Rocket Propulsion

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Lecture No. # 08

Examples Illustrating Theory of Rocket Propulsion
and Introduction to Nozzles

Good morning: First let us recap what we have done so far in a couple of minutes. And then, we will solve one or two small problems, such that we are fully aware of the theory of rocket propulsion and then move over to a new chapter on nozzles.

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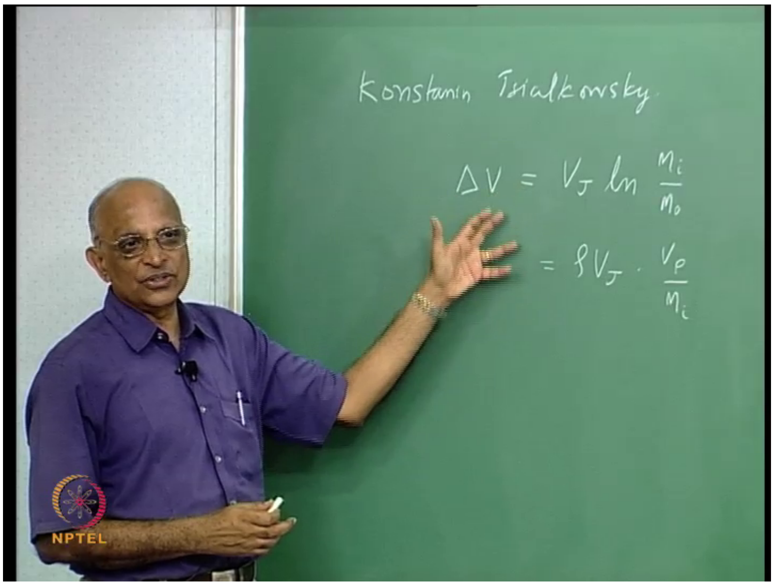

We discussed the Rocket Equation first formulated by the Russian school teacher Konstantin Tsialkowsky. We derived the rocket equation and found that the ideal velocity increment of a rocket is given by the efflux velocity or the jet velocity multiplied by the natural logarithm of the ratio of the initial mass to the final mass of the rocket. We also found that for a rocket with a smaller amount of mass of propellant compared to the total mass of it, we found that this equation gets slightly simplified and density of propellant times the jet velocity becomes important parameter and this multiplied by the volume of the propellant divided by the initial mass of the rocket gives the velocity increment by the rocket. Therefore, we will try to do one or two small problems, such that we illustrate how this ideal velocity works. We have seen what the payload mass fraction is, we saw what is the propellant mass fraction and we saw the structural mass fraction. We related these fractions in the earlier classes. Let us go ahead and solve one or two small problems.

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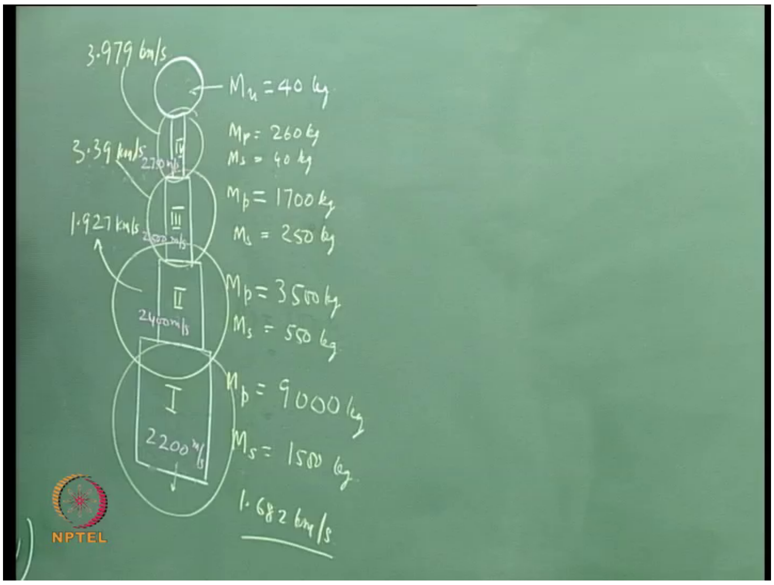

I take a problem of the 4 stage rocket to begin with similar to SLV 3 about which we discussed in the last class. Let us say, I have a booster stage which is the first one, which first takes off from the ground and then on top of it I have the second stage, then I have the third stage and then I have the fourth stage and on top of it fix my useful mass which we call as payload. Let me take a typical example, in which the top mass that is the useful mass, which we called as useful mass, is equal to 40 kg.

The propellant mass of the first stage, is equal to 9000 kg and the mass of the structure; that means, the casing, insulation and inert of the first stage is equal to 1500 kg. The jet velocity of the first stage is equal to 2200 meters per second. Now, on top of this stage is the second stage.

The second stage has a propellant mass equal to 3500 kg and the mass of the structure of this stage is equal 550 kg. On top of the second stage, you have the third stage, for which the mass of the propellant is equal to 1700 kg, and its structural mass is 250 kg. Then we said a fourth stage whose mass of propellant is 260 kg and the mass of the hardware including the structure is equal to 40 kg.

The jet velocity of the first stage is 2200 m/s, the jet velocity of the second stage is slightly higher at 2400 meter per second. For the third stage, it is 2500 meter per second and the fourth stage the jet velocity is 2750 meters per second. Therefore, we have this 4 stage rocket, on top of which you have a useful mass or payload mass of 40 kg. It takes off from the ground. We would like to determine the value of the delta V of this rocket or the total value of the ideal velocity, which the rocket gives.

