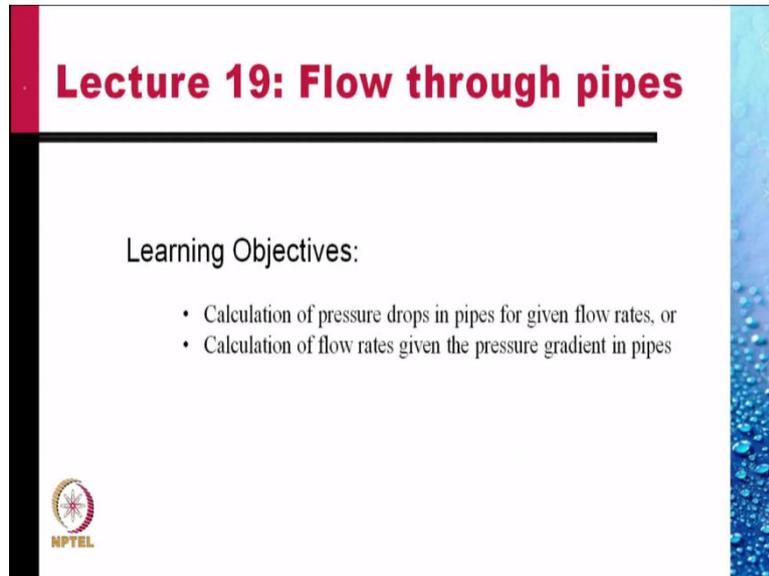**Fluid Mechanics and its Applications**
Professor Vijay Gupta
Sharda University
Indian Institute of Technology, Delhi
Lecture 19
**Flow Through Pipes**

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Welcome back.

Today we will discuss flow through pipes. Flow through pipes is one of the most common applications in industry. Almost all process industries, that include chemical industries, pharmaceutical industries, involve some reactors in which fluids are moving in and moving out. The fluids are supplied through pipes.

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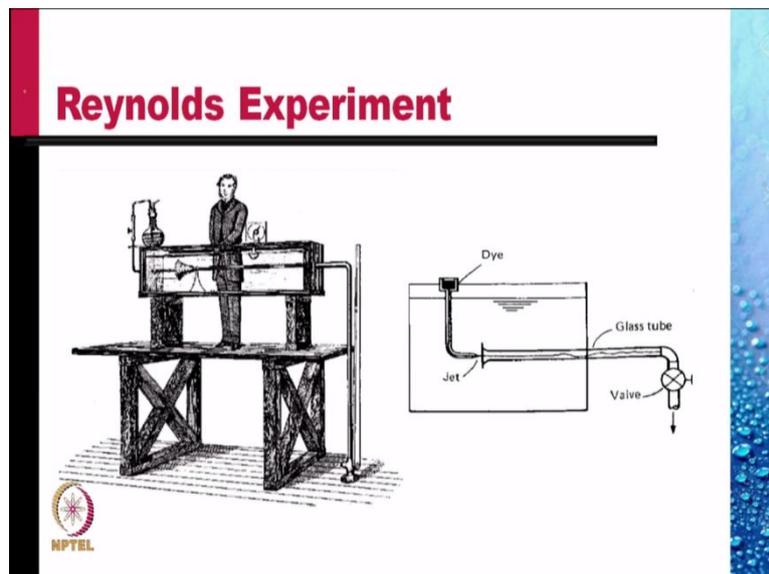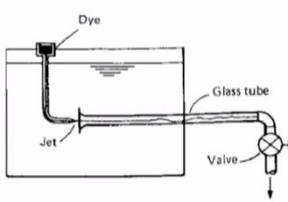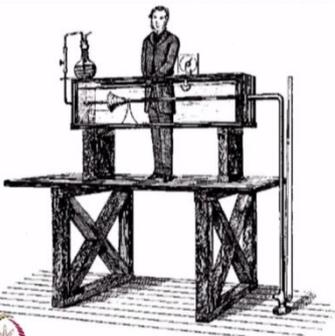
So, we need to worry about the pumping power required. The first reported experiments connected with the flow of pipes are due to Reynolds that we have discussed earlier. Reynolds, a British scientist conducted experiments on a glass tube submerged in water through which the fluid, that is water, flowed through and the rate of flow was controlled by a valve. To study the behaviour of the fluid Reynolds injected a dye at the centre of the tube. The dye came on as a streak and he studied how that streak behaved as the time passed on.

