Space Flight Mechanics
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology – Kharagpur

Module No # 16
Lecture No # 80
Trajectory Transfer (Contd.)

Okay so in the last lecture we have been discussing about the basic idea about the trajectory transfer and also we looked into the rocket equation that is used for the satellite. Now in this trajectory transfer we have seen that it can be based on the same center of force or different center of force.

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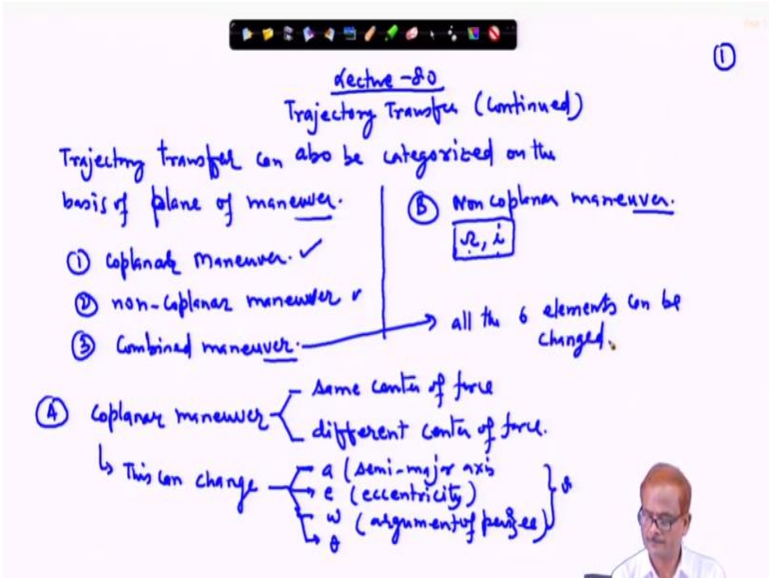

And another is the category for that transfer so the trajectory transfer can also be categorized in the basis of number of impulse of number of impulse already we have mentioned. On the basis of co planner or non-co planer on the basis of plane of maneuver accordingly we will have the co-planer maneuver and non-co planer maneuver and the combined maneuver which involves both co-planer and non co planer maneuver,

Okay so co planer maneuver we can have same center of force different center of force this maneuver can change a e and ω means the semi major axis eccentricity and argument of perigees. So these are the 3 things which can be affected by co-planer maneuver non co planer maneuver in this rest of the things can be changed. So non coplanar maneuver means maneuver not in the same manner and therefore the Ω i can be changed.

And of course here we have not put the θ is also there equally we can write here the θ so in the non-co planer maneuver Ω and i both of them can be changed.

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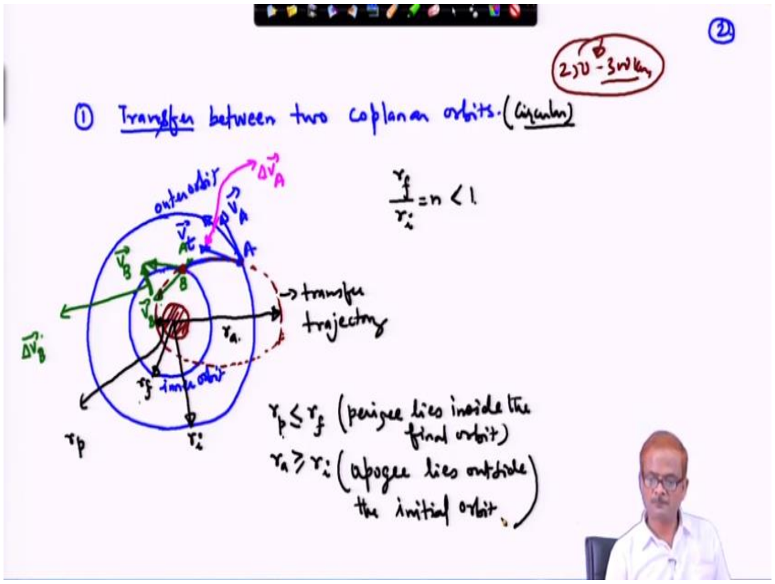

So now we discuss about the transfer between the orbits we have got the basic idea transfer between 2 co planer orbits. And in the combined maneuver as it implies you are doing both co planer and non-co planer maneuver and therefore all the 6 elements can be affected all the 6 elements can be changed. So what we are going to study here first we will look into a general parametric view of the transfer equation this trajectory transfer equation.

