Engineering Physics – II

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Module No. # 01
Lecture No. # 01

So, welcome all of you to this course of lectures on electricity, and magnetism. There will be around 20 lectures the subject is vast there are ever, so many phenomena that come under this subject matter of electricity and magnetism, and twenty lectures are too short in number for us to discuss them. However we know that this is only a supplement to what you are going to study in your classroom, and the experiments that you are going to do in your laboratory. So, it is good for you to remember, that what these lectures aim at is to provide you with the perspective with an understanding, and the way one has to think in order to understand the subject matter of electro dynamics.

Now, apart from gravity which binds us to the earth, almost every aspect of our life is governed by electro dynamics phenomena. Starting all the way from atoms, and molecules to chemical reactions, and then we have the mental processes, the biological processes, all the electrical appliances that you have in your home, in your laboratory, in your office, in your lab, in your class room; all of them are governed by a set of laws which are called as Maxwell's equations which summarize all of the known electrodynamics.

So to that extent there is no other interaction which is more important important for us, which is more essential for us, in this particular universe, because even life would be impossible for us impossible on this earth without electromagnetic phenomena. It is quite remarkable actually, that although there are ever ever, so many phenomena, all the way starting from atomic physics to let us say huge power stations, and transmission lines that we see, all the electromagnetic phenomena can be summarized in a set of merely 4 equations known as the Maxwell's equations.

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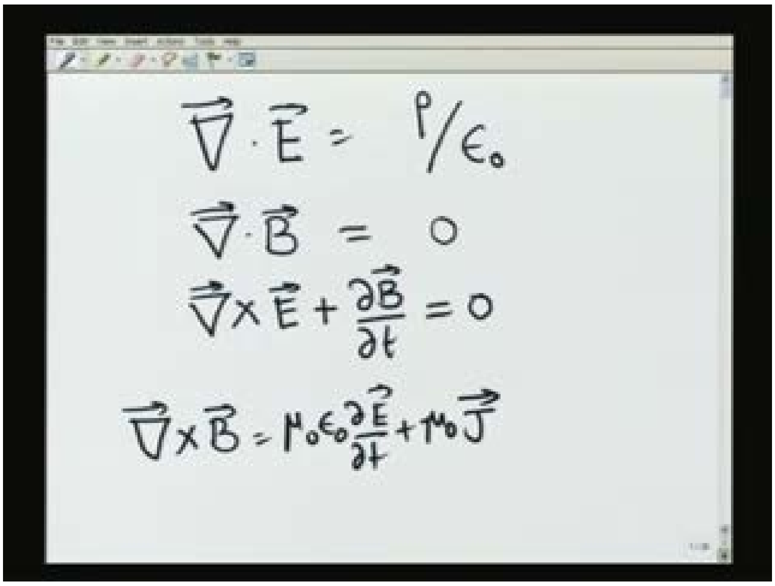

So, let me write down them for you, you are already aware of them, the first equation is the so called, coulomb's law which tells you that divergence of the electric field is equal to the charge density divided by a constant which is called as the electrical permittivity or the dielectric constant of the vacuum. The second equation tells you, that there are no magnetic monopoles. So, which we write by saying divergence B equal to 0; the third equation is the famous Faraday's law of induction, which also give rise to Lenz's laws and eddy currents so on and so forth; which tells you how a change in magnetic field – A magnetic field is changing in time as you can see, he is going to produce an electric field.

So, here you should imagine that the magnetic field is the source, and the electric field is the effect; whatever is being produced by the magnetic field, and the last equation is a generalization of the Ampere's law which tells you, how currents actually produce the magnetic field. So, that is written as mu naught epsilon naught delta E by delta t plus mu naught J. Here j is the current density which is produced by the moving charges, and the other current mu naught epsilon naught delta E by delta t is the displacement current, which is actually the changing electric field which can produce the magnetic field.

Of course, this is not the end of electromagnetic equations, because these 4 Maxwell's equations have to be supplemented by what is called as the Lorentz force equation. Now, what is the difference between Maxwell's equation, and Lorentz force equation.

Maxwell’s equations tell you how sources, namely charges and current densities are going to produce electric, and magnetic fields for you.

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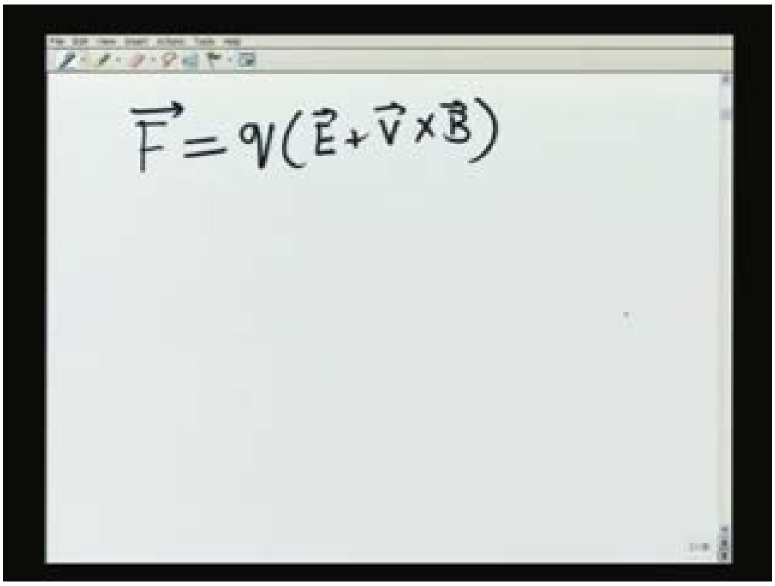

On the other hand, the Lorentz force equation which I am going to write, we will tell you how the electric and the magnetic fields are going to act on a charge particle. So, that is given by F is equal to q E plus V cross B. So, all in all we have a set of 5 equations; before Maxwell’s equation that I wrote in the previous page, and the fifth equation that I have written now which explain all the known electro dynamic phenomena.

In other words, when we start a course called electrodynamics; here we have 2 options - either we climb up all the way, understand phenomena as you go through by looking at various experimental facts; and then set up these equations 1 by 1 or if you are doing an advanced course, we would say that we will start with Maxwell’s equations, because we know them to be correct laws of nature. We start with the Lorentz force, because we know that they are already its already verified experimentally to a great extent, and then we work out the consequences.

Here, since it is an introductory course for you; since, it is the ever first course on electrodynamics for you, what we shall do is to take the first view point; we start with simple experimental facts, we start with simple mathematical techniques; try to build the whole subject matter of electrodynamics step by step. In other words, we are going to climb up the hill to reach the peak of the laws of electro dynamics, rather than climb down the hill as we are quote various applications.

However, before we do that as you might have noticed, if you were to go to the Maxwell's equations again, you see that these equations involve the kind of mathematics which you to, you would not encounter when you looked at Newton's mechanics. In Newton mechanics, what you have is ordinary differential equation; whereas, what you have here is a partial differential equation, a set of partial differential equations; and these partial differential equations involve the divergences, the curls, and so on and so forth. And therefore, what we should first of all do is to get our basics right. We should know, what the meaning of these divergence mean is, what the meaning of the curls? What is the meaning of the equation for example that curl E plus delta phi by delta t equal to 0. In other words, we have to understand, we have to pick up the basic vector analysis, and the vector operations that we do.

So, before we embark on the journey of learning electro dynamics, what we shall do - is to take a short break; start with vector analysis, vector algebra, we develop all the basic mathematical apparatus that is required by us, and then we shall return to electrodynamics phenomena. However, before I do that what I need to do is to set up my notations. So, let us start with the most basic prerequisite of everything that we have to do, and that is setting up a co-ordinate system.

The charge particles are going to be located at various points; if you are interested in a static case or if you are interested in stationary magnetic fields, there will be steady currents which are located in various regions of space. If you are not interested in a steady situation of course, there are charge particles which move from point to point; in order to discuss these phenomena, in order to discuss how they move we have to set up a co-ordinate system.

What we shall do is to start with the most well known of the co-ordinate systems, namely the cartesian co-ordinate system. The rectangular cartesian co-ordinate system, consist of a rigid scaffolding of 3 basis vectors which are orthogonal everywhere, and which are universal. So, let me show them that now for you.