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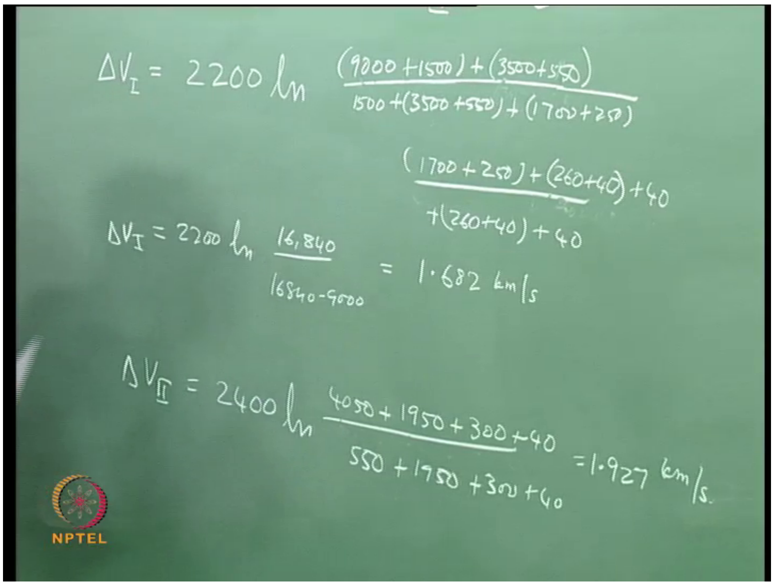

The first slide at 05.59 gives the velocity increment (ideal velocity) of ΔV_I, that is available to the rocket when the second stage starts and so on and therefore the net ideal velocity of the rocket is equal to what is provided by the first stage plus what is provided by the second stage plus what is provided by the third stage plus what is provided by the fourth stage. Therefore, our main effort has to find out what is the delta V of the first stage, second, third, fourth stages. I just arithmetically add it all up and I will know what will be the ideal velocity given to the payload by this fourth stage vehicle.

Let us do the calculations. Let us do for the first stage and similarly we can do for the second, third and the fourth stage. The first stage, you find that the exit or the jet velocity is equal to V_J is equal to 2200 m/s. Therefore, we have 2200 meters per second into logarithm of the initial mass of the rocket to the final mass of the rocket. What is the initial mass of the rocket at this point in time? It includes all the mass of propellants, all the mass of structures plus the payload.

Therefore, if we were to write the expression for the initial mass, which as we saw in this expression for ΔV to be the numerator, it is equal to for the first stage, the propellant mass is 9000, the structural mass is 1500, now we add the second stage which is equal to 3500 plus the structural mass is 550, we keep adding up for the third stage, the propellant mass is 1700 plus we have 250 for the mass of the structure plus the last stage we have 260 plus 40 plus and the useful payload which is plus 40. Now, what is the final mass of this rocket after the first stage is performed? What must, I remove or what must be done?

First stage propellant gets burnt out and therefore, the total mass minus the mass of propellant in the first stage which is knocked off. Alternatively, I have 1500 kg as the mass of the first stage structure plus we add the mass of the other stages viz., 3500 plus 550 for the second stage plus we have for the third stage which is equal to 1700 plus 250 plus we have 260 plus 40 for the fourth stage plus we have the value of 40kg for the payload. In this way we can get the value of ΔV_I. Let us substitute it. We therefore get, ΔV_I is equal to 2200 ln of the masses in the numerator 9000 + 3500 + 550 + 1700 + 250 + 260 + 40 + 40 and this gives me the total initial mass. We get this value to be 16840 kg. If we were to subtract from it the mass of propellant in the first stage, we get the final mass of the rocket at the end of the first stage of operation as subtract the value of 9000, that is 16840 minus 9000 which is 7840 kg. Hence ΔV_I is therefore 2200 ln(16840/7840) and this gives the velocity to be 1.682 kilometers per second or 1682 meters per second. Let us put the value here. We get the velocity of the first stage ΔV_I as equal to 1.682 kilometers per second.

Similarly, we do for the second stage. I know what the initial mass of the rocket at the start of the second stage is. Similarly, we get the final mass when its propellant is burnt out. We start with the initial mass of the rocket which should be the second stage, plus third stage, plus fourth stage, plus the payload weight. And what is the mass of the second stage? 3500 plus 550 that is 4050. Now, the third stage 7700 plus 250 that is 1950, the stage fourth stage 260 plus 40 which is 300 kg plus useful 40 kg, And what is the mass at when the second stage has stop functioning? It would be less by the second stage propellant mass of 3500 kg. Hence, ΔV_{II} of the second stage, is equal to the jet velocity which is 2400 meters per second multiplied by the logarithm of the ratio of these mass before and after. This will give me a value of 1.927 kilometers per second. Therefore, the second stage gives a ΔV_{II} of 1.927 km/s. Now, we repeat the calculations for the third stage and how do we get the calculations for the third stage? The jet velocity is 2500 into logarithm of the mass of this plus the fourth stage plus the payload which is 1700 + 250 + 260 + 40 + 40 which is the initial mass. At the end of the third stage I have depleted this amount of propellant for the third stage. I am left it 250 + 260 + 40 + 40. This gives me the velocity increment of the fourth stage which gives velocity of let us put this over here. We have a slightly higher velocity of something like 3.39 kilometers per second.

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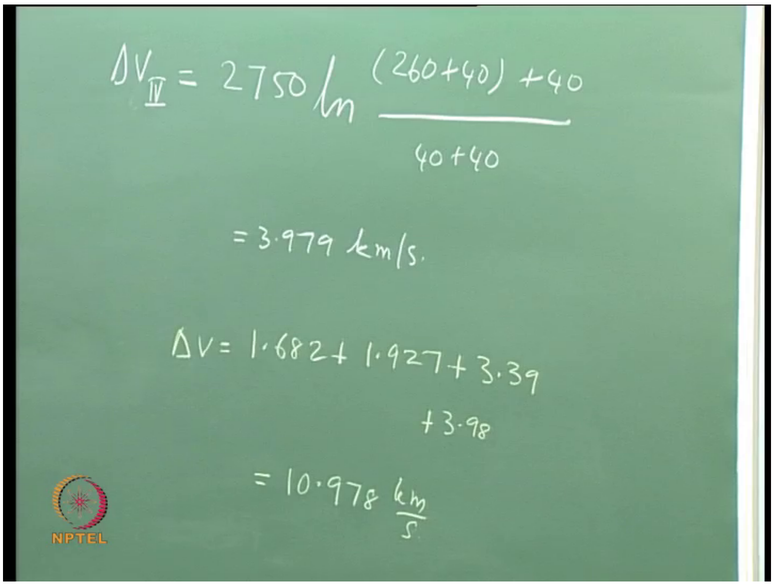