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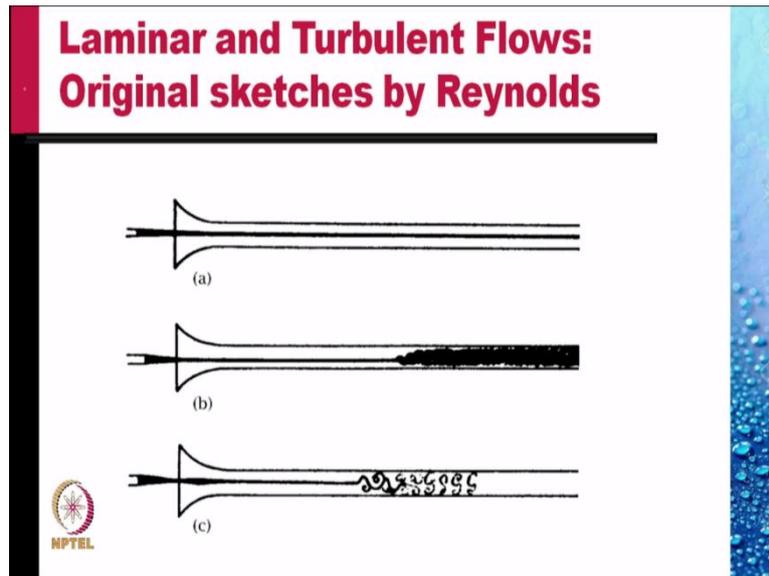
These are the sketches from the first paper of Reynolds. He found out that when the speed of flow was low, the streak appeared as a smooth filament and went down the tube. As the flow speed increased, the streak started diffusing outwards, and for larger speeds, the streak became turbulent. So, he concluded that low speeds the flow through a pipe is laminar, and is turbulent at high speeds.

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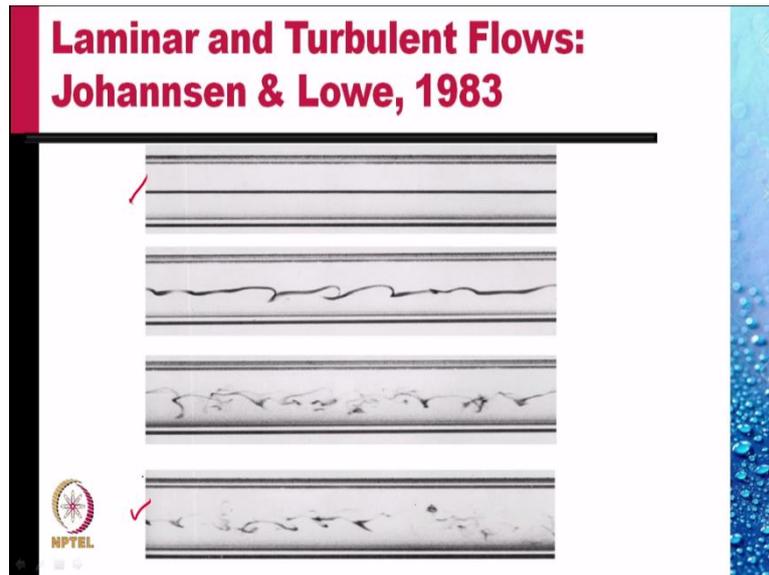
These are some recent pictures of the flow through the same equipment. Here this first picture is at a low speed. Then as the speed increased, the streak of dye started changing character, and by this speed, there is there was no longer a smooth streak, but largely dispersed dye.

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This transition from laminar to turbulent is interesting. While laminar flows are dominated by viscous forces, turbulent flows are dominated by inertial forces associated with large-scale mixing of the fluid. It stands to reason that in geometrically similar situations, the transition from laminar to turbulent flow should occur at a fixed ratio of inertial and viscous forces. Since the Reynolds number measures this ratio, the transition should be marked by a fixed value of the Reynolds number, which is defined as $\rho V_{average} D / \mu$ of the liquid.
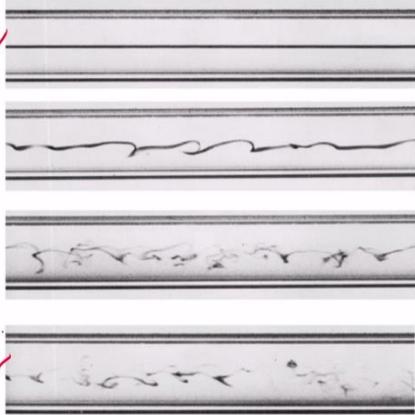
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The value of Reynolds number at which the transition from laminar to turbulent flow takes place is called the critical Reynolds number, Re cr. When the Reynolds number is below 2300, the flow is always laminar, and any disturbance in the flow is quickly damped out. However, when Reynolds number increases, the flow may or may not be turbulent, depending on the conditions at the inlet and the levels of vibration in the experimental setup.

Johannsen and Lowe did the experiment in 1983 on the same equipment as Reynolds used in his laboratory at the University of Manchester. Reynolds had reported that the transition took place at Reynolds number as high as 100,000, but in the later experiment, a century later, they could never take the transition beyond about 8,000. That was largely because the roads near the laboratory in University of Manchester are now busy roads with lots of traffic. The traffic

causes disturbances in the building, and because of the vibrational environment around the laboratory, the transition now takes place earlier.

If great care is taken in making the pipe walls very smooth, and in suppressing the turbulence in the incoming fluid, and in isolating the experimental setup from external vibrations, the transition can be delayed to as high a Reynolds number as 100,000. But for most engineering applications, a value of 2300 represents a good choice for the critical Reynolds number.