So say the satellite is in the inner orbit going into this circular orbit and then what is required that there is an inner orbit. So this is the outer orbit and here this is your inner orbit and it is recorded to put your satellite from this place into this orbit the maneuver how it will start. You can start the maneuver say you want to start here at A and terminate it B so then you need to go like this V_A is the velocity in the circular orbit and V_t is the velocity in the transfer orbit.

So impulse required will be this much this is your delta v shown by the incline at this point again the absolute velocity in this orbit should be here in this direction this will be tangential to this point. Say this is point B V_B is the velocity here while you get here v transfer at we can tag it with orbit A V_{At} and here in this orbit velocity will be here in this direction in the transfer orbit V_{Bt} the transfer orbit at B.

So how much extra impulse is required this is extra velocity direction so impulse is required here in this direction. ΔV and thereby you can achieve but to do this exercise completely you will see that this orbit should look like this. And remember the orbit should be such that here say in the case of the earth once we are planning so also we have to take care of that while we miss to thrust at the point B so it should not go and dip into the earth.

So trajectory as to be such that it is not going into the atmosphere if it goes into the atmosphere interact with the atmosphere then it may burn out which severally it will get severally damaged. So minimum altitude of 250 to 300 kilometers is required and immediately after launching you have this much of altitude which is just for the. Okay we are going to study this particular aspect here. So what we can see this is our transfer trajectory and for the transfer orbit this acts as r apogee and this point acts as r perigee.

Immediately from here it is visual to us that r perigee this will be less than equal to r final what is r final? r final is this radius is this is r_f and what is r_i this is r_i the initial orbit radius and this is the final orbit radius and both are circular let us say. For convenience just assume it is a circular so r perigee will be less than r_f and r apogee will be greater than r_i. So this is the condition imposed here including we assume here $r_f / r_i = n$ and therefore because the r_f is less than r_i here this will be less than equal to 1.

So we assume strictly this is less than 1 because it cannot be the same orbit so this are the information's given to us and we have to process it further okay.

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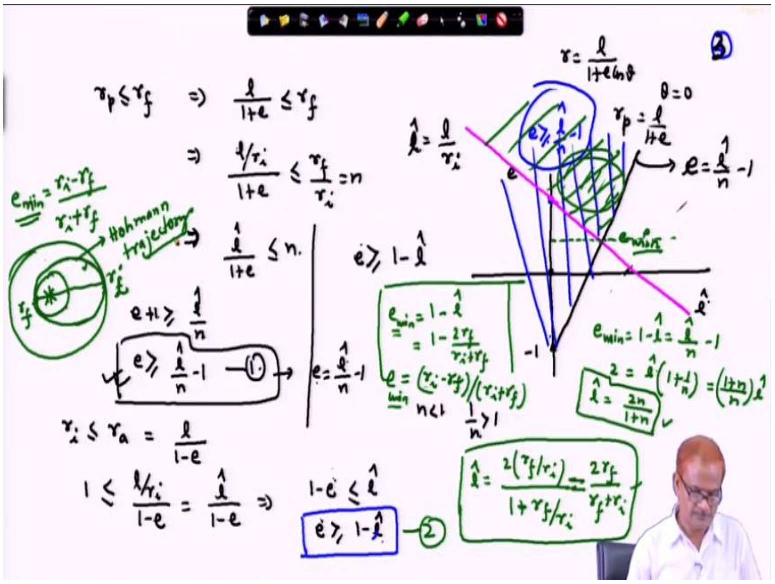

$$r_p \le r_f$$

$$\Rightarrow \frac{l}{1+e} \le r_f$$

$$\Rightarrow \frac{\frac{l}{r_i}}{1+e} \le \frac{r_f}{r_e} = n$$

So once we have got it that r_p is less than r_f and r_p we note that quantity will be l / (1 + e) r = l / 1+ e Cos θ at perigee θ = 0 and therefore r_p = l / (1 + e). As we have done earlier so this quantity will be less than equal to r_f we divide both side by r_i this becomes (l / r_i) / (1 + e) is less than r_f / r_i. This we write as n as we have written and this again will write as $\frac{\hat{l}}{1+e}$ is less than n where $\hat{l}$ is l / r_i.

