Transport Phenomena of Non-Newtonian Fluids
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Lecture - 04
Viscoelastic Non-Newtonian Fluids

Welcome to the MOOCS course Transport Phenomena of Non-Newtonian Fluids. The title of this lecture is Viscoelastic Non-Newtonian fluids. In previous couple of lectures, what we have seen? We have seen a different characteristics of Newtonian fluids and based on those characteristics, we have defined a non-Newtonian fluids and then, we have classified a non-Newtonian fluids. We have seen these non-Newtonian fluids are a grouped as a time independent non-Newtonian fluids, time dependent non-Newtonian fluids and viscoelastic non-Newtonian fluids.

So, we have already seen enough details about the time independent and time dependent non-Newtonian fluids, their models and then a few example problems as well. In the previous lecture, we have also seen a few details about the viscoelastic fluids. Now, in this lecture we will be discussing in detail about a viscoelastic fluids once again along with a few possible mathematical models that are well-known for the case of a viscoelastic non-Newtonian fluids.

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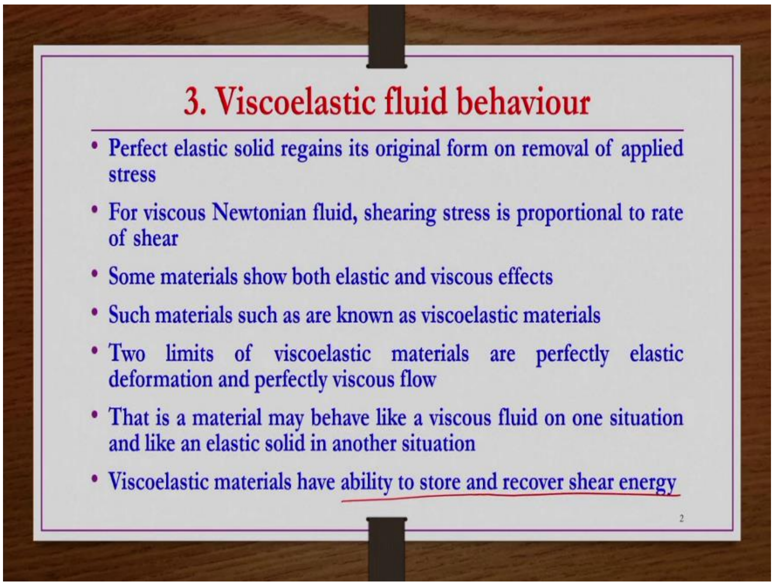

Viscoelastic fluid behaviour, as the word viscoelastic indicates these fluids will have both viscous as well as elastic behaviour. Under certain limiting conditions, the elastic behaviour may be predominating; whereas, under some other conditions or some other range of shear rate, the viscous effect may be more important right. But given a same material may be having both elastic and then viscous behaviour, then such fluids are known as the viscoelastic fluids.

So, what is elastic body? Perfect elastic body, it regains its original form on removal of applied stress. When we apply external force to elastic material, what will happen? It undergoes certain kind of deformation. However, when you remove the applied force, what will happen?

The material will try to gain its initial position. If it is perfectly elastic, then it will come back to its initial position; otherwise it can only get back partially to its previous state ok. So, such materials are you know elastic materials. So, if the completely they are coming back to initial position, then they are perfectly elastic solid materials.

However, in the case of Newtonian viscous fluids, what happens? Shear stress is proportional to the shear rate, where the applied force when you remove the material will never come back to its initial position; it cannot even partially get back to its previous state ok. So, such materials such viscous for such viscous Newtonian fluids, the shear stress is proportional to the shear rate.

However, some materials have found to be display both elastic and then, viscous effects and these materials are known as the viscoelastic materials. Some textbooks you may also find the word elastic-viscous materials as well. It both of them are same indeed in general, but you know some people refer these materials as elastic-viscous material, if the elastic behaviour of the material is more compared to the viscous behaviour ok. Otherwise, they are same; one can use either a viscoelastic or elastic-viscous term for these materials.

Obviously, what we can understand? Two limits of viscoelastic materials are possible and these are perfectly elastic deformation one limit and another limit is perfectly viscous flow. Also, this material may behave like a viscous fluid on one situation and may behave like elastic solid in another situation. That is also quite possible. That is what is one of the main characteristics of this viscoelastic fluid behaviour.

So, viscoelastic materials thus, they have ability to store and recover shear energy ok. So, that is basically about viscoelastic fluid behaviour. We are not just talking about viscoelastic material; but we are in general, trying to have a kind of more information about viscoelastic fluid behaviour.

So, we see a few examples, or real life situation, where you can realize this viscoelastic behaviour in a given fluid ok. So, we start with couple of examples and then we go into the other details of you know mathematical details of you know representing this viscoelastic fluid behaviour etcetera.

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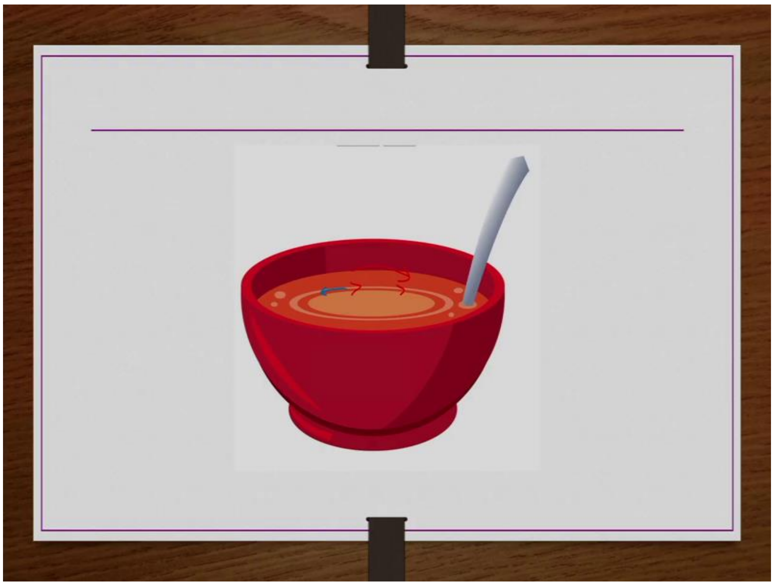

So, some proprietary soups, they do have an elastic behaviour, viscoelastic nature. How we can realize? Let us say you have soup bowl and then, you are rotating with a stirrer; rotating with a stirrer. So, then what you can see? You can see these circles are internal circulations kind of things are possible. They are forming like this if your rotation direction is this one right.

So, but sometimes what happens you know when you rotate it for some time, you rotate it for some time and then, what you do? You stop rotating. Then, what you can see that material whatever the viscous fluid that you have taken or the soup that you have taken that may be found like a halting or the internal circulations are found to be halting and then, they try to come back to previous locations or previous positions.

That means, rotations the winning whatever the internal circulation that you can see on the surface, they will move in the other direction because after stopping the, after stopping the rotation, what happens? They try to go to the backwards you know circulation may go in a reverse direction. Why? Because this material whatever is there, if it is having viscoelastic behaviour, then only it is happening. And then because of that elastic nature, the material remembers or memorizes remembers its previous position and then, it tried to come back right.

So, it partially come backs because it is a; it is also having the viscous behaviour. So, then, partial recovery is possible for the kind of you know viscoelastic fluid behaviour. This is what one can realize easily in some of the you know situations experimental situations ok where you can realize the elastic behaviour and fluid if having or let us say the soup is having elastic behaviour, so then you can have a such rewinding of internal circulations is possible.

It is not necessary that all the soups are having elastic behaviour, viscoelastic behaviour; only some of them are having viscoelastic behaviour. So, you can try doing this experiment yourself at home or in restaurant and then, you may observe this kind of observation in general.

And then, you may observe it, only if that particular given soup solution is having viscoelastic behaviour otherwise not right. Only a few of proprietary soups are having this viscoelastic behaviour, but not all the soups. Now, we take another example.

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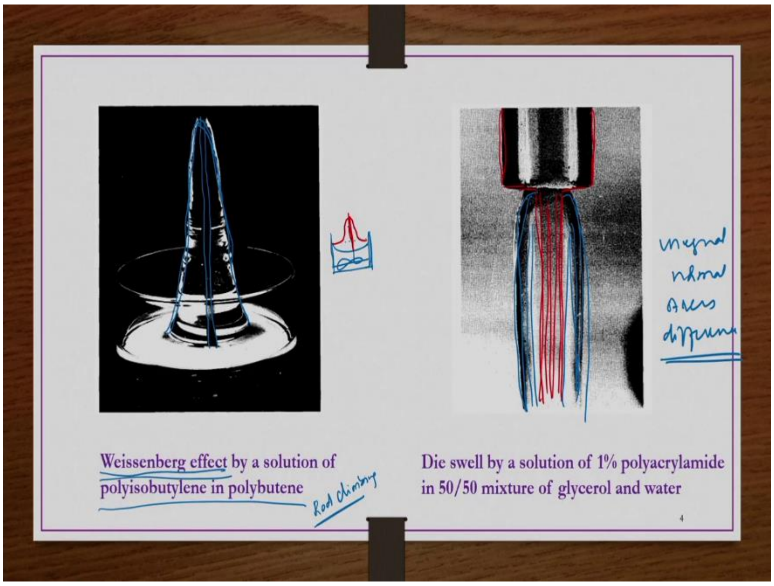

Rod climbing effect which is also known as the Weissenberg effect right. So, you take a solution of polyisobutylene in polybutene right. So then, now you take the solution in a bowl and then you try to rotate. You have a kind of a stirrer and then that is stirrer you are trying to rotate so that to mix the solution.

