk about drag and lift. this is again i mean one can think about drag is essentially a resultant force and, lift is kind of an force which will normal i mean resultant force normal to the flow direction. So, here is an example of an airfoil and this shows the pressure distribution so the pressure at the top surface is lower and the pressure and the higher surface is the higher so this is called the i mean typically, this is called the pressure side this is called pressure side and this is called suction side and, then when the flow passes by that then you have the distribution drag is resultant force in the flow direction direction of the flow we have pre-stream velocity far away from the body. There are some other here this is an aircraft so this is streamlined body this is called drag reduction this is also streamlined body where there is a drag reduction but this is a black body so depends so it depends on the geometric nature of the body things are classified that whether these are streamlined bodies, so obviously, streamlined bodies means which are aligned with the flow field and there will be less separations and, these are the block body that means it's just like so what it happens is that when you have streamlined body obviously drag would be less when you have block body drag would be less i mean high because it's just like an how these forces are calculated so, if i have a airfoil like this, then this is the resultant force the drag forcing acting and, this is the lift force then the Fd will be resultant for of cos phi and, now, over the airfoil if you see this is the pressure distribution and, this is the normal force and, the shear stress distribution then the drag force would be the integration over the surface of the pressure and the shear stress component, lift forces, the pressure and shear stress component and whatever lift and drag force we obtain once we scale it with the dynamic head and the projected area then, we get the drag coefficient and liquid. So, when you talk about this most of this airfoil and external bodies we very often use this drag coefficient lift coefficients and things like that, so these are very common terminologies that continues.



$F_D = F_R \cos\phi$
$F_L = F_R \sin\phi$



Drag: $F_D = \int_A \left(-p\cos\theta + \tau_w \sin\theta\right) dA$

Lift: $F_L = -\int_A \left(p\sin\theta + \tau_w \cos\theta\right) dA$

Drag coefficient: $C_D = F_D \Big/ \left(\frac{1}{2}\rho u^2 A\right)$

Lift coefficient: $C_L = F_L \Big/ \left(\frac{1}{2}\rho u^2 A\right)$

Projected area against the force

Again, you look at this so this is the angle then once you look at the drag forces so you have a pressure component this is known as the from drag, pressure drag this is the shear stress component known as the friction drag. As we have already seen in the dimensional analysis so you can see when we go to low Reynolds number regime this drag, the viscous drag is, so the drag force has two components. So, once you move to the low Reynolds number regime, this is going to dominate. And once you go to high Reynolds number regime, then this form that is going to, that means the pressure force is going to dominate. So the domination of the force, so that one can also identify what is the physical processes that is going to take place.

okay, so I mean but when you calculate the complete drag force you have to have both the components due to pressure and the shear stress and then depending on the situation, so if we look at back so this component from the drag force I am getting the friction and drag and this side I am getting the pressure drag and then I can get the if I again scale it with the dynamic heat and the area. So I'll get this equation in this format. So what one can say, the drag coefficient, because this is my drag coefficient, that's a total drag coefficient on that complete drag coefficient that will have two components. One contribution coming, the drag coefficient due to viscous forces, another due to pressure forces.