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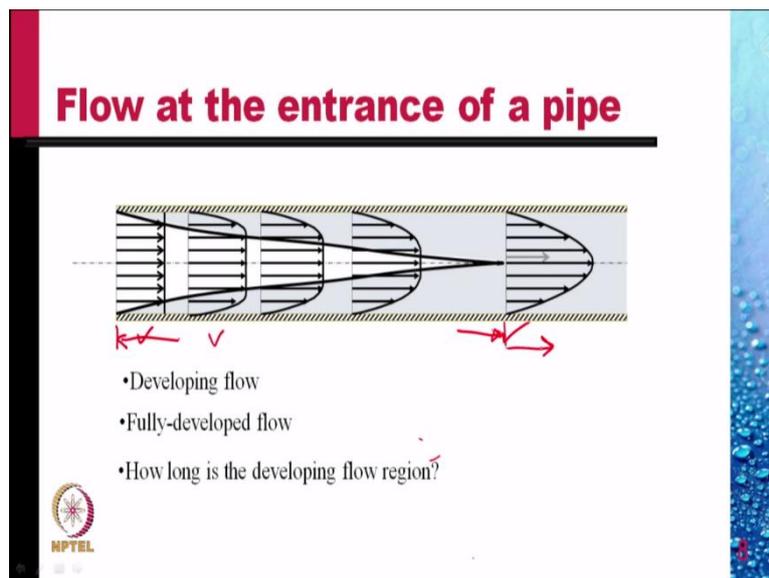
We had earlier discussed the flow at the entrance of a pipe. When the flow enters a pipe with a uniform velocity across a section, soon the viscous action at the wall takes over and slows the fluid down at the wall. So the flat velocity profile at the inlet changes into velocity profile where the velocity is 0 at the wall, by the no slip condition, but the velocity is uniform in the centre.

As the flow proceeds down the tube, more and more of the flow area is affected by viscosity till at this location, all of the area is now affected by viscosity. Beyond that, the profile does not change. This is the region over which we say that the flow is developing, and beyond this is what is termed as the fully-developed flow. How long is the developing flow region? Again, this region should depend upon the inertial and the viscous forces. So, this should depend only on the Reynolds number.

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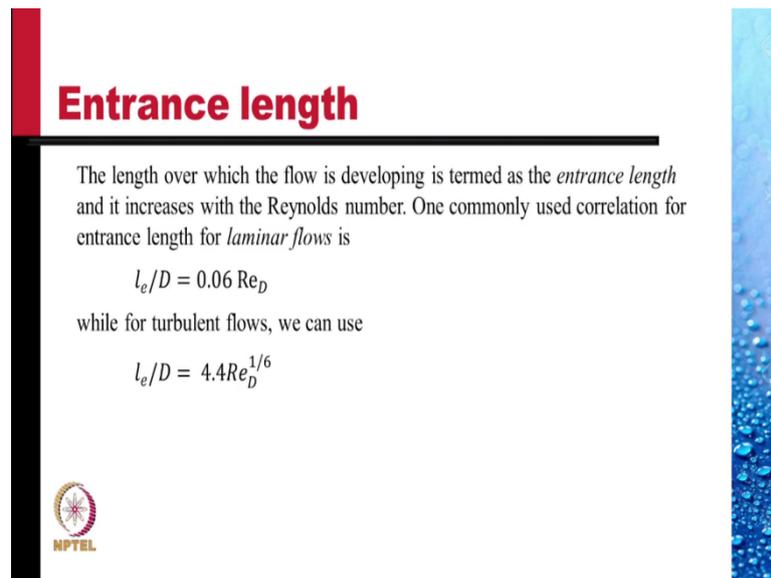
The length over which the flow is developing is termed as the entrance length, and it increases with the Reynolds number. One common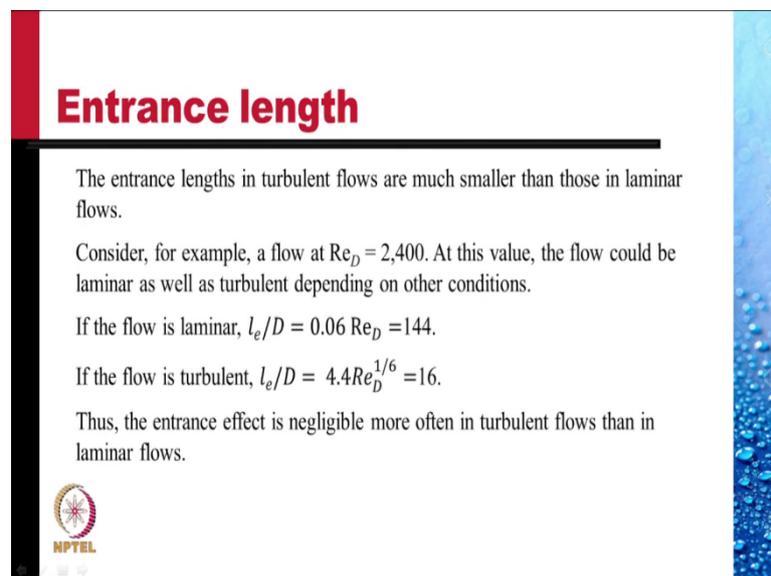ly used correlation for entrance length for laminar flows is $l_e/D = 0.06$ Re_D. While in the turbulent flows, this correlation is $l_e/D = 4.4Re_D^{1/6}$.