What happens? So, in general you know the fluid surface you know when you are rotating, then you can say conquer, these kind of surfaces in general you see if you rotate any kind of material that you take right. If it is not having viscous behaviour, when you rotate the initial surface was like dotted line let us say, but when on rotation, the top surface of the liquid will drop like this right; concave upward direction like this ok.

So, but now if the material is having any elastic behaviour, it will try to climb the rod like this. It will try to climb this stirring rod like this. So, this is what known as the, you know Rod Climbing Effect, right.

Why does it happen? Because when such material which when such materials which are having elastic behaviour also along with the viscous behaviour or for viscoelastic behaviour, when you rotate, these rotations produces a stress which is perpendicular to the direction of the shearing, right. Because of that stress perpendicular to the direction of shearing, so a kind of normal stress would be developed and then, that normal stress leads this material to climb the rod like this right.

So, rather finding rather having kind of you know the surface like this, if it is having elastic behaviour, the material will try to climb like this, ok. So, because the rotation, the shearing direction in the perpendicular to the shearing direction, stress would be developed that is known as the normal stress because of that normal stress, the material tries to climb the surface like this.

Another effect die swelling effect, we can have. So, let us say you have a container in which you have die and then you try to open it or take it through the outlet. So, what happen? It rather have falling down straight away like this in general which you find for a normal viscous fluids right. So, it is showing some kind of swelling effect. What kind of swelling effect it is showing?

You know it swells like this at the outlet and then, expands like this slightly and then, falls out rather than a kind of a straight path out like red colour lines; blue colour lines indicates some slightly, it is swelling kind of thing. This swelling is also because of the stresses that are developed in a direction normal to the in a direction perpendicular to the shearing direction or normal to the shearing direction. So, here again, we can see the normal stress are playing the role ok.

So, now, recollecting one of the characteristics of Newtonian fluid; Newtonian fluids what we have seen? We see only shearing stress are possible. Shear stress are only possible; normal stresses either identically 0 or their differences are 0. That is what we have seen.

But in non-Newtonian fluids, those normal stresses may not be 0. Indeed, in the viscoelastic fluids, those normal stresses or their difference are not 0 ok or unequal normal stress differences are possible. Unequal normal stress difference is possible in the case of viscoelastic fluids. So, that is the reason now we see a few details about the normal stresses.

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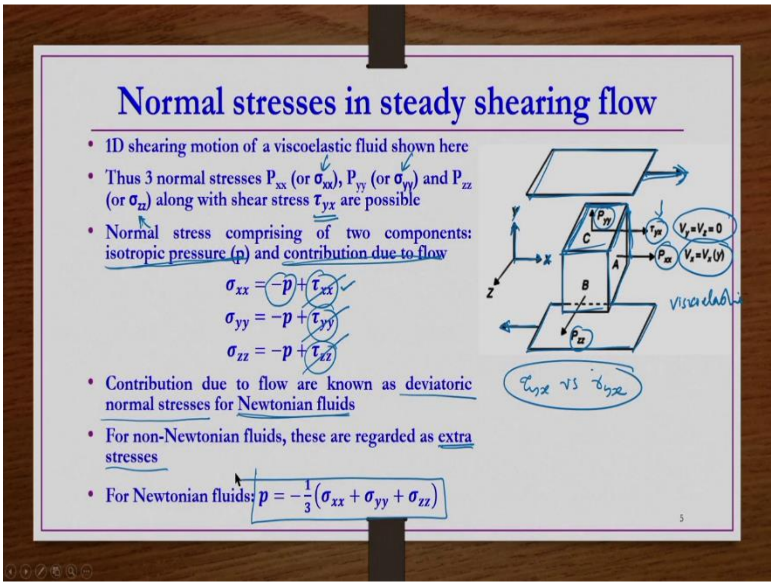

So, normal stresses in steady shearing flow. Now, 1D shearing motion of a viscoelastic fluid, we are considering. We have a fluid element like this here confined between two planes. Now, we are a shearing this plane such a way that shearing action takes place; this fluid element undergo shearing action.

So, if it is only a viscous material which is not having elastic fluid, so then what we have seen? Only stress developed that will be shearing stress and then, that shearing stress would be in the direction of the shearing and then, plane on which it is working that plane would be normal to the shearing direction.

So, τ_{yx} because we are shearing in x direction and then, in the other y direction only the velocity is changing. So, V_x velocity component is there and then that is function of y, so then τ_{yx} that is the only stress is possible because V_y, V_z are 0 here that is how the limiting condition we are taking it ok; such a way that we are designing this experiment shearing experiment ok.

But if this material whatever the fluid element that you have taken, if it is not simply viscous fluid element, but it is a viscoelastic fluid element, then what happen in addition to the shearing stress? You will also experience normal stresses; P_{xx}, P_{yy}, P_{zz} normal stresses are also possible. They also possible indeed because of that those normal stresses only so called rock climbing or you know die swelling effects that we have seen in previous slides, they are possible because of existence of a these normal stresses.

If the material simple Newtonian fluid, you will you would get only τ_{yx} ok; simple viscous fluid, you will get only τ_{yx}. You would not get any other kind of other normal stresses there, you will be having those values only if the material is viscoelastic.

So, in addition to the shear stress τ_{yx}, you will also have a three normal stresses that is P_{xx}, P_{yy}, P_{zz}. These are also represented as σ_{xx} σ_{yy} and then, σ_{zz} in some books ok. And then, these normal stresses whatever are there, they are comprising two components; isotropic pressure and then, contribution due to the flow on the viscous normal stress ok. This normal stress is having isotropic pressure as well as the viscous normal stress that is nothing but contribution due to the flow right.

So, how do we define them? σ_{xx} is minus p plus tau xx. What does mean by $-p + \tau_{xx}$? This these are acting in though both of them are in a normal direction, but in a different I know they are opposite to each other ok.

So, let us say the pressure is acting you know towards the fluid surface; towards the fluid element that you have taken whereas, the viscous stress because of the flow is that viscous normal stress because of the flow that is acting towards the you know out outer to outwards the fluid element, whatever we have taking ok that is what this $-p + \tau_{xx}$.

So, similarly, σ_{yy} σ_{zz} also you can write like this. So, this p indicate the isotropic pressure whereas, the τ_{xx}, τ_{yy}, τ_{zz} are known as the deviatoric normal stresses, if the viscous component is having only Newtonian fluid behaviour. That is if the τ versus $\dot{\gamma}$ is linear behaviour, then whatever this pressure; whatever this $\tau_{xx}, \tau_{yy}, \tau_{zz}$ that are existing because of the elastic behaviour elastic part, they are all they are known as the deviatoric normal stresses for Newtonian fluids ok.

However, for this non-Newtonian fluids, these are regarded as extra stresses; these are regarded as extra stresses. So indirectly, in the case of Newtonian fluids what we have? This τ_{xx} is 0, τ_{yy} is 0, τ_{zz} is 0; identically they are 0 or their difference is 0.

So, they when they are identically 0, how pressure can be defined? If you add them together, you will get $\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = -3p$; that means, isotropic pressure for the case of Newtonian fluid, you can define it as $-\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$right. But, such kind of definition for the isotropic pressure is not possible in the case of a non-Newtonian fluids.

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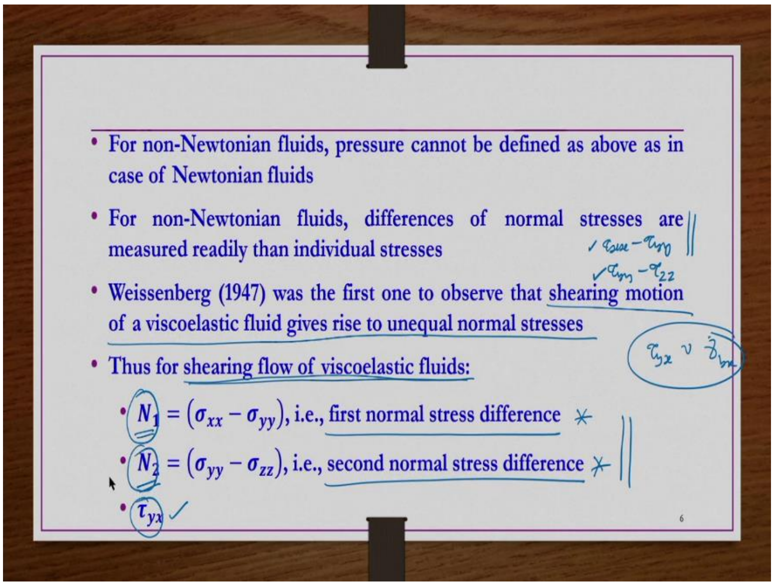

In the case of non-Newtonian fluids, having such kind of definition for a this isotropic pressure is not possible because for such kind of materials, whatever the τ_{xx}, τ_{yy}, τ_{zz} or their differences are not identically equal to 0; they are unequal, right.

For non-Newtonian fluids, differences of normal stresses are measured readily than individual stresses. So, then, we will be seeing in the subsequent weeks, where we will be measuring this rheological behaviour τ versus $\dot{\gamma}$ or τ_{yx}versus $\dot{\gamma}_{yx}$ etcetera. Those information shear that is indirectly shear stress versus shear rate information etcetera we are going to measure. We are going to see the working principles how to measure such a rheological behaviour.

