Gas Dynamics
Prof. T.M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module - 16
Lecture - 39
Non Isentropic Flows - Rayleigh Flow

Hello every one welcome back, we discussed friction and it is effect on supersonic cos of sonic flow as when a disk compressible, we figure that it was going towards m equal to 1 that point.

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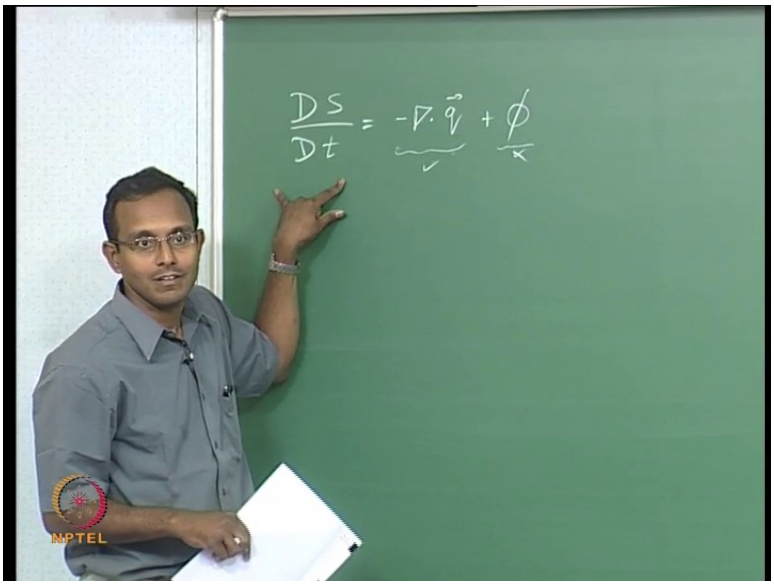

Now, we already did this analysis where we said that change in entropy was going to be either due to convergence of heat in dock load element or because, of decimation of heat due to friction. We currently we used the situation where we said this was 0, and we said this exists that was still last class, from today we are going say this does not exist and only this exists. We going to say only heat transfer exists there is no friction invest flow is what they going to think about right now these also a non-isentropic process as we can see.

Because, I am adding heat to the system and entropy increases, but that also says that if this whole term happens to be negative my entropy decreases. That means, heat is going out of my fluid and my entropy will decrease for the fluid that may also happen, now we will go think about that can entropy decrease et cetera later. So, now, what kind of problem is will I have to think about, when I say I heat transfer.

I am thinking about heat transfer should I consider heat transfer something like it is temperature of the gas is increasing by 1 Kelvin or 2 Kelvin etcetera or should I consider really big changes that the idea we have gone discussed first. I would say it depends on what is a already existing energy in the flowed element, and how much is the change. Only based on that I can now tell, whether the heat tradition is actually important or can I neglect it hence related to what is already exist energy in the fluid, and is it gone to significantly change.

Because, I added some extra heat or removed some extra heat and whatever, added or removed that is going to change by energy in the system, if it is changing significant and I am worried about that change if it is significant enough. Then I will have to worry about that heat transfer also, and they have take into account whether that heat transfer is going to affect my problem or not otherwise I will neglect it. Typically the example which people give will be in combustion chamber, there is flow through a duct and it is burning inside.

Typically in gas ropy engines there is going to be a duct, which is called a combustion chamber. And they going to inject fuel inside and it is going to burn, by the time of it is out of the chamber it is fully burnt, and it is hot products coming out that chamber if that is a situation. Now, what should be my exit condition will it become mark one that kind of question is what you are going to ask, and slightly jumping ahead of myself in here, but any ways typical example, where heat transfer matters a lot happens to be combustion.

One very common example for aerospace application happens to be this in us, now in actual combustion of gases, we will have composition changing. Of course, you know that if I have methane and air mixing, and then burning I am going to produce carbon dioxide and water there is not going to any of methane or oxygen present; if I think about psychometric mixture in the products, which means my composition completely change which means my heat capacity the C P value the C V values the gamma the ratios C P by C V, we all were going to changed.

Also the molecular rate of the gas, which means my r for the gas specific gas constant will also change. But, in this simple gas dynamic scores we going to neglect all of these changes which I just now talked about, I am going to say there is no composition change. So, my C P values are exactly going to be the same before and after heat addition, I am going to say gamma exactly remains the same r exactly remains the same. These are my assumptions currently this is the simplifying assumption of course, you can take into all these changes, and still work through the problem then just more tedious we would not go through that right now.