And let us do for the last stage i.e., the fourth stage gives a velocity increment of ΔV_{IV}. And we find that the jet velocity of the fourth stage is 2750 meters per second. That is 2.750 kilometers per second into logarithm of the initial mass is 260 is the mass of the propellant + 40 is the structure + plus 40 and what is left after the fourth stage burns? 40 plus 40 is what is left. And this gives me the velocity as equal to something like 3.979 kilometers per second. The fourth stage gives me a velocity of 3.979 km/s.

Now, let us try to draw some inferences from the above. We find that the first stage gave a velocity increment of 1.682 kilometers per second; the second stage gives me a slightly higher value 1.927; third stage gives a significantly higher value 3.39, while the fourth stage gives even higher value of 3.979 km/s. That means, the upper stages contribute more ΔV than the lower stages: the reasons being, one is the jet velocity of the lower stages is generally smaller than the upper stages though larger quantities of propellant are burnt. And th second point is we have the benefit of a lighter rocket towards the end which can give you a higher ΔV. And therefore, what is that total delta V of the rocket we just add the velocities.

That means, the total delta V which is the increment provided by the rocket, that is the addition of 1.628 km/s provided by the first stage + the second stage which gave us 1.927 + the third stage which gave us 3.39 + the last stage which gave us something like 3.98 kilometers per second. And the total velocity what we get is therefore equal to 10.978 kilometers per second. We must remember, that we have neglected the velocity gains when the discarded stages are removed. And this is the velocity which is available to you for orbiting plus the potential energy or the velocity to increase the height of the rocket from the ground to the particular orbit. And how do you match it with the orbital velocity and the total velocity?

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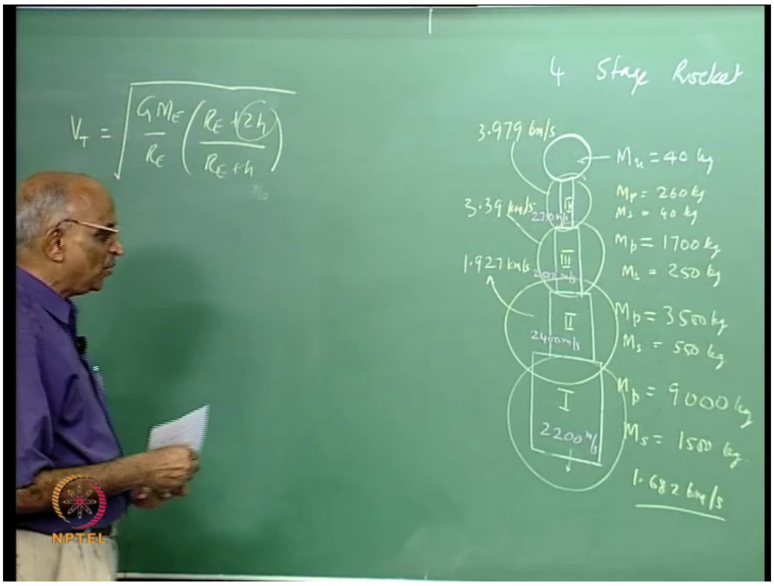

You already derived the expression, that the total velocity provided by the rocket can be written as $\sqrt{GM_E/R_E \times (R_E + 2h) \div (R_E + h)}$ and this will tell you at what height this particular rocket can launch the particular satellite. And this is how we do any velocity increment for any multistage rocket.

To be able to illustrate about initial acceleration and the need of a strap-on or cluster of rockets, I give you some more data and let us do a small problem as an extension of the above problem. Suppose, we want to find the acceleration of the first stage rocket, when it takes off from the ground.

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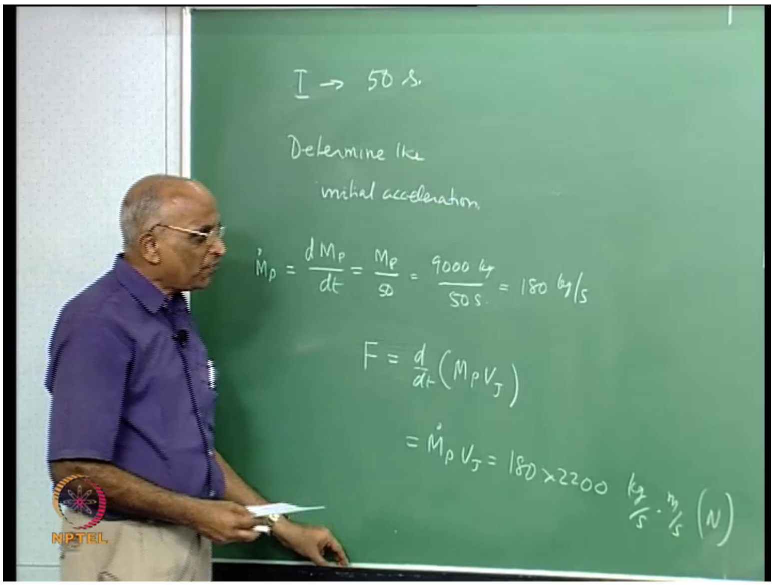

The first stage operates for let us say, 50 seconds. And the question posed now is; if the first stage operates for 50 seconds determined the initial acceleration of the vehicle or this particular rocket. Now, how could we do this? That means, each stage operates may be, the first stage operates for 50 seconds, the second stage for 35 seconds and so on. May be the final stage operates for let say another 80 seconds. But the data which is given to you is that the first stage which has a propellant mass of 9000 kg, operates for a time of 50 seconds. The flow rate should be given; let us presume that the flow rate of the mass leaving the rocket is a constant.