In addition to them, we will also see the methods how to measure the normal stresses or normal stress differences. So, there we can see these normal stress differences whatever are there, they can be easily measured rather than the; rather than the individual normal stresses like a σ_x like τ_{xx}, τ_{yy}. So, you can easily measure these differences rather than individual normal stresses.

And then, whatever the rod climbing effect due to the Weissenberg was seen in one of the previous slides. So probably that is the first one to have a kind of information that shearing motion of a viscoelastic fluid gives rise to unequal normal stresses; unequal normal stresses.

So, when you are defining shearing motion or shearing flow of a Newtonian fluid or a fluid which is not having any elastic behaviour, so then what you need to give you get? You need to have simply τ versus $\dot{\gamma}$ information is only required right. Only τ versus $\dot{\gamma}$ information is only required.

But if you wanted to define a shearing flow of viscoelastic fluids, in addition to this τ_{yx} versus $\dot{\gamma}_{yx}$, you also need to have a N_1 versus $\dot{\gamma}_{yx}$ and N_2 versus $\dot{\gamma}_{yx}$ that information is also required or N_1 is nothing but first normal stress difference and then, N_2 is nothing but second normal stress difference.

So, in addition to the shear stress versus shear rate information, you also need to have information about the first normal stress differences versus shear rate. And second normal stress, second normal stress difference versus shear rate information also in order to completely define rheological behaviour of viscoelastic material right.

So now, this N_1, N_2 we see a few more details now. Some examples if you see, so let us say I have taken polystyrene in toluene solution. So, the first normal stress difference N_1 is plotted against a shear rate.

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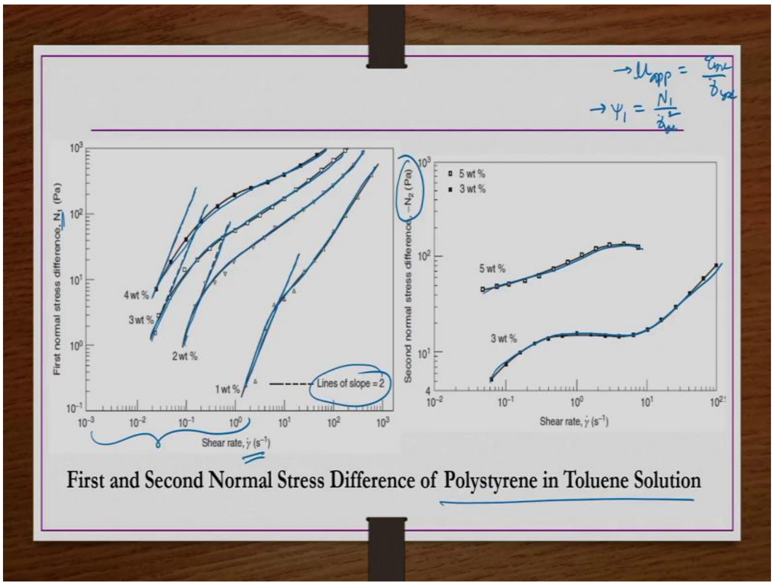

So, you can see the for different weight percent of a polystyrene in toluene solution, you can see different curves like these are possible. Now, the shapes are also changing, they are not having you know they are appear to be similar, but they are significantly different from other that is what you can see.

And then, also τ versus $\dot{\gamma}$ whatever the curves that rheological curves that you have seen for time independent or time dependent non-Newtonian fluids, they are having a particular shape right. The shape, the size etcetera are maybe changing, but they are having for a given type of material they are having particular shapes. So, but for viscoelastic fluids, it is not necessary that you may get all these kind of shapes only for all the fluids ok, right.

So, this is the first normal stress difference. And then, under the limiting conditions of a low shear rates, under the limiting conditions of low shear rate, what we can see? This data whatever is there, N_1 versus $\dot{\gamma}$, you can see a kind of you know approach you can see there we can have we can draw a kind of straight line of slope 2; of slope 2 that is what we can see, ok.

So, because of this region only. In general you know viscosity when you define, you know viscosity you define or apparent viscosity you define, what you do? Shear stress divided by shear rate only simply you do. But similar stress coefficients like you know viscosity that we have defined. Likewise using the normal stress differences, if you wanted to define the stress coefficients that we are going to do in next slides, you know they are let us say ψ_1, ψ_2 like this they are given.

So, there you will be seeing this ψ_1 is $\frac{N_1}{\dot{\gamma}_{yx}^{\,2}}$ because under the limiting condition of low shear rate, N_1 is expected to have a you know proportionality behaviour with respect to the square of the shear rate; but not individual shear rate ok. That is the reason you can see. N_1 is proportional to the square of shear rate at low shear rates that is what we can see from this graph ok.

So, second normal stress differences, you can see the negative of second normal stress difference is shown here. So, that is having you know different nature depending on different weight percent of the polystyrene toluene solution. Just in order to have a kind of feel about their normal stress differences, I have shown these graphs ok.

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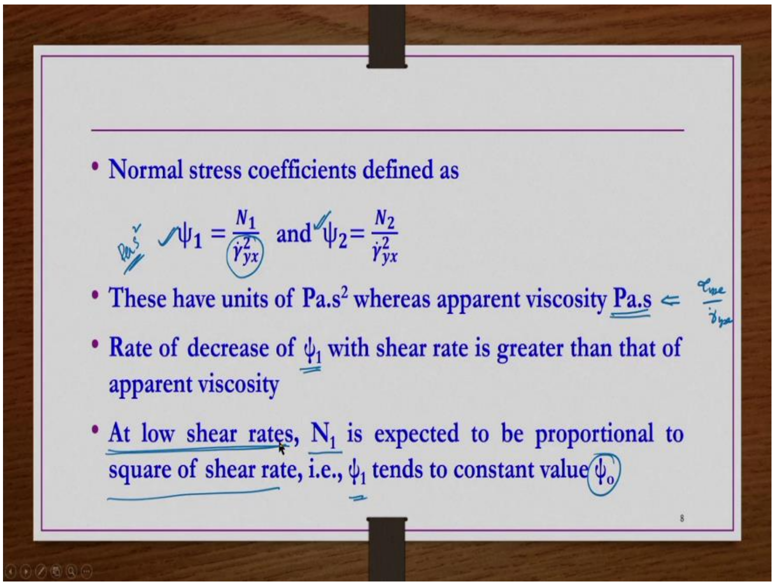

Now, the normal stress coefficients, they are defined as first normal stress coefficient. N_1 is the first normal stress difference; ψ_1 is the first normal stress coefficient right. So, that is $\frac{N_1}{\dot{\gamma}_{yx}^2}$. So, why square, because you know my previous slide, we have seen these stresses at low shear rate are having a kind of limiting nature that N_1 proportional to you know $\dot{\gamma}_{yx}^2$ ok.

So, similarly, second stress coefficient ψ_2 is $\frac{N_2}{\dot{\gamma}_{yx}^2}$. So now, the units if you see, you can have pascal second square from here because gamma dot is having second inverse. So, the stress differences are having pascal. So, pascal by 1 by second square. So, then pascal second square you will be getting right.

Whereas, the apparent viscosity or the viscosity whatever we have seen before the viscoelastic materials, they are having units of pascal seconds. Because there whatever this apparent viscosity is there, so we simply had $\frac{\tau_{yx}}{\dot{\gamma}_{yx}}$, not the $\dot{\gamma}_{yx}^2$ ok. So, that is the reason this difference is there.

And then, rate of decrease of ψ with shear rate is greater than that of apparent viscosity. This is another experimental observation and then, as we have already seen at low shear rates, N_1 is expected to be proportional to the square of shear rate. So, obviously, the ψ_1

tends to a constant value of ψ_0 at low shear rates. This is valid only at low shear rates region, not for all the ranges of shear rates.

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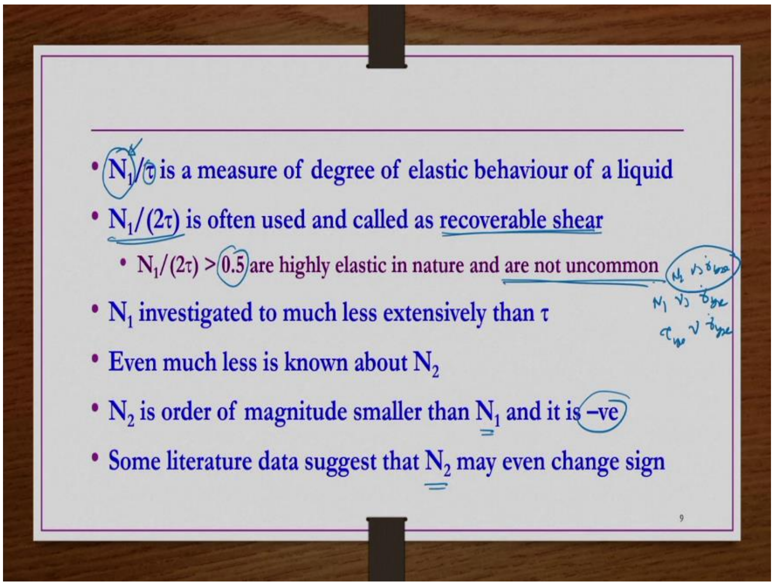

So, further this $\frac{N_1}{\tau}$ is found to be a measure of degree of elastic behaviour of liquid because this N_1 is the first normal stress difference which comes into the picture, if the material is having elastic behaviour only. So, that divided by the shear rate whatever is there, it gives the relative contribution of these two. So, that means, what we can understand with respect to the shear rate? Whatever $\frac{N_1}{\tau}$ information is there that gives the degree of elastic behaviour of a liquid.