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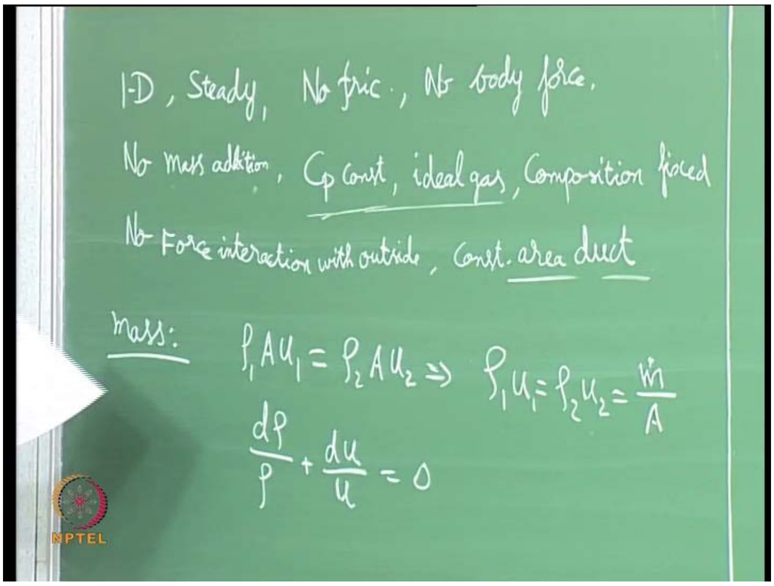

So, we will go into assumptions for my flow with heat addition, flow with heat addition I am going to say it is 1 D flow, steady, no friction, no body force, no mass addition by the way it is no mass addition means negative or positive. So, now, addition or subtraction of mass there, I am going to assume C P constant I am going to assume ideal gas, so when these two are together what is that is mean, C P constant and ideal gas what is this, something more than just perfect gas.

Calorically perfect gas that is what it is going to be, composition does not change composition I will call at fixed, I do not want to write does not change there. And no external forces, no body force, no any force interaction, no force interaction external with outside. Other than the force on the wall due to presser that is always going to be there, other things are not existing say I do not want to put a fan inside an add work to my gas or I do not want have anybody force, which is always going to add energy to the flow or remove energy from the flow that point any of those situations that kind of force interactions we do not want have. So, with all these assumptions whatever we did as 1 D simplified, 1 D steady flow whatever equations we used long back, when we derived the very first time the equations of motion, they will be obeying most of these assumptions. So, I can start using those equations here, I am going to say mass consideration I will have one more thing I want to add here is constant area duct also constant area duct one more assumption I want to add.

So, I will have rho 1 A u 1 equal to rho 2 A u 2, so now, I can write rewrite this thing as rho 1 u 1 equal to rho 2 equal to m dot by A. The same as what we did for final flow not much of difference, by the way this particular flow is called a Rayleigh flow, and in differential form we know that this particular equation can be written as d rho by rho plus d u by u equal to 0, these two we already derived any ways.

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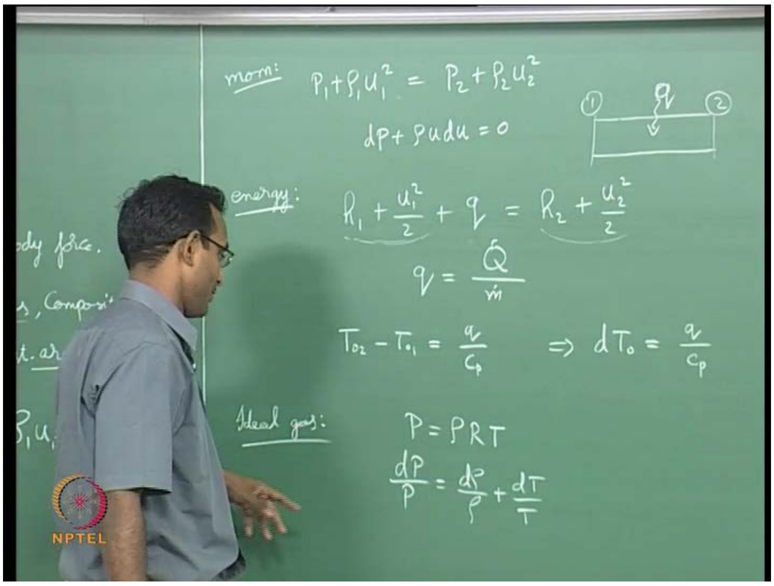

Now, we will go for momentum consideration, since it is 1 D I do not want to say x momentum or y momentum, it just momentum p 1 plus rho 1 u 1 square equal to p 2 plus rho 2 u 2 square by the way this is the very special case of our regular momentum equation. Where I am saying there is no force on the side wall because, it is a constant area duct, the areas going to be constant there is no p 2 minus p 1 into d a by 2 kinds of terms do not existing or we called it as just effects we going to have any of that term in here in this simple situation.

This in differential form will give me d p plus rho u d u equal to 0, I also use that rho u as a constant in here already. We did this kind of derivations long back I just reminded now, energy conversation this is the case where it will change from all the previous assumptions. Now, I am going to say energy is going to coming into the system, so that is not the same as what we had, before previously we have been assuming adiabatic, now energy is coming into the system.

So, my expression will be slightly different h 1 plus u 1 square by 2 plus some q the net heat coming in per unit mass. Because, everything else is per unit mass energy per mass equal to h 2 plus u 2 square by 2, I am assuming that there is a duct the inlet section as 1, the exits section as 2 that kind of system I am assuming currently. And I am assuming saying the net heat inside this whole duct happens to be q, small q I am using as a net heat content per unit mass of fluid going through that is what I am having here.