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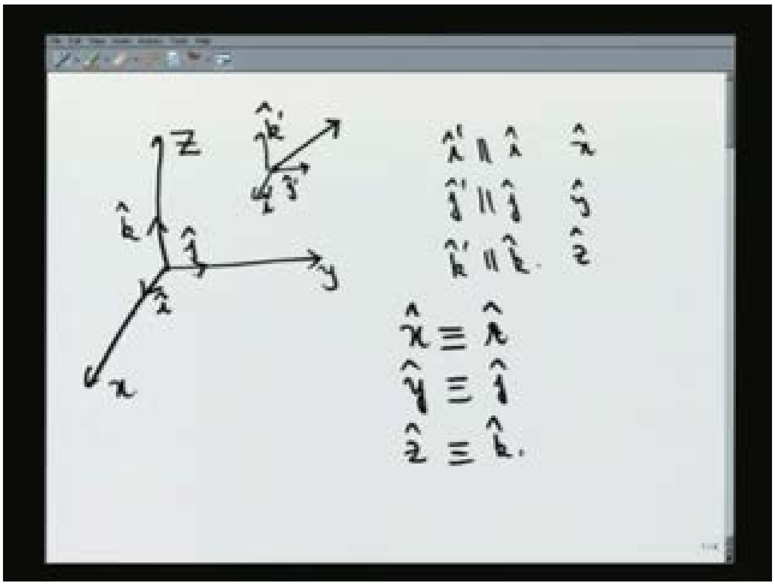

So, I will draw a co-ordinate system; let me label the axis. I have my x axis here, I have my y axis here, and the z axis here. So, when I say that these forms are rigid scaffolding, and the basis vectors are independent of the location. What I mean is the following. To start with let me note the unit vector along the direction x, by the notation I hat, the unit vector along the direction y that is along the y axis by j hat, and the unit vector along the z axis by the notation k hat. These are the 3 unit vectors, which are orthogonal to each other; and they form my basis.

When I say, that I have a rectangular cartesian system what I mean is, if I take a point p here, and if there is a vector here; it could be the velocity vector it is in this direction, in order to take the components of this vector; what I do is to look at the basis vectors written as the following, namely my i is along this direction, j is along this direction, and k is along this direction. What do I mean by that - By that I mean that the unit vector i is parallel to the unit vector i here, the unit vector j is parallel to the unit vector j here, the unit vector k is parallel to the unit vector which I originally defined. So, in that sense the orientation of the unit vectors, for that matter even by relative orientation does not change as I move from point to point, and this is what defines a rectangular cartesian co-ordinate system.

Therefore, what we mean is that, if I about to denote it by i prime, j prime and k prime. I know that, I prime is parallel to i, j prime is parallel to j, and k prime is parallel to k.

Sometimes, it is convenient not to use the notation i, J, k for these unit vectors; which do in fact, form a basis. But we can also use the notation, they be given by x prime, x hat, y hat and z hat. What do you mean by that? By that I mean, that x hat is inter used inter changeably with I hat; y hat is used interchangeably with j hat; z hat is used interchangeably with k hat, is that ok.

What we so do, is to see how we can actually move on to other co-ordinate systems. The simplicity in rectangular cartesian co-ordinate system is that the unit vectors do not change their orientation. Taking the components, taking the (( )), evaluating divergent, curl, setting up differential equations all these are extra ordinarily simple.

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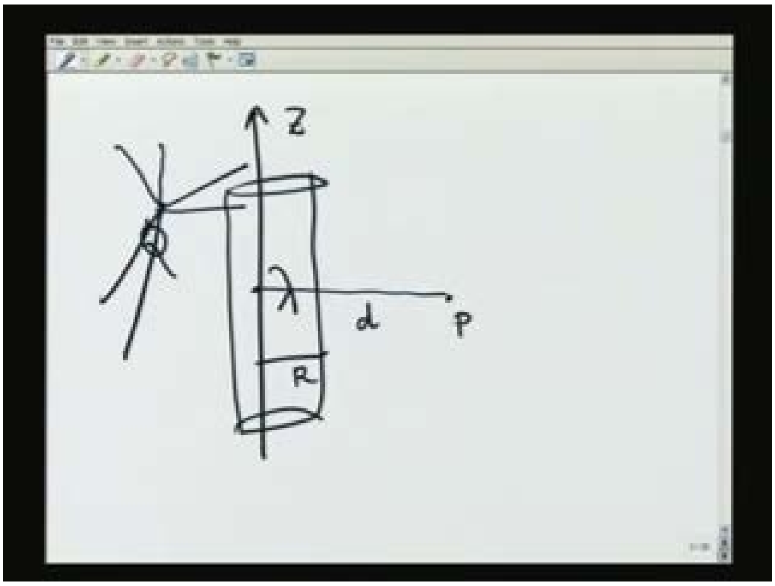

However when we look at a given physical problem; like for example, I am interested in the field produced by joint point charge particle or I am having infinite line charge with a uniform charge density lambda, and I am interested in the electric field produced at a point p, at a distance d; for example, by this infinite line charge lambda.

Now, if you look at this former case; namely the point charge or if you look at the infinite line charge, what is it that you find. For the point charge we find, that there is a s spherical symmetry. All that you have done is to choose a special point in the space, that is the location of the charge; whereas, once it is done all the directions in the space are completely equivalent. In other words, it is immaterial whether I am going to construct a rectangular cartesian co-ordinate like this or I am going to construct a rectangular cartesian co-ordinate like this or by any other rotation, they are all completely equivalent.

In a similar manner, if I were to look at the infinite line charge lambda, you see what we have now is not spherical symmetry, but what we have now is a cylindrical symmetry. I have not only chosen your preferred point, that is let us say the center of this very, very long line charge, but I have chosen a whole direction, the direction along which the line charge is distributed.

Now, what I have is a cylindrical symmetry. Because if I were to draw a cylinder of a given radius R let us say, all points on this cylinder are are completely democratically to be treated with respect to the point charge lambda. For simplicity say, I could have taken this to be along the z axis. In order to describe this kind of very special, but frequent apparent situations; it is convenient for us not only to use rectangular cartesian co-ordinate system, in fact, not to use rectangular cartesian co-ordinate system, but to use what are called as the other co-ordinate systems namely, the cylindrical co-ordinate system, and the spherical polar co-ordinate system.

Now, I will start with the simpler of the two. What is closest to the rectangular cartesian co-ordinate system is the cylindrical co-ordinate system, then I move onto the spherical co-ordinate system is slightly, which is slightly more complicated. There is no great deal about these things, because you people have already studied it, in your course on engineering mechanics. So, I am not going delve into all the great details of these spherical polar co-ordinate system or cylindrical polar co-ordinate system, but we shall get into the essentials, whatever is required for our purpose in order to understand the laws of electrodynamics. What shall we do, when we want to set up the cylindrical co-ordinate system.

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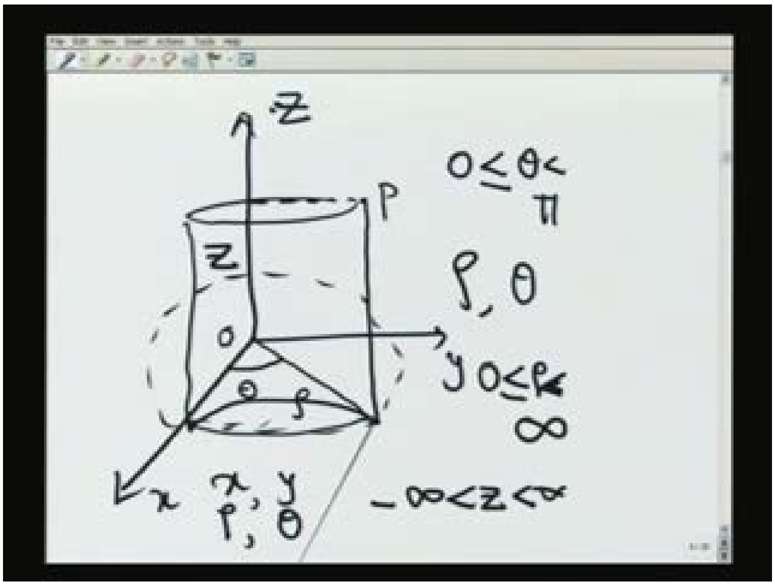

Let me draw the cartesian co-ordinate system again. So, what I have is the x axis in this direction, I have the y axis in this direction, and I have the z axis in this direction. Now, imagine for reversals that the x, y plane is the horizontal plane, and the z axis is coming out of this particular plane; that is what we have.

Now, let me take a point for example, p what I shall do is to not to give the coordinates of p in terms of x, y, z. But I shall project this p in the x, y plane. So, this is the projection of p in the x, y plane, and then in the x, y plane I am going to draw the distance at which this projection is going to lie from the origin o; let me call this distance plus rho. Now, obviously when I look at the projection, where that is respect to the origin o, this makes an angle which I shall denote by theta.

So, in the x y plane, I have been able to set up 2 coordinates namely rho and theta; and for any fixed value of rho what I shall do is to draw a circle which will actually go like this, and theta will vary from 0 to 2 phi, so which I shall write here. So, I shall say 0 less than or equal to theta less than phi; that means, we have been able to completely characterize the projection of the point p in the x, y plane, in terms of 2 equivalent parameters namely rho and theta.