So, this $\frac{N_1}{\tau}$, if it is very small tends to 0; that means, we can say that the elastic behaviour is negligible or the material is not having elastic behaviour, it is only having the viscous behaviour. If this $\frac{N_1}{\tau}$ is tends to infinite or large values, then we can say that material is having large elasticity or you know highly elastic behaviour is having that is what we see.

However, rather $\frac{N_1}{\tau}$, it is $\frac{N_1}{2\tau}$ which is called as a kind of recoverable shear ok. And then, this value greater than 0.5 is also possible and then it is not at all; it is not at all uncommon. Having $\frac{N}{2\tau}$ greater than 0.5 is not at all uncommon, but however we call them are highly elastic in nature.

N_1 information: N_1 versus $\dot{\gamma}_{yx}$information whatever is there that is very much lesser information compared to the τ versus $\dot{\gamma}$ information whereas, N_2 versus $\dot{\gamma}$ information even lesser information is available because of the limited studies have been carried out.

Sometimes it is also found that N_2 is order of magnitude smaller than N_1 and even it is negative. As we have seen as one example N_2 negative of N_2 versus $\dot{\gamma}$ shear rate that we have seen. Sometimes, it is also found that N_2 may also change the sign such kind of examples are also found. So, these are the few basic details about these viscoelastic materials. So, now, what we see? We see mathematical models for viscoelastic behaviour right.

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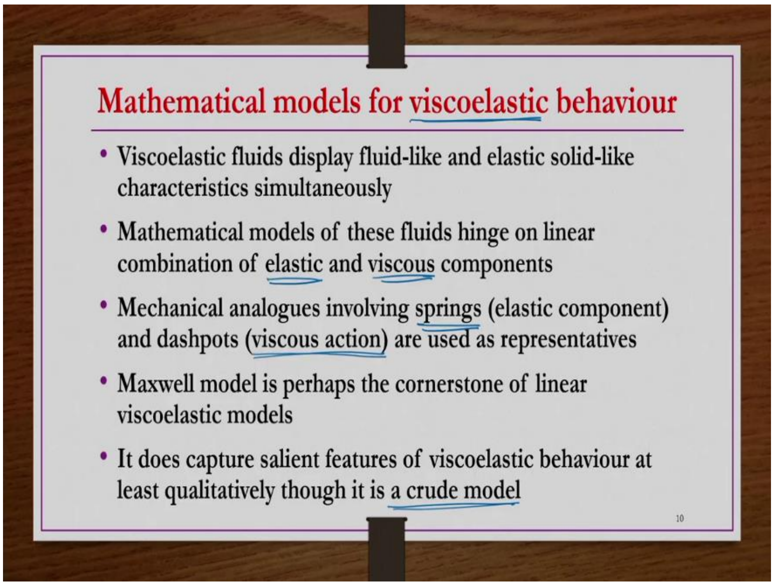

So, mathematical models, when we talk about mathematical models for a given material, so we need to have a components which represent true physics or physical behaviour of the material. So now, here what we have in this viscoelastic material? We have both elastic and then viscous nature so then we have to have a component which represent the elastic behaviour mathematically and which also have a component which mathematically represent the viscous behaviour of the material.

So, viscoelastic fluids display fluid like and solid like characteristics simultaneously that we already know. So, that is mathematical models of these fluids hinge on linear combination of elastic and then, viscous component. So, linear combination, nowadays non-linear combinations have also been found. And then several different types of you know viscoelastic fluid behaviour models are available.

But we take a few cases a couple of cases only, where we have a linear combination of an elastic and then viscous components. These linear combinations, again we can have them in series and parallel. So, accordingly, we have two models, we are going to see these two parallel to we are going to see these two linear viscoelastic fluid models.

So, analogously mechanical represented representation of this material, we can have like a springs now representing elastic components and then, dashpots representing the viscous actions. So, when we have these two in series, then we call that model Maxwell model and it is perhaps, the cornerstone of the linear viscoelastic models. And then, it does capture salient features though it is a crude model.

So, now, first we see this Maxwell model, where we have the linear combination of viscous nature as well as the elastic nature represented by dashpot and elastic component spring component ok, respectively.

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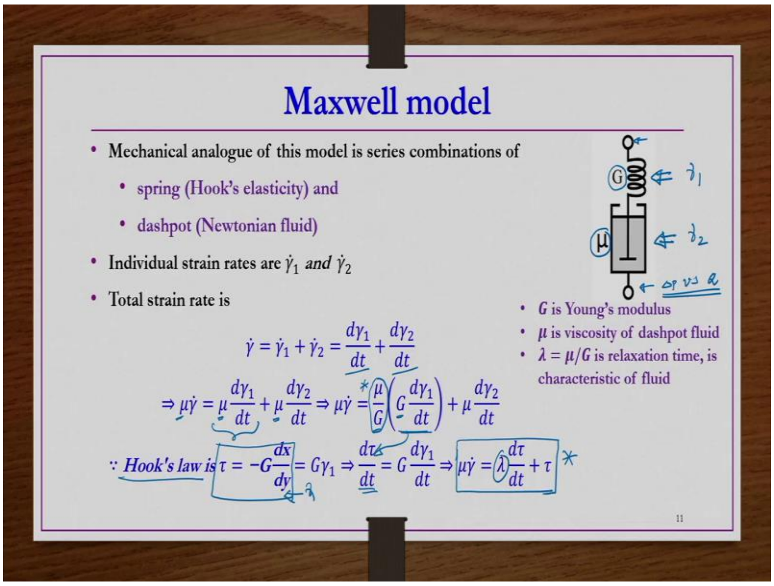

So, Maxwell model, mechanical analogue of this model is series combination of spring and dashpot right. So, they are arranged in a series like this. So, now, we have a spring here whose Young's modulus is G and then, we have taken a dashpot, so which is a vessel containing a Newtonian fluid.

To the outlet of this container, we have a flow constrictions so that you know whatever the ΔP versus flow rate information instead, they are proportional ok; ΔP pressure drop is proportional to the volumetric flow rate, such kind of flow constrictions are there.

So, the viscosity of the Newtonian fluid is μ that we have taken in this vessel, which we are now taking as a kind of representation of viscous behaviour as a dashpot and then, we are taking a spring to have a kind of elastic component elastic behaviour.

So, now, when you apply certain force to this top portion here, so that will go to the spring and hence, because of the moment in the spring whatever the piston that is connected to the spring that is inserted into this vessel. So, because of this piston, the deformation in the Newtonian fluid will take place ok. So now, whatever the strain rate are there, so they are different in each of them; they are not equal because it is passing first through the spring, then it is coming to the viscous fluid ok.

So, then whatever strain in the spring is there, let us say if you take γ_1, whatever the strain in the dashpot is there so that if you take a γ_2 right; so, then the total strain whatever is there would be the summation of these two. That makes a basis for this model and then subsequent mathematical simple equation takes place.

So, now, individual strain rates and $\dot{\gamma}_1$ and $\dot{\gamma}_2$; strains are γ_1 and γ_2 respectively, strain rates are $\dot{\gamma}_1$ and $\dot{\gamma}_2$ for these two components. So, total strain rate would be nothing but $\dot{\gamma} = \dot{\gamma}_1 + \dot{\gamma}_2$. $\dot{\gamma}_1$, we are writing $\frac{d\gamma_1}{dt}$ and then $\dot{\gamma}_2$, we are writing $\frac{d\gamma_2}{dt}$.

Now, either side, what we do? We first multiply by viscosity μ. So, $\mu\dot{\gamma} = \mu\frac{d\gamma_1}{dt} + \mu\frac{d\gamma_2}{dt}$. Next step, what we are going to do? To this elastic component, we are dividing and then multiplying by Young's modulus so that $\frac{\mu}{G}\left(G\frac{d\gamma_1}{dt}\right)$, we can have one term and then this $\frac{\mu}{G}$ is nothing but the relaxation time which is the characteristics of the material, which is the characteristic of the viscoelastic fluid ok. Plus second term $\mu\frac{d\gamma_2}{dt}$ is as it is right.

So, now, Hooke's law what we have? We have $\tau = -G\frac{dx}{dy}$. This d, this $\frac{dx}{dy}$ is nothing but γ_1. So, $\tau = G\,\gamma_1$ according to Hooke's law right. Now, if you take a differentiation either side with respect to time, so we have $\frac{d\tau}{dt} = G\frac{d\gamma_1}{dt}$.

So, that in place of here wherever you have $G\frac{d\gamma_1}{dt}$, you can simply write $\frac{d\tau}{dt}$. So, we have left hand side, $\mu\dot{\gamma}$ as it is, right hand side first term is $\gamma\frac{d\tau}{dt}+\tau$, this is what we are having. This is the primary equation for the Maxwell model. Now, this equation, you can solve for different γ values and then see the behaviour of this material with respect to the time.