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The entrance lengths in turbulent flows is much smaller than those in laminar flows. Consider for example, a flow at the Reynolds number 2400. In this value, the flow could be laminar as well as turbulent, depending upon other conditions. If the flow is laminar, the Reynolds number is 2400. So, the entrance length is 0.026 times the Reynolds number, which gives us 144 diameters.

But if the flow is turbulent, this entrance length is only 16 diameters. Thus, the entrance effect is negligible more often in turbulent flows, then in laminar flows. But as you remember, most commercial flows are turbulent, so the entrance effect can be neglected in many situations.

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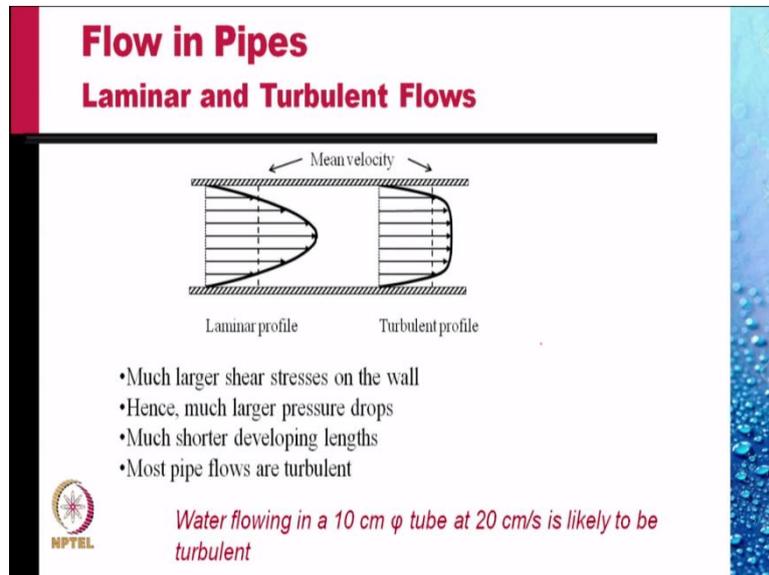
This illustrates the difference in the profile through a laminar flow and in a turbulent flow. The flow profile in laminar flows is parabolic with mean velocity one-half of the maximum velocity, while the velocity profile in turbulent flows is quite flat at the centre. It is much sharper at the walls. And because of this, there are larger shear stresses on the wall than in the case of laminar flows. And because of this, there are much larger pressure drops, much shorter developing lengths as we discussed, and then we know the most flows are turbulent. Water flowing in a 10 cm diameter tube at a speed 20 cm/s is likely to be turbulent.

A few lectures ago, we obtained the laminar velocity profile for fully developed flows in a pipe. We considered an annular control volume. In this element, we wrote the forces, the pressure forces and the shear forces. Because of full development, these two forces should be balancing. There should be no net force, and the force balance gave us this relation for w, the velocity component in the z direction.

We showed that p' is constant, that is, there is a linear pressure gradient down the pipe. From this we obtained the velocity profile. And the velocity profile is parabolic in R, capital R is the radius of the tube. This velocity profile. And on integrating, this we found out the flow, and then from the flow we found out the average velocity, which was $-\dfrac{p'R^2}{8\mu}$.

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The volume flow rate was determined as $\frac{\pi R^4}{8\mu}(-p')$. This is known as Hagen Poiseuille law and depends upon two conditions: the flow is laminar, and it is fully developed. That is the length of the pipe is much larger than the entrance length, so that the entrance length the end effect can be neglected.

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This is for laminar flows but laminar flows are few. So, we need pressure-drop relations for turbulent flows. Since, the analytical solutions for this problem is not available, we resort to experimentally-developed correlations for the purpose. Since the pressure drop depends on several parameters, namely, the density and viscosity among the fluid properties, and the length, diameter, and the roughness of the pipe wall, we need to develop pi numbers to reduce

the number of independent parameters in the problem using the techniques that we learned in last few lectures.

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So, what is our problem now? Given a pipe of length L and diameter D, find out the total pressure drop in the pipe when the fluid of density ρ and viscosity μ flows through this at a given volume flow rate.

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There are two forces that are important, the pressure forces and the viscous forces, and in addition, we write the inertial forces. So, we developed the scale factor of all of these three types of forces, and equate them, since there should be only one force scale factor, and from this, we obtain two pi numbers. We have $k_{F,i}$ for the inertial force which should be $k_M \times k_a$,

where m is the mass and a is acceleration. Mass is like ρ times volume, so, $k_M = k_\rho k_L{}^3$, and k_a, as we have done a number of times earlier is $k_V{}^2/k_L$. So, this gives us $k_{F,i} = k_\rho k_V{}^2 k_L{}^2$, and from this we get a pi number, $\frac{F_i}{\rho V^2 L^2}$.

