**Space Flight Mechanics**
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**Lecture – 92**
**Interplanetary Mission (Contd.,)**

In the last lecture as we have seen the figure become very complicated but we have fairly good idea about what is happening when the satellite is living from the sphere of influence of the earth ok and entering into the orbit about the sun. Now, it will cost in the orbit around the sun according to Hohmann transfer ok, and once it reaches the sphere of influence of say the other planet which is in this case Mars so then what is going to happen?

So it will follow the same rule as for the exit from the sphere of influence the same rule applies for the entry into the sphere of influence. I will just draw the figure for that rest other things remain the same.

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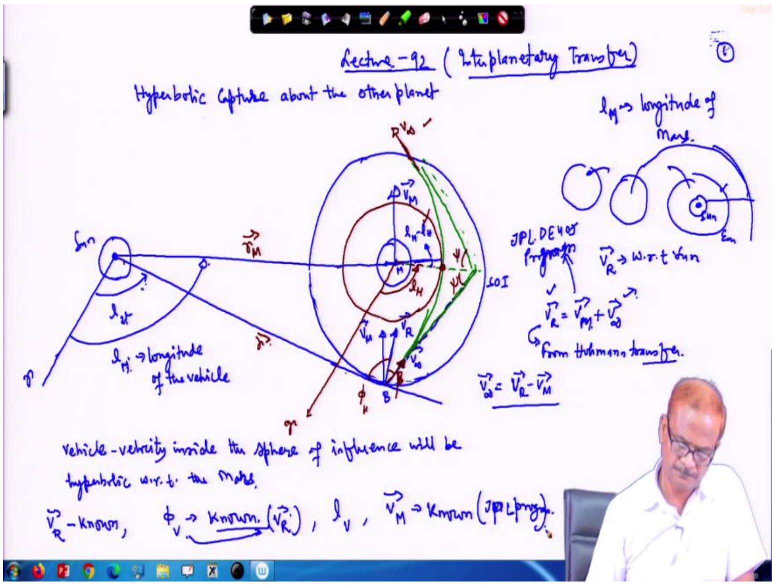


Here this is the sun. This is the vernal Equinox we will write this as a $l_M$ to indicate this is the longitude of $l_M$ is the longitude of Mars. This is the longitude of the vehicle ok we will make some changes here. What will do, we will keep the vehicle here Mars is here and then it is entry into the Mars from the lower side. Because we have taken earlier that it is exiting from this site once it goes to the Mars we assume that this is entering from this site that is the from the lower side.

This is your Earth here here in this place. This is Sun. Here this is the Earth and the Mars is here. Once you are departing from this place and it will go off into the Mars anyway, whatever the way we made the figure if it does not matter. So I will show it here the this to be the l vehicle and this is to the longitude of the Mars and I will make the Mars figure here this is Mars and about this then we have the sphere of influence.

See this is the sphere of influence. And this we are indicating as the Mars. Let us say this is $V_\infty$. The hyperbolic excess velocity of the entry and this is the actual velocity which we can calculate from the Hohmann transfer. This is the $V_R$ is the where is the velocity with respect to the sun. And this is $V_\infty$ and V mass line here in this place. This is Sun and Mars is here velocity will be here in this direction.

So we can see here $V_M$ like this. So $V_R$ equal to $V_M + V_\infty$. $V_R$ is known ok this quantity from the previous step from Hohmann transfer it will be known. V Mars, this is known from the JPL, JPL D 405 program. From the JPL program this is known and $V_\infty$ that can be calculated from this place. This is $V_R – V_M$ hyperbolic excess velocity this is known to you from this state.

$$\overrightarrow{V_R} = \overrightarrow{V_M} + \overrightarrow{V_\infty}$$
$$\overrightarrow{V_R} - \overrightarrow{V_M} = \overrightarrow{V_\infty}$$

And then the rest of the things is as usual we have defined earlier. So, $V_\infty$ is the velocity we draw the asymptote and from here the velocity vector as it goes it takes turn like this. And this is the point where you would like to capture the satellite. At this point the retro impulse is applied and the vehicle orbit can be circularize. Otherwise it will escape from this side. Ok. So at this point again, the velocity vector will be here $V_\infty$.