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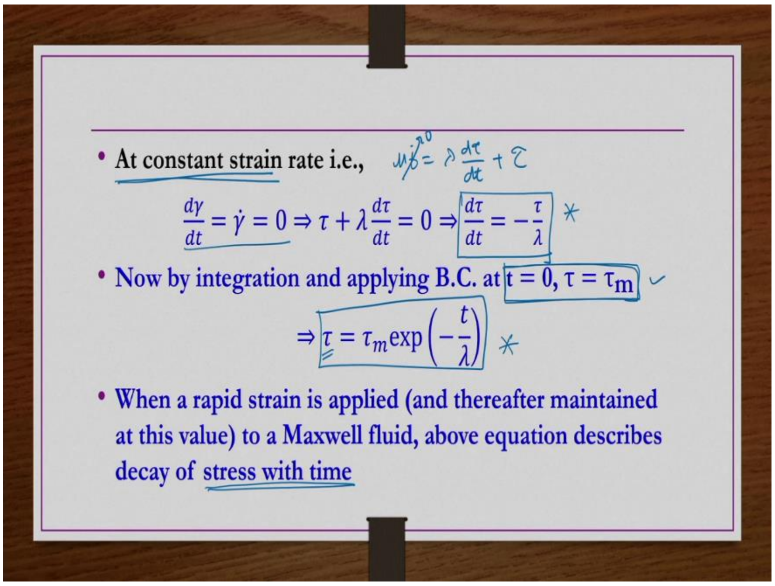

So, a constant strain rate. So, that means, when $\dot{\gamma}$ = 0, when $\dot{\gamma}$ = 0 that is $\frac{d\gamma}{dt}$ is equals to 0. So, in the previous equation whatever that we have this equation, $\mu\,\dot{\gamma} = \gamma\,\frac{d\tau}{dt} + \tau$ this what we have.

So, if the at constant strain rate that is $\dot{\gamma}$ = 0 or at constant strain; at constant strain rather than rate at constant strain, what we have? This γ is constant. So, then $\dot{\gamma}$ should be 0. So, $\frac{d\gamma}{dt}$ is equals to 0. So, then $\tau + \gamma\frac{d\tau}{dt} = 0$ that we can have a simply $\frac{d\tau}{dt} = -\frac{\tau}{\gamma}$.

So now, you can integrate this equation and then, apply at t = 0 whatever this stress is there so that is the maximum; whatever the stress at t = 0 is there that is the maximum because that is gradually decaying as the spring is moving up and down. So, the piston connected to that spring is you know dissipating that stress to the Newtonian fluid that has been taken in the vessel as a dashpot.

So, that is gradually decaying. So, that is what physically also we understand. So, at initial time, t = 0, this stress has to be maximum for this model. So, then when you integrate this equation and then apply this boundary condition, you will get simply $\tau = \tau_m exp\left(-\frac{t}{\gamma}\right)$, right.

So, here when rapid strain is applied and thereafter maintained at this value to this maximum fluid, this above equation what indicates? It indicates the stress whatever the tau is there that stress decreases, decreases with time or decay of stress with time that is what we can see from this equation right. So, graphically, also we are going to see this behaviour of this particular equation now.

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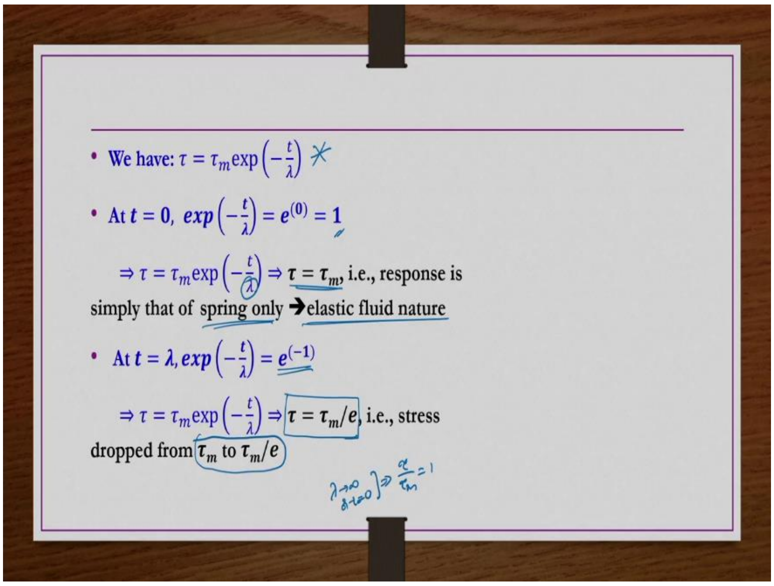

So, now just now we derived for Maxwell for viscoelastic fluid, whatever the Maxwell model is that we have $\tau = \tau_m exp\left(-\frac{t}{\gamma}\right)$ right. Now, when t = 0, $exp\left(-\frac{t}{\gamma}\right) = exp^{(0)}$. So, whatever this exponential term is that is equals to 1.

That means, $\tau = \tau_m$ that is response is simply that a spring alone, the elastic fluid nature is only showing. Because this whatever the $\frac{t}{\gamma}$ can be 0 for two cases, when t = 0 or when $\gamma = \infty$. So, γ is ∞; that means, purely elastic behaviour. γ indicates the relaxation time of the material, which is the characteristic property of the viscoelastic material. If it is large, that means, elastic nature is very large for that material.

Like γ is like a property of the material like density, the viscosity, thermal conductivity etcetera are we are having physical properties. Like, similarly one of the rheological property of this viscoelastic material is the, this relaxation time γ ok. It does not change with the condition, it is the material property that we are having.

So, at t = γ, if you do the same thing $exp\left(-\frac{t}{\gamma}\right)$ would be nothing but $exp^{(-1)}$. So, then what we have? We have $\tau = {}^{\tau_m}/_{e}$ that is stress dropped from τ_m to ${}^{\tau_m}/_{e}$ value as the time progresses from t = 0 to t = γ. So, now from here, what you have? ${}^{\tau}/_{\tau_m}$ = 1, when γ tends to ∞ or t is equals to or t tends to 0, this is what we have.

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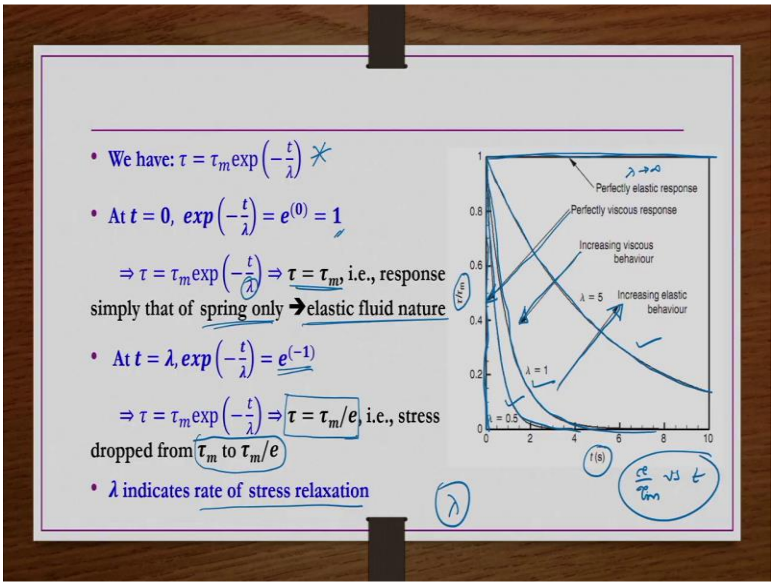

So, pictorially, if we have; so here whatever the γ is there that indicates a rate of stress relaxation. So, pictorially, if you see when γ is very large, when γ is very large γ tends to ∞; that means, $exp\left(-\frac{t}{\gamma}\right)$ is there that is $exp^{(0)}$. So, that is 1. So, then ${}^{\tau}/_{\tau_m}$ should be equals to 1. Because now this we are plotting not τ versus t, but we are plotting ${}^{\tau}/_{\tau_m}$ versus t.

So, when γ tends to ∞, that is, material is fully elastic; perfectly elastic response is there. So, then ${}^{\tau}/_{\tau_m}$ would be having this line like this. But as the γ value decreases, the elastic responses gradually decreases and the viscous and then, viscous responses gradually increases ok. And then, at certain position, what we have? We have a kind of perfectly elastic; perfectly viscous behaviour when γ tends to 0 right.

So, as γ decreases, you can the whatever the τ/τ_m is equals to 1 is there, so that you know we will start showing these kind of behaviour for decreasing γ values with respect to time. With respect to time, τ/τ_m maybe having these kind of behaviour are shown here; τ/τ_m versus may have this kind of behaviour you know for different γ values.

γ tends to ∞, we have this nature, when γ decreases, so then you have a this kind of curves like this and whatever drawn here like this. Further when γ equals to 1, you can have this behaviour. When γ is equals to 0.5, you can have this inform this behaviour right.

So, what we can see? When γ is decreasing, viscous behaviour is increasing; when γ is increasing, elastic behaviour is increasing for that material. So, γ is a kind of a characteristic of a material. So, which indicates the how strongly elastic nature that material is having. If it is long, if it is large value then it is highly elastic material.

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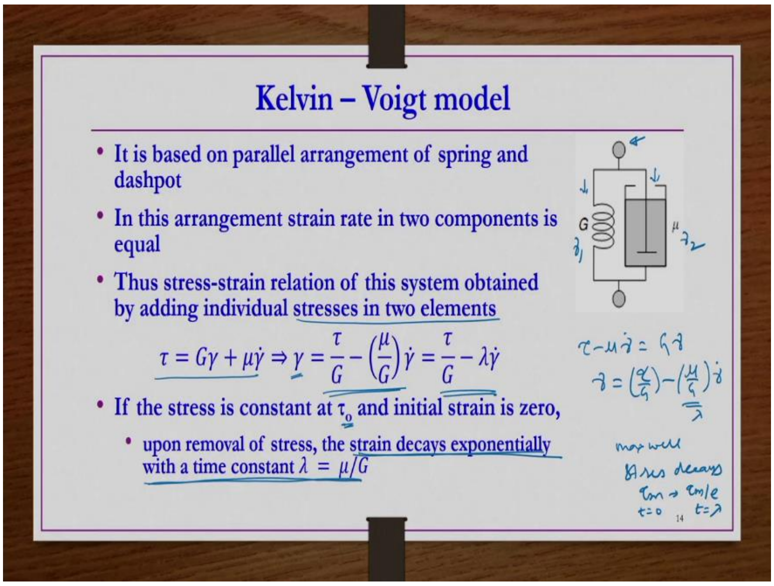

So, now we see Kelvin-Voigt model which is the other model, where we have the linear combination of again spring and dashpot, but now these are parallel to each other. So, now, we have a spring, whose strain rate whose strain is γ_1 and then, whatever the strain in the viscous fluid or the container having viscous Newtonian fluid, the strain is γ_2 alright.

