Fluid Mechanics & its Applications
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Lecture 20

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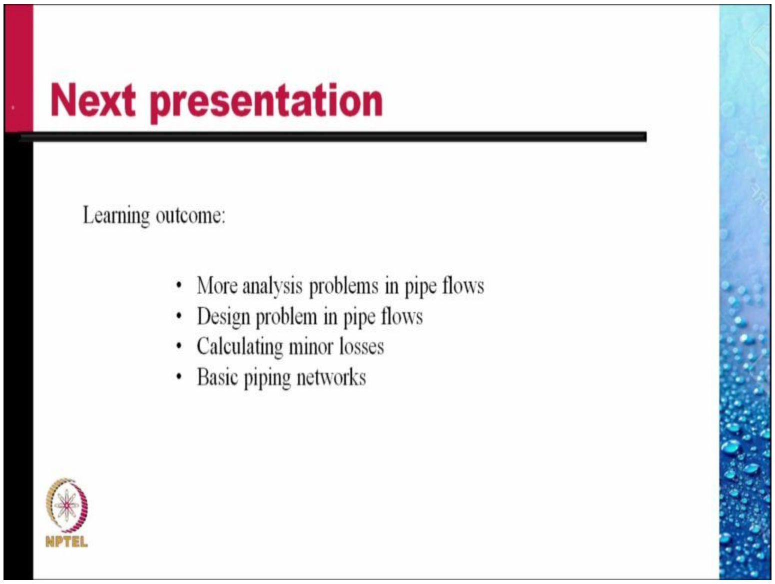

Welcome back.

In this presentation, we will do more problems of the analysis in pipe flows. We will also do a few problems on the design in pipe flows, and then we will cover some basic piping networks.

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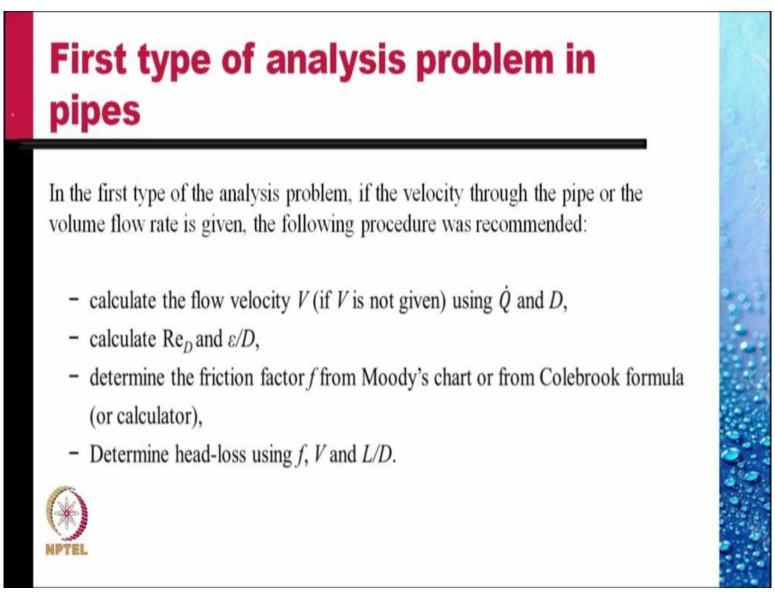

In the first type of analysis problems, if the velocity through the pipe or the volume flow rate through it is given the following procedure to calculate the pressure difference or the pressure loss through the pipe is recommended.

Calculate the flow velocity, if the velocity is not given, using the volume flow rate and the diameter of the pipe.

Next, calculate the Reynolds number and the roughness factor. Then determine the friction factor from the Moody's chart or from a Colebrook calculator. Once we know the value of f, we can determine the head loss using f, V and L/D.