Rho is the distance of the projection from the point o. So, clearly rho takes values from 0 to infinity; whereas, theta takes values all the way from 0 to 2 phi as I have written earlier. Now, what about the z axis; no problem about that, because what we are going to do is to project the point p along the z axis, and we shall retain the z axis as it is, and clearly the z variable will take all the way, all the way from minus infinity to plus infinity; it can go all the way from minus infinity to plus infinity therefore, I write minus infinity less than z less than plus infinity.

So, what have we done? Instead of, going through the usual rectangular cartesian co-ordinate system, in terms of x, y, z; I projected my p to the x y plane, and in the x, y plane I replace the variables x and y by 2 equivalent variables, please mind namely rho and theta; you give me any value of rho and theta, I will give you, I will be able to give you x and y; give me any value of x and y, I will be able to give you the value of rho and theta except of course, at the origin, and that you shall be able to write down in the following equation which is clear from the geometry.

So, how do we write; we simply write x equal to rho cos theta, and y equal to rho sin theta, and z is equal to z. So, the new variables are therefore, given in terms of rho theta and z. Clearly from the geometry that we had in our earlier figure, you see what we have done is to erect, the whole series of cylinders that is what we did; for every given value of rho there is a cylinder you keep on moving along the z axis, and then you change the value of rho, you draw one more cylinder, you change the value of rho you draw one more cylinder, eventually you filled the space with a whole number of infinite number of concentric cylinders, and you look at the point on any one of those cylinders in terms of rho theta and 5; that is what we have done; therefore, it is an equivalent description.

There is a small minor point which is of not any great interest to this particular course, but for the sake of completeness, we should be able to we should be mentioning that and that is if I put rho equal to 0, that is if I sit at the origin or if I sit along any point along the z axis theta is not defined.

On the other hand, if I go and look at x, y, z there is so much ambiguity; x, y and z are going to neatly fix the coordinates, but that is a matter of not very great importance to as in this; therefore, we need not worry about this particular aspect, but please remember that, because eventually when you write down the volume element or whatever you might start wondering what happens to the volume element that you are going to write; the so called, integration measure. That is something that we have to remember.

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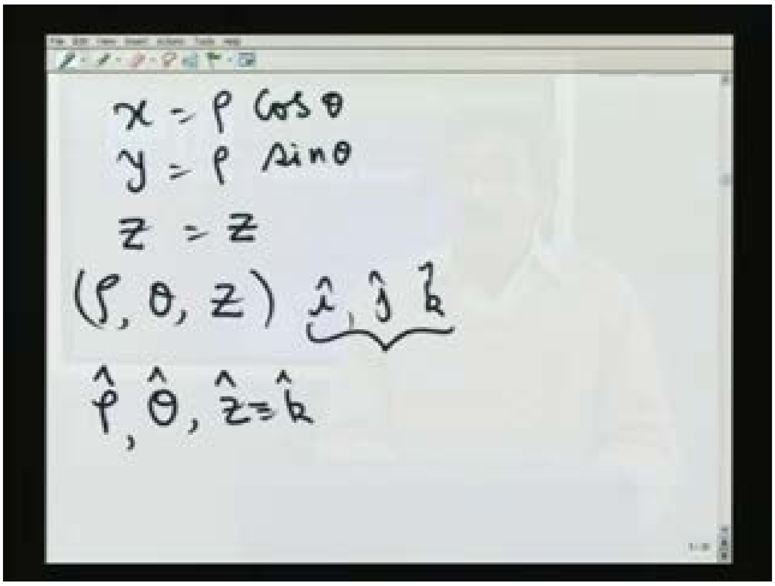

Now, having done that once we introduce the new coordinates rho theta and z, we have to introduce the analogs of i, j and k; we have to introduce the analogs of these that is, I have to introduce a unit vector which changes only when rho is change, which will denote by the symbol rho hat. I have to introduce a unit vector theta hat what does it do, I keep rho and z fix, and I only vary my theta and ask how that vector is going to change that I shall denote by theta hat; and in a similar manner, I have to introduce unit vector z hat which is identically the same as k hat as we said earlier; and if you did that then we have erected the co-ordinate system completely, we know how to go from one co-ordinate system to another without any trouble; and that we shall do in the next time.

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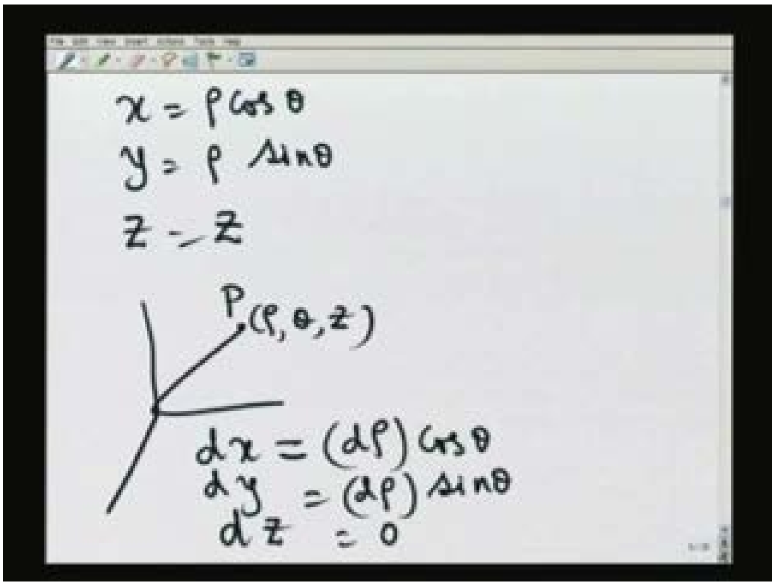

How do we do that; in order to do that it is good to go back to the basic definition. And what is the basic definition - I will again write this, x is equal to rho cos theta, y is equal to rho sin theta, and z is equal to z mean; it is a relenting thing, but it is good to write that. Now, let me imagine, that I have a point p in this co-ordinate system; and this is having the coordinates rho, theta and z, the point p. What I shall now to its to hold theta and z fixed, I will change only my value of rho; I ask what is the result in displacement in the point p.

That is not difficult to find out at all, because as you can see the x co-ordinate will change by a quantity dx, which is given by d rho cos theta; the y component will change by a quantity which is given by dy is equal to d rho sin theta. And z of course, will not be able to change, because they are only moving in the x, y plane, because they are changing rho which is defined only the x, y plane. Therefore, we shall write dz is equal to 0, that is what we shall do.

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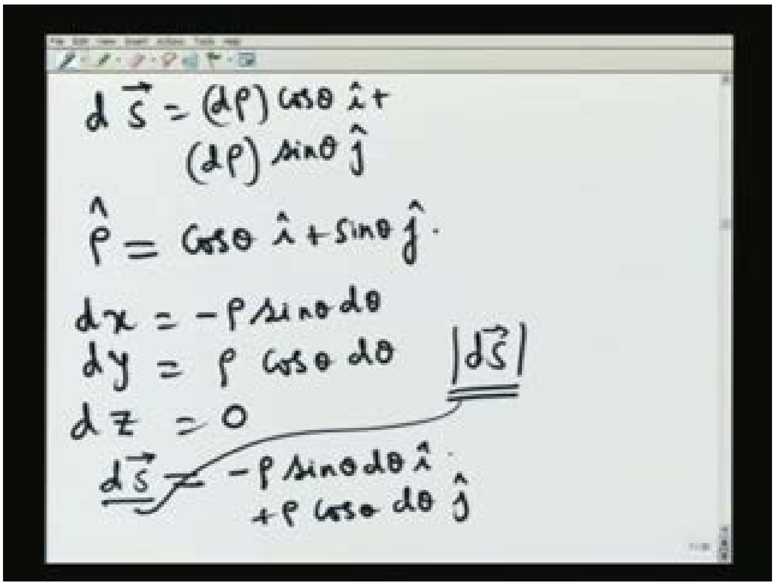

So, since I know dx, dy and dz; and since, we know that they are all along perpendicular directions, I should be able to write my infinitesimal displacement as d rho cos theta into I hat plus d rho sin theta into high j hat. Well, how do I find out the unit vector. Calculate the magnitude of this infinitesimal displacement divided by the magnitude, and to get back to your unit direction. So, I do not have to work it out, it is a very easy thing to work out, we shall therefore say that the unit vector rho is simply given by cos theta i plus sin theta j.

Now, clearly as you people can see the direction of rho will change as we keep on changing theta, because rho is always radically outwards in the x, y plane. So, let me show that for you here, in the previous picture when we come back, it is a very previous picture, you see my rho was defined there and it was a radically outward quantity. So, if I am along the x axis rho is along the x axis, if I am moving along the y axis rho is along the y axis. If this theta our phi by 4 for example, rho would be along the direction that y axis x, y plane; therefore rho is going to change as I keep on changing the direction which is indicated by this.