So now, again here also the material whatever the Newtonian fluid is there that is taken or the Newtonian part viscous part is that is taken in this container which is again at the bottom is having the flow constriction with the pressure drop is proportional to the volumetric flow rate.

So, now what happens here when you apply external force at the top? So, then whatever the strain rate is there that is going to be same for both of the components, because it is equally transmitted; but the stresses are different, but the stresses are different. So, the stresses are added together in general to get the total stress right.

Whereas, the in the Maxwell model, the strain rates are different, stresses are same so then, we added the strain rates together to get the total strain rate. So, that is the difference between these two models. So, in this arrangement to; in this you know parallel arrangement whatever the strain rate into components is there that is equal. So thus, stress strain relation of the system obtained by adding individual stresses in two elements. So, that is $\tau = G\gamma + \mu\dot{\gamma}$ right.

So, now, again what we can do? We can write this gamma one side so that $\frac{\tau}{G} = -\left(\frac{\mu}{G}\right)\dot{\gamma}$ we can have the other side. This equation what I am trying to do? I am writing $\tau - \mu\dot{\gamma}$ first $\tau - \mu\dot{\gamma} = G\gamma$.

So, $\gamma = \frac{\tau}{G} - \left(\frac{\mu}{G}\right)\dot{\gamma}$ and then, this mu by G, we can write it as γ so that this equation γ = $\frac{\tau}{G} -$ γ $\dot{\gamma}$, we can write. This γ is nothing but again relaxation time of the viscoelastic fluid; relaxation time on the viscoelastic fluid.

So, if the stress is constant at tau naught and the initial strain is 0, then what we can see? Upon removal of stress, the strain decays exponentially with the time constant γ $\frac{\mu}{G}$. In the previous case, Maxwell model, what we have? Stress decays exponentially from τ_m to ${}^{\tau_m}/_{e}$ as time progresses from t = 0, t = γ right. But here, this strain decays exponentially with a time constant γ = $\frac{\mu}{G}$. That is what difference between these two models.

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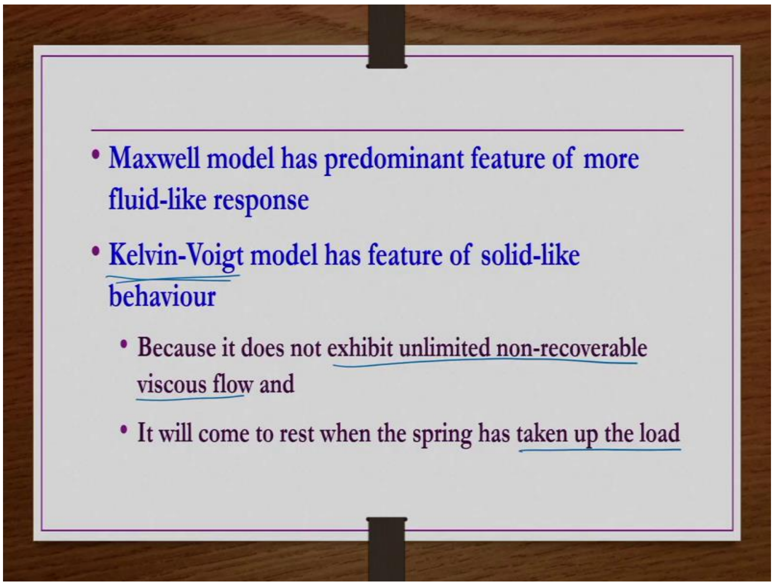

So, by comparison, Maxwell model has a predominant feature of more fluid like response whereas, the Kelvin-Voigt model has feature of solid like behaviour because this material does not exhibit unlimited non-recoverable viscous flow. Further, it will come to rest, when the spring has taken up the load. So, this Kelvin-Voigt model has a feature of more solid like behaviour, whereas, the Maxwell model having the feature of more fluid like response.

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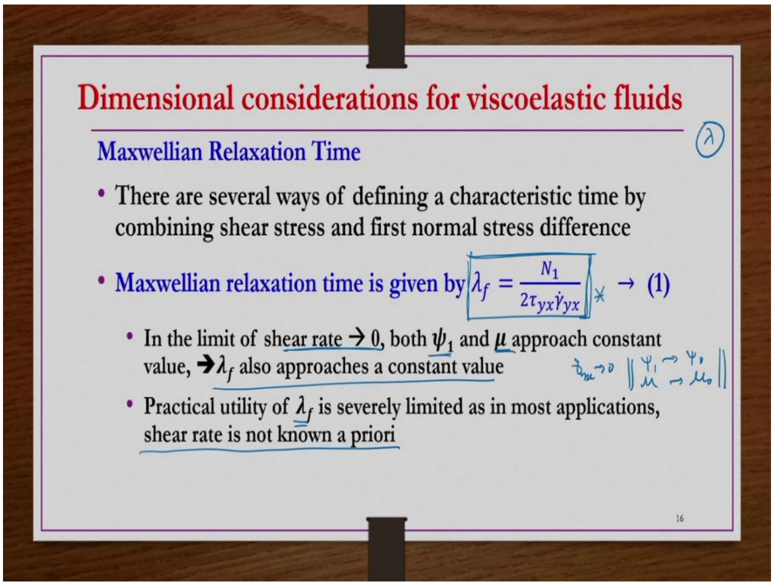

Now, the dimensional considerations for viscoelastic fluids we see; two different possibilities we see under the dimensional consideration for viscoelastic fluids. So, first one is the Maxwellian relaxation time. So, whatever the relaxation time γ is there, so can we have a kind of a dimensionless relaxation time.

That is the point in a basic in one single word if you want to say, single line if you wanted to say. If one single line if you wanted to say whatever the relaxation time γ is there, so can we have a kind of dimensionless representation of the same time constant or relaxation time?

So, first we see the Maxwellian relaxation time. So, there are several ways of defining a characteristic time by combining shear stress and first normal stress differences, ok. So, Maxwellian relaxation time is taken that information into the consideration that whatever the N_1 versus $\dot{\gamma}_{yx}$information is there, similarly whatever the τ_{yx} versus $\dot{\gamma}_{yx}$information instead that has been taken in the consideration such a way that we can have this γ_f is equals to ${N_1}/{2\tau_{yx}.\dot{\gamma}_{yx}}$ which is known as the Maxwellian relaxation time; which is known as the Maxwellian relaxation time.

So however, in the limits of zero shear rate or the low shear rate, what we know? This ψ_1 and then μ both approaches a constant value; ψ_1 approaches ψ_0. And then, μ approaches a whatever the μ_0 ok; ψ_1 approaches constant value ψ_0 and then, μ approaches zero shear viscosity μ_0. So, both of them are constant; both of them are constant that we have already seen.

So, obviously, if these two are under $\dot{\gamma}_{yx}$ tends to 0, if these two are the having limiting constant value. So, then whatever the γ, we have defined based on that information that will also have a, you know constant limiting value.

However, practical utility of γ_f is see is severely limited as in most applications shear rate is not known a priori. So, if you wanted to know this one, you have to know what is this $\dot{\gamma}_{yx}$ and then all that for a given application, then only you can use it.

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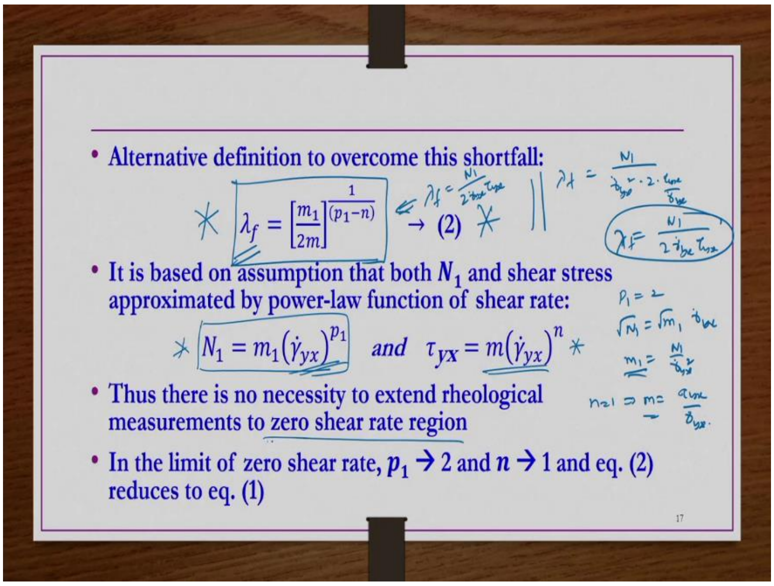

So, in order to overcome, there are different ways of you know defining a relaxation time, dimensionless or relaxation time have been carried out and one of that representation is $\gamma_f = \left[\frac{m_1}{2m}\right]^{\frac{1}{(p_1-n)}}$ that is what we have.