If I think about q it is going to be my heat flux Q dot divided by m dot, if think about continues flow with some particular mass fluoride. This will be the heat fluorite or power spent on this it is not heat flux release power this coming in that, so I will rewrite this expression in terms of T naught. Because, this you know that this is your h naught 1 and this is your h naught 2, I can write that assuming C P is constant.

I can rewrite this whole thing as T naught 2 minus T naught 1 equal to q by C P is simple enough, and this is going to give me d T naught is equal to q by C P. The differential form this is the integral form and that a differential form these are the expressions, we have next equation as of now we use mass momentum energy, next one is ideal gas law that still going to be the exact same thing p equal to rho R T. So, I can write my d p by p equal to d rho by rho plus d T by T that is going to be this expression.

From here on we are going to do similar exercise as what we did for my fan of flow, where I want to write every one of these quantities d p by p d rho by rho d T by T etcetera in terms of d u by u. And see what if velocity changes by a small fraction, we want to write everything else in terms of that, so easiest thing to do d rho by rho from mass equation, will be minus of d u by u. Now, I will have the point d T by T in terms of d u by u and now I can go and look at my momentum equation I can get the d p by p from here, and terms of d u by u we did this last time.

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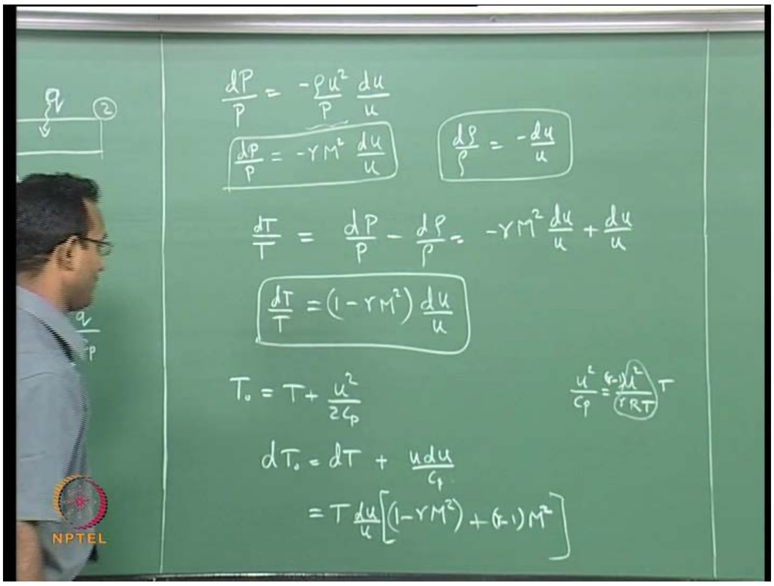

I just go through it little fast d p by p is equal to minus rho u by p du, now I will put u square by u here. So, this is the term I have now I want to express this in some other way, we know that this going to be equal to minus gamma M square d u by u how do I know that, p equal to rho R T. So, I will just put rho R T here I just have R T at the denominator I will put gamma above and below. And I will have gamma times u square by gamma R T then that becomes ((Refer Time: 12:51))

This is one expression I have d p by p, now I will put this along with 0 by rho equal to minus d u by u from mass equation, into my p equal to rho R T expression. So, I am going to have d T by T is equal to d p by p minus d rho by rho, which is equal to minus gamma M square d u by u plus du by u. So, I am going to get 1 minus gamma M square times d u by u, previously when we derived these kind of expressions we never ended up with this relation.

This is now different this because, previously we assumed that T naught was a constant, and that was the current strain in condition and from there we got a different expression for d T by T. Now, I cannot do that, so I am going through some other path and I am getting to some other expression because, of that based on the assumptions is the same set relations any way, mass, momentum energy, entropy it is definition, mass number definition, ideal gas these are standard set of expressions. I will just put different conditions on now on the expressions change because, of that.

Now, I will next go and do I also did d rho by rho I just write it here, d rho by rho equal to minus d u by u this is coming from mass equation is not anything special, I just wanted to write all these expressions together. Now, I will go and use T naught equal to T plus u square by 2 C P, I am again assuming C P is constant, and H naught is this particular definition we having this. So, I am going to write d T naught equal to d T plus derivative of this will be u d u by C P that expression I have there.

Now, I want to substitute for d T from my d T by T here, so I will take this T that side and that will become my d T directly. So, it is going to be T I will pull out the d u by u together, so I will have an expression d u by u times T is outside 1 minus gamma m square is inside, this is for the first term plus the next term I want to d u by u. So, I want look at d u by u will become u square by C P as inside term, I will write u square by C P separately, this can be written as u square by gamma r gamma minus 1 goes to the numerator this expression, I will multiply and divide by T.