$$r = \frac{l}{1 + e\cos\theta}$$

$$r_p = \frac{l}{1+e}$$

So this is non-dimensionalized earlier this things were very important because once the computer was not there and so much computation has been done so this was the method of doing it plotted do parametrical study and the depth the plot of that and then from their end for the result. So various other cases okay so this implies e + 1 greater than $\hat{l}$ / n or e will be greater than $\hat{l}$ / n – 1 this we term as equation 1.

$$e + 1 \ge \frac{\hat{l}}{n}$$

Similarly we have seen that r_i is less than r initial r oppose is greater than r_i so r_i is less than r apogee and r apogee is nothing but l / 1 – e okay dividing both side by r_i. So this becomes 1 and l

/ r_i 1 – e $\hat{l}$ 1 – e and this implies 1 –$\hat{e}$ this is less than $\hat{l}$ or $\hat{e}$ is greater than equal to 1 – $\hat{e}$. So the, another equation we are getting this is not $\hat{e}$ this is just e so e is greater than equal to 1 – $\hat{l}$. Now we plot both of them will plot here e on this side and $\hat{l}$ on the x axis this is the origin here.

So once $\hat{l}$ is 0 from here can see this equation if we just equate the with the equality sign so once $\hat{l}$ is 0 e = -1. So somewhere it cuts here in this point here this is -1 and slope of this curve is 1 / n so it goes like this n is greater than n is less than 1 here in this case this particular and therefore we will draw accordingly. We are assuming one is present here so it will be little like this.

N is less than 1 therefore 1 / n this will be greater than 1 so slope of the curve will be more than 45 degree and then using this part. If we use this is e greater than $\hat{l}$ n -1 so this is the equation for e = ($\hat{l}$ / n) – 1 okay. So $\hat{e}$ will be greater here on this side so here your equation e greater than equal to l cap divided by n – 1 this is satisfied. Now take the, another equation this one once $\hat{l}$ = 0 we can see that this will be equal to 1. $\hat{l}$ equal to 0 e = 1 this is just on the opposite side in the slope of the curve is – 1 okay.

$$e = (\hat{l} / n) - 1$$

So from minus here just it goes like this and then e greater than 1 lies here on this way is the common area is your dashed line with green and blue okay. So this is your common area and this is e minimum this is the minimum e possible okay. For this transfer orbit because both this equation 1 and 2 this both of them need to be satisfied and it can be satisfied only in this common area where which is I have shown it shaded with green and blue.

$$\hat{l} = \frac{2(r_f/r_i)}{1 + \frac{r_f}{r_i}}$$

So e minimum can be computed as writing here 1 – $\hat{l}$ equal to $\hat{l}$ n -1 from equation 1 and 2 by equating them. And therefore from this place this will be equal to 2 = $\hat{l}$ and this is 1 + n divided by n times $\hat{l}$ is 2n divided by 1 + n. So therefore this can be written as n we have already written here r_f / r_i and this is 2 times r_f / r_i divided by 1 + r_f / r_i. And what is the value of e so then the value of e can be computed from anyone of them.

$$e_{min} = 1 - \hat{l}$$
$$e_{min} = 1 - \frac{2r_f}{r_i + r_f}$$

So e minimum equal to $1 - \hat{l}$ and we can write either of this or this so $1 - 2r_f / r_i + r_f$ now this is your e minimum. And what this quantity is e minimum is nothing but your $r_i - r_f$ $r_i + r_f$ so this implies that you have the minimum inner and outer orbit and your transfer orbit will looks like this. This is your final position and this is the initial orbit r_i and this is r_f. You get minimum eccentricity for this condition where this orbit will be touching the external and the internal orbit the inner orbit and the outer orbit.

