Introductory Mathematical Methods for Biologists

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Lecture - 15
L'Hopital's Rule and Partial Derivatives

Hi, welcome to this lecture. In this lecture we would discuss some 2 topics associated with calculus which are important to understand and they will be useful in some particular context. So, first I will tell you the title and then we would explain what they are. So, there are 2 topics, which you would quickly discuss one is call L'Hopital;s rule.

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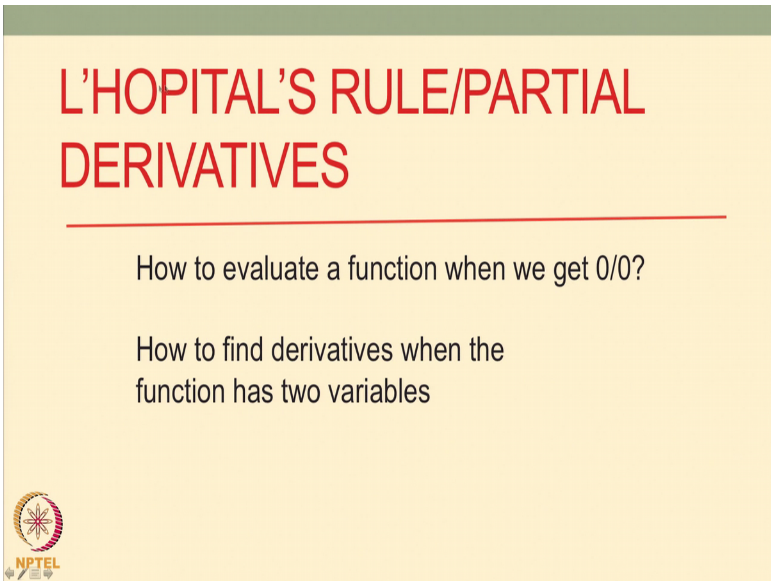

So, it is also called L Hospital's rule should be read as L'Hospital's rule and second part is partial derivatives. These are 2 different topics in this lecture we will answer 2 questions; first using the L'Hospital's rule like we would answer, how to evaluate a function when we get things like 0 by 0 and the second question is how to find derivatives when the function has 2 variables, these are the 2 things that we would discuss in this lecture.

So, first let us think of this rule call L'Hospital's rule.

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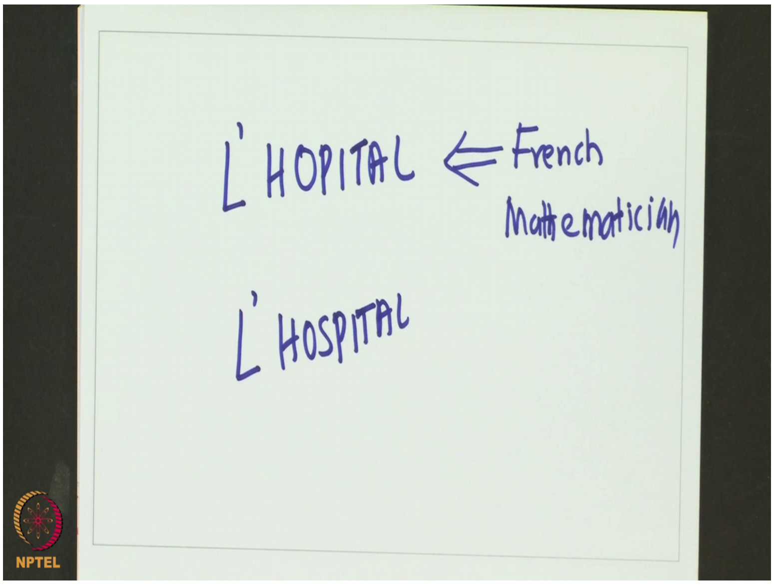

So, this first of all about the name a little bit I wanted to say, this is a person's name is called Ghim L'Hospital is the person's name, it is a French name he is a French mathematician. So, a French mathematician called Ghim L'Hospital found a rule often in some books is written as L'Hospital. I think it is because the law L'Hospital would mean the hospital. So, this rule by this mathematician name L'Hospital is very useful in mathematics and that is something which is used when we have to find some if we evaluate some function it is a particular point, but let us consider the function first let us consider the function sin x by x.