In a similar manner, it is also not a difficult thing for us to find out what the theta direction is, the prescription is very simple hold rho and z fixed, and simply change the theta; well what do we do for that again we can write dx will simply given by given by minus rho sin theta d theta; what I have done, I have done written dx is equal to rho d of cos theta which is nothing, but minus sin theta d theta. In a similar manner dy is given by rho cos theta d theta and dz of course, is equal to 0, because the theta is also defined in a plane which is perpendicular to the z axis; namely the x, y plane.

Again I can write down the infinitesimal displacement, because of this in the point p which we had defined earlier, and what will it turn out to be; it will simply turn out to be minus rho sin theta d theta I hat plus rho cos theta d theta j hat, that is what it is. Now, they were to find out, the unit vector along the theta direction is to calculate the modulus value of this d s and divide this vector by this modulus value. So, take this value bring it here, and put it here then we would have done our job.

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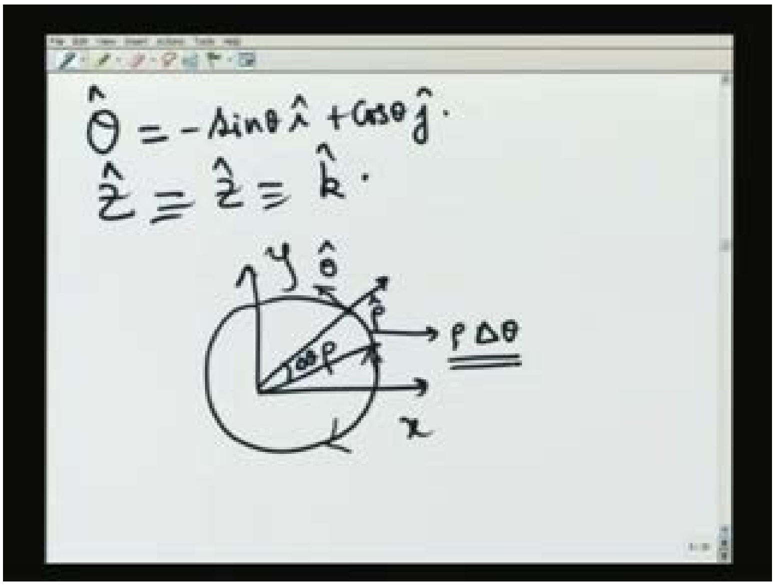

So, we shall do that, and if we did that you will find that the theta cap is simply given by minus sin theta i plus cos theta j; so, we succeeded in constructing the 3 unit vectors along the theta direction, the rho direction, and the z direction there is nothing nothing much to do in the third direction, but for the sake of completeness, I will write z identically z equal to k hat. But at this point it is good to pause, good to pause and ask what is it that we did geometrically.

So, let me write down the figure here again; since, z axis is a kind of uninteresting variable, let me restrict myself only to the plane here. So, I have my x axis here, I have my y axis here, and I have my circle here. This circle is the projection of the point p if you feel like, and this is my rho. So, what is it that is meant by the unit vector rho - the unit vector rho is nothing, but the unit vector which is radically awake in this particular sense; whereas, theta if you look at the formula that I have written just here, is simply given by the tangent to this circle at this particular point.

So, as you people can easily see rho and theta continue to remain perpendicular to each other. So, in going away from rectangular cartesian co-ordinate system to the cylindrical co-ordinate system, we did not abandon the orthogonality of the unit vectors. The unit vectors in the rectangular cartesian system were orthogonal, the unit vectors in the cylindrical co-ordinate system are also orthogonal, the one and the only difference between the 2 co-ordinate systems is that there the unit vectors, their orientation did not change as I move from place to place; whereas, as you people can see here as I move along the circle rho, and theta continuously change their orientation such that, they remain perpendicular to each other.

Please also notice that in this particular definition, there is a specific prefer direction that has been chosen for theta, namely theta would be positive if I went in the anti clock wise direction; that is I move from x to y and theta would be negative, if I move in the clock wise direction which is the usual convention that we hold. So, what is the summery? The summery is that whenever, you have a cylindrical symmetry we will be able to use the cylindrical co-ordinate system to our advantage, it is characterized by 3 unit vectors rho, theta and z; rho is readily outwards in the x y plane, theta is tangentially in the x, y plane, z is simply parallel to the z axis or perpendicular to the x, y plane.

Once we have done this, it is clear from the geometry that if I were to move along the theta direction. So, let us say that I move the direction by a certain angle delta theta, then this arch length is nothing, but rho delta theta. This is one and the only fact that we have to remember, if you want to construct the line element, the area element or the volume element.

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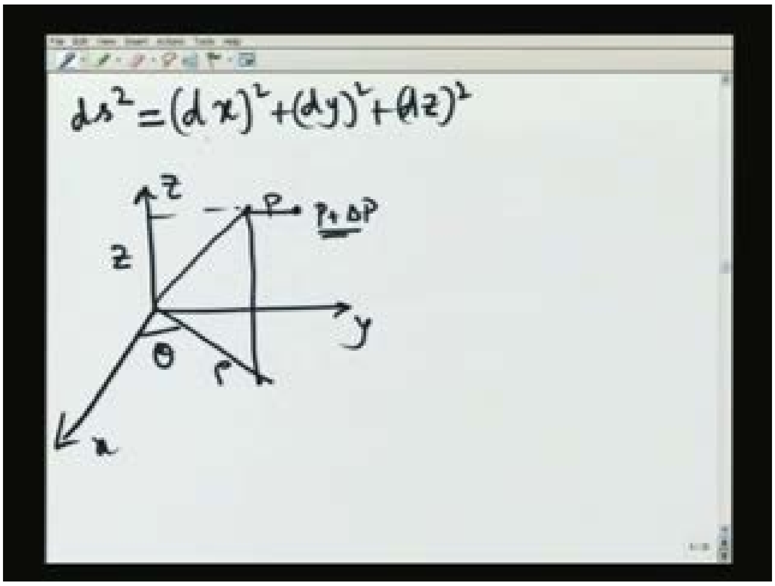

So, given this fact, it is not difficult for us to construct the line element and the volume element at all, because we can immediately write that d s squared; which is the displacement which comes, because of moving the point p to another point p by a small by a very, very, very small distance, will be simply given by dx square plus dy square plus dz square. That is what I did all that I have done is to make use of the Pythagorean law.

Let me show that for you again, in terms of this co-ordinate system. So, I have this co-ordinate system, I have z, I have y, I have x here; I have taken the point p here, and then what I have done is to project it; this is my theta, this is my rho, and this is my z; I move it from a point this point which has coordinates x, y, z to nearby point let us say p plus delta p; I have moved to this point. I am not interested in finding out what the change is in in terms of dx, dy and dz, but I am interested in expressing it in terms of rho, theta and the z axis. That is what I am interested in, because rectangular cartesian co-ordinate system is something that is familiar to all of us.

But then, we know how to write down dx in terms of the infinitesimal changes in rho and theta, dy in terms of the infinitesimal changes in rho and theta, and dz of course, is given in terms of d z.

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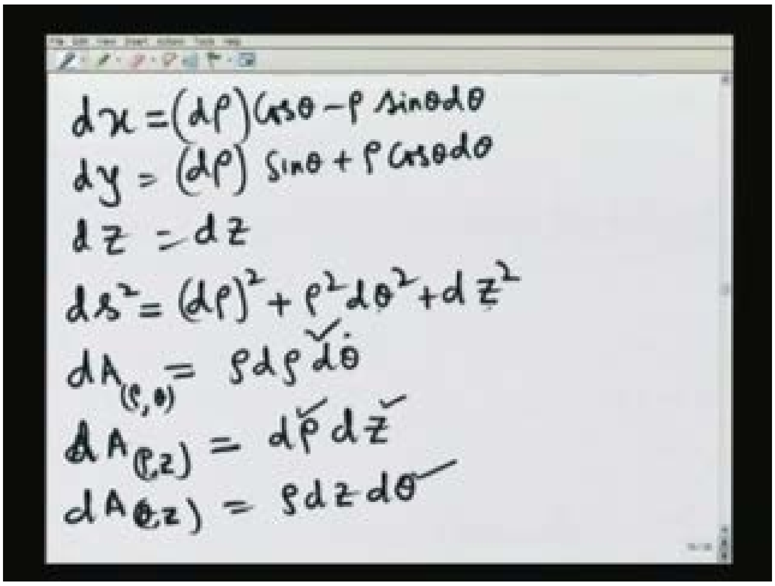

So, if I were to do that it is very easily done, because now I am going to give independent displacements both in terms of rho and theta, unlike what I did earlier; therefore, dx will be given by d rho cos theta minus rho sin theta d theta dy will be given by d rho sin theta plus rho cos theta d theta, and dz will be given by dz.

So, now let us split this equations and find out, what is the result and expression that I am going to get for my length element; that length element will therefore, be given by simply d rho square plus rho square d theta square plus dz square. Of course, if I were to move radically, you will need a certain d rho; as I showed you with the arch length in the previous diagram here, arch length is given by rho delta theta. Therefore, if you hold rho and theta fixed, and if you move along the theta direction you cover a distance rho square d theta square, if you move along the z direction by a quantity dz, you move along the dz square.