$$e_{min} = (r_i - r_f)/(r_i + r_f)$$

Especially this kind of trajectory we call as the Hohmann trajectory and we call this as the Hohmaan transfer so, this is the minimum eccentricity transfer not necessarily the minimum energy transfer. We will discuss later on to, went this moment transfer can be greater than other transfer. So we have one more transfer we call it as the bioelectric transfer it will come due course of time once we progress.

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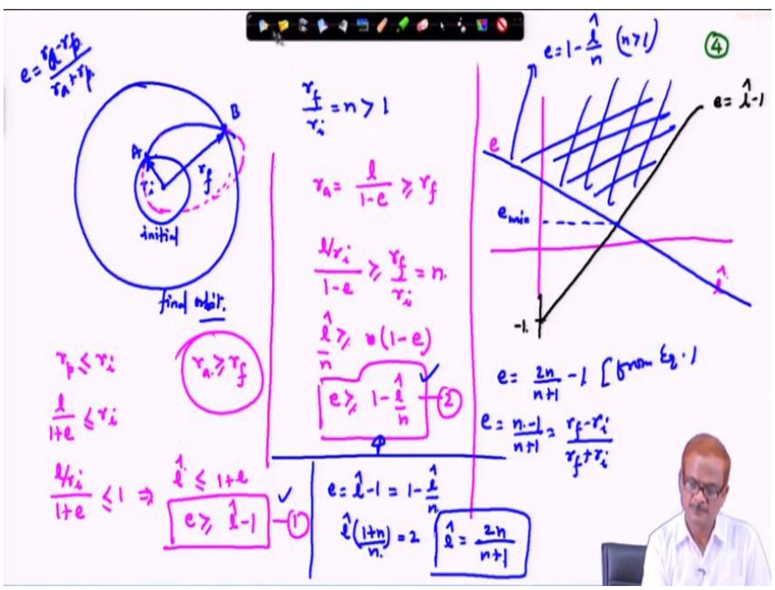

Okay once we are done this same exercise can be done for going from inner to the outer orbit. So similarly we have inner orbit here and some satellite is here and transfer is required to go from this place to this place. Here in this case this is the final position and this is the initial position this is r_i and r_f in this case r_f / r_i this quantity we have written as m and this is greater than 1 this is A and B. The initial orbit and this is the final orbit and what we have done for the other part the same exercise can be done here also.

$$r_p \leq r_i$$
$$\frac{l}{1+e} \leq r_i$$
$$\frac{\frac{l}{r_i}}{1+e} \leq 1$$

Now you can see that r perigee you can extend it you can look like this so r perigee this is going to be less than equal to r_i. And similarly r apogee is greater than r_f so this 2 conditions must be satisfied and therefore this if l / 1 + e should be less than ri. We follow the same process dividing by r_i and this implies $\hat{l}$ is less than equal to 1 + e or e is greater than or equal to $\hat{l}$– 1. This is equation 1 similarly this condition we can utilize here r apogee is equal to l / 1 – e is greater than r_f divide both side by r_i.

$$r_a = \frac{l}{1-e} \geq r_f$$
$$\frac{\frac{l}{r_i}}{1-e} \geq \frac{r_f}{r_i} = n$$

So l / r_i is 1 – e is greater than $\hat{l}$ this is greater than equal to m times 1 – or $\hat{l}$ / n this is 1 – e n is a positive quantity and therefore e is greater than 1 – $\hat{l}$ / n this is equation 2 and plot both of them e on this side and $\hat{l}$ here on this side so the equation 1 $\hat{l}$ is 0 = -1. So this is -1 here and slope is 1 It goes like this is e = $\hat{l}$ – 1 another equation we will get for this one e = 1 – $\hat{l}$ divided by n slope is negative and this is less than 1 because n is greater than 1.