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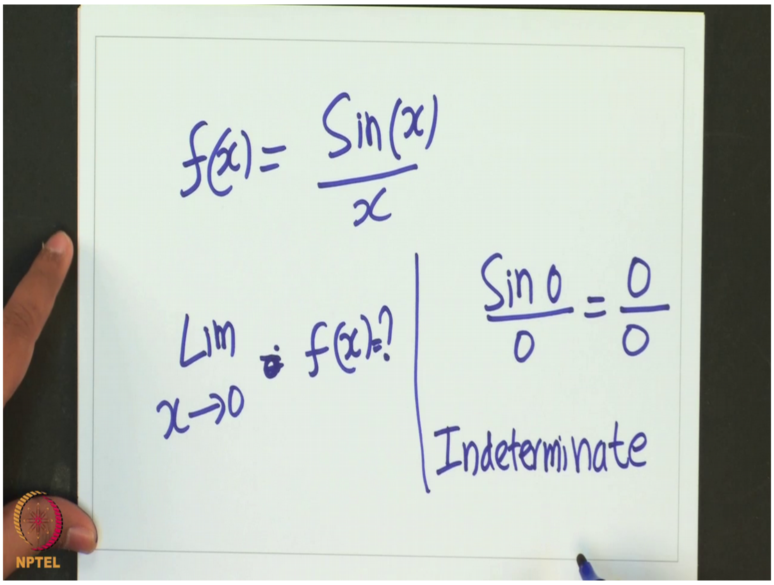

So, let us think of this function f of x is sin x divided by x.

Now, if you want to plot this function for example, you would want to evaluate this function at different points. So, what is the value of the function when x is 0. This is something that you would want to think about it. So, what is when the function goes to as x goes to 0, what is f of x? This is something that you would want to know. So, in this case if I put sin 0 by 0 sin 0 0 this is the x is 0. So, you would get 0 by 0 and 0 by 0 is indeterminate we cannot determine what is 0 by 0 is therefore, because 1 by 0 is infinity. So, infinity times 0 we do not know what their answer is.

So, since this is indeterminate. So, whenever you have such situations, sin x by x and you want to evaluate this at x equal to 0, you would get sin 0 by 0 which is 0 by 0, which we cannot determine.

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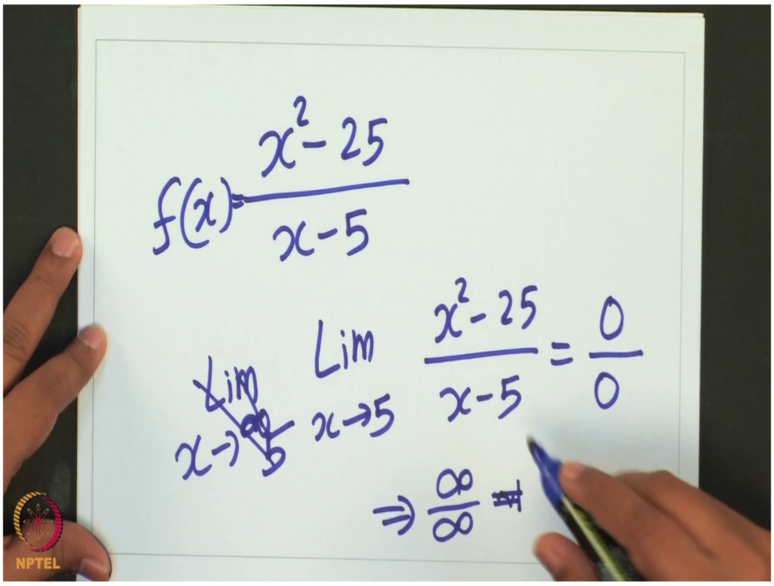

There will be some other examples like many other examples you could think of many other function, like a simple function you could think of for example, x square minus 25 divided by x minus 5. This is some function that you have f of x, when x is in the limit x is near equate to 5 if the limit x is going to 5 in the limit, x going towards 5 x square minus 25 divided by x minus 25, x minus 5 would be.

So, x square is 25, 25 minus 25 0 x minus 5 is also 0. So, you would get 0 by 0. So, whenever we have such 0 by 0 this is indeterminate. Sometime you would also get infinity by infinity this is also indeterminate, but of course, one by infinite get right 0 as. So, this is essentially. So, this is also like indeterminate.

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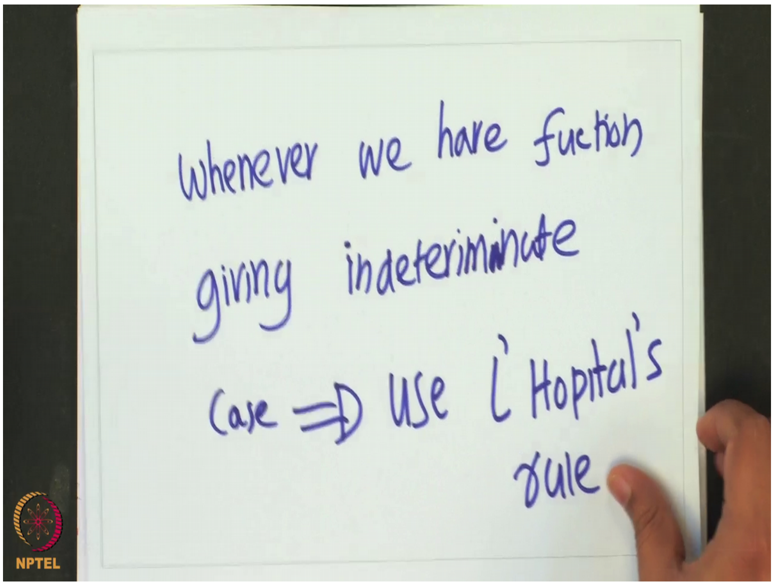