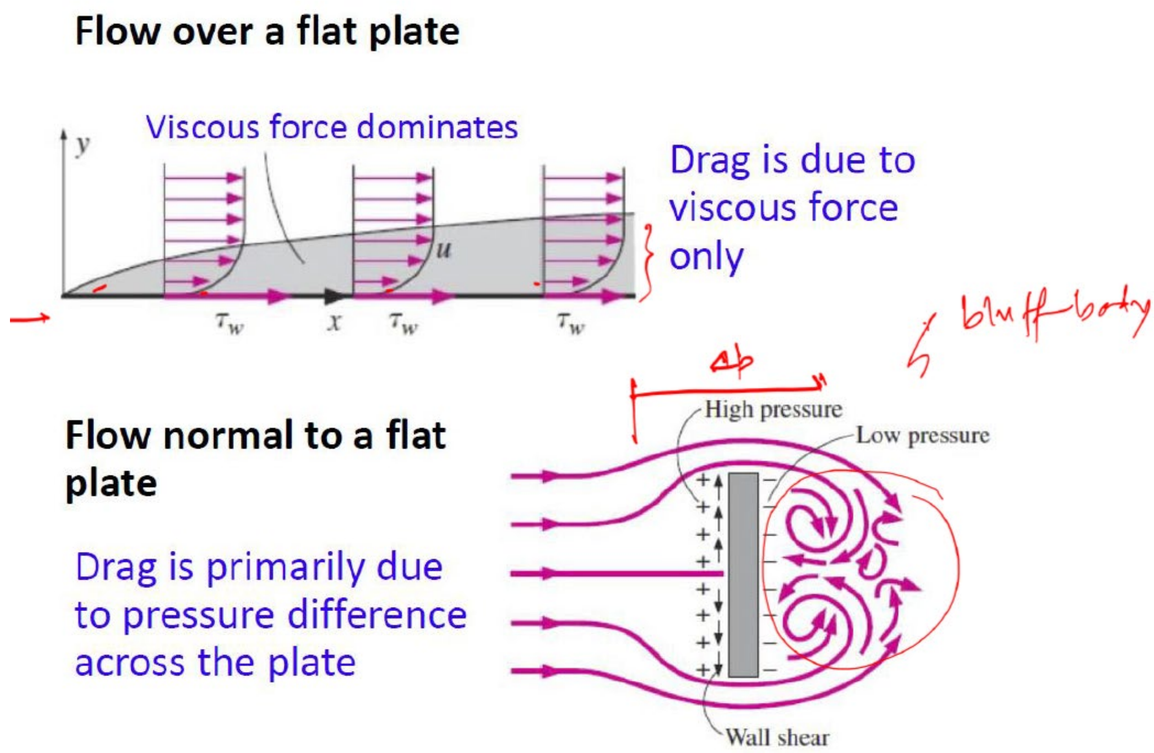
Okay. So, that's what is going to happen okay so when you talk about these forces. Now, similarly moving to quickly touch the boundary layer things, so, that means when the flow over I mean flow over flat plate obviously here you see this is a example of a bluff body okay, this is a flat plate where things are when the flow flows over the flat plate, then what happens is that, the boundary layer grows and obviously the velocity profile also grows over there, so, this region there will be more viscous domination and this is along the plate and this is perpendicular to the plate so here there is a lot of recirculation zone which is created or location zone and because of this side and this side there is a delta p that is going to create some drag forces because i mean there is a pressure differential across that vertical plate which is going to generate the drag force when the flow passes. So, this one can say that this is a flow which is passing over a black body.

Drag: $$F_D = \int_A (\tau_w \sin\theta)\, dA + \int_A (-p\cos\theta)\, dA$$

Viscous (friction) drag   **Form (pressure) drag**

$$\frac{F_D}{\frac{1}{2}\rho u^2 A} = \frac{\int_A (\tau_w \sin\theta)\, dA}{\frac{1}{2}\rho u^2 A} + \frac{\int_A (-p\cos\theta)\, dA}{\frac{1}{2}\rho u^2 A}$$

$$C_D = C_{D,\text{viscous}} + C_{D,\text{pressure}}$$



So, there are some experimental flow visualization which shows when the flow over a flat plate, obviously there is a drag due to viscous, but in the normal plate you can see that region behind the plate is quite high. So, that is going to, as I already said, the drag force is going to be primarily due to the pressure differential across these things.

Okay. So, what we can go back and recall that the dimensional analysis that, if it is a laminar flow then Cd is a function of Reynolds number and if it is a turbulent flow then the drag coefficient will be function of Reynolds number and then epsilon by d. This epsilon by d is the factor, which is the smoothness factors and so low Reynolds number range the cd is inversely proportional to the Re which is a creeping flow or stroke flow zone, high Reynolds number which is a constant laminar or epsilon by d for the turbulent cases. So external flow which is close to the smooth body usually remain laminar up to Reynolds number 10 to the power 5 unlike internal flows or beyond that they are going to be turbulent then you can actually use this roughness factor or non-smoothness factor to estimate the drag coefficients. There is a chart which is available in any textbook which is known as the Moody's chart. So, typically when you try to calculate the drag force for the given set of parameters you can always refer to Moody's chart and estimate the drag coefficients and things like that.

So, this is what the advantage you have using this kind of dimensional analysis or things like that okay! we'll stop here and continue that other discussion in the next session.