So the slope will be less than 1 so it will go like this and this curve is for equal to 1 – $\hat{l}$ divided by n where n is greater than 1. And therefore slope becomes less than 1 and both this equation 1 and 2 will be satisfied in this region. And this is your minimum so as usual we solve for e minimum equal to $\hat{l}$ – 1 this equal to 1 – $\hat{l}$ divided by n. And this implies $\hat{l}$ 2n n + 1 – 1 from equation 1 e = 2n – 1 so n – 1 / n + 1 and this is r_f – r_i divided by r_f + r_i.

$$e = \frac{2n}{n+1}$$

And eccentricity equation e is basically for any ellipse is given by r_f – r apogee – r perigee divided by a r apogee + r perigee.

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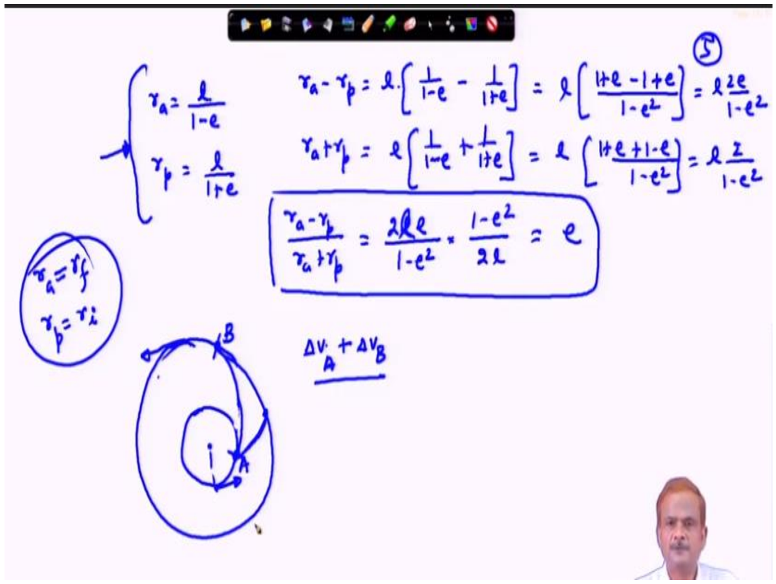

$$r_a = \frac{l}{1-e}$$

$$r_p = \frac{l}{1+e}$$

$$r_a - r_p = l\left(\frac{1}{1-e} - \frac{1}{1+e}\right) = \frac{l\,2e}{1-e^2}$$

$$r_a + r_p = l\left(\frac{1}{1-e} + \frac{1}{1+e}\right) = l\frac{2}{1-e^2}$$

$$\frac{r_a - r_p}{r_a + r_p} = e$$

We go to the next page you just look here in this equation r apogee equal to l / 1 – e and r perigee equal to l / 1 + e. So r apogee – r perigee l time 2e – 1 – e^2 and similarly r apogee – r perigee l times 2 / 1 – e^2 here r apogee – r perigee divided by equal to 2 le divided by 1 – e^2 times 2 l times l – e^2 and this gives you e. So while we are solving for the minimum case so there it implies that minimum eccentricity here ra corresponds to r_f and r perigee corresponds to r_i in this case and vice versa.

In the previous case we are just opposite of that here r_f corresponds to r perigee and ri corresponds to r apogee. Okay so just now we have looked into the basic things required now we can derive for 1 general trajectory transfer in this parametric terms. That means this is at point A here another point B and you just want to go from this orbit to this orbit. We want to send the satellite from here to here from point A to point B okay.

So how much total impulse will be required so total impulse will be given by $\Delta V_A + \Delta V_B$ this we need to compute okay. And general derivation can be done so this case what we have done the minimum eccentricity refers to there you initiate the transfer just at tangentially to the orbit okay. And then you also merge into the other orbit tangentially so that is the minimum eccentricity transfer also we call as the moment transfer.

Here in this case this is a generalized transfer you want to just go from one place to another place you start here to go here in this point it is possible only thing that you require larger amount of impulse. So this general treatment we will do in the next class thank you very much.