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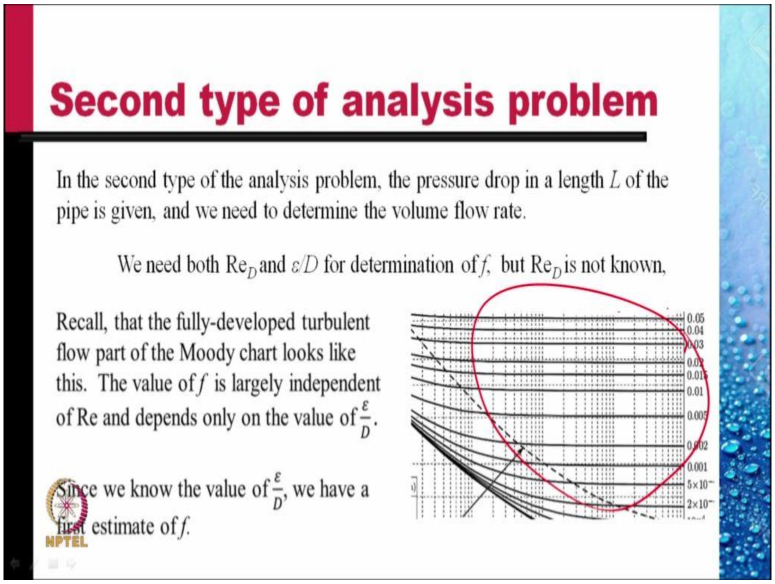

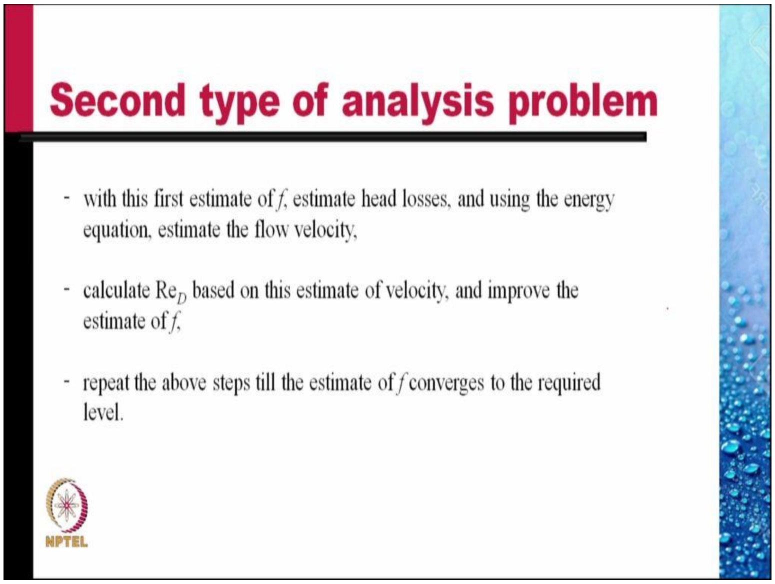

In the second type of analysis problem, pressure drop in a length *L* of the pipe is given, and we need to determine the volume flow rate through the pipe. The pipe geometry is also given. But, since we do not know the velocity, we can not calculate the Reynolds number. And therefore, we cannot calculate *f* directly, because the Reynolds number is not known.

Recall, that in fully-developed turbulent flow part of the Moody chart, the value of *f* is almost constant for a given value of ε/D. Since we know the value of ε/D, we have a first estimate of *f*. With this first estimate of *f*, we estimate the head loss, and using the energy equation, estimate the flow velocity. We calculate the Reynolds number based on this estimate of velocity, and improve the estimate of *f*. We repeat the above steps till the estimate of *f* converges to the required level.

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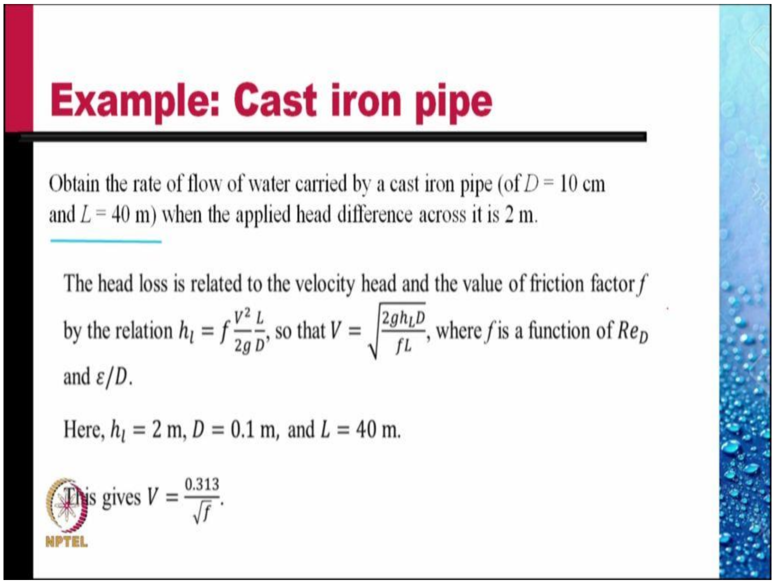