So, whenever you have such indeterminate quantities, we have to use a rule prescribed by this French mathematician called Ghim L'Hospital. So, whenever we have function giving indeterminate and the function was indeterminate in situation right indeterminate case use a rule which is called the L'Hospital rule and what is the rule I am going to describe what is the rule.

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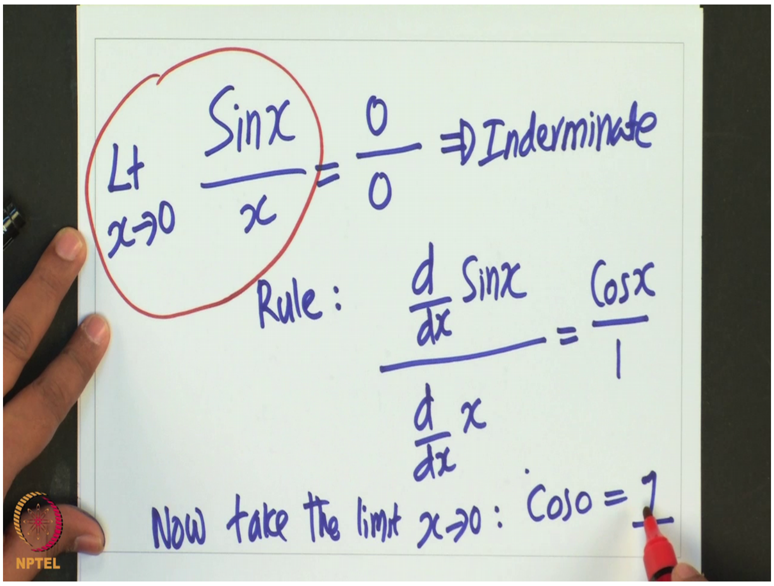

So, rule let us take the example of sin x by x. So, what we want? We want to calculate sin x by x when x is 0. So, since this is sin 0 is by 0 is 0 by 0 sin 0 is 0, x 0 this is indeterminate. So, what we want? The rule says this is the rule take the derivative of numerator and denominator.

So, you calculate d by dx of sin x and d by dx of x and then take the limit and then take the limit. So, this is nothing, but cos x divided by 1 and then now take the limit. Now take the limit x going to 0. So, when cos x this, this is cos x. So, when x going to 0 it is cos 0 and this is 1. So, the answer of this sin x by x in the x going to 0 the answer is 1. So, this sin x by x in the limit x going to 0 will give us this answer which is 1, because this is the way to do is find the derivative of the numerator find the derivative of the denominator and then take the limit here.

So what did we do? So, we did basically limit.

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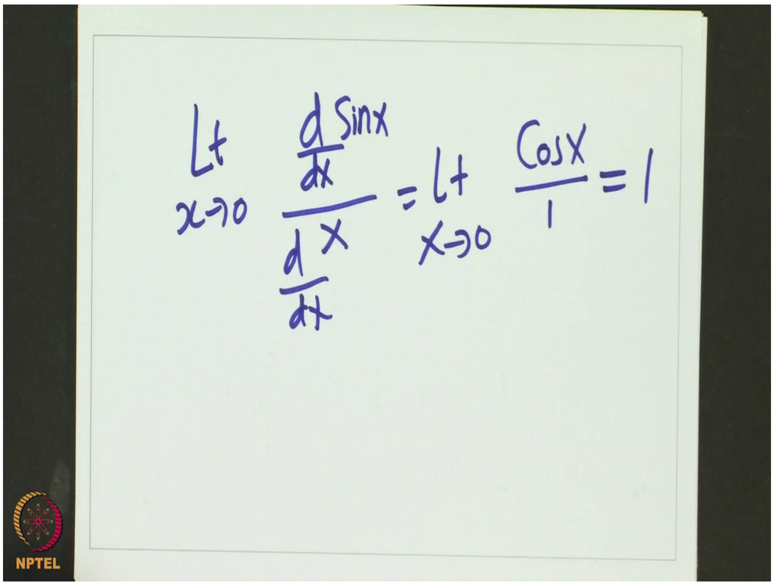

So, we just took the limit x going to 0, d by dx of sin x by d by dx of x, which is limit x going to 0 cos x by 1, which gave us one this what we did . So, the rule the L'Hospital rule says if so, L'Hospital rule what is it say.