So, I am getting this as M square, so gamma minus 1 M square times T that is what I am getting here write. And, now I have d u by u times this expression and I am pulling out T times d u by u out, so I will get gamma minus 1 M square if you derive this once it will make clear life comfortable anyways. So, now, I can simplify this a little bit I know minus gamma M square will cancel it this plus gamma M square, I will have minus M square there and 1 here that still remains.

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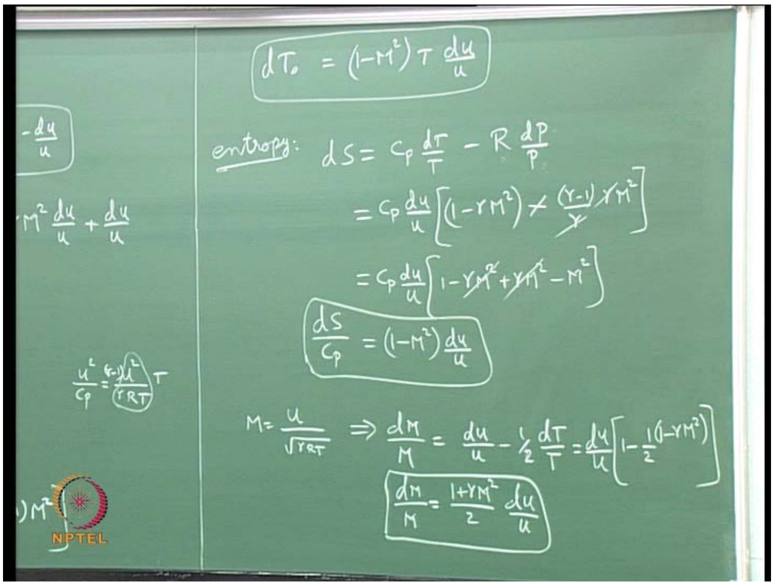

So, I am going to end of with d T naught is equal to 1 minus M square T times d u by u is one more relation. Now, the next thing I want to think about is entropy expression, entropy relation we note d s is equal to C P d T by T minus R d p by p this we derived in the first two classes somewhere, we have this relation. Now, I know d T by T in term of d u by u and d p by p in term of du by u, so I can just substitute them directly, I pull out C P continuously out of everything and d u by u outside.

Then I will have 1 minus gamma M square that is coming from my d T by T, and then I have minus this T has to be rewritten in terms of C P while let be gamma minus 1 by gamma. So, I can rewrite that it will come to be gamma minus 1 by gamma times d p by p is minus, so I am going to cut this minus and gamma M square d u by u I have already taken out this what I will have, now will cancel this gammas. So, I will just have gamma minus 1 times M square there.

I will write it once more 1 minus gamma M square plus gamma M square minus M square called these two get cancel, so I have 1 minus M square times C P. So, I will write this expression as d s by C P is equal to 1 minus M square times by d u by u one more important relation. Now, as it is always customary for us we will convert everything in terms of d M by M that is the only thing left.

So, I want to write mach number relation M equal to u by square root of gamma R T, so this is going to give me d M by M equal to d u by u minus half d T by T I think it is very a simple derivation you can derive it one more time, we want to do it right now. So, I have this, now I substitute this d T by T in terms of d u by u that is again a simpler thing to do. So, I just put it here d u by u common factor 1 minus half times d T by T was 1 minus gamma M square times d u by u, so that is this.

So, now, if you think about it 2 minus 1 will get cancel you will just have 1 plus gamma M square by 2 that will be a final relation d M by M equal to 1 plus gamma M square by 2 d u by u one more relation. So, we had a whole bunch of relations I want to write all of them in place, so that it will in one place in your notes really and it is easier to discuss after that point.

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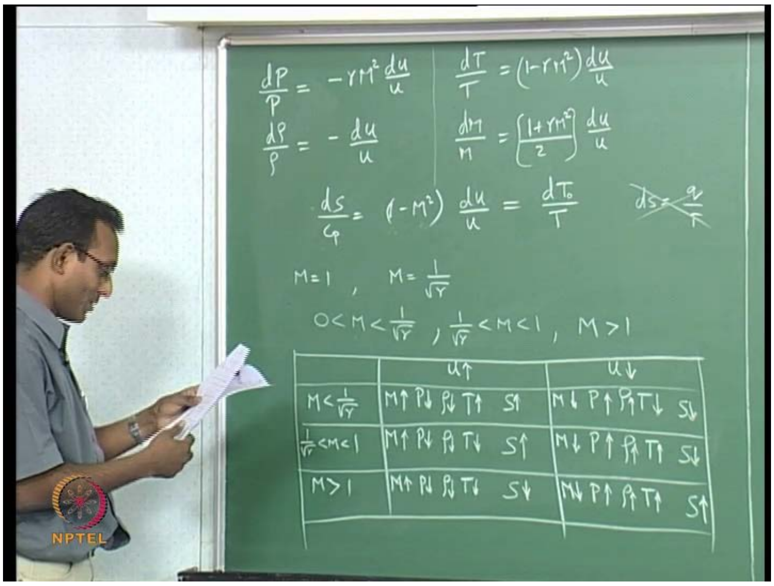

So, I will write it all together d p by p is minus gamma M square d u by u d rho by rho equal to minus d u by u d T by T equal to 1 minus gamma M square d u by u d M by M equal to 1 plus gamma M square whole by 2 times d u by u. And there is only one more thing left d s by C P equal to 1 minus M square times d u by u this is equal to d T naught by T this is one thing I did not talk about till now. It, so happens I derived separately d T naught I derived separately d s, it, so happens that they both will give you this kind of expression.