Since, rho theta z, x, y, z are both rectangular cartesian, I am able to write it as orthogonal to each other, I am going to I am able to write it as d rho square plus rho square d theta square plus dz square.

In a similar manner, I should be able to write down what the area elements are. Now suppose, I am interested what the area element in the x, y plane is in terms of the rho theta Co-ordinate; well hold dz fixed and simply look at this two terms. In other words, the area element in the rho theta surface, let me not call it a plane at this particular point will be simply given by rho, d rho, d theta. In a similar manner, the area element in the rho's at surface; the rho's at surface actually correspond to move moving on the cylinder, holding the theta fix; therefore, my area element on the rho's at surface is simply given by from this, we can easily see that it is d rho dz, and the area element in the theta z surface is given by rho dz d.

So, if I had an integration to perform for example, in the theta z plane corresponding to a cylinder, you would be able to you you would be using this area element or if I had a function that I had to integrate freezing the value of theta, and you have to vary only rho and z; you would use d rho dz. And on the other hand, if you had to integrate in the x, y plane, in the polar coordinates which are all familiar with then you would end up with rho d r d theta; that is what you would do.

In a similar manner, I should be able to construed what my volume element is, well it is simply given by the product of this quantities which multiply what d rho dz and d theta in my length element.

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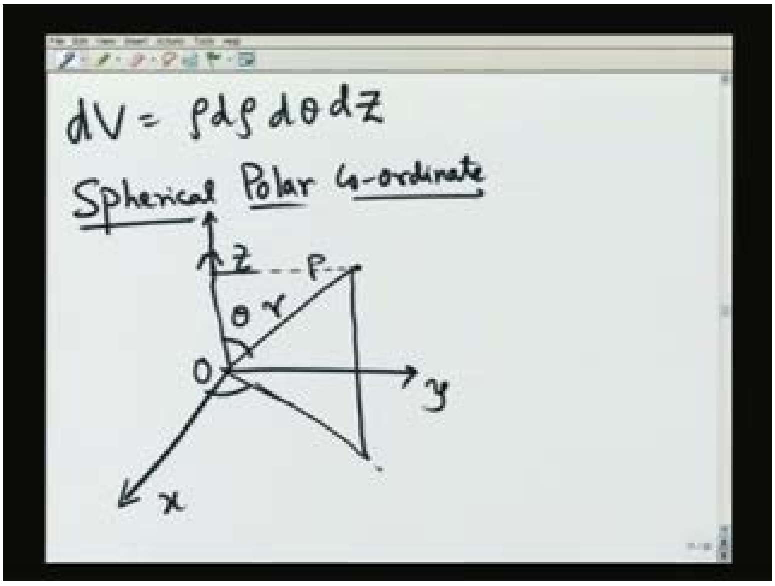

Therefore, my volume element is simply given by rho d rho d theta dz. So, what have we done? We have not only constructed an orthogonal co-ordinate system in terms of rho theta z, we have found out how to measure the unit vectors, how to determine them; we have found out what the various area elements or in each of the 2 surfaces - There are 3 surfaces corresponding to rho theta, rho z and theta z. And we are also able to write down the volume element, and this will be very useful for us, later when we try to determine the potentials, the fields, the forces and what on so on and so forth, in electro dynamic phenomenon. We should sort of conclude, the discussion of the cylindrical polar co-ordinate system; what I shall now do is to get into this spherical polar co-ordinate system.

The spherical polar co-ordinate system is also not any great complications for us; once we get the geometry right. And the geometry for us is all familiar, because of whatever we studied in in geography, what is it that we did in order to locate various cities or places in geography, we characterize them not in terms of x, y co-ordinate or for that matter cylindrical Co-ordinate, we characterize them in terms of latitudes and longitudes that is exactly what I am going to do right now.

However in order to be on firm ground; let us start with the rectangular cartesian system again. So, you have this z axis along this particular direction, you have the y axis along this direction, you have the x axis along this direction, and I now write the point p. Now, when I am interested in this spherical polar co-ordinate system, as I told you this could be useful whenever there is a spherical symmetry; therefore, the first question that I ask is what is the distance that this point p makes with respect to this origin; more often then or this origin is chosen based on convenience depending on the problem.

If I am interested in the field produced by a point charge, the origin will be located at the center; if I am interested in this scattering of a charge particle, because of a nucleus. Like for example, in rather for scattering the origin will be located in the scattering center then with the nucleus. So, there is a certain physical significance attached to this choice in a practical problem. So, let me write this origin, and ask what is the distance which is this point p is going to describe is going to possess and I shall denote it by r.

Now, what I shall do is to ask what is the angle made by this point p with respect to the z axis. So, this angle will be simply given by the theta. Now, let us again project the point p to the x y plane, take the coordinates of the point p in the x y plane, I am not interested in the length of this projection, because I already given the total length; I will only ask what is the angle made by the projection with respect to the x axis, and I shall denote it by phi.

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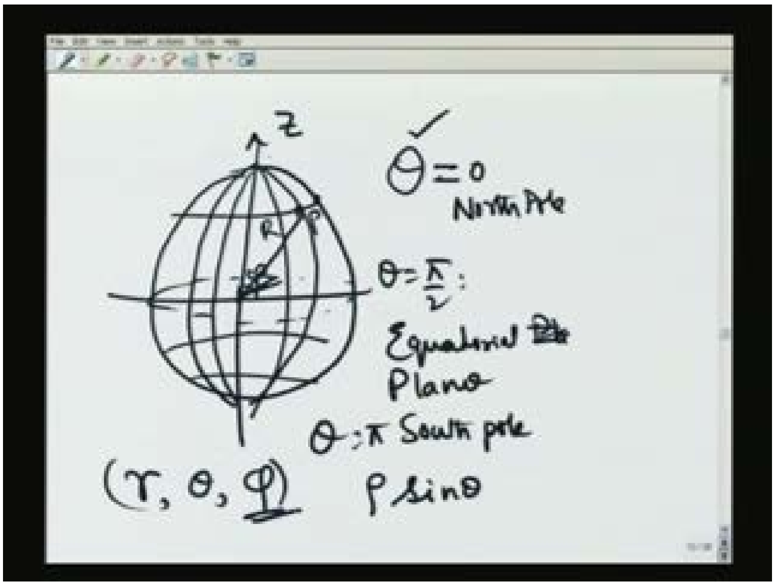

So, what have I done; that shall be indicated by a figure now that I am going to write in terms of a sphere. So, I have a sphere here, and there is a point p here, for the time being if I imagine that I have my radius fix. So, these are my latitudes, and these are my longitudes. So, if that is what I am going to write down. So, let us say that, I am taking a point p here, I am asking what the angle made by this particular point p is with respect to the z axis. That is what I am asking.

Now, of course, I can complete the circle at this particular point. So, this is of course, my equatorial circle that is what I have here, this is my origin and if this is the point what I am asking is, if I projected the x y plane; there is going to be unit vector in this direction and I am asking for the angle phi made by the point.

In other words, the variable theta is nothing but the angle at the point p makes with respect to the north pole, the angle phi is nothing but the angle made by the projection with respect to some prefer direction in the equatorial plane; that is all what we are doing. So, what does it mean; theta equal to 0 what correspond to the north pole, so let me write it down, it will correspond to the north pole. Theta equal to phi by 2 would correspond to equatorial plane plane, and theta equal to phi would correspond to south pole. This is for a given value of r.

Now, imagine that I keep on changing my value of r starting from very very small values of r namely r equal to 0, what have I done, I have filled the space with a series of concentric spheres. So, when I describe my location of my point in terms of spherical polar co-ordinate system, I first ask at what distance is it from the origin. So, that will be given by the distance r, and then I ask what is the angle made by that with respect to the z axis; and the z axis is the line that connects origin with the north pole of the sphere. And that angle will be denoted by theta then I project the point on to the x, y plane which is the equatorial plane which of course, it depends on where the point is located, and ask what is the angle made with respect to the x axis and that would be denoted by the phi.

So, all that we have done is to take over the concept of, whatever we have studied in geography made minor changes in our notation, and introduce this co-ordinate system. So, in the spherical polar co-ordinate system, the variables are given by r theta and phi. Once I fix my values of r and theta, one can easily see that the freedom in phi corresponds to moving along this circle. This circle has a radius which is given by, this circle has a radius which is given by rho sin theta rho sin theta. However phi itself can take values going all the way from 0 to 2 phi. So, if you remember that we shall be able to write down the ranges for r theta and phi.