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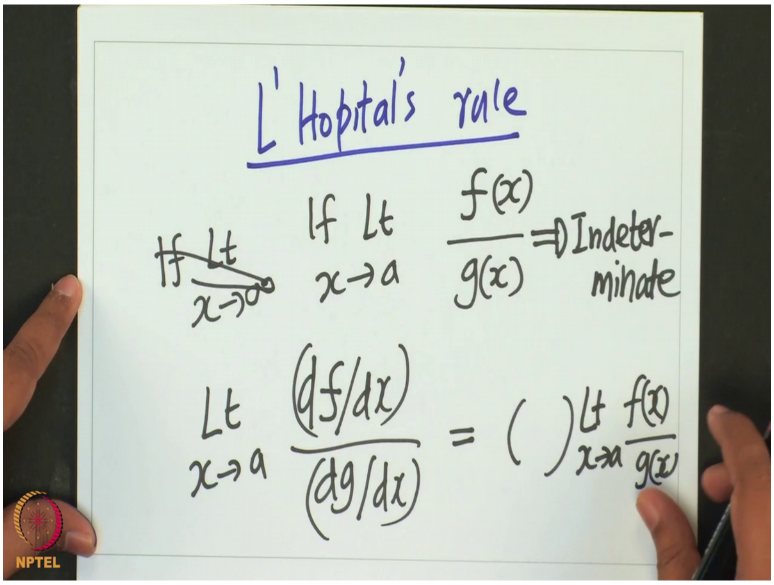

So, let me write down this would essentially say, that if you have a function, if the limit x going to some a x going to some a. So, if limit x going to a, you have f of x by g of x if this is indeterminate, then you take limit x going to a df by dx and dg by dx. So, you calculate the derivative of the numerator and derivative of the denominator separately, and then take the limit and this would be the answer whatever we get would be the answer for this. This would be this would be equal to this would be equal to limit x going to a, f of x by g of x. This this would be the answer to this question that is what this rule would specifically say. So, now, let us take the other example that we just discussed.

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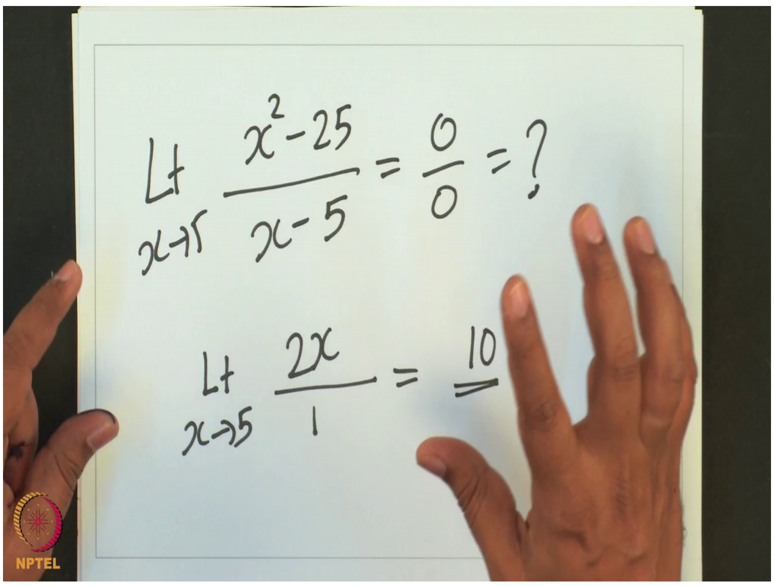

If we have x square minus 25 divided by x minus 5, and you want the limit x going to 5 and this is going to be 0 by 0. So, we cannot determine. So, if I take the derivative of this derivative of the numerator. So, derivative of x square is 2 x and derivative of 25 is 0, and derivative of denominator is 1. The derivative of denominator is 1 and you have to apply the limit x going to 5 here this would give you 10. So, the answer to this is going to be 10 as x going to 5 this value will go to 10 this function would be how a value 10.

The rough idea is that how does the slope vary how fast this goes slope derivative would mean slope and how does the slope would help us to determine where it reach is what is being used the idea is being used in this, which I would urge you to read about and think about how this can be done, how what is the what is going on while we while this is the rules implemented, while why this is rule is useful and appropriate. but in this for the purpose of course, enough to do that whenever we would use some function, we might get 0 by 0 or infinity by infinity immediately take the derivative of numerator denominator and apply the limit you would get their function.