So if it is an escape then this is a flyby care. So, whatever we have discussed here is the fly by goes according to the same rule. All the calculation done here it is applicable to flyby if the velocity increases in the case of the flyby we say this is the gravitational slingshot. OK what is the gravitational slingshot it is not the usual word, we always use of planetary flyby. The planets ship flyby the velocity can increase that absolute velocity with respect to the Sun the velocity can increase or either it can decrease also that means you can reduce the velocity of the satellite with respect to the sun using planetary flyby.

But you are to plan right in the beginning. Ok and passports while it passes through the; it goes on in the trajectory then the correction has to be applied because these are very fine calculation which needs to be done on the computer. As usual this is your side angle $\psi$ angle and all others thing that remain the same $\phi_v$ already we have written here $\phi_v$ I will write this as the this is the radius vector r and this is $r_M$.

So with this angle write as $\phi_v$ which is the; I will not say this is a longitude of the resultant velocity or the velocity with respect to the sun. The $\phi_v$ is indicated here. $V_\infty$ is your; in this direction and then this becomes your corresponding $\beta$ angle as we have indicated in the last case. Again this way whole thing can be constructed. So all the angle calculation what we have done last time everything is applicable.

Here in this direction this is your vernal Equinox direction longitude of the perigee where it is aligned that you can write here let us say a in this case we write as this is $l_M$ so this we write a $l_H$. Ok according to the notation we have used earlier. So they will differ like this. This line goes straight here in this direction this angle real these differences is $l_M$ - $l_H$. So what we are doing that the planetary velocity once it goes inside the sphere of influence the vehicle velocity inside the sphere of influence will be hyperbolic respect to the Mars as we are considering here in this case.

Ok therefore we have the $V_R$ is known this quantity is known then $\phi_v$ will be known because heliocentric velocity of the vehicle is non $V_R$, $V_R$ is known and therefore the $\phi_v$ is known all these angles can be computed. Longitude of the vehicle this is known and the velocity of Mars this quantity this is also known it is a known quantity from JPL ephemeris program.

This angle we write as $\theta$ as usual ok so things need to be properly programmed. These are the idea which goes in the actual implementation. So using this all these information and $\Psi$ angle this can be determined. Whatever we have worked earlier if $V_\infty$ becomes known this is $V_\infty$ becomes known. So rest other things can be worked out using the information we have gathered here.

So using this information and hyperbolic equation asymptotic of angle $\Psi$ can be determined So $\Psi$ can be determined as we have written earlier in terms of; we have written in terms of $V_P$ and

$V_C$ velocity in the circular orbit and in terms of this we have defined it. So $\Psi$ can be known from here moreover peri centre longitude this we have written as $l_H$ peri centre longitude, $l_H$ distance, $\rho_0$ which is the parking Orbit radius and velocity $V_P$ which is the velocity at the perigee can we determined making possible computation of Retro impulse.

So the same equation is to be applied only thing here in the last case we assume that it was already in the parking orbit and from there the $\rho_0$ was known to us here in this case we have to determine the $\rho_0$ and what will be the velocity of the $V_P$ where we want to capture ok that can be determined. So, it depends on at what angle this is entering into the orbit and accordingly we will from the symmetry.

You can see that where the perigee will lie and all these things are obvious from this place. For this we will not have any problem in the exam because it is mathematically little complicated in terms of angles and other things this is just an exposure to you for this technology how the things are done. Ok so lastly, we have the planetary capture this planetary flyby which is based on the same concept.

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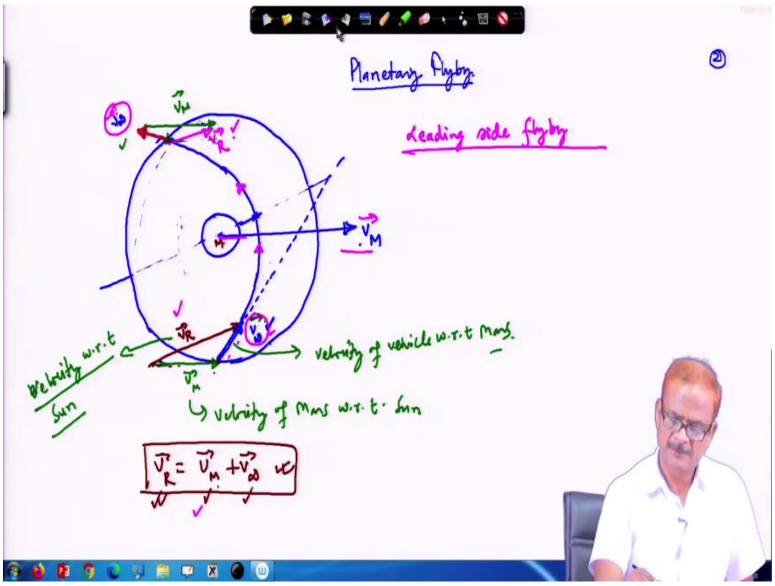