We do an example. Obtain the flow rate of water carried by a cast iron pipe of diameter 10 cm and length 40 m, when the applied head difference is 2 m. The head loss is related to the velocity head, and the value of friction factor f by the relation $h_l = f\frac{V^2}{2g}\frac{L}{D}$. So that we can write $V = \sqrt{\frac{2gh_LD}{fL}}$, where f is a function of Reynolds number and ε/D. Here in this relation for V, we know everything except the value of f. So, we plug in all those values, and we get a relation $V = \frac{0.313}{\sqrt{f}}$.

So, this should give you a hint directly. If we start with an estimate of f, we calculate a velocity V; from that velocity V, we calculate the Reynolds number. And once I have a Reynolds number and the value of ε/D, I get the value of f from a Moody chart or from a Colebrook calculator.

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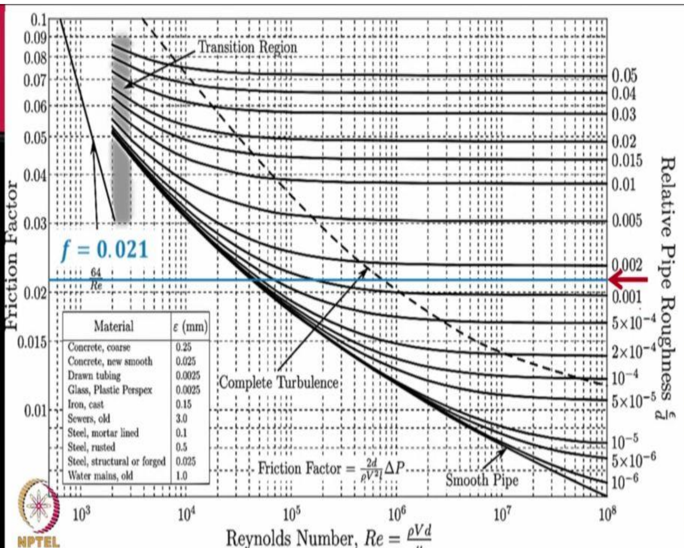
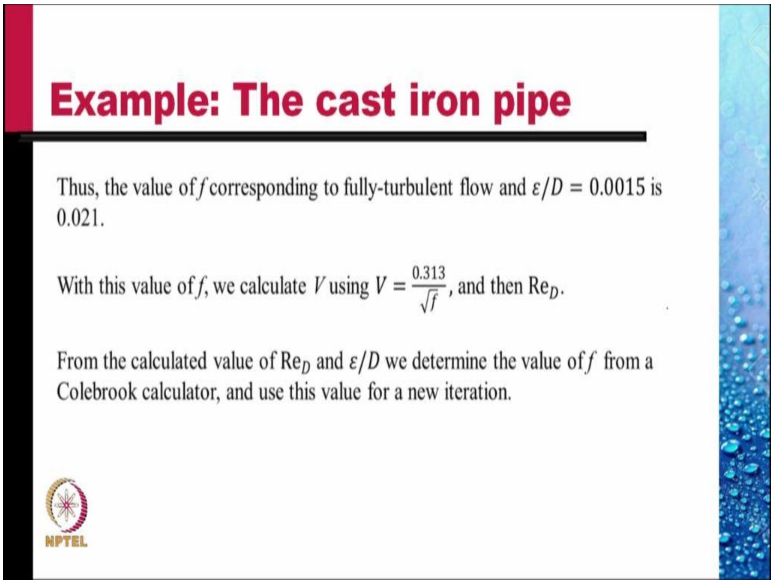
Example: Cast iron pipe

The value of ε for cast iron pipes is 0.15 mm so that ε/D is 0.15 mm/10 cm = 0.0015.

We start by assuming the flow to be fully turbulent so that f can be estimated without knowing the value of Re_D.





The value of ε for the cast iron pipe is 0.15 millimeter, so that the ε/D is 0.0015. We start by assuming the flow to be fully turbulent; so that f can be estimated without knowing the value of Reynolds number. From the chart, we enter at 0.0015, the value ε/D, the relative pipe roughness. In this we get a value of f as 0.021. We start with this value and we calculate V from $\frac{0.313}{\sqrt{f}}$, then Re_D, and do iterations.