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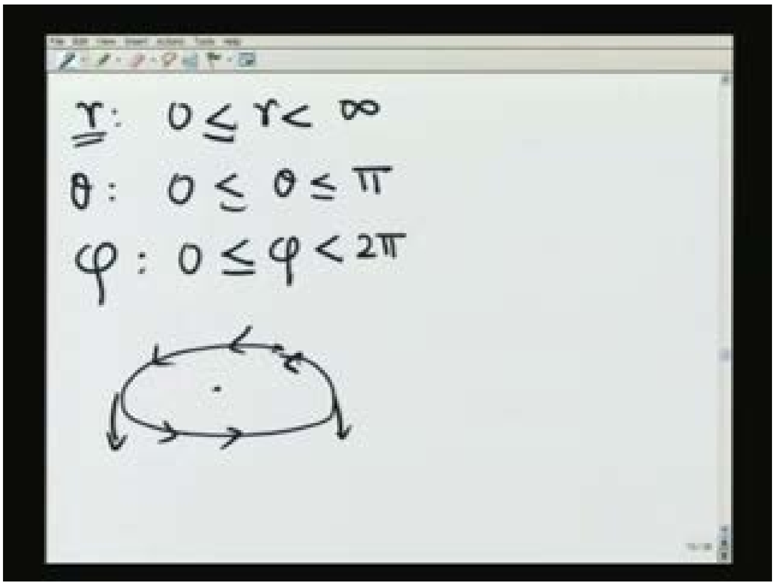

So, what are they going to be, r will take values 0 less than r equal to r r less than infinity, theta will take values 0 to phi remember we said that theta equal to 0 corresponds to the north pole, theta equal to phi corresponds to the south pole, that is the fastest that you are able to go; therefore we write 0 less than or equal to theta less than or equal to phi, phi of course, shall move on a circle which is in the equatorial plane; therefore, we write 0 less than or equal to phi less than 2 phi I have been. So, what extra careful in writing this, I did not write less than or equal to 2 phi, because we all know that we identify phi equal to 0 with phi equal to 2 phi, because when you complete the full circle you come back to your original point

You say people were to look at a globe in your house, and look at the radial direction, the theta direction, and the phi direction that I have defined. You can easily check that the 3 directions are mutually perpendicular to each other. The variation in the theta phi restricts you on the surface of a sphere; whereas, the radiation in or a variation in r is going to take you out of the surface of the sphere. So, even without drawing any figure it is clear, that the unit factor in the direction of r has to be necessarily perpendicular to theta and phi.

For convenience state, if you were to look at the equatorial plane for example, so this corresponds to the origin z is equal to 0, my theta will be everywhere along the minus z direction. Because I said, if I were to draw a longitude it will be along pointing along the minor that direction; whereas, my phi will be moving along the curve. So, thus we see that theta and phi are also perpendicular to each other. This is not only true of the equatorial plane, it is true everywhere; in other words all that we have done is to do a little bit of a minor generalization, it is not too great a generalization again from cylindrical co-ordinate system to spherical polar co-ordinate systems where we have 3 orthogonal vectors. So, if you could only construct the unit vectors in the direction of r theta and phi, then we would be home we would write down the length element, we would write down the various area element, we would write down the volume element, and then whatever we would have to do right for example, say setting up the divergence or curl or gradient in this co-ordinate systems can be easily done.

Speaking algebraically it is no great complication. In fact, it can be worked out geometrically also, but let us look at it algebraically, because geometrically you can work it out by drawing nice area elements later.

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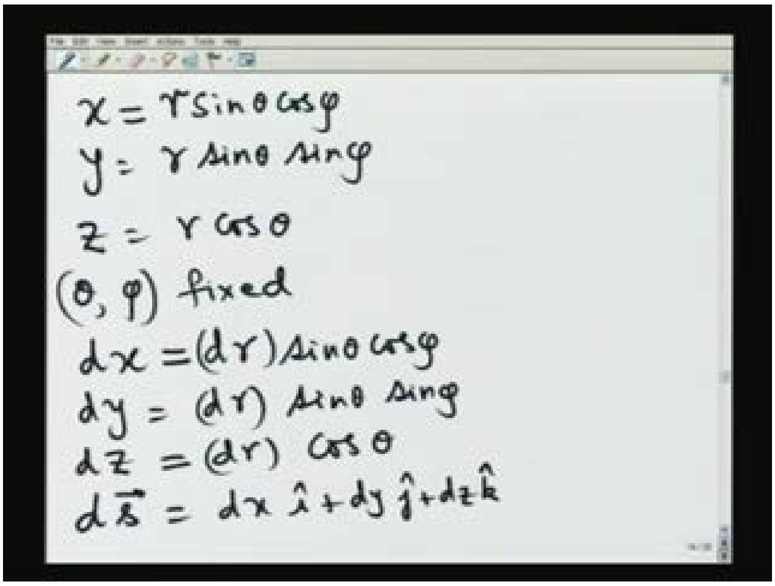

So, again let me rewrite x y and z in terms of r theta and phi. So, from geometry it is clear that x is simply given by r sin theta cos phi, y is simply given by r sin theta sin phi and z is given by r cos theta. Theta is the angle that the point p makes with respect to the z axis. Therefore, x and y lie in a plane perpendicular to that which is the reason why I have r sin theta, and r sin theta in both these places which is essentially the value of rho; cos phi and sin phi gives you further the projections along the x axis, and the y axis; that is what is said. It is a slight generalization over what we did in the case of the cylindrical co-ordinate system.

If you remember that what do, we do well hold theta and phi fix vary r and find out what your corresponding change in the point p is. So, let me write that down theta phi fixed. When you keep theta and phi fixed you either go, you either inverse this sphere or outside this sphere depending on the direction in which you move, and in that case my x component would change by a quantity d r sin theta cos phi, the y component would change by the quantity d r sin theta sin phi, and the z component would change by a quantity d r cos theta.

In the cylindrical co-ordinate system, when we move along the rho direction my z component did not change, because it was not the x, y plane, but when you move radically all the components are going to change, unless you are on some special points. For example, if theta equal to 0, you are along the z direction you move only along the z direction the x and y component should not change, but since we are interested in a general point p, all the 3 components are going to change.

In other words, the displacement of the point p which I shall again write, you will be simply given by dx i hat plus dy j hat plus dz k hat; where dx, dy, dz are written in this 3 expressions. Now, all that we need to do is to take the modulus of this vector displacement, and then divide the displacement vector by that.

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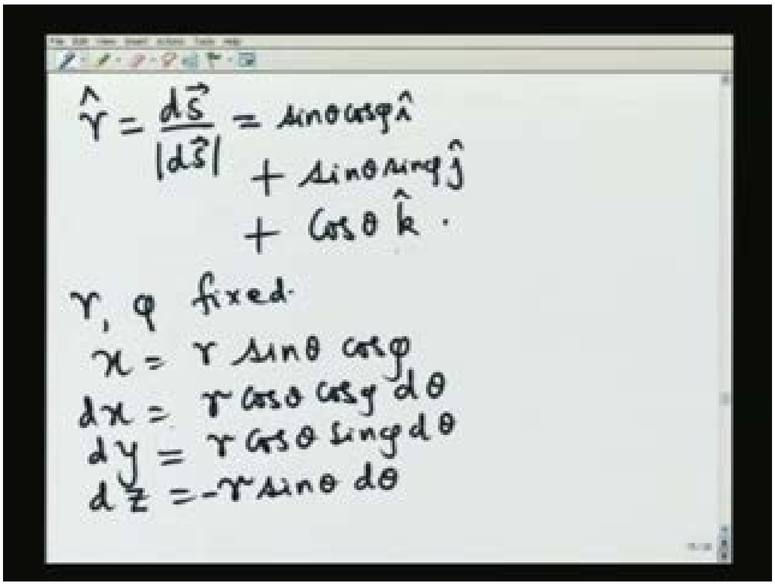

So, in other words my r hat or r cap is simply given by d s by mod d s. well if I substitute, what are we going to get? We are simply going to get that, it is given by sin theta cos phi i hat plus sin theta sin phi j hat plus cos theta k hat. As a special case, imagine that you are sitting along the x axis and you are moving radically; that means, you simply move along the x axis which is indeed correct, because if I put theta equal to phi by 2 and phi equal to 0 sin theta sin phi vanishes, cos theta vanishes, and only sin theta cos phi survives that is equal to 1 or is parallel to the x axis.

If I sit along the y axis and move radically, it is equivalent to moving along the y axis indeed theta equal to phi by 2, phi is equal to phi by 2; therefore, the coefficient of I hat vanishes, coefficient of k hat vanishes, only the coefficient of j hat vanishes and that is equal to 1; and in general, if you were to sit anywhere on the sphere and move radically this is the distance that you would be moving. So, the unit vector is simply given by this, and you should again check for you own sake that modulus of this object is equal to 1; once again, so that you are 100 percent sure that you have not committed any mistake. The conception of theta and phi is also not very difficult, because we have sort of evolved a general procedure all though I did not explicitly mention it.