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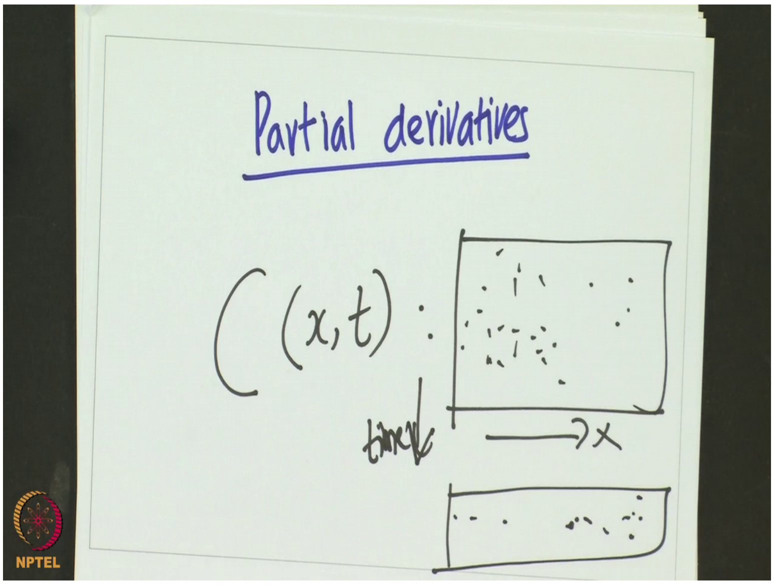

So, it is the all that is what is wanted in that part. Now we would discuss the second part, this part is called partial derivatives concentration c which is a function of x and t. X is space and t is time both the c varies. So, at it would vary if we have a cell.

The concentration would vary of course, in space, but it will also vary in time like this is t equal to 0, in the next minute or next hour this concentration would be more here and less here. So, the concentration could change in time and along the space. So, whenever you have some function, let it be concentration or anything you want, which is some function of 2 things x and t.

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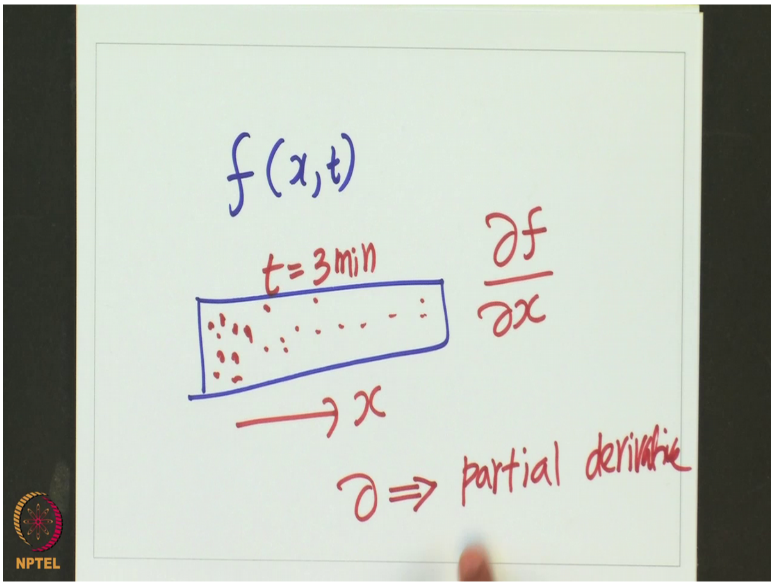

Very often you would want to take the derivative only with respect to 1 of it or you want to keep time constant and desired how is it changing in space. So, that is think of you have a cell or you have a tube, in which you have concentration you want to know at t equal to 3 minute at the third minute.

How is the concentration varying along the x that is all I want to know. I keep the time constant I want to know how is the concentration varying along x. This quantity this thing to write mathematically is not if I write df by dx it will be wrong, the way to write this is del f by del x. So, this symbol like this is called the partial derivative what does that mean? It means exactly what we just said I keep the time constant at t equal to 3 minutes I want interested to know how is this function, how is concentration varying along the x if the function is concentration here, how is this concentration function varying with respect to x keeping the time constant at 3 minute I just wanted to know this.

So, then I would calculate del f by del x. So, let us think of an example. So, let us say your example is. So, let us take let us imagine they take an example.