$$\overrightarrow{V_R} = \overrightarrow{V_M} + \overrightarrow{V_\infty}$$

So we right here the planetary flyby this we need to discuss. See this is exactly the same thing what we have been doing for the; in the previous case what we have been discussing the same thing is applicable here. Let us say this is $V_\infty$ by entering here and this is the velocity of Mars is here in this direction. So we show it by another colour for this is $V_M$ this is the velocity of Mars this is $V_\infty$ from where this will come?

This will come from $V_R$, $V_R$ is known to us. This is the velocity about this one as it comes to this sphere of influence of the Mars. Ok. This is your Mars. Ok. So with respect to Mars velocity how much angle this we are is making that will decide what will be the $V_\infty$ and once $V_\infty$ decided from this step then we can extend this. You can extend line like this and so velocity will go here asymptotically and will exit from the other side.

So it will go here like this and exit from the other side. So this becomes your perigee position here. Now whatever the $V_\infty$ is here the same $V_\infty$ you will get in this place. So this is your $V_\infty$. This will not change and V Mars as usual, this is the V Mars also it does not change in magnitude and it remains the same. So, what will be the resultant velocity? $V_\infty$ is here in this direction and mass is moving like this.

So we have to show the resultant velocity here in this place. So, with this velocity; this is the velocity with respect to Sun. This is the velocity of Mars with respect to sun and $V_\infty$ this becomes velocity of vehicle with respect to Mars. You can see that the V resultant is given by $V_\infty$ by $V_M$. The same technique we have to apply here in this place to get the result.

What will be the corresponding resultant velocity here in this case that we need to work out? So we have here $V_R$ equal to $V_M + V_\infty$ this is the basic equation. We are going to use and we have used earlier also. If we add to the excess hyperbolic velocity, the velocity of the planet we get the resultant velocity with respect to the sun. If we follow this notation for this implies that I should not put here V Mars in this place rather I should locate Mars velocity from this place.

So that both of them can add up for this is your V Mars the following discussion notation V Mars + $V_\infty$, $V_\infty$ goes from this direction to this direction. This is your $V_\infty$. And then V Mars to the resultant velocity then will be $V_R$. So this is the resultant velocity at the exit $V_\infty$ remains same whatever it is because of because this is the velocity with respect to the Mars or with respect to the planet.

And therefore, inside sphere of influence we have the conservation law if you apply; it will apply with the same velocity with whatever velocity it is entering here in this place. So this is your $V_\infty$. So $V_\infty + V_M$ is equal to $V_R$ as usual. We can see that $V_R$ is a larger vector here and $V_R$

here it becomes smaller vector. Ok that means the velocity of the vehicle with respect to the sun had reduced here in this place on the exit.

Means that the damping is taken place in the velocity and this week all the leading side leading side by flyby why? Because your velocity it is a; velocity is shown here in this direction its trajectories passing in front of the Mars. Velocity is in this direction of the Mars of the passing through this. If it passes from the backside college the trailing side bypass planetary flyby. So we look for that also, how does it look like?

The concept is very simple and based on the previous whatever the work we have done the same equation applies here in this place and everything can be calculated. So over all because of this the velocity of the vehicle it gets reduced with respect to the sun.

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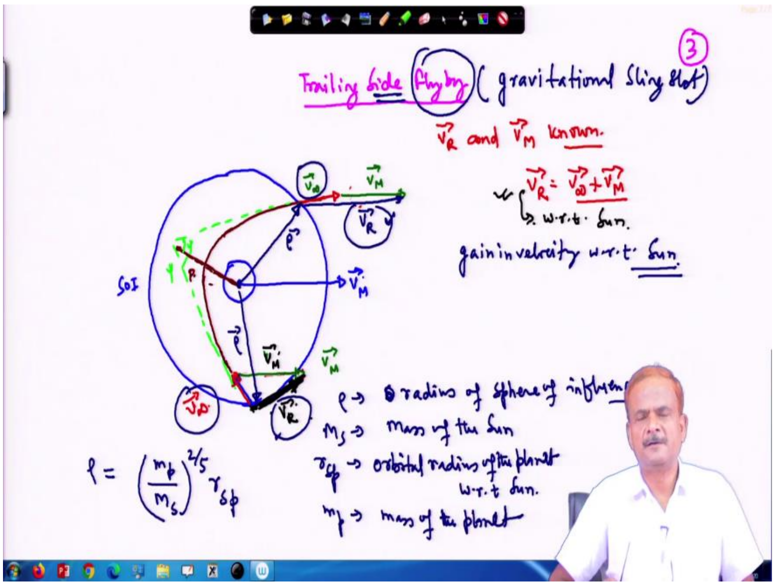