So, how do I find out what the theta vector is well hold are n phi fixed; and just vary theta. Remember x is given by r sin theta cos phi which tells you that dx will be given by r cos theta cos phi d theta. In a similar manner, dy will be given by r cos theta sin phi d theta whereas, dz will be given by r minus r sin theta d theta; (( )) along the theta direction. So, we found out dx, dy and dz when we varied only theta and held r n by fixed.

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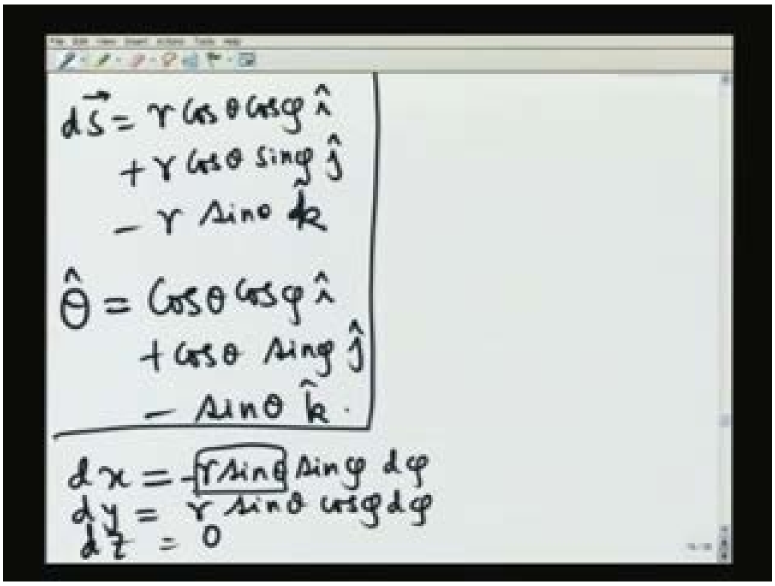

So, the displacement will be given by r cos theta cos phi i plus r cos theta sin phi j minus r sin theta t k that is what we have; please multiply these quantities by d theta which is missing in the expression. Again all that we have to do is to take the modulus of this vector, and then divide d s by this modulus of this vector and get the theta direction.

Well it can always almost be return on inspection, it will turn out to be cos theta cos phi i plus cos theta sin phi j minus sin theta. I mentioned that if you were to sit in the equatorial plane, the theta should be pointing along the minus that direction, because that is how the longitudes come when you start from the north pole; that is substantiated with this expression, because if I put theta equal to phi by 2, the components along the I and j vanish the component only along the k survives and there is a correct minus and sitting here.

Similarly, you can look at what the value of theta should be at various other points on the surface of the sphere, it will agree with it; and all that we are left to do is to consult the unit vector in the direction of phi - the unit vector in the direction of phi is somewhat simpler, because we know that the z component does not depend on phi, z was simply given by r cos theta; therefore all that we need to do is to worry about the changes in the values of the x and y, indeed phi is the various angle that you produce as you move in the x y plane on that circle of radius r sin theta; that is something that we already mentioned. Therefore, what is it going to be my dx for this particular k s will be then given by r minus r actually sin theta sin phi and dy will be given by r sin theta cos phi. I have to put a d phi here and a d phi here.

Because d of sin phi is cos phi d phi and d of cos phi is minus sin phi d phi that is what we have. D z is identically equal to 0. So, the displacement is entirely in the x y plane along the tangential direction, exactly like what happened in the case of this indicial polar co-ordinate system, except that there the radius was denoted by rho, here the radius is denoted by the quantity r sin theta.

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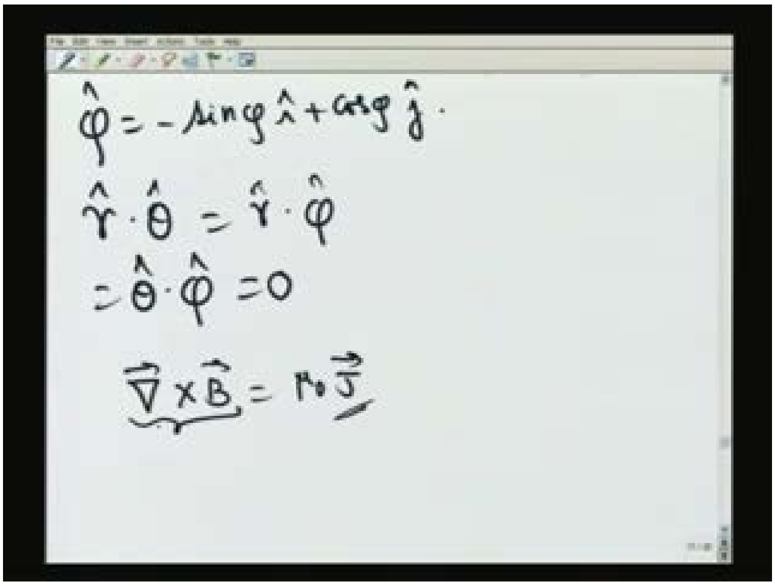

So, if you remembered it, your phi hat will be simply given by minus sin phi i hat plus cos phi j hat. So, without doing any great complicated algebra or manipulation or for the matter in involved geometry (( )), we have succeeded in consulting all the 3 unit vectors. These 3 unit vectors have a complementary dependence as they move on the surface of the sphere of any radius, they keep on changing their orientation, but even if they keep on changing their orientation, they remain orthogonal to each other.

That is something that can be verified experiment explicitly; I am not going to work it out for you, because of I have already given you the expression for all the 3 unit vectors, please go back and verify that r dot theta equal to r dot phi is equal to theta dot phi is equal to 0; that is something that has to be verified. In other words what we have done is to erect locally orthogonal co-ordinate systems, they are not globally orthogonal they keep on changing as they move from point to point that might sound like a disadvantage, but on the other hand the advantage is that, when there such specific symmetry this will be particularly useful.

Now, before I windup the discussion of this co-ordinate systems, because I have discussed practically whatever is required for this course, because this are the only 2 co-ordinate systems that I am going to use throughout, all these 20 lectures or so that I am giving; there is 1 more point that we have to be made made and that is about the sense or they handedness of the co-ordinate system. That is very very important; because if you go back to the Maxwell's equations, we have equations like curl of B is given by mu naught J, let us say I have drop the time dependent term.

Here the right hand side the current density is a polar vector; whereas, the left hand side in was a curl, very soon we will see when I will do the curl operation, if then it acts on a polar vector it is going to give an axial vector when it acts on an axial vector, it is going to give a polar vector. And the order to define what is a polar vector or an axial vector, and to steer clear of troubled waters what we have to do is to define our sense once, and for all.

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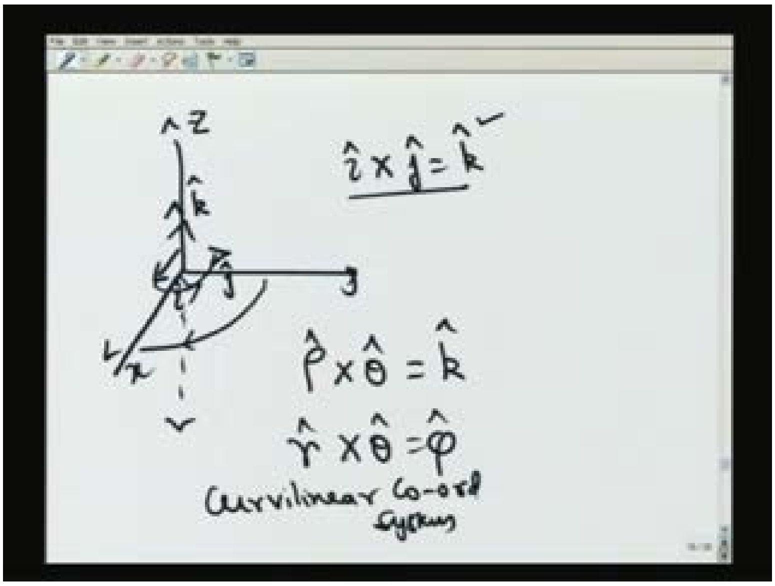

So, let us do that that is something that all of us are familiar with, but let us be very, very explicit about that, and let me draw the co-ordinate system again. I have the x axis, I have the y axis and I have the z axis. Initially I told you that when I draw these x, y, z; they are mutually orthogonal to each other. That is not enough in order to define the sense; what we shall know further stipulate is that this z axis is further characterized by an additional relation. In order to specify the addition relation, let me write the unit vector I here, let me write the unit vector j here, and let me write the unit vector k here.

We now demand that is an additional requirement that we are going to make, that I cross j must be equal to k. In other words a positive displacement along this z axis which I am going to achieve, should come from i cross j; and what is the geometric meaning of the cross product of i cross j. If you have for example, a screw which turns all the way from the x axis to the y axis, let us say anticlockwise direction, the pitch should be positive that is you should move along the positive z direction. On the other hand, if you have a screw which turns from the y axis to the z axis, then the that is actually like unscrewing the whatever your nut or whatever it is, then you move along the negative z direction. So, we have defined the sense.