Therefore, we can say the rate at which propellant gets depleted that is dMp/dt is constant and is Mp divided by 50 seconds which is equal to 9000 kg is the mass of the propellant and it is getting depleted over 50 seconds. Therefore, the rate at which the propellant is leaving the nozzle is equal to something like 180 kilograms per second. You find the mass flow rate from a nozzle is quite high. We are talking of several hundred kilograms per second and when the rocket is huge, like what we said is the moon rocket it will be very much higher. Therefore, the mass propellant mass flow rate or rather dMp by dt, which I can also write as m° is equal to this value.

Now, we know the value of V_J to be 2200 m/s. I want to find out the initial acceleration of the rocket. To be able to find the acceleration, I need the force produced or the thrust of the rocket and what is the force? The force F which the rocket gives is equal to d by d t of change of momentum, and the change of momentum is equal to Mp into V_J. The thrust is equal to m° into V_J, which equals 180 kg/s × the value of V_J of 2200 m/s. what is the unit? Kg per second into meter per second; kg meter per second^2; this is in Newton.

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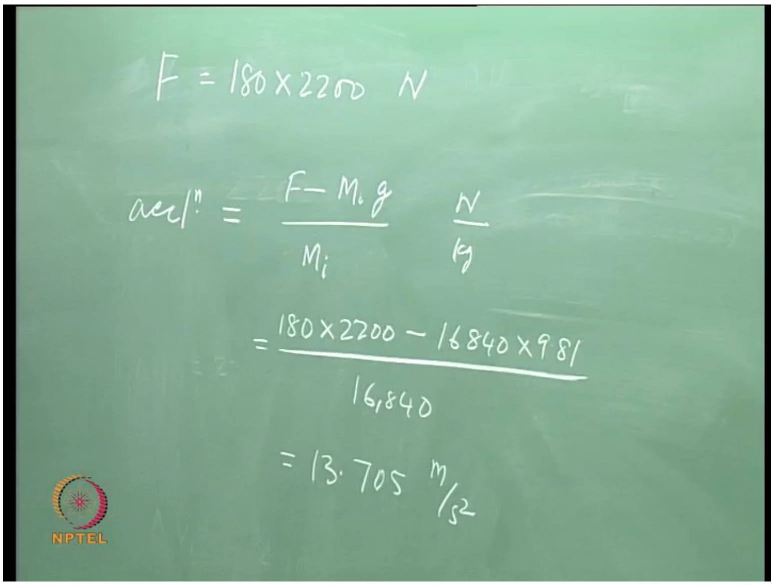

We find that the force which the rocket develops or the thrust is equal to 180 into 2200 Newtons. Therefore, what is the acceleration at take off? We find acceleration of this particular vehicle at take off is equal to the force minus the initial mass which is subjected to the initial gravitational field of the earth, divided by Mi. Mass into acceleration is equal to the net upward force. And therefore, the acceleration at take off for the rocket is equal to 180×2200 − the initial mass of the rocket 16840 kg × the gravitational field of 9.81 m/s^2 divided by 16840. This gives me the value of acceleration as equal to 13.705 meter per second square. This is Newton divided by kilogram and the unit is meter per second square.

Therefore, you find that the rocket is leaving the ground with an acceleration which is something like may be 1.4 to 1.5 g. This is the level with in g with which it is leaving. Is there anything else we could do?

Supposing, I want the acceleration when the fourth stage just starts operation i.e., it fires. Let us say, I know the time over which the fourth stage operates and once I know the value of m°, I can calculate the force. I correct for the attraction of this initial mass by the Earth, divided by the total mass that will give me the acceleration. I can therefore, find out the acceleration at any particular time and this is how we determine the acceleration.

The last thing, which I would also like to know is what is the payload mass fraction of this rocket?

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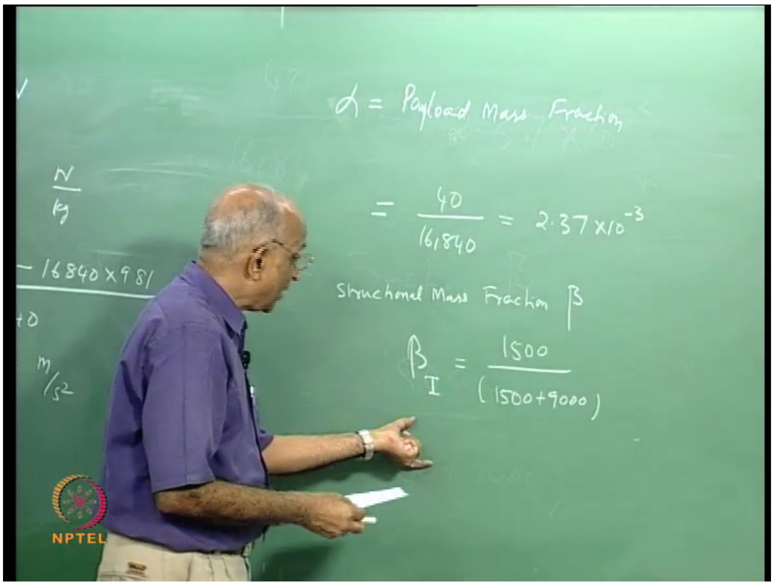

Let us say, what is the value of α which we said is the payload mass fraction? Can you tell me what it should be? Useful payload, 40 kg divided by total vehicle mass and that we said is 16840 kg. It comes out to be a very small number. Let us put down this number. It is 2.37 into 10^{-3}. Or rather the net useful fraction that comes out of this rocket is something like 0.2 percent. You find that in rockets lot of energy gets expended.