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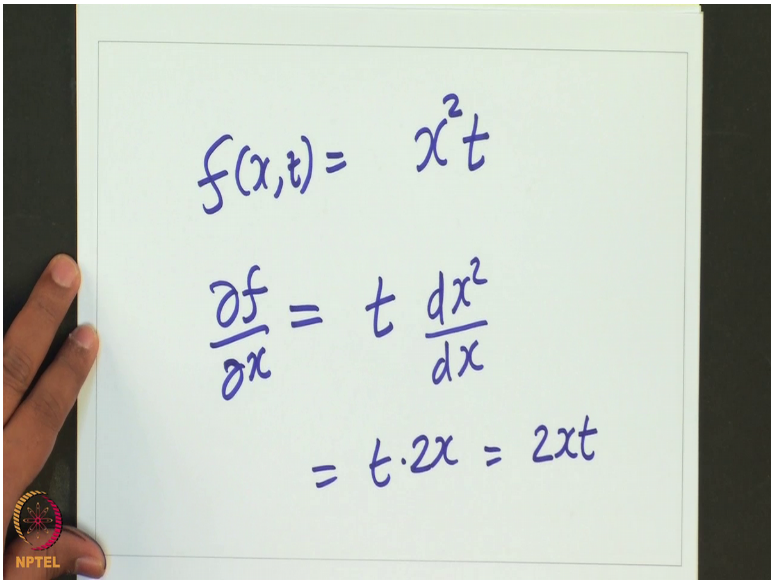

So, let us take f of x comma t this function will be concentration or anything. So, let us call this function as x square times t. So, this is my function now I want to know only at a particular time how is this varying in space. So, I will then do del f by del x, I would do partial derivative what does this mean? I keep t as a I imagine that t is a constant, then I find the derivative of x square alone, this is what this means. So, this would mean keep t constant and find the derivative x square alone. So, that is t times 2 x. So, this is 2 x t. So, this is what is happening. So, at I fix the time this time could be 3 minutes or whatever the time, and I find the derivative of x the x part only with respect to x alone and this is called del f by del x. Same thing I could find del f by del t.

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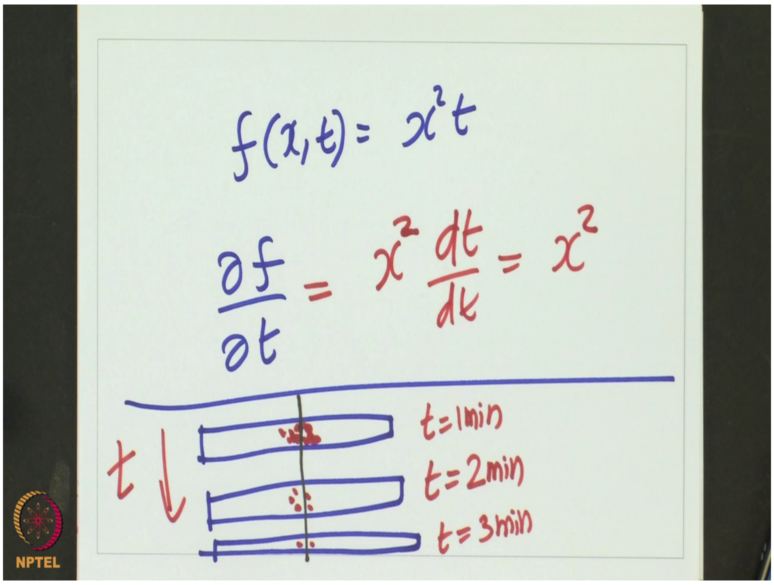

So, let us take the same example I have f of x comma t, which is x square times t and I want to calculate del f by del t. What is that equal into the in the case of concentration what is it mean; let us think of what is it physically mean.

I have a cell at different times, I am interested what is the concentration at the centre of the cell that is all I am interested. So, I will look at only one x location and I look at the concentration here was very large at t is equal to 1 minute, at t equal one minute the concentration was large, then it little bit reduced and then it is further reduced. So, the concentration was increasing or decreasing as the time goes I do not know. So, this is equal 3 equal 3 minute and so on and so forth. It could increase or decrease depending on it, the function. So, the point here is that I look at only at one particular position, but at different times.

So, this is time. So, at different times I look at just one location and measures the concentration there and if the I am interested how does this change with time. So, the how does the concentration change at x as the function of time, then I would do derivatives like this. In this case if I have a function x square t, if I do del f, I did to the partial derivative with respect to t, I would assume x square is a constant and then I would take the rest of it and find the derivatives with respect to t which is 1. So, the answer is just x square. So, this is very useful because we have things changing in x and t some time you would have things changing along x and y.

So, you would have like if I take a cell very often, you would have things varying both along x and y.