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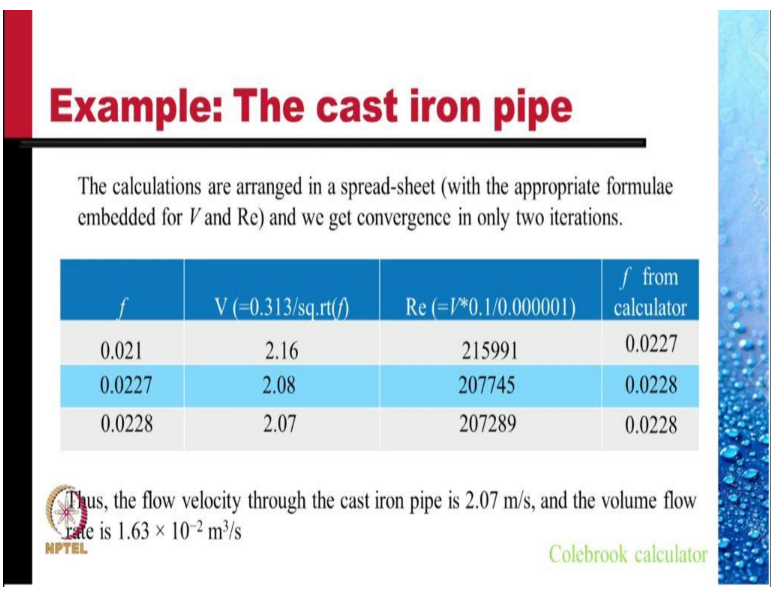
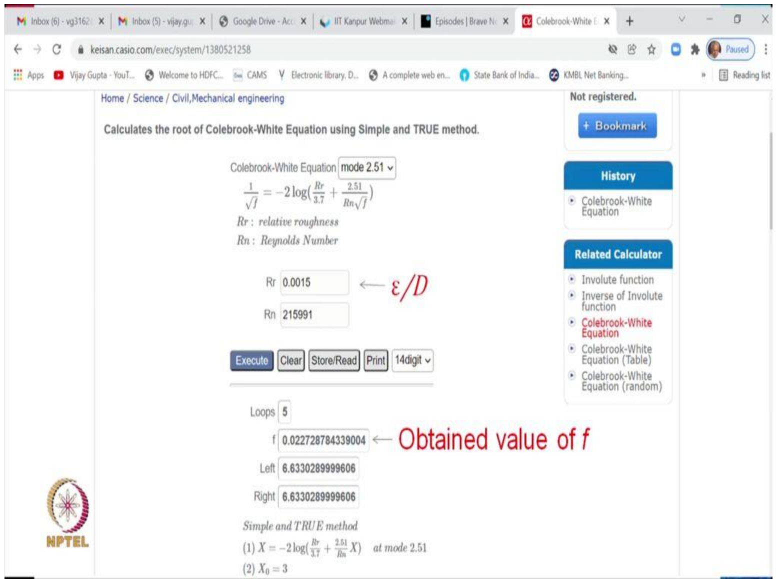


f	V (=0.313/sq.rt(f))	Re (=V*0.1/0.000001)	f from calculator
0.021	2.16	215991	0.0227
0.0227	2.08	207745	0.0228
0.0228	2.07	207289	0.0228



So, the first value of *f* we start is 0.021. The value of V from the formula $V = \frac{0.313}{\sqrt{f}}$, we get the value of V to be 2.16 m/s; from which we calculate Reynolds number as 215991 from Colebrook calculator. So the value of *f* we get from the calculator is 0.0227. This value can also be obtained from the Moody chart. The next round we start with this value, 0.0227. And we get a value of V as 2.08, and the value of Reynolds number 207745. And the value of *f* from this is a slightly larger, 0.0228.

And with this value when you put in there, I get velocity as 2.07; and the Reynolds number as 207289, slightly lower, but the value of *f* is the same at 0.0028. Thus, the value of *f* has converged, and we get the flow velocity 2.07 as an estimate of the flow velocity through the cast iron pipe. The volume flow rate corresponding to this velocity is obtained by multiplying this velocity with the cross-sectional area of the pipe, and we get 1.63×10^{-2} m^3/s.