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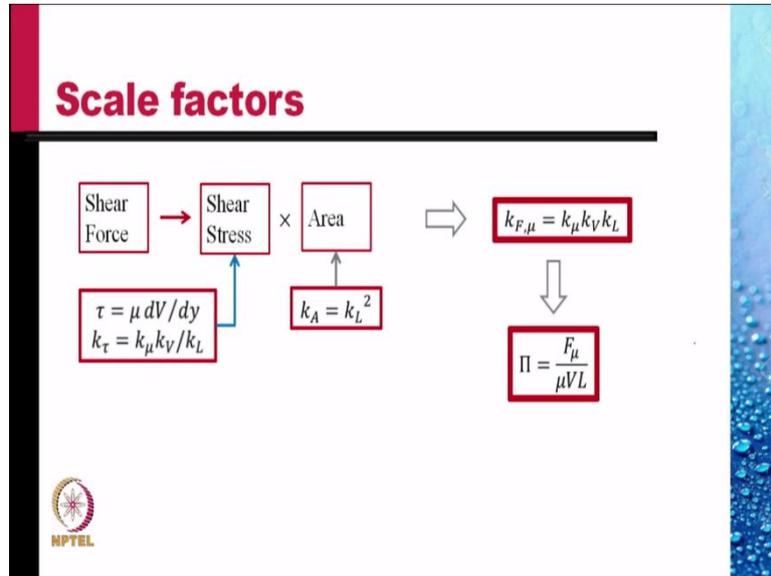
Similarly, we develop the pi number for the viscous force, and pi number for the viscous force is obtained as $\frac{F_\mu}{\mu V L}$, the same pi number that we obtained in the last class.

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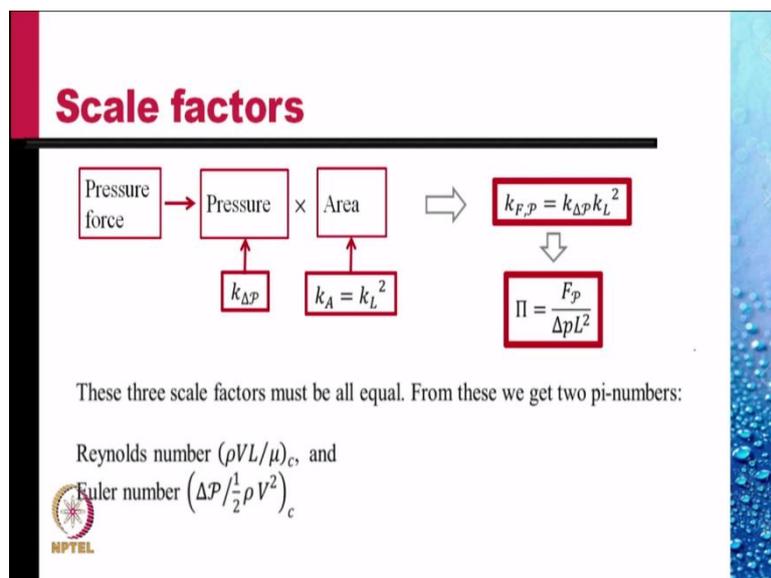

The third pi number we deal with is a pressure force, which is pressure into area. For pressure or rather pressure difference, we have $k_{\Delta\mathcal{P}}$, and for area we have $k_L{}^2$, and so, $k_{F,\mathcal{P}} = k_{\Delta\mathcal{P}} k_L{}^2$,

or pi is equal to $\frac{F_{\mathcal{P}}}{\Delta p L^2}$. Notice that we have converted p into script $\mathcal{P}$ which is the non-gravitational pressure discussed earlier.

These three scale factors must be equal. From these we get two pi numbers, the Reynolds number $(\rho V L/\mu)_c$. It is assumed that all values are characteristic. The Euler number $\left(\Delta\mathcal{P}/\frac{1}{2}\rho V^2\right)_c$. One half is there because of convention.
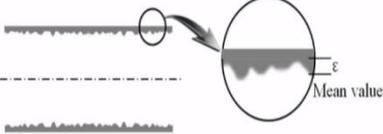
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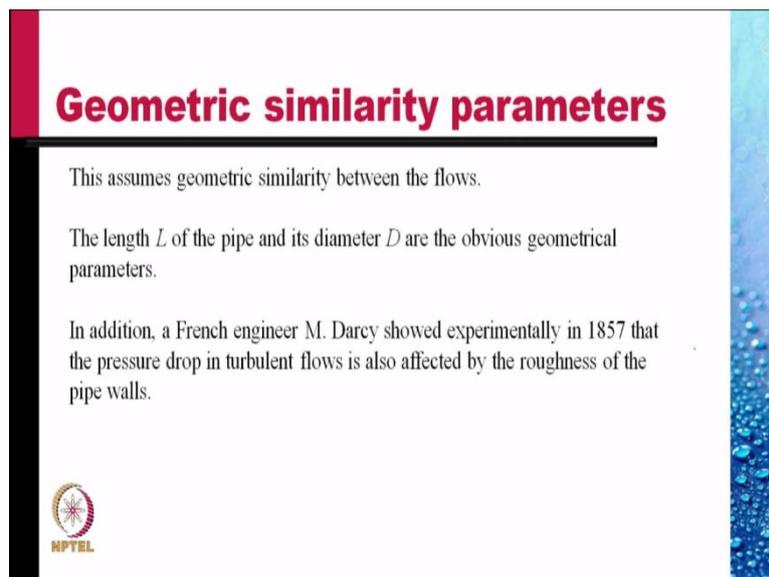
This assumes a geometric similarity between the flows. The length L of the pipe and its diameter D are the obvious geometrical parameters. In addition, a French engineer by the name M. Darcy showed experimentally in 1857 that the pressure drop in turbulent flows is also affected by the roughness of the pipe wall.