And then, it is based on assumption that both N_1 and shear stress approximated by power law functions of shear rates. So, that is N_1. This is the assumption and defining this particular you know alternative relaxation time γ_f; alternative dimensionless relaxation time γ_f is represented by this way, right.

So, for in order to define this one, these are the two assumptions have been taken. What are these assumption? That normal stress, first normal stress differences versus shear rate. So, whatever the first normal stress difference versus shear rate and then, shear stress versus shear rate information whatever there, experimentally obtained information instead they are represented by some power law model like this, right. Then they obtained this one.

So, however, thus, there is no necessity to extend rheological measurements to zero shear rate region, if you are using this equation; if you are using this equation ok. But in the limit of zero shear rate, p_1 tends to 2 and then, N_1 and n tends to 1. This above equation whatever is there that will again get back to our Maxwellian relaxation time of a $\gamma_f = {N_1}/{2\tau_{yx}.\dot{\gamma}_{yx}}$, this is what you get.

How? Let us say from this equation N_1 information, if p_1 = 2, if p_1 = 2, then $\sqrt{N_1} = \sqrt{m_1} = \dot{\gamma}_{yx}$. That means, m_1 is nothing but ${N_1}/{\dot{\gamma}_{yx}}^2$. This we already know. And then, similarly when n = 1, then we know m = ${\tau_{yx}}/{\dot{\gamma}_{yx}}$.

If you substitute this m_1 and then, m here under the limiting conditions, then we have γ_f is equals to m_1 is ${N_1}/{\dot{\gamma}_{yx}}^2$. And then, m is nothing but ${\tau_{yx}}/{\dot{\gamma}_{yx}}$ so that is nothing but; because whole power 1 by p_1 - n; p_1 is 2, n is 1 so that power is 1. So, then what we have? ${N_1}/{2\dot{\gamma}_{yx}.\tau_{yx}}$ this is what having. So, which is same as which is nothing but Maxwellian relaxation time.

So, this is better way. What is the difference between that equation number 1 and 2 that two different forms of Maxwellian relaxation time is that if we use this one, you do not need to extend the rheological behaviour to zero shear rate region.

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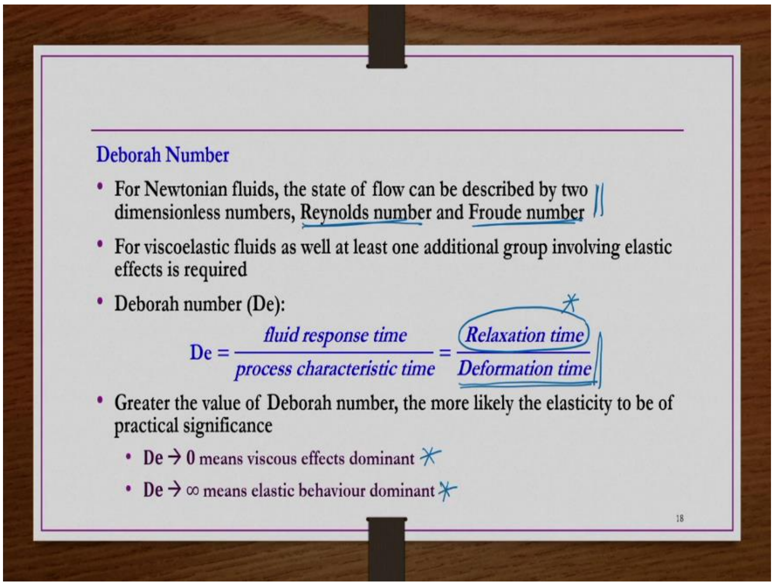

So, another dimensional consideration for a viscoelastic fluid behaviour, we take in the form of Deborah number. What it is that; is what we are going to see. So, in general, for Newtonian fluids the state of flow we can describe by two Reynolds two dimensionless numbers; one is the Reynolds number, another one is the Froude number.

Reynolds number is nothing but ratio between inertial force to the viscous force. And the Reynolds number is nothing but the ratio between inertial force and viscous force. So, whereas the Froude number is nothing but ratio between inertial force and gravitational force. So, these two are known. So, then you can you know define the state of a flow for a given Newtonian fluid ok.

So, for viscoelastic fluids as well at least one additional group involving elastic effects is required. So, that is what we call Deborah number. So, this Deborah number is nothing but the ratio between relaxation time and deformation time. This relaxation time is nothing but the material characteristic, it is the material characteristic. Deformation time depends on the flow situation, depends on the flow situation right.

So in the, in obviously, so relaxation time also that is you know indicates you know if it is large so then obviously, its elastic behaviour is more significant. That is what we can see. That is what we understand. So, if it is this Deborah number is very large, so then it represent highly elastic behaviour in the flow system right. If it is very small, so that means, we can say that so called viscous effects or so called elastic effects are having less influence on the flow situations.

So, greater the value of Deborah number, more likely the elasticity to be of practical significance in that flow situation. Why flow situation? Because the deformation time depends on the flow situation, right. So, that we take one example and then, realize quickly.

So, if Deborah tends to 0, Deborah number tends to 0, we can say more viscous effects or viscous effects are dominant. If Deborah tends to ∞, we can say that elastic behaviour is dominant. So now, we see couple of problems to conclude viscoelastic fluid behaviour ok.

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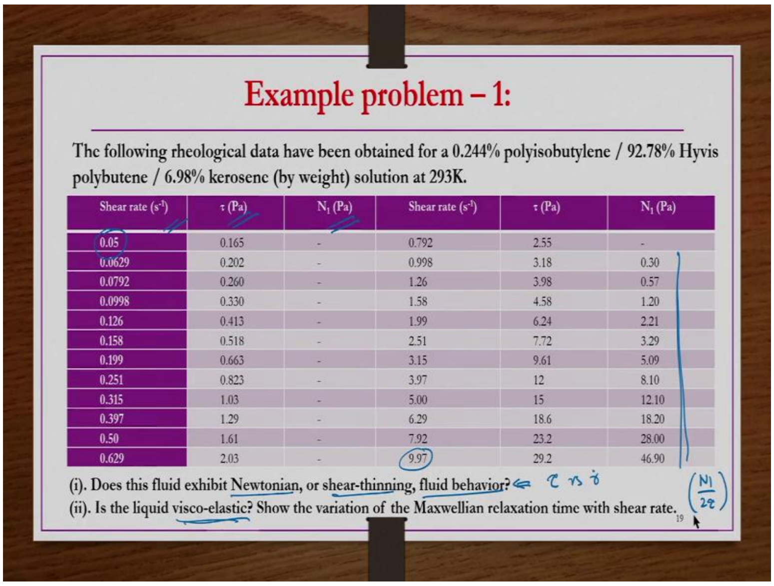

So, and, so fluid rheological behaviour has been measured and then that is given. Shear stress versus shear rate information and then first normal stress; first normal stress difference versus shear rate information is given. Shear rate is changing from 0.05 to 9.97 that is almost 0 to 10 second inverse ok. When shear rate is in this range, so corresponding shear stress and then, first normal stresses are given here; first normal stress differences are given here.

So, the first part of the question does this fluid exhibit Newtonian or shear thinning fluid behaviour? If you wanted to know Newtonian or shear thinning behaviour, what you have to do? You have to plot τ versus $\dot{\gamma}$ right. If it is passing through origin and linear, it is a Newtonian behaviour. If it is a passing through origin, but you know it is not linear, then we can have a shear thinning or shear thickening behaviour ok. So, that we can see.

Second part of the question; is the liquid viscoelastic? Obviously, it is viscoelastic because first normal stress difference is given here lot right. So, we can see the Maxwellian relaxation time and then, ${N_1}/{2\tau}$ calculations also we can see, because we know that ${N_1}/{2\tau}$ indicates the recoverable shear for the case of viscoelastic fluids right. So, then, we also need to do Maxwellian relaxation time versus shear rate plotting.

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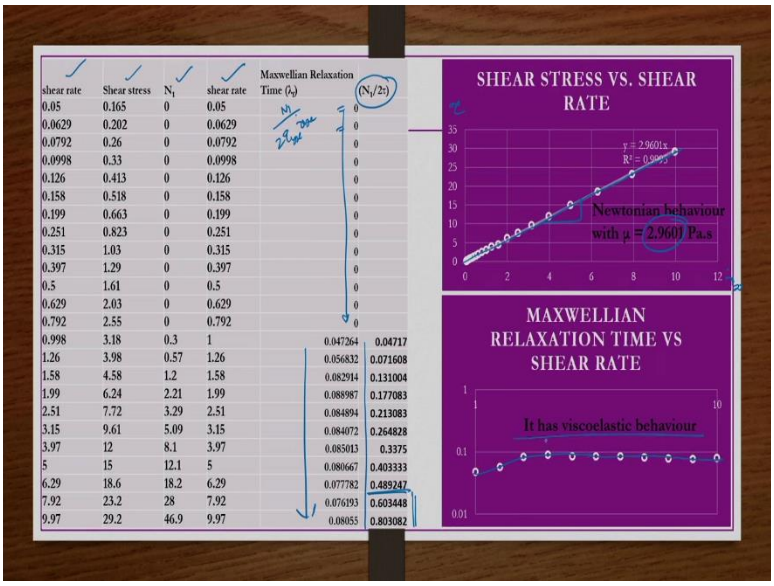

shear rate	Shear stress	N_1	shear rate	Maxwellian Relaxation Time (λ_f)	($N_1/2\tau$)
0.05	0.165	0	0.05	0	
0.0629	0.202	0	0.0629	0	
0.0792	0.26	0	0.0792	0	
0.0998	0.33	0	0.0998	0	
0.126	0.413	0	0.126	0	
0.158	0.518	0	0.158	0	
0.199	0.663	0	0.199	0	
0.251	0.823	0	0.251	0	
0.315	1.03	0	0.315	0	
0.397	1.29	0	0.397	0	
0.5	1.61	0	0.5	0	
0.629	2.03	0	0.629	0	
0.792	2.55	0	0.792	0	
0.998	3.18	0.3	1	0.047264	0.04717
1.26	3.98	0.57	1.26	0.056832	0.071608
1.58	4.58	1.2	1.58	0.082914	0.131004
1.99	6.24	2.21	1.99	0.088987	0.177083
2.51	7.72	3.29	2.51	0.084894	0.213083
3.15	9.61	5.09	3.15	0.084072	0.264828
3.97	12	8.1	3.97	0.085013	0.3375
5	15	12.1	5	0.080667	0.403333
6.29	18.6	18.2	6.29	0.077782	0.489247
7.92	23.2	28	7.92	0.076193	0.603448
9.97	29.2	46.9	9.97	0.08055	0.803082

So, the same information plotted here. So, this shear stress versus shear rate and then, first normal stress difference versus shear rate is given. So, Maxwellian relaxation time that is $\gamma_f = {N_1}/{2\dot{\gamma}_{yx}.\tau_{yx}}$ information that you have when you do it, so then you get these values right.