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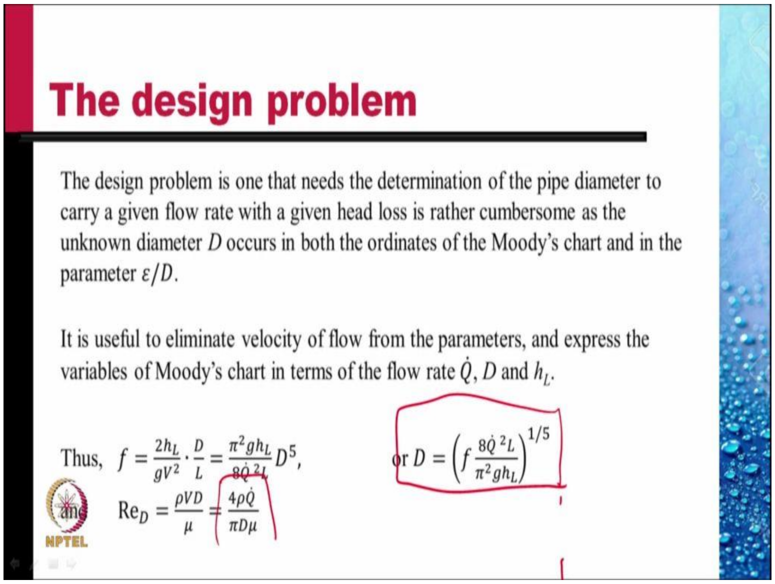

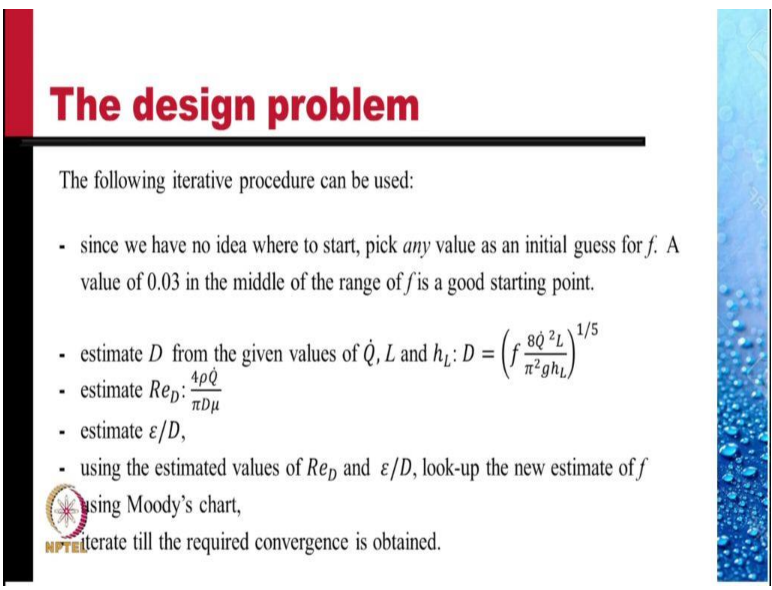

Next we do a design problem. The design problem is one that needs the determination of the pipe diameter to carry a given flow rate with the given head loss. And it is rather cumbersome as the unknown diameter D occurs in both the ordinate of the Moody chart and in the parameter ε/D. It is useful to eliminate velocity of the flow from the parameters, and express the variables of the Moody chart in terms of the flow rate $\dot{Q}$, D and h_L. Then, out of these $\dot{Q}$ and h_L are known and D is the unknown. So, the friction factor which is $\frac{2h_L}{gV^2}\cdot\frac{D}{L}$ becomes $\frac{\pi^2 g h_L}{8\dot{Q}^2 L} D^5$.

And from this we can express D like this in terms of $\dot{Q}$ and h_L. The value of f is not known here, all other quantities are known. And then the Reynolds number can be expressed as $\mathrm{Re}_D = \frac{4\rho\dot{Q}}{\pi D\mu}$.

The following iterative procedure can be used.

Since we have no idea where to start, pick any value as an initial guess for *f*. A value of 0.03 in the middle of the range of *f* is a good starting point. Estimate D from the given value of $\dot{Q}$, L and h_L from the formula for the diameter that we just derived, and then estimate Reynolds number. Once we know D, we can calculate *ε/D*. Then using the estimated value of Reynolds number and *ε/D*, we can look up a new estimate of *f* from the Moody chart; and iterate till the required convergence is obtained.