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The wall roughness may be characterized by the mean height, ε, of the roughness elements on the wall, and therefore, the non-dimensional geometric parameters are $\frac{L}{L_c}$, L_c is the characteristic length, L is the length of pipe. So, L, D and ε are the three geometric parameters. So, we get three non-dimensional geometric parameters $\frac{L}{L_c}$, $\frac{D}{L_c}$, and $\frac{\varepsilon}{L_c}$.

We still have to choose what L_c we use.

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We can then write any non-dimensional dependent parameters like pressure drop as a function of Reynolds number, Euler number in these geometric parameters. The most common problems encountered in pi flows is to evaluate the piezometric pressure drop $\Delta\mathcal{P}$ for a given flow rate.

The non-dimensional piezometric pressure drop is $\frac{\Delta \mathcal{P}}{(\Delta p)_c}$ can then be written as a function of the Reynolds number, Euler number, $\frac{L}{L_c}$, $\frac{D}{L_c}$, and $\frac{\varepsilon}{L_c}$.

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It is convenient to use diameter of the pipe as the characteristic length in these problems, and we use the average velocity of the fluid through the pipe as the characteristic velocity. There is no characteristic difference in the pressure in the independent parameters, and so we can choose this arbitrarily. If we choose the characteristic pressure difference $(\Delta p)_c$ as $\frac{1}{2}\rho V_{av}^2$, the value of $\left(\frac{\Delta \mathcal{P}}{\frac{1}{2}\rho V^2}\right)_c$ is rendered as unity, a constant and so, it drops out from the functional relationship. If we treat $\Delta \mathcal{P}$ as an dependent parameter, then the functional relation becomes $\left(\frac{\Delta \mathcal{P}}{\frac{1}{2}\rho V^2}\right)_c$ is a function of Reynolds number, $\frac{L}{D}$ and $\frac{\varepsilon}{D}$.

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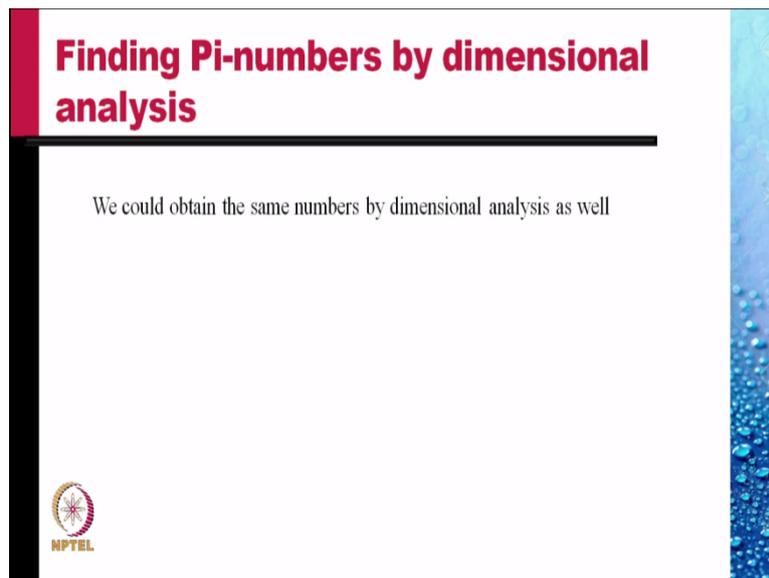
We can find the pi numbers by dimensional analysis also. The dimensional analysis was introduced in the last lecture.

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We start with the list of parameters. The basic group as we defined there are ρ, D, V_{av}, the independent parameters which contain all the dimensions involved. There are three dimensional involved, M, L and T here, and so, we choose three independent parameters as the basic group, and ρ, D, V_{av} involve all these three dimensions and do not form any non-dimensional parameter amongst themselves.

The other independent parameters which are not in the basic group are μ, viscosity, length L of the pipe, and ε, the mean roughness height, and we write one dependent parameter. In this case

it is $\Delta\mathcal{P}$, the piezometric pressure drop. We write the dimensions of all these parameters. Then we form for other parameters and variables that has this group. We take one of these and multiply them by ρ^a, L^b, and V_{av}^c, and then determine the values a, b and c which will make this non-dimensional.