Which will see right now we just derived this is this correct, it is correct just simply by looking at this I want tell this d s is equal to C P d T naught by T C P d T naught is hear net heat added to your system right. So, I am getting an expression that looks like q by T d s equal to q by T was the basic definition for entropy change right, so I am getting back to that expression from here. So, everything is matching, so I want to worry about this whatever we have written till now is all correct.

Now, when I look at this expressions typically we always had this 1 minus M square term, which why my behavior changes may across M equal to 1 and that always be in the case. But, this time there having a term 1 minus gamma M square that is going change things now right, so I have a term 1 minus M square I also have a term 1 minus gamma M square. So, that is going to be change when M equal to 1 one of the critical points where things change is M equal to 1, and another critical point is 1 minus gamma M square equal to 0 which will happen when 1 by gamma square root.

So, M equal to 1 by square root gamma these are the two critical points, where the behavior of the flow might change, other places it is going to be same trend no difference. Now, is this more than 1 or less than 1 gamma is always greater than 1, so now, I am going to say that this always subsonic value, so in subsonic range there is going to be I have to think about 0 to 1 by square root gamma is 1 case 1 by square root gamma less than M less than 1 as another case, and M greater than 1 is third case or 1 less than M is a third case whatever.

So, these are the cases we have now have to study each of these separately that is a basic idea. Now, because of this I am going to put a big table, similar to what you have been doing all other cases, I just put a table saying u increases, u decreases and here I am going to talk about various mach number ranges M less than 1 by square root of gamma that is a first range. The second is 1 by square root gamma less than M less than 1, third case M greater than 1 these are a three cases I want to look at. Now, we will pick variables 1 by 1 I will pick mach number first.

So, when u increases where is d M by M that is here for whatever M value I am always going to have this number positive, it is never going to change sign. So, if u increases M increases if u decreases M decreases, so is going to be same direction, so I just look at all these arrows and just put the same thing as u for M that is what happens for mach number. Next I look at pressure d p is negative of some positive quantity times d u by look at this then I am going to say M square is always positive of course, whatever is a change in u, the opposite will be the change in p.

If d u is positive d p is negative, pressure decreases when velocity increases, so it is straight opposite of this relation just go and put the opposite arrow of these, oops these two are the other way, this is the what happens there. Next is density, I will just write density and temperature together, so will first look at density of course, d rho by rho as minus d u by u, such going to be inverse relation with respect to velocity. What this says is presser and density goes the same way.

Temperature, this is where the funny step happens 1 minus gamma M square changes sign M equal to 1 by square root gamma that will happen when M equal to 1 by square root gamma, and more if it is more than the value will be negative. So, if mach number is higher than 1 by square root gamma u and T are inversely related, otherwise they are directly related. So, when it is lesser it is directly related u increases T increases, here u increases T decreases.

But, all the other cases it is more than 1 by square root gamma, so it is going to go the opposite direction u increases T decreases here u decreases, so T increases this is what happens there. Now, there are two more variables left, and you know that they are s and T naught and they are directly related to each other, so I will just put s and you know it is same for t naught also. If you want you can put comma T naught next to it also, I will just do one of them s and T naught are exactly going the same way.

Now, I look at this expression d s is 1 minus m square times d u, so this is having a critical point across M equal to 1. So, I am saying if velocity increases, entropy increases for M less than 1, so M less than 1 has the first 2 rows, so entropy increases for these two cases if velocity decreases entropy decreases. Now, if it is M greater than 1 this 1 minus M square will become negative. So, if velocity increases entropy decreases, so this will go the opposite direction this is what I am seeing. And I just wanted to make a few commons on this, the first one I want to see is d s same as d T naught.

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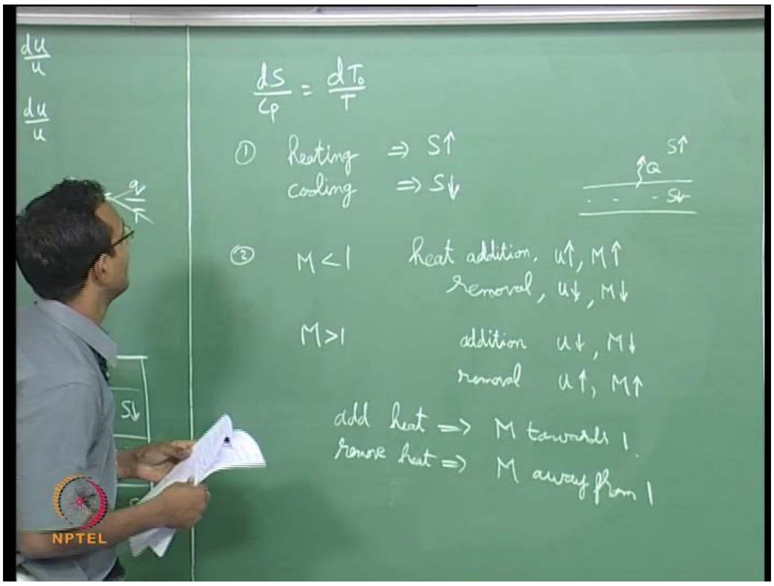