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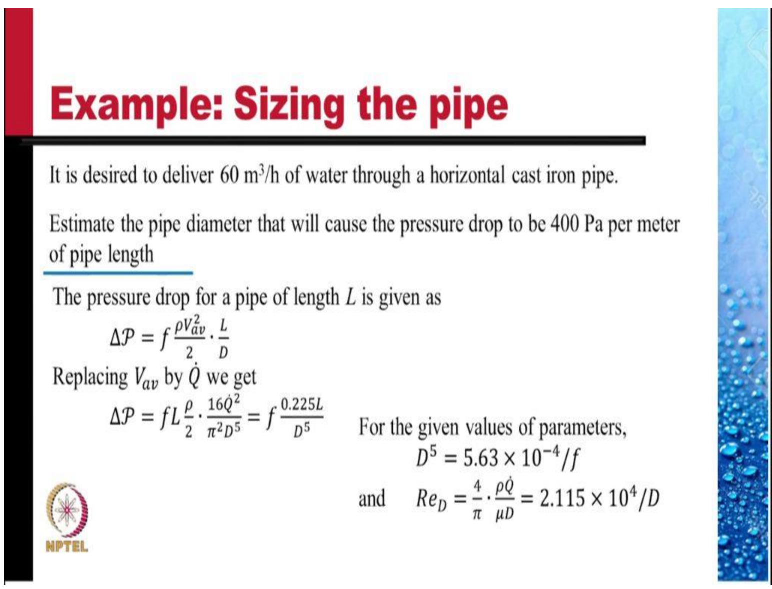

We will show this process through an example. It is desired to deliver 60 m^3/hr of water through a horizontal cast iron pipe. Estimate the pipe diameter that will cause the pressure to drop at 400 Pa/m of pipe length. The pressure drop for the pipe of length L is given by this formula. And replacing V_{av} by $\dot{Q}$, $V_{av} = \frac{\dot{Q}}{\frac{\pi}{4}D^2}$. We get $\Delta\mathcal{P}$ in terms of $\dot{Q}$, D and f. And we plug in the value of the known quantities, I get $\Delta\mathcal{P} = f\frac{0.225L}{D^5}$. For the given value the parameters, D^5 can be written as $5.63\times10^{-4}/f$; and the Reynolds number can be written as this.

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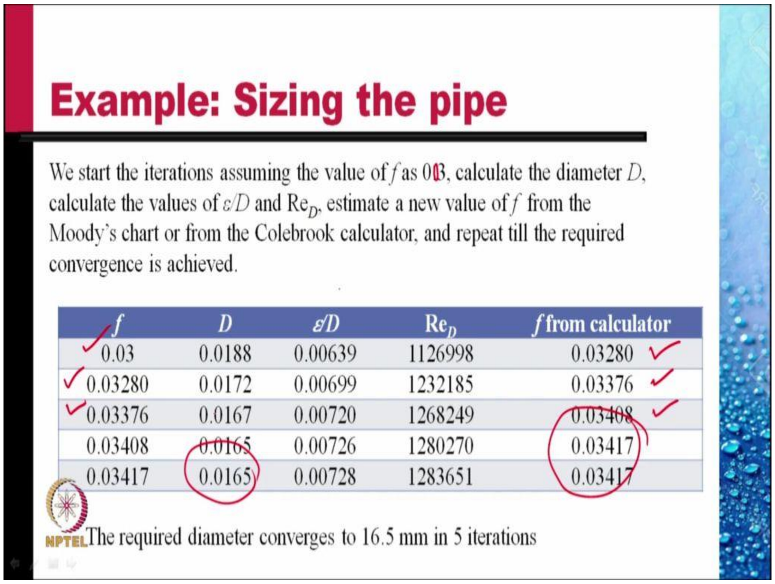

So, now, we are ready to start iteration. We start iteration assuming the value of *f* as 0.03. Calculate the diameter D, calculate the value *ε/D* and ReD. Estimate a new value of *f* from the Moody's chart or from the Colebrook calculator, and repeat till the required convergence is achieved. So, we start from *f* is equal to 0.03, and we get *f* from calculator as 0.03280.

These calculations can be done very easily on a spreadsheet like Excel with formulae embedded; so, the repeated calculation do not take much time. Then using this value in the second iteration, we get a new value as 0.03376. Value *f* is increasing. I plug in this value next, and I get the value as 0.03408; it is not converged yet. And now I do it two more times, I get the converged value of *f* as 0.03417. And then this is the converged diameter of the required pipe. That is, we need a pipe of 16.5 millimeter diameter to carry the required flow with the given pressure drop.