If we use viscosity, we get a parameter $\frac{\mu}{\rho V_{av}D}$ as non-dimensional pi number. The inverse of this is the Reynolds number. With L, you get L/D as a non-dimensional parameter. With ε, you get ε/D as a non-dimensional parameter. With $\Delta\mathcal{P}$, the dependent parameter, we get $\frac{\Delta\mathcal{P}}{\frac{1}{2}\rho V^2}$ as the dependent non-dimensional parameter, Π_4.

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Simplification for full-development

$$\frac{\Delta\mathcal{P}}{\frac{1}{2}\rho V_{av}^2} = \mathcal{F}\left(\frac{\rho V_{av}D}{\mu}, \frac{L}{D}, \text{ and } \frac{\varepsilon}{D}\right)$$

If the pipe is very long compared to the entrance length l_e we may assume the flow to be fully developed along the entire length L of the pipe.

Since the conditions in a fully-developed flow do not change along the length of the pipe, the shear stresses at the wall will be constant, and therefore, we expect the pressure drop per unit length to be constant.

In other words, $\frac{\Delta\mathcal{P}}{\frac{1}{2}\rho V_{av}^2}$ should vary linearly with L. Thus,

$$\frac{\Delta\mathcal{P}}{\frac{1}{2}\rho V_{av}^2} = \frac{L}{D}\,\mathcal{F}\left(\frac{\rho V_{av}D}{\mu}, \frac{\varepsilon}{D}\right)$$

So, we can write the equation just like before. $\frac{\Delta\mathcal{P}}{\frac{1}{2}\rho V_{av}^2} = \mathcal{F}\left(\frac{\rho V_{av}D}{\mu}, \frac{L}{D}, \text{ and } \frac{\varepsilon}{D}\right)$. Now, if the pipe is very long compared to the entrance length l_e, we may assume that the flow is fully developed along the entire length of the pipe. Then the pressure gradient is constant in the pipe. If pressure gradient is constant in the pipe, then $\Delta\mathcal{P}$ should increase linearly with L. And so, we can write $\frac{\Delta\mathcal{P}}{\frac{1}{2}\rho V^2}$ is L/D, the length parameter non-dimensional, is taken out, linear $\Delta\mathcal{P}$ is linear with L. So, it can be written as L/D times a function of Reynolds number, and ε/D.

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The dimensionless function of Reynolds number and ε/D is termed as the Darcy friction factor, and is denoted by f, lowercase f. So, in terms of f, the pressure gradient parameter $\dfrac{\Delta P}{\frac{1}{2}\rho V^2}$ is simply $f\dfrac{L}{D}$, Or in terms of the head loss, we can write head loss h_l is which is $\Delta P/\rho g$ is nothing but $f\dfrac{V_{av}^2}{2g}\cdot\dfrac{L}{D}$.

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For laminar floor region we know $V_{av} = -\dfrac{p\prime R^2}{8\mu}$, and if we cast this in the form of f, we get $f = \dfrac{64}{\text{Re}}$. So, in the laminar flow region the friction factor, the Darcy friction factor f, varies like $\dfrac{64}{\text{Re}}$.

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Many researchers at the beginning of the 20th century did extensive work on the determining the dependence of f on Reynolds number and $\frac{\varepsilon}{D}$. The prominent among them were Nikuradse, who worked with pipe walls roughened with sad grains of different sizes. Hunter Rouse, a very famous fluid dynamicist, R J S Pigot. Rouse and Pigot worked on developing a representation chart of the data. Colebrook, who experimentally developed the correlation of the friction factor with Reynolds number and $\frac{\varepsilon}{D}$, and Louis Moody who is credited with putting the work of all these people together into a chart which is famously known as Moody chart, a Moody diagram.

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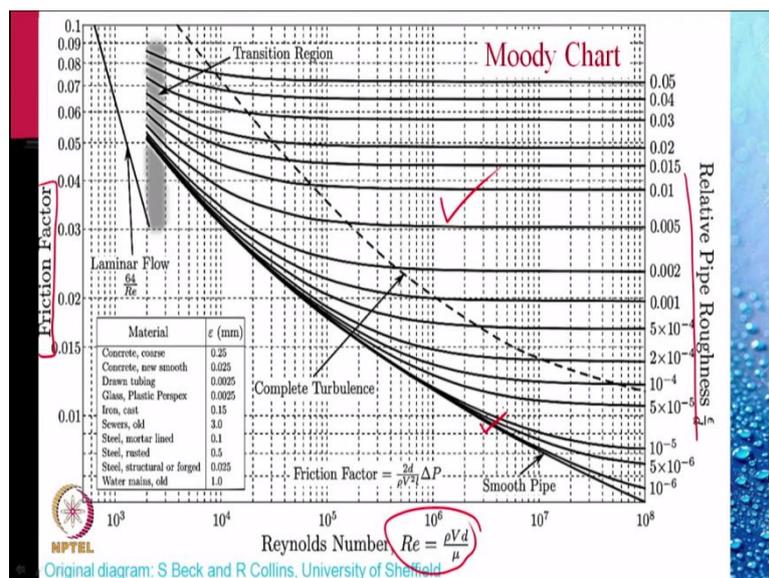
This is what a Moody chart looks like. Moody charts help us determine the friction factor f, the Darcy friction factor, as a function of Reynolds number with the relative pipe roughness

parameter, ε/D, as a parameter. Thus, given a Reynolds number and given a roughness parameter, ε/D, we can locate a point on this chart, and read from the left hand scale, the effective friction factor.