And the aim of a rocket designer is to improve this fraction to something like 2 percent, 3 percent, which is what we will be considering in the subsequent classes. We talked of the payload mass fraction. Supposing we ask in terms of the structural mass fraction let say β. But then, we have four stages, first stage, second stage, third stage, fourth stage. Let me say, I am interested in the structural mass fraction of the first stage. What will be the value? Yes, the structural mass is 1500 kg divided by what: here we should be a little careful, I am asking for the first stage. The first stage consist of 1500 kg as structural mass and the balance 9000 kg as the propellant mass. Therefore, the total mass of the first stage is 1500 plus 9000 = 10,500 kg and the structural fraction mass of the first stage is 1500 divided by 10,500.

If I consider the second stage well it is going to be something like 550 divided by 3500 plus 550. We can calculate for this third stage and for fourth stage. And this is how we calculate the payload fraction, structural fraction, may be a propellant fraction. We also calculated that the total velocity and the acceleration.

Having done a problem for a rocket let us do one for the satellite using the same theory. We had said that a satellite carries some amount of fuel or propellant and the movement propellant is consumed, the useful life of a satellite is over. Therefore, let us take an example.

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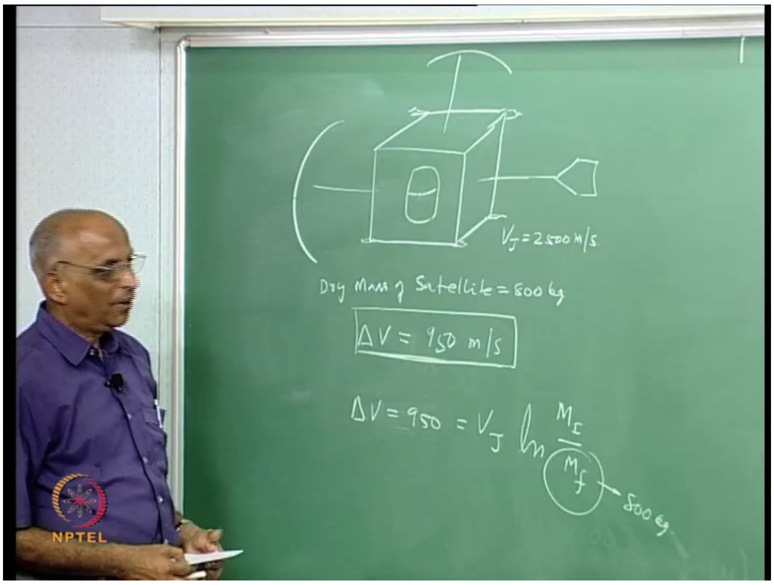

Let me take an example of let say INSAT. The INSAT satellite consists of something like a cube or box like structure: this is the basic structure, inside this is housed the electronics and propellant tank. And we use the propellant for correction of the orbit of the satellite. We will get into details of this when we do the liquid propellant rockets. We attach a series of rockets at the edges over here, a lot of them something like 16 of them, such that, we can make the small corrections in velocity whenever required in space.

Supposing, we need to determine the quantity of propellant to be carries in the INSAT spacecraft. The dry mass of the satellite is given as let us say 800 kg. What is this dry mass? Mass of the structure, mass of the tank, maybe I could have something like an antenna, solar panels, sensors, etc.

To be able to maintain this satellite let say for 10 years or 15 years, we need to be able to make the necessary corrections for its attitude, orbit and also to push it out of the geosynchronous orbit once its life time is over. I need to be able to configure the rockets such eventualities and the total ΔV required for the corrections and eventualities is given. Let us assume that this velocity is 950 meters per second. This is equivalent to the incremental velocity provided by the rockets.

I should qualify the correction velocity that is required to be provided. Whenever you have the satellite not pointing correctly, I must give a small impulse to the satellite; that means, I have to give some change of momentum to the satellite to deflect it. I know the mass of the satellite therefore, I can find out what is the corresponding delta V which I must give to the satellite. I keep on adding the different ΔV for attitude control, for station keeping, etc., including the final push away out of the geosynchronous orbit. I find that during the total life period of this satellite (over several years) I need to give a delta V of 950 meters per second. Now, as a designer we should know how much fuel or how much propellant do we carry in this space craft? This is what I want to determine.

In other words, all what we are saying is, the ΔV to be provided by the rockets is equal to 950 meters per second. And then, we need to know, what is the jet velocity of the different rockets and then we say of the initial mass divided by the final mass of the satellite. Initially we have some mass while after propellants are exhausted we have a final mass. What is given is that dry mass? Dry mass is the final mass when there is no propellant. Therefore Mf is given as equal to 800 kg. The rockets are designed to give a certain V_J. Let us presume that the value of V_J is equal to 2500 meters per second.

In the next class we will find out how we calculate this value; Therefore, now my question is what must be the mass of propellant which I carry in this particular rocket? The initial mass of the satellite will also contain the propellants in it.

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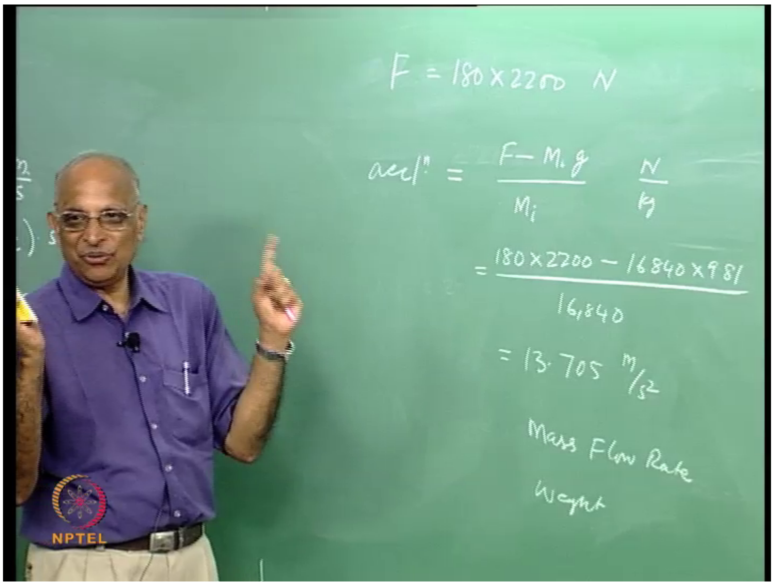

Therefore, the initial mass is equal to m_i kg, dry mass of 800 kg is the final mass. The initial mass is therefore 800 plus the mass of the propellant. And we find that ΔV what is required is 950 meters per second should be equal to the jet velocity is 2500 into logarithm of 800 + the mass of the propellant ÷ 800. And we know, what is the mass of propellant to be taken in the satellite. Let solve this equation, we get 800+Mp (mass of propellant in kg) / 800 kg is equal to $e^{950/2500}$. We take exponential on both sides to get the above. 950/2500 = 1.462. Or rather, I get the value of Mp that is the propellant mass as 370 kg.