You would have perfectly legitimately, perfectly legally use the other convention, you could have demanded i cross j equal to minus k that is, I will not put this z axis outside the plane, but it should come outside the plane, but below and not above. But would define what is called as a left handed co-ordinate system. There are actually text books in physics which were return earlier employing the left handed co-ordinate system, but by now it is a universal convention to use the right handed co-ordinate system, and that is what we have and that is what we stick to.

Now, this handedness is not lost when we go from the rectangular cartesian co-ordinate systems to the other co-ordinate systems. As an example, you people should work out and check that in the cylindrical co-ordinate system rho cross theta is indeed given by the k; that is how it is.

In a similar manner, you can go back and check in the spherical polar co-ordinate system that or cross theta is equal to phi. That is although, I am going to keep on changing the orientation of my 3 unit vectors - the basis vectors as I move from point to point, this relation of right handedness will not be lost; by the way in completing this discussion of these co-ordinate systems, I should also mention that whenever the basis vectors; these unit vectors analog i, j, k; they remain global. They do not change their orientation from point to point, that would be what is called as a cartesian co-ordinate system; whereas, the minute these basis vectors acquire a dependence on the point itself, those co-ordinate systems are called curvilinear co-ordinate systems. No, surprise about that, because when we moved on the surface of a sphere, we are actually moving either along the latitudes or along the longitudes.

When we erected the cylindrical co-ordinate system, we were actually moving around the circle; we were not moving along the straight line; whereas, in a cartesian co-ordinate system which is not curvilinear, what is it that we do you find out whatever displacement to work by 3 linear displacements, and not curvilinear displacements. So, let us remember that a curvilinear co-ordinate system is one, where the unit vectors have their orientation which changes from point to point.

Of course, we could have a curvilinear co-ordinate system which is more complicated then what we have, the 3 co-ordinate, the 3 unit vectors need not be orthogonal to each other; that would be a very, very general co-ordinate system. For that matter, we could have such a thing, even in a plane for example, this would be an example of what is called as an oblique co-ordinate system, but we shall not deal with curvilinear co-ordinate system which are not orthogonal nor shall we deal with oblique co-ordinate system, we should deal only with orthogonal either rectangular cartesian co-ordinate system or the curvilinear co-ordinate systems. In fact, we are going to restrict ourselves either to cylindrical (( )) spherical in most of the cases fine.

After having said that, we are going to look at these co-ordinate systems this is not the end of the tale for us. Because it might so happen, that there is a certain phenomenon; you are interested in finding out the potential, because of the charge distribution. Let us say…

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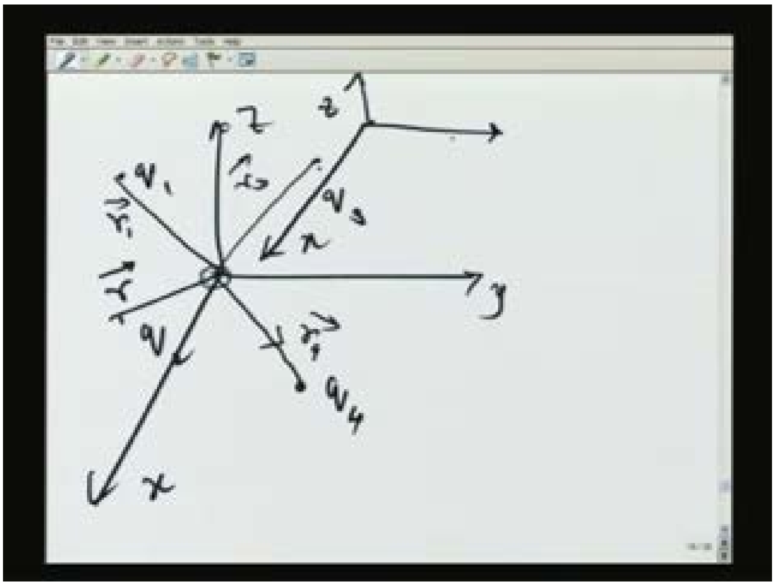

Let us say that there is a charge located when 1 located here, there is a charge q 2 located here, there is a charge q 3 located here, there is a charge q 4 located here. Let us say that, they are all distributed on this plane there is no complication about that, and each of them is a physical point, but on the other hand if I want to give it a mathematical description I have to give their positions.

So, even if I were to restrict myself only to a rectangle cartesian co-ordinate system, I will have to erect 1 system co-ordinate system. So, I shall call it x, I shall call it z, this will be the y, this will be the x. Now, with respect to this co-ordinate system, I will have for example, a position vector which I shall call r 4, because I am going to measure the location of each of these charges, the charge q 3 is at a describing a position vector r 3, the location of charge q 1 is simply given by r 1, and the location of some q, some very arbitrary generate point will be simply denoted by r.

Now, your question arises; whereas, whether there is any preferred choice in having this point was the origin. The answer is; obviously no, there is no great reason. Unless of course, there is a special symmetry concentration, but right now we are not interested in the symmetry concentration; it is perfectly possible, that you could have chosen a co-ordinate system which starts at this point; the z axis would be in this direction the x axis would be in this direction, and the y axis would be in this direction; z, x and y.

Now, not only that there is no reason, why the new co-ordinate system should in fact be parallel displace with respect to the original co-ordinate system. Let me display that in the next figure.

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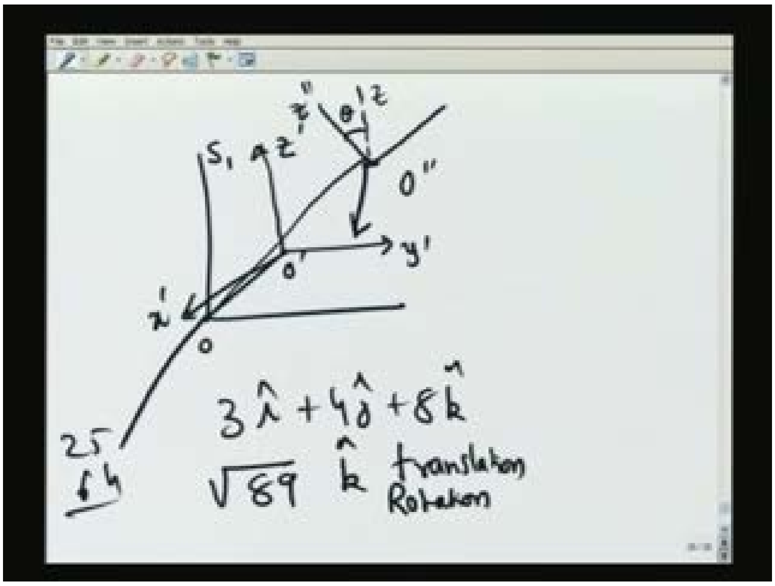

So, let us say I have a co-ordinate system, we shall denote by S 1. What I shall do is I will not rotate my co-ordinate system at all, but I shall move the origin and all the 3 coordinates parallely, so this is o let me call this as an o prime, and now I will erect another co-ordinate system let us say. So, this is my x prime, this is my y prime, this is my z prime. On the other hand, what I can do is I can move it to another point, let us say somewhere here; this is my o double prime and what shall I do. Here I shall erect the co-ordinate system which is still rectangular Cartesian, but then what do I do, I move from this point o to o double prime and I do not simply keep it there rigidly, I rotate it by a certain angle theta. Therefore, if my original z axis were here, my z double prime is here; it makes an angle theta with respected to the z axis. So, what am I saying - what I am saying is that, if you give me one rectangular cartesian co-ordinate system, I shall produce an infinite number of them.

How do I produce an infinite number of them, by translations and by rotations; and that is something that we shall do continuously, because we need to have a meaningful dictionary with respect to various observers who are going to do various observations. Somebody says the force had a component for example, 3 i plus 4 j plus 8 k force on a charged particle, because of this charge distribution; which I wrote earlier. Some other observer tells me no no this is wrong, the force had a magnitude which was simply given by let us see 3 squared is 9 plus 16 25 plus yeah yeah 25 plus 64 is 101, let us see no 25 plus 64 is 89; root 89 let us say along the k direction; suppose somebody said that.

Now, there is absolutely no contradiction between these two; although they might appear to be giving two independent two results which do not agree with each other, because they might have actually chosen two different co-ordinate systems. We do not want a quarrel between two observers, who have chosen two different co-ordinate systems - either in terms of the location of the origin or in terms of the orientation - the relative orientation. Therefore, if you know how to go from one co-ordinate system to another co-ordinate system, either under the shift of the origin that is called translation or by rotation, then we will be able to compare results perform in various co-ordinate systems, and we shall take that up in the next lecture.