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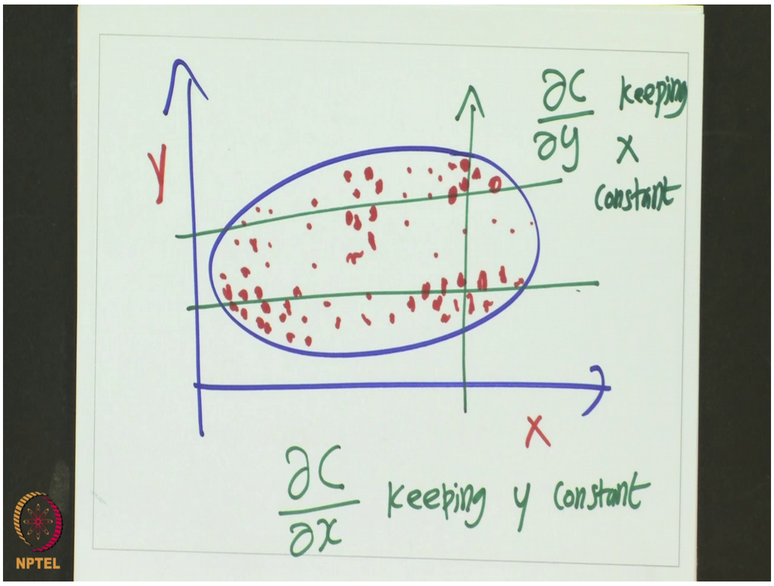

So, if I have a cell. So, this is my x y axis, and if I look at in the cell the concentration would be very small here very high here and small if I go. So, very high here and it could reduce and then further again increase and also if when I go x I could reduce and then increase here, and then again reduce the little small concentration again high concentration here, and then small concentration here.

So, that is the concentration changes with x and y. Very often we would want to just take one particular value of x and a right. So, just to I is or y. So, I just want to take this particular value of y and I want to see how the way concentration changes. It first decreases then increases. So, I want to know how does this concentration C changes with respect to x keeping the y constant. So, I want to do del c by del x keeping y constant. So, I take a y value and I look at how does the concentration change along x. I can take another y value then I look at how does the concentration change as I go along the x axis. So, this is represented by del C by del x. Sometime you would want to have the opposite where you would want to take some particular x value.

So, at this particular x value, you would want to see how does the concentration increase as I go along the y. So, here I would come to calculate del C by del y. As I increase y how does the concentration change for a particular x value. So, this x value I go along this x. So, x is same I am not varying x, but I would vary y and I just wanted to calculate how does del c by del y, keeping x constant. So, this partial derivative this kind of idea of partial derivative is going to be very useful both to understand things varying spatially x y z and things varying temporally which is x y z and in general x y z and t.

So, there would be many example, concentration is the most studied most useful example in biology, where concentration of proteins vary across the cell. Any heterogeneous things like in space and time any event around us would vary both in space and time. So, that is always useful to describe any real life phenomena, anything that we see around us would be varying in space as well as in time. So, therefore, this partial derivative is going to be very useful. Another small thing which is something that we learnt, but I want to clarify something which is slightly separate which is derivative.

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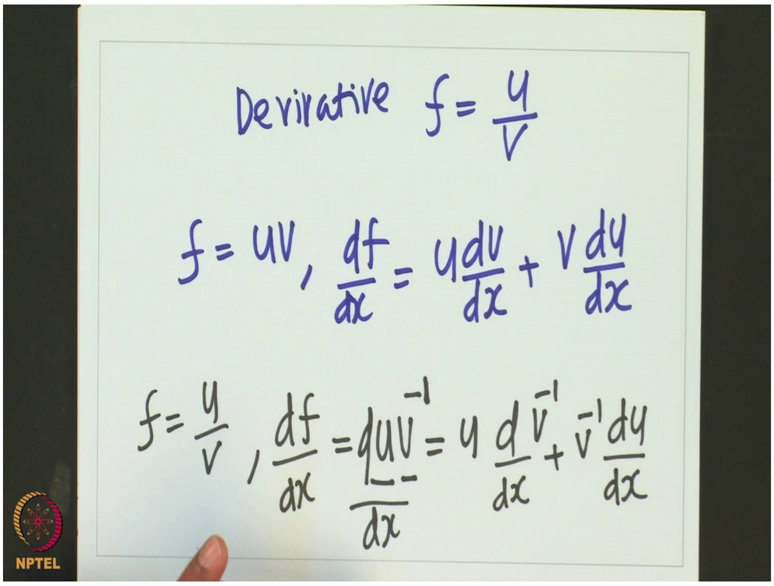

This is the third thing, which is derivative if you have a function which is like u by v. So, we learnt if we have a function u times v and we want to calculate df by dx, we said first you keep the u constant and calculate dv by dx. Then you keep the v constant and calculate du by dx. So, this is something that we discussed. Sometime you would have this function which is like a ratio of 2 functions. So, you have in this case as we said if f is u by v then if you want to calculate df by dx how will we do that? The idea here is again I do not have to learn if you know this you can use the same thing because u by v, I can write as u v power minus 1. So, then this became product of 2 terms, and if you have a product of 2 terms I first find I first keep the u constant.