If the satellite has to be operational for 20 years or more, we may require more of fuel because more corrections are required. If we want it for one year I can have much lower mass. Whenever we read in the news that the initial orbit has a significant error and therefore the life time of this satellite comes down. Why does it come down? Initially, itself I take some of this fuel and use it for the corrections and therefore, the propellant available for the station keeping of this satellite and the attitude orbital corrections are decreased. Are there any questions?

Your question is, how did I get 950 meters per second over a given period? See you know that, the satellite is slightly getting drifted, because I have the gravity of the moon, may be gravity of some other planet which is there or else some aerodynamics itself. Therefore, I know the mass of the satellite, I know what is the force which is required to give the orientation. Therefore, I can calculate what is the value of impulse for particular correction, once I know the correction at that movement of time, I know what must be the initial mass. Therefore, I know what is the value of correction required; may be for the first correction like that you know daily or once in a few days we require corrections. Therefore, I keep on adding all these corrections and the sum of all these things is what gives me a value of 950 meters per second. We do not really do a force balance; we just specify the ideal velocity required for the corrections

Let summarize once again. In order to appreciate the points made so far, I show a scale model of the GSLV rocket of the Indian Space Research Organization. Here we see that, we have a core stage at the bottom followed by the second stage over here, followed by the third stage over here. The core stage is surrounded by 1 2 3 and 4 straps; that means, you have a core stage followed by 1 2 3 4 rockets, these 4 rockets are a cluster and are known as straps.

At the beginning of the mission, the core and the straps either fire together or the straps fire before and immediately the core fires so that, you get a huge thrust, which can carry the entire mass of the launch vehicle. Once, the firing of the straps are over, they are discarded and thereafter the core fires for a small additional time and then this is also removed. The second stage then operates and once the second stage operation is over, it is also removed and it is thrown out and then the third stage takes over. The third stage fires and takes this space capsule which is sitting on the top of the third stage and puts it into orbit; but before putting into orbit, the third stage is also removed.

Therefore, we see in this particular rocket you have something like a core rocket, you have four straps, you have the second stage rocket, you have the third stage rocket on which you have the space capsule i.e., the satellite and it is this satellite which is put into orbit. The satellite, which is put in orbit, is the INSAT satellite. I show a scale model of it; the satellite consists of a box like structure, which is shown in brown over here. You have the solar panel here, which takes the energy from the sun converts it to electricity. You have a balance for the solar panel mass - a solar boom over here and you have the antennas.

But what I really wanted to show was, you have in red 1 2 3 4 similarly, you have 1 2 3 4 something like 16 to 18 thrusters which are small rockets which are mounted at the edges of the satellite, which will correct for the position of the satellite, may be the orbit and also the position of the satellite. Therefore, even a satellite in orbit has rockets attached to it which give it some impulse.

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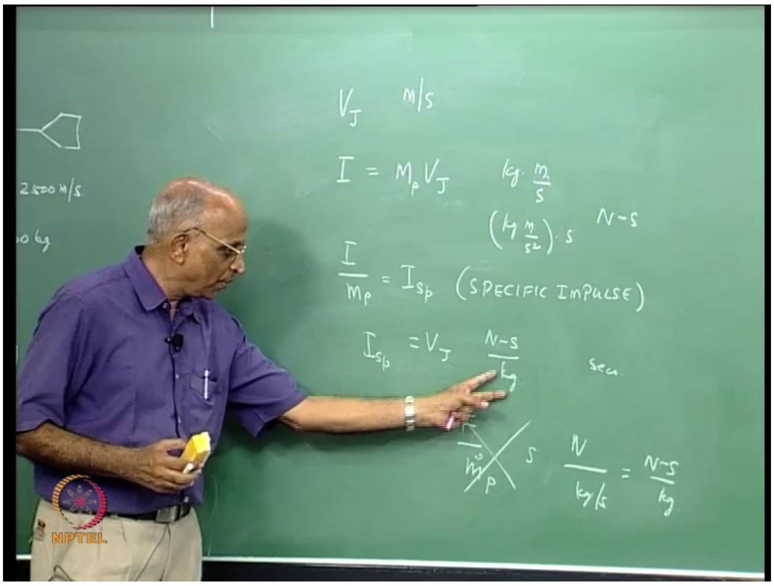

V_J is a very important quantity that is the efflux velocity, unit is meter per second. We talked in terms of Tsialkowsky's equation or Rocket equation, which told ΔV is equal to V_J into the natural logarithm of initial mass by final mass. We had the term impulse of a rocket, what is impulse? The change of momentum. This equals mass of the propellant into V_J and is the impulse, what is the unit here? We are talking of kilogram meter per second. This could also be written as kilogram meter per second square into second, which is same as Newton second.

Let us be very clear about units: We have kilogram meter per second which is impulse. We can also write it as kilogram meter per second square into second which is nothing but Newton second. Therefore, impulse could expressed in kilogram meter per second or

Newton second. This is the total impulse which is given by the $M_p \times V_J$, because this is what is moving out. Now, I ask myself a question. Can we say impulse per unit mass of propellant, which could call as specific, instead of calling as impulse call it as specific impulse that is impulse per unit mass and that gives me the value as V_J. And what is the unit I get? Newton second by kg and therefore, you find that the jet velocity and specific impulse are the same.