This says that if my T naught increases; that means, I am giving heat to my flow right, if I add heat then my T naught increases. So, if I give heat to the flow my first common happens to be heating causes entropy to increase, cooling causes entropy to decreases. So, can I have cooling you can still have it why, entropy is decreasing will it happen in nature that is what we need to think about, all this time in all other problems before last class or including last class, we had the situation where there was no other interaction with this surroundings.

The only system that I have was my gas flow and there was nothing else outside, but this time I am opening out thermodynamic world outside this gas also. So, now I have a system and the surrounding around it, if I have this gas flow duct and I am having heat into it that was easy problem right, we will pick the tougher problem heat flowing out of it if flowing out of it. Then entropy inside here is decreasing, but I am heating this surroundings that entropy increases.

And, since I am loosing heat from a small place and giving it to a large volume, typically I going to increase entropy. Because, most of the molecules are happy there while little less, some small amount of molecules are sad we think about it way right, we already discuss this sad versus happy for entropy right. So, it is going to become something like that, so overall I am going to have net entropy increasing a little bit.

So, this whole system is still possible, this is the reason why were refrigerators work still that is also a read case where I am cooling the items inside my refrigerator, and it still works because, entropy overall is increasing. The next main point I want to tell, when I say overall it is entropy of the universe that is including system and surrounding net entropy change is greater than 0. Next main point I want to say, M less than 1 heat addition I am going to have velocity increasing, mach number increasing, heat removal velocity decreases, mach number decreases.

Now, I will put M greater than 1 by the way these are observations from this table, I am saying M less than 1 are the top two rows, I am finding that if velocity increases I am adding heat to the system. Entropy increases, T naught increases, I told T naught is same as s, may be I will just write it also T naught increases, T naught increases, T naught decreases.

And the same thing and the same thing other side I did not ask space, so I did not write it and any ways, entropy and T naught are going to go exactly same way. If my T naught increases I am heating that flow, I am heating the flow here and I am finding that my mach number increases, my velocity increases for M less than 1. And it is M greater than 1, when I cool then my velocity increases.

So, these arrows are going to be flipped across here, if I have heat addition then u decreases, M decreases, heat removal then I am going to have u increases, M increases. Now, you want to look at this again when look at this, if I am subsonic, if I add heat I am increasing mach number, if I am supersonic, if add heat I am decreasing mach number. So, it looks like there both converging towards M equal to 1, if I add heat, if I add heat this going to go M towards 1.

If I remove heat, then I am going to go M away from 1, main thing you need to remember are these two, if I add heat my system is going to 10 toward M equal to 1 that is what these four lines are saying basically. And if I remove heat, I am going to have my mach number away from M equal to 1, very important things we have to remember, and we have some special thing across M equal to 1 by square root gamma. Where I will just point it here itself, the only changes it is going to switch the relation between T and u that why you got that relation anyways.

When I am subsonic, and below that critical gamma if I add heat my velocity and temperature both are going to increase. But, beyond some point velocity increases, but temperature decreases, the reason being if you go think about it, the reason being the density drop is much more when I am going close that M equal to 1 that the compressibly they effects are beginning, so serious that they have to go the opposite direction to match p equal rho R T and rho u equal to constant.

To do that, it has to go the other way that is the basic idea in here, I am still adding heat, but it is going the other direction. Because, density change is going to be too stronger on that region, actually it is pressure and temperature if you look at the changes you will see that density changes going to true that.

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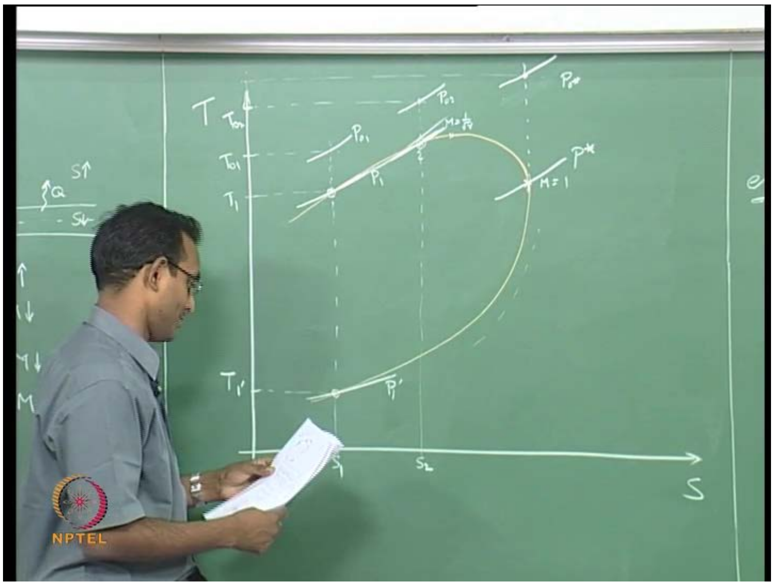