So, this is basically d by dx of u v inverse which means I keep u constant and find the derivative of V inverse, plus I keep the V inverse constant and find the derivative of u. So, this would give us exactly the same idea. So, whenever you have derivative of a ratio of 2 functions this is the idea to use write it as u times V inverse and do this case.

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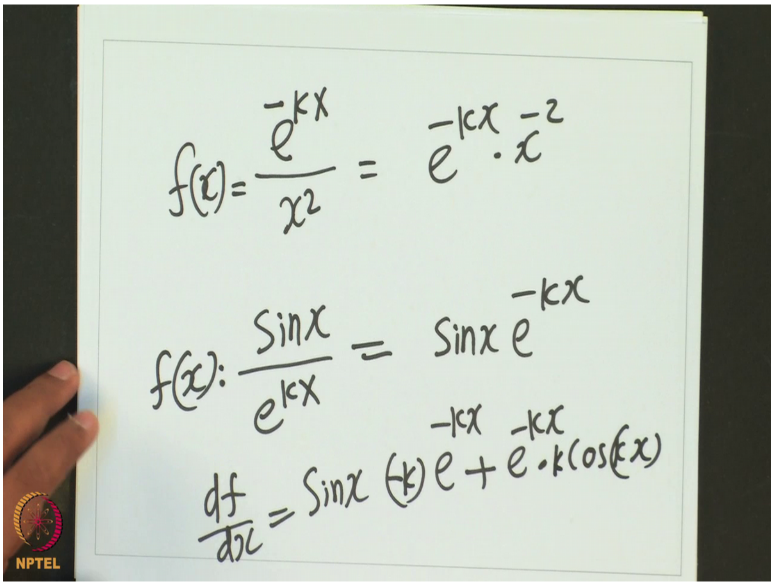

For example let us take again let us take e power k x divided by some function. So, let us write e power minus k x divided by some other function. So, let us say x square.

I am just writing some function if this is my f of x, I would write this as e power minus k x times x power minus 2. If I have sin x divided by e power k x this if this is my f of x, I would write this as sin x e power minus k x, and then I will find the derivative of this. So, if I want to find the derivative of this df by dx I would keep sin x constant and find the derivative of e power minus k x which is the minus k e power minus k x, then I would plus I would take e power minus k x constant and find the derivative of this which is cos k x times k.

So, I would do this kind of thing and solve this. So, 3 things we learned these are 3 miscellaneous things; first thing we learned is L'Hospital's rule. So, let us summarize what we 3 things we learned.

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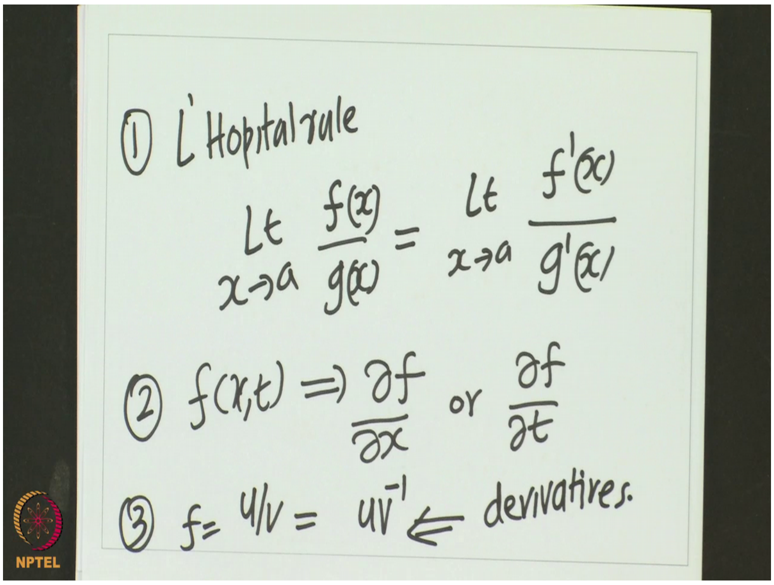

So, one we learnt the L'Hospital rule, which said that in the limit if f of x by g of x is indeterminate, this is equal to limit x going to a f prime of x this prime is derivative and g prime of x. So, you take 2 derivatives, and then you apply the limit this would be equal whenever there is intermediate these are 0 by 0 or infinity by infinity, that is the first thing we said that.

Second thing that if you have a of some function x comma t you can calculate del f by del x or del f by del t these are called partial derivatives, and the third thing is said that if you have f is equal to u by v, then you can write it as u v inverse and then calculate the derivatives of this u inverse v. So, these are the 3 things I want to save which is very useful in the lessons ahead and we would use these ideas very often. With this I would stop this lecture and continue in the next class.

Bye.