We will make some corrections for the mechanism of flow taking place in a rocket. We will find that it may not be identical, but this is the way to go about it. You know, in many text books, they express the value of specific impulse in seconds which is really not correct. We are talking of force divided by mass flow rate; because I could have also written Isp as equal to force divided by m°. Why do I write it? Force into time is impulse and therefore, this is same thing as impulse over total propellant mass.

Therefore, if we write instead of mass the weight or rather the weight rate of flow of the propellant or we alternatively express force in kg instead of in Newton, I am left with a unit of Isp in seconds which is really not right because the unit of force should have been Newton and the units should be Newton divided by kilogram per second, which gives me Newton second by kilogram. In fact, units are important. Therefore, please be careful when you read a book. If the unit is specified as seconds for specific impulse may be the unit being considered for force is kilograms. Therefore, it becomes necessary for us to multiply it by 9.81 and then have this unit of Isp as N s per kg.

May I will take an example as I go long. You know, see there is always a problem. People talk in terms of mass flow rate, they talk in terms of weight flow rate as you are mentioning, but can we say weight flow rate? It is actually mass flow rate. You cannot have weight flow rate since you cannot have force which is flowing out. We must distinguish between mass and weight correctly. Whenever I measure a mass by a spring balance, it is the attraction and therefore, we measure a force that is a weight, where as when we consider quantity of matter it is mass or quantity of matter.

Let us keep our definitions clear for as masses always imply quantity of matter which is Kg; impulse is Newton second, impulse per unit mass of propellant is Newton second by kilogram or force by mass flow rate over here. I think these definitions are important.

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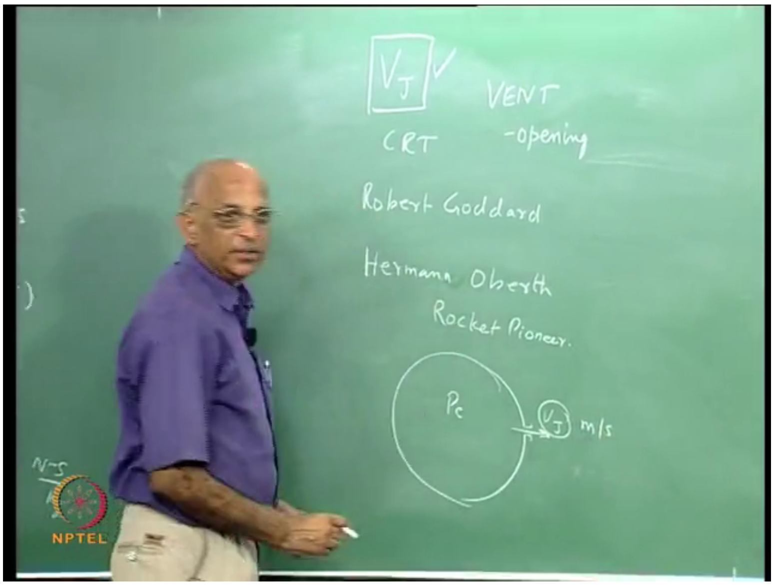

Let us now come back to the next part, namely we ask how can we obtain a high value of V_J? We said that it essential to get a high jet velocity. We had mentioned that when Tsiakowsky derived the rocket equation, since the cathode tubes were coming up in those days, his idea was to use the high jet velocity of electrons. In fact around that time Robert Goddard in US was also thinking in terms of high velocity electrons to be used in a rocket. But then electrons have a very low mass and therefore force that we get is small.

In fact, you also had that the third rocket pioneer by name Herman Oberth at about the same time. He was from Austria. He wrote a beautiful book on rocket propulsion and you will be surprised many of the things we do in rocket today remains exactly similar to what he suggested at that point in time. And in fact, what he said was you put one stage after other and I can get a high velocity like what we did in staging of rockets. And he is also a great rocket pioneer.

I would like to now address on how to get the high jet velocity V_J.

Consider a chamber in which I have a high-pressure gas filled with pressure Pc. Now I make a hole in the chamber. I know the gas will escape out. I want to find out the jet velocity or the efflux velocity of the gas. Let me take you through a small example, I have a balloon, because this tends to be somewhat easy to illustrate. Therefore, what I am trying to consider is I have a balloon filled with air, at a pressure Pc. I make a small opening here, and I want to find out the velocity with which the gases are going out. And this is exactly what we do in rocket propulsion. In other words, we allow high pressure to be built up in a chamber and allow the high pressure gas to escape through a vent. The gases escape at some velocity and the balloon goes up. It is pushed up similar to a rocket being pushed up. If I could have a controlled opening, I could get continuous thrust.

And therefore, let us again fill the balloon with air by closing this vent or hole. I now open this vent, and I find the air going out at a certain velocity. I want to calculate the jet velocity of this particular air, which is leaving the balloon. Let us do this problem. We may increase the pressure; we increase the flow rate and the velocity. Our aim is through this small opening what we have here, what is the value of Vj that we get in meter per second. We have done such problems in the thermodynamics course in the first year engineering, but let us repeat it.