So, the next thing is I want to draw T S diagram for this, I want see the state change on a state diagram. So, I will draw a really big plot this stay it is entropy and this is temperature these are may access, so I am going to say I am starting with some T 1 and that will have a particular T naught 1 this is the current situation. And I am going to say this is my actual state location, this happens to be my s 1.

So, I can draw a pressure line through this that is my p 1, and if I go the same entropy isentropic conditions and I meet that T naught that becomes my p 0 1 curve. This is my actual state location that p 0 1 is a imaginary locations stagnation condition, now from here if I heat the flow I know my entropy increases, if heat the flow d S is d T naught, so it is going to increase heat. It is going to go to the right, now I want to look at what happens to temperature.

Let us assume that my mach number is really, really small I am in the first row, my temperature should increase, entropy should increase, which means I am going up and to the right. From here, I am going to go up and to right something like this I will try and use an another color for me actual Rayleigh flow curve is going to do something like this. And my state points are going to move along this curve, and if I am add that particular mach number, where this term become 0 M equal to 1 by square root gamma.

Then temperature does not change when velocity increasing, when velocity increasing I am going to have heat addition for subsonic conditions, from this relation I can tell that. If my velocity increases I am adding heat for subsonic condition, but temperature does not change; that means, I am going to have a slop has going to be horizontal, first temperature does not change, but my entropy increases is going to go plot without changing temperature. That happens to be that special point M equal to 1 by square root gamma that is my special condition.

If I go beyond that point, now I am on the second row the main change is only in this term d T by T is equal to 1 minus gamma M square d u by u, everything else just obeys the same thing as before, if it subsonic it behaves a same way. That one thing now, velocity increase, when I heat velocity increases, temperature decreases, entropy increases, temperature decreases that is what I need for T S diagram. So, I want to look at this and draw my T S diagram, entropy increases temperature, decreases.

So, it is going to drop down how for there will be a point where I will reach M equal to 1, I am continuously increasing mach number I will reach M equal 1, at M equal to 1 we will go back and look at this expression where 1 minus M square is present. At M equal to 1 I am going to have a condition where d S is 0, which means entropy cannot change any more at that location. So, I will have to have a vertical slop for that condition, so I will make it vertical slop and I will mark that particular critical point M equal to 1.

Now, I will start from the other side supersonic section, I will say it is same p naught I will start with supersonic section that will be somewhere here let say, this is my one prime I will call it, I should call it T 1 prime. So, I will have a p 1 frame going through this, now I want see if it is supersonic condition M greater than 1, I am going to go from here.

Of course, if I heat my entropy increases I want to see what happens to temperature M greater than 1 this decreasing T naught I want increasing T naught that is this I find that temperature increases, entropy increases, temperature increases when I heat this particular case. So, I am again going to go up and to the right, and this keeps on happening till M equal to 1 and the curve looks something like this, do not could me on this particular shape exactly the shape can be anything depending it can even being something like this.

Depending on whatever constraints we want to worry about that currently, will keep one particular curve we would not pick another curve currently. So, we have this particular curve, so now, if I think about another point where it reaches, let us say I pick this point and call it 2 of course, I went from p 1 to p 2 from here to hear. What should happen I should know that I am adding heat to a subsonic system, in the first row right adding heat to subsonic system in the first row pressure decreases.

Which means now my p 2 must be slightly less than p 1 it, so looks like there all in the same curve, they should know that this curve is slightly below the other curve if I extended it that is why it should look. Should be slightly below that, I am not drawing very exactly currently, and p naught also we will find that will drop I did not write that in a table, p naught also drops that will also be a little lower is will be my s 2. And that particular point is your T naught 2 is what happens there.

Ideally, if I drawn it correctly I will find that the kinetic energy the gap between these two is going to be increasing that T naught and T 1 the gap is here u square by 2 C P that be suppose to be increasing, it looks like there almost the same. That means, I have not drawn it correctly it should have dropped a little earlier the orange curve, but any ways if I keep on adding heat further, I will go towards M equal to 1. So, will have a condition where I am having a p star, what are you finding here my pressure is continuously dropping to lower and lower and lower, lower curve just coming to a lower curve.

And my p naught also will be this is my p naught 2, my p naught star will be a still lower curve somewhere here, this is p naught star and I will find that my T naught must have increased for that. Let us say that is my T naught I should drawn in a straight, I missed the star point, so I will draw it the reverse way this is my p naught star curve that will be means stagnation state p naught T naught condition.