And then, we also know wanted to know whether this material is having sufficiently elasticity or not. So, then ${N_1}/{2\tau}$ calculation, we have to do. When we do these things, so we can see ${N_1}/{2\tau}$ is having more than is having more than 0.5 also. If it is more than 0.5, we have already seen you know that material is a having highly elastic behaviour.

And then, that elastic behaviour is showing more significance; when the shear rate is larger from when shear rate goes 8 or higher so then we can see ${N_1}/{2\tau}$ is going more than 0.5 values. That means, you know the elastic behaviour is having large influence on the flow situations.

So, coming to the rheology of Newtonian or shear thinning behaviour to know, so the shear stress τ versus $\dot{\gamma}_{yx}$ is plotted here. So, then, we can see this curve in the all these data points you know forming a straight line providing the viscosity 2.9601 pascals and then, it is passing through the origin. So, it is a Newtonian behaviour right.

So, we also asked to plot γ_f Maxwellian relaxation time versus shear rate. So, then that you plot so you can see this curve here. It is not 0, so then, it is having significant value. So, the elastic behaviour this material is also having elastic behaviour. One last example problem on Deborah number.

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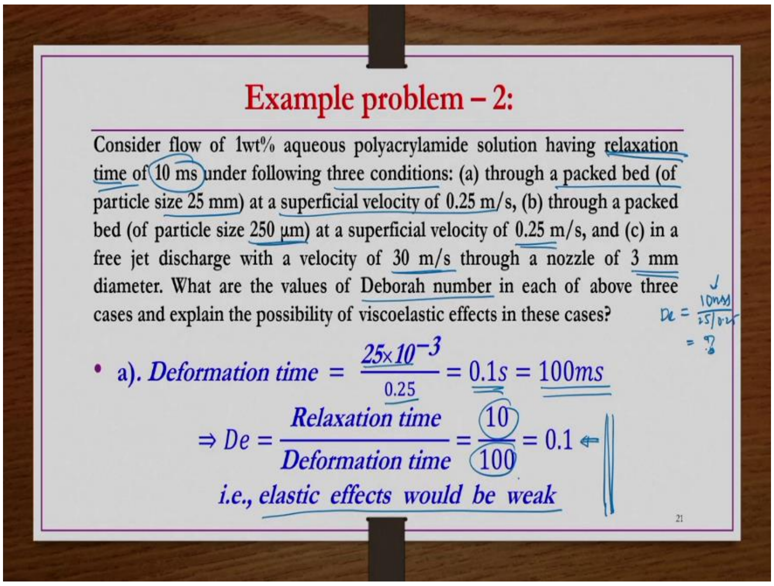

So, consider a flow of 1 weight percent aqueous polyacrylamide solution having relaxation time of 10 millisecond. So, relaxation time is given that is the characteristics of the material, it is like a property of the material. So, this material is flowing under three different condition; one condition is a packed bed of particle size 25 mm at a superficial velocity 0.25 meter per second.

So, Deborah number if you wanted to know, you need to know this relaxation time divided by the deformation time. This deformation time is nothing but whatever the particle size is there that divided by the velocity if you do, you will get you will get the so called deformation time.

So, then, this overall is 10 milliseconds also you convert into the second. So, then you will get the Deborah number. So, you have to calculate Deborah number like this and then accordingly, you have to say whether the material whether the viscous effect is important or not. So, that is.

Same is the second situation also, the same material is flowing through a packed bed a particle size 250 microns, but it is superficial velocity of 0.25 meter per second. And third case, in a free jet discharge with a velocity of 30 meter per second through a nozzle of 3 mm diameter. So, for each of the case, we have to find out the Deborah number and then, explain the possibility of viscoelastic effects in these cases.

So, first case deformation time is nothing but whatever the particle size 25 mm divided by velocity 0.25 meter per second. So, you get 0.1 second that is nothing but 100 milliseconds alright. So, then Deborah number would be relaxation time by deformation time; relaxation time is 10 milliseconds divided by the deformation time is nothing but the 100 milliseconds, just now we calculate it. So, it is 0.1.

So, it is very small value. So, the elastic effects would be weak or small in this case first case, where this material is flowing through packed bed of particle size 25 mm at a superficial velocity of 0.25 meter per second. Now, what is the physical importance of this calculation?

So, though the material that you are processing is having elastic behaviour. So, your condition your velocity if you maintain less than 0.25 meter per second and then, particle size is not less than 25 mm or more than 25 mm, you will have a situation like you know the elastic effects would be negligible.

So, in your design equation for the packed bed, you do not need to consider so called elastic effects. You can straightaway use the Ergun’s equation or Kozeny Carman equation depending on the flow situation, flow condition and then, get the pressure drop information and all that without worrying about the elastic behaviour of this material. That is the physical importance of you know getting these numbers calculated.

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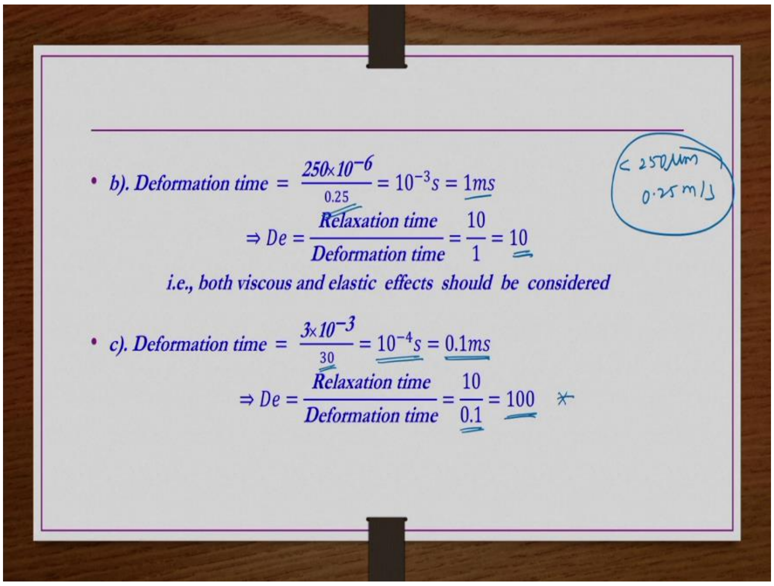

Now, similarly for the second case deformation, you see it is 250 microns. So, 250 into 10 power minus 6 meters of particle size divided by 0.25 you know meter per second of the velocity. So, then you get 10^{-3} seconds so that means, 1 milliseconds.

So now, Deborah number, relaxation time by deformation time that is $\frac{10}{1}$ is 10. So, that is it is neither small nor large, nor very large. So, both viscous and then elastic effects would be important; both of them should be considered. So, then if you have the particle size something like 250 microns and then, velocity like you know higher than the 0.25 meter per second something like this if or your flow conditions, in the design of packed bed, you have to worry about incorporating the changes in the pressure drop relations because of a elastic behaviour.

That should also be incorporated or if you cannot change the design already existing packed bed, so then you have to change your operating conditions such a way that this Deborah number should come down. Like increasing the particle size or decreasing the velocity such kind of thing things, you have to do and then find out like combination where Deborah number is very small so that you can avoid the elastic contribution in the overall flow situation.

If you cannot do that one in the design, you have to incorporate the effect of elasticity for this packed bed case for this flow conditions. So, both viscous and elastic effects would be considered for the first case. So, the last case 3 mm nozzle dia. So, 3 x 10^{-3} meters and then, 30 meter per second is the velocity.

So, 10^{-4}seconds that is 0.1 milliseconds. So, now Deborah number would be relaxation time 10 divided by the deformation time 0.1, it is 100. So, it is very large value. It is very large value so that you know elastic effects would be very strong, you cannot avoid it any circumstances, you have to incorporate in the design also, the elastic effects should be incorporated.

You have to incorporate in elastic effects as well in the design as well as the operational situations for this condition, third condition right. That is the point you know having these calculations. So, this is all about the viscoelastic fluid behaviour including the possible mathematical models and then, example problems. So, the references for this lecture are given here.

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These two references are very essential for this viscoelastic fluid behaviour right. So, other references are also provided here ok, as a kind of reference.

Thank you.