You will notice a few things. On the left-hand side, laminar flow is a straight line represented by $64/\text{Re}$. Since it is a log log plot, a relation $f = 64/\text{Re}$ would plot as a straight line. This is for laminar flow up to a Reynolds number about 2000. Then this grey area, to something like 3000, is an area which is the transition region, and one is not very sure of what the friction factor is in this region.

After that there is turbulent region, and the turbulent region is divided in two portions by this broken line. On the right of the broken line, the flow is called fully turbulent, and you will notice that in this region, the value of f is quite independent of the Reynolds number. It depends only on ε/D. We will exploit this fact in solving the problems. To the left of this, the flow is not fully turbulent, and the friction factor depends both on the Reynolds number and ε/D. This limiting line is for $\varepsilon/D = 0$, that is for a smooth pipe.

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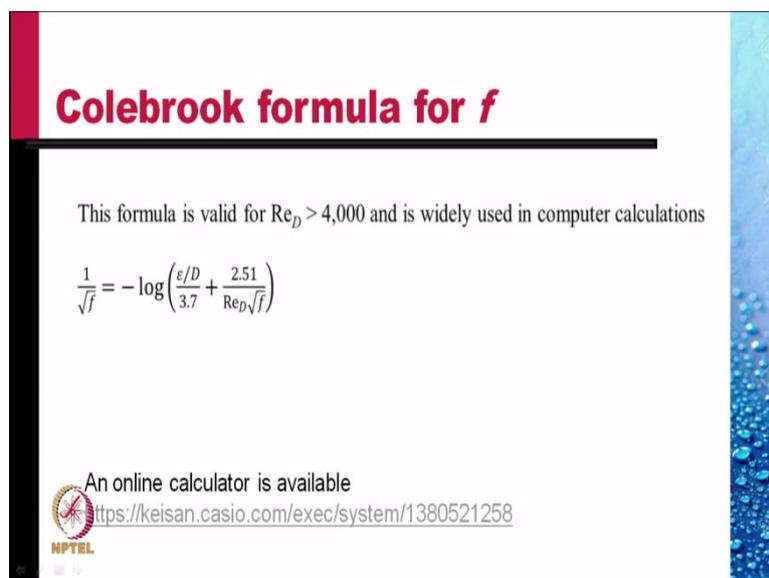
Colebrook developed a formula for Reynolds number greater than 4000, and which is widely used in computer calculation, $\frac{1}{\sqrt{f}} = -\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}_D\sqrt{f}}\right)$. There are various online calculators available. One of these calculator is at this address https://keisan.casio.com/exec/system/1380521258. If you Google search a Colebrook formula calculator you will get a number of calculators there.

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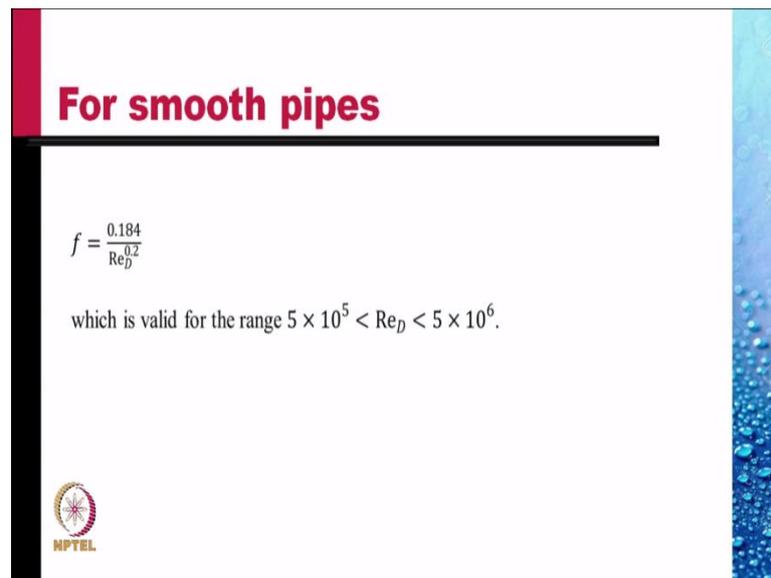
For smooth pipes, the line which was the lowest, the friction factor is given by $f = \frac{0.184}{\text{Re}_D^{0.2}}$. This is valid for the range of Reynolds number between 5×10^5 to 5×10^6.