Now let us look into another condition, which is the trailing condition flyby. So, in this case, what is the result? Again, this is your sphere of influence; Mars is going here in this direction. Now let us assume that the $V_\infty$ is in this direction some another colour will use this is $V_\infty$. Add to this and $V_R$ is known to us. Ok $V_R$ is known to us $V_M$ is also known to us.

So $V_R$ and $V_M$ these are known from the previous result $V_R$ equal to $V_\infty + V_M$ that means I show here in this direction $V_M$. I will use green colour for this. This is your region. This indicates that $V_R$ should be in wrong direction so this is your $V_R$ so this is the, velocity; $V_R$ is the velocity with respect to the sun. With respect to the Mars what will be the velocity? Immediately it can be determined from this place.

$$\overrightarrow{V_R} = \overrightarrow{V_M} + \overrightarrow{V_\infty}$$

This is your $V_M$ this is following this equation $V_\infty + V_M$ equal to $V_R$. Now it happens now it goes passes through the back side of the Mars. So it passes through this path somewhere this is the perigee this is the periapsis; this the periapsis position. Corresponding asymptote can be shown like this. This is $\Psi$ and $\Psi$ ok now while it is coming from this place again the velocity will be $V_\infty$.

So here you have the velocity I shown by the same magnitude as $V_\infty$. Now $V_\infty$ and $V_M$ this is your $V_\infty$ and VM is in this direction. I use the same magnitude of the vector. This is V Mars. So what will be the resultant result will be this circle so this is $V_R$. So you can see that here in this case. $V_R$ is much larger as compared to this case. Here this is the resultant velocity with respect to the sun this has reduced in magnitude. This may be $V_R$ was longer this was reduced.

Here in this case $V_R$ is shorter and this is growing in magnitude. So that means it is gaining velocity. So there is mutual exchange of energy and the momentum and because of this we get a higher velocity, but remember this $V_\infty$ and this $V_\infty$ they are the same because this is the velocity with respect to the Mars and the according to the conservation law this has to be the same.

Ok so this we call as trailing side flyby and the result of the trailing side flyby is that there is gain in velocity with respect to the sun. So using this manoeuvre can be done for going to some other planet that means I have lesser energy for the satellite and if we do the trailing side flyby manoeuvre then we gain the velocity. This is beneficial and therefore going from one planet to another planet if we do the flyby mission, it helps.

And this particularly this part as I told you that tailing side flyby this is called the gravitational slingshot. But I will always use the flyby ok planetary flyby this is the technology involved in this but we are not going to do any problem on this particular aspect. So this topic is over only think what remains that our sphere of influence that we need to work out. So for working with the sphere of influence I will do that because we go back and look here in the previous lecture to you see here this is the $\vec{\rho}$.

$\vec{\rho}$ is nothing but your; the radius of sphere of influence so from where this is coming. So that $\vec{\rho}$which we have indicating here this is also your $\vec{\rho}$. On this side this is also $\vec{\rho}$. So this is the sphere of influence. So from where this is coming this particular part? This sphere of influence ρ can be written as; it is simple as defined in terms of the planetary mass and the Sun mass.

And this particular was derived by a particular person name Tissaran but I give just equation here in this place. So ρ can be written as M planet divided by mass of the sun to the power 2 to the power 5 times $r_{s\phi}$. So $r_{s\phi}$ is orbital radius of the planet with respect to the sun. $M_P$ is the mass of the planet and $M_s$ is the mass of the sun and ρ this is the radius of a sphere of influence, thank you very much.

$$\rho = \left(\frac{M_P}{M_s}\right)^{\frac{2}{5}} r_{s\phi}$$

So if the time permits then I will take the derivation of the radius of the sphere of influence towards the end of what we called as the transfer manoeuvre. We are going in now into the Landau intersection all those things so at that time or at the end of that I will consider it if the time permits, thank you very much.