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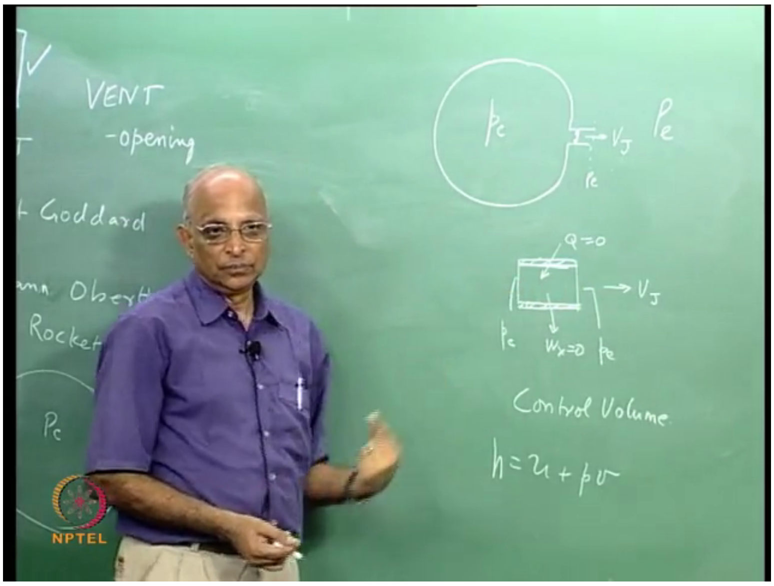

I draw a huge reservoir, which I call as a chamber. The pressure in it is Pc; I provide a vent over here. I allow to this particular area of opening for the vent. I want to calculate the value of velocity of the jet Vj, when the pressure at the exit of the vent is pe. The vent or hole is spoken of as a nozzle.

What do I have to solve for: I know the pressure here, I know the pressure at the exit, I want to know the jet velocity. I am interested in this particular vent or nozzle, which has a given shape. I do not know the shape, what I just showed you something like this. Air enters at the pressure Pc; air leaves at a pressure Pe I am interested to calculate the value of Vj.

This is a control volume, why do I say a control volume; I have a fixed volume in space. And what is happening in the control volume is that air is entering at a high pressure and leaving at a low pressure. And therefore, I solve for a control volume. And to be able to solve this problem, I have to make some assumptions. What are the assumptions I could probably make?

Let us say this is the vent. Let us assume the flow to be adiabatic, that means the vent is such that there is no heating of the air in the vent or no heat comes from outside into the nozzle. In other words, I say Q which is entering this particular boundary of the control volume is zero. Let us for the present also assume that this vent is rigid; if it is rigid, it cannot expand; it cannot do any work. Therefore, the work done by this vent that is Wx is equal to zero.

See, I could have something like a flexible vent, which could move and it can do some work. But I assume that it is rigid, when it is rigid I have the work done is zero since there is no displacement. Therefore the assumptions are that the vent is such that it is adiabatic, that heat transfer is zero and the work done by the vent is zero. Across the surface what we get the work done is zero. The flow can also be assumed to be steady. Now I have to write the steady flow energy equation for the control volume.

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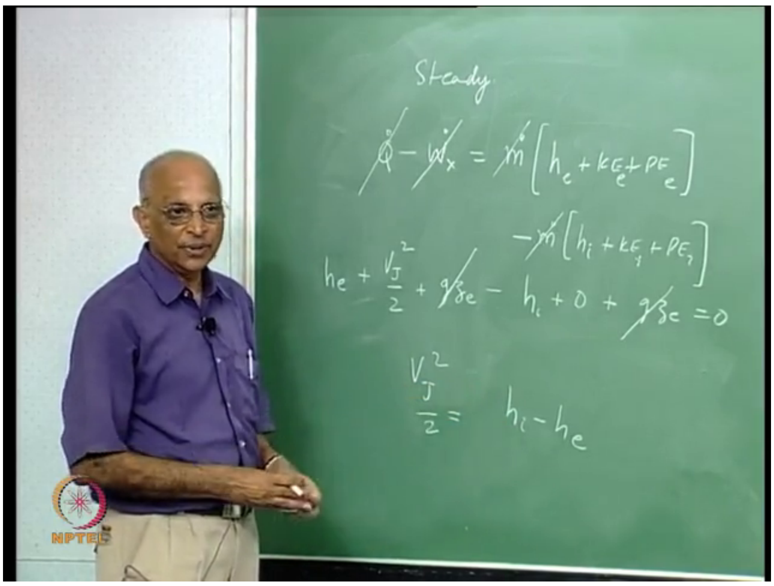

The flow has been assumed as being steady. What do you mean by that the flow is steady? The mass, which is entering by vent and the mass leaving the vent is the same. If we have Q the rate at which heat is entering the vent and Wx rate at which work is done by the air, what would happen? Let us say that the mass of gas, which is entering the vent and leaving the vent is m°. You have enthalpy, which is entering and which is leaving and because you have some heat which is coming in and the work is done at the vent surface. The energy entering is the enthalpy hi plus kinetic energy plus potential energy at the entry to the vent and what is leaving is the enthalpy he at the exit plus I have a kinetic energy at the exit plus I have a potential energy at the exit.

Why do we write enthalpy here to consider the heat energy, because you have internal energy and it also has a specific volume that has some flow work? Enthalpy is equal to internal energy plus p into specific volume. Therefore, we have for the energy balance: Q minus Wx is equal to what is the enhancement in its energy; it has got in the enthalpy, kinetic energy and potential energy during its motion through the control volume.

Q and Wx being zero are dropped and this left hand side is zero; since the flow rate at the entry and exit are the same, m° also cancels out. .

So if we have he plus the kinetic energy per unit mass, it is equal to $V_J^2/2$ where Vj the exit velocity. I have potential energy, potential energy is equal to g into ze or the height above the datum at the exit. At the entry, we have hi plus kinetic energy at entry plus potential energy at the entry. What is the velocity with which the gases leave the chamber? See, the chamber is huge, this is small; therefore the velocity what we get at entry into the vent is almost zero, that means I can I can neglect the velocity at the entry. The potential energy per unit mass is g zi where zi is the height above the datum at the entry.

If the nozzle or the vent is small, the change in height will be very small and I can cancel out the g zi terms containing potential energy. And we get V_J^2 divided by two is on the other side and we get it equal to hi enthalpy at the entry minus enthalpy at the exit he.

We will continue with this in the next classes and try to see under what conditions can I get a high jet velocity.

To summarize, we looked at two problems illustrative of the principle of rocket propulsion. And then we went ahead and try to find out what is the jet velocity with which a pressurized gas will squirt out of a vent.