Now, I am finding that from here as I added heat I am also accelerating the flow and now your kinetic energy is much higher, T naught star minus T star is your kinetic energy and that is very high currently all that you have seen. Now, if I did the same thing from supersonic condition, your finding that my velocity will decrease T naught 1 is here, T 1 prime is here from here, all the way to here is may kinetic energy currently u square by 2 C P. And as I go towards M equal to 1 I am decreasing the amount of kinetic energy prism.

If we look at p naught 1 that is more than p naught 2, which is more than p naught star that all about is going to be the case, this is going to be the general trend, now we can start talking about p naught variation, and similar to here fan of flow conditions what will happen, if I start from one mach number and I keep on adding heat. It is going to go along this curve if it is subsonic, going to go like that and when it is going to go reach this point that is what we saw right.

If it is heating it going to go towards M equal to 1, if I remove heat it is going to go in the opposite direction of this curve from M equal to 1 it is going to go away, but it is going to be on this line. Why will it be always on this line alone, if I am having I am already having mass flow constant, if I had some other mass flow then it will be shifting the curve will shift. If I have lower mass flow, then the curve shifts to outside envelop it goes one row out of this that is what happens here.

If I draw another curve outside, this will be higher mass flow condition that is what happens, similar to fan of low conditions. Anyway will go look at what happens numerically some other time, today I just still have give some more expression to you, maybe I want finish all of them. So, I want to have every result in terms M, so that I can have a table generated, I want p 2 by p 1 in terms of mach number 1 and mach number 2 like that.

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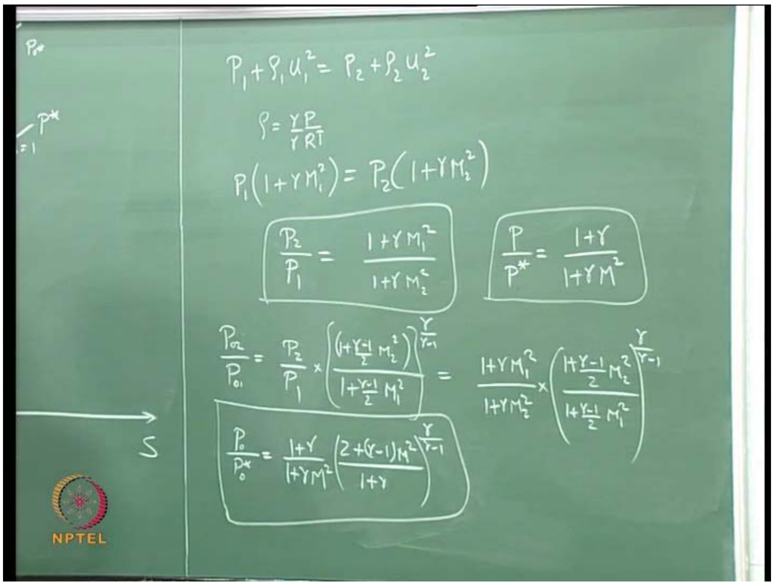

So, I will just look at my momentum equation now of course, you know that I can rewrite things like rho equal to p by R T multiply and divide by gamma. Now, I can rearrange this such that I can get gamma M square in here, if I put u square along with it I will get gamma M square also we will keep it like this. Now, from here you can rearrange this row substituted in there and you will get that expression to be p 1 multiplied by 1 plus gamma M 1 square equal to p 2 multiplied by 1 plus gamma M 2 square.

This is not new by the way this is the same expression you got for shocks normal shocks, p 2 by p 1 is 1 plus gamma M 1 square divided by 1 plus gamma M 2 square, this is one relation you have. And of course, now I can make this simplified version I can give this as p by p star I am substituting one of them as M equal to 1 condition, and I can get a p by p star also from here. If do a p by p star I am substituting a M 1 as 1 1 plus gamma divided by 1 plus gamma M square, I am just having M square now I am interested in one particular mach number and I am comparing with it star value.

So, it is just M square and not M 1 or M 2 this is one relation, the next immediate relation is an easily get is p naught 2 by p naught 1S another easy relation to get. So, think about p naught 2 by p naught 1 I just have d u is p 2 by p 1 multiplied by isentropic relations. So, it is p 2 by p 1 into 1 plus p naught by p 2 will be in the numerator gamma minus 1 by 2 M 2 square divided by 1 plus gamma minus 1 by 2 M 1 square, this whole thing to the power gamma by gamma minus 1 this is your isentropic relation for p naught by p.

So, now, I will substitute this inside here and you can get one relation, just write it this is another relation I have. And this again I can write p naught by p naught star, and that is going to be 1 plus gamma divided by 1 plus gamma M square times I will rewrite this expression a little bit, I have something else in my notes, but it is gamma minus 1 M square divided by 1 plus gamma to the power gamma by gamma minus 1 I have this relation. Another important expression given in tables, one more expression now we will look at next variable actually I think I will stop here next variable take a little longer. So, I will derive the remaining two temperature density T naught, and the entropy later next class we will do it see people next class.