Transport Phenomena of Non-Newtonian Fluids
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Lecture - 07
Capillary Viscometers - Errors and Corrections II

Welcome to the MOOC's course Transport Phenomena of Non-Newtonian Fluids. The title of this lecture is Capillary Viscometers Errors and Corrections part 2. We have been discussing how to measure rheology of an unknown fluid using a capillary viscometer that is what we have been discussing in last two classes right. So, then we have a recapitulation of what we have studied in last two classes.

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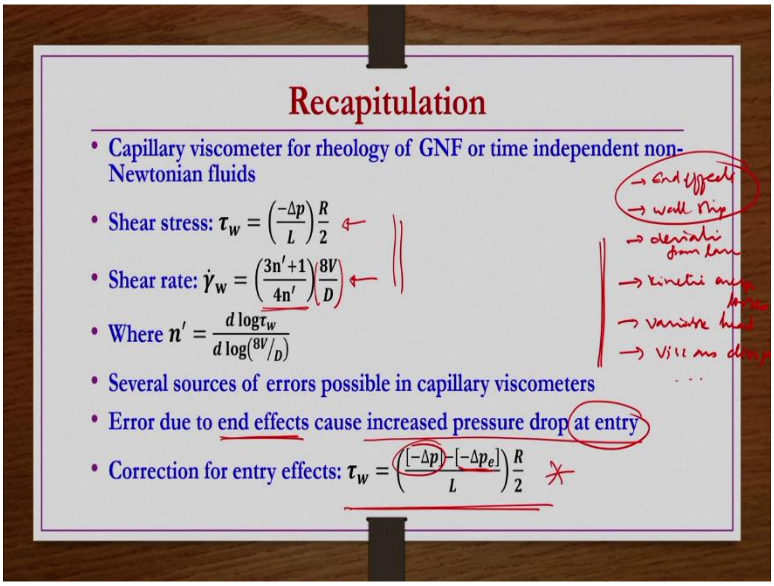

Capillary viscometer for rheology of generalized Newtonian fluids or time independent non-Newtonian fluids, when you apply capillary viscometer or when you use capillary viscometer for rheological measurements of a time independent non-Newtonian fluid. So, then how to design or how to develop the working principles, working principles in the sense the equations that are required to obtain the shear stress, shear rate etcetera we have already discussed.

So, what we have find? We find that you know shear stress using capillary viscometer that you can get it as $\tau_w = -\left(\frac{-\Delta p}{L}\right)\frac{R}{2}$, right. So, the wall shear stress, so; that means, if you know the pressure drop so then you can obtain the wall shear stress without any difficulty. And then in obtaining this relation we have seen that the fluid nature and nature of the flow whether laminar or turbulent or transition region which region of the flow that we have not considered.

And then nature of the fluid also not brought into the picture until we find in this relation $\tau_w = -\left(\frac{-\Delta p}{L}\right)\frac{R}{2}$; that means, as long as the flow is fully developed, incompressible and steady then we can use this relation for any fluid that is flowing through a capillary, right.

So, then specific to the fluid then shear rate expression we have obtained; the shear rate wall shear rate expression for a time independent non-Newtonian fluids or generalized Newtonian fluids we found it as $\dot{\gamma}_w = \left(\frac{3n'+1}{4n'}\right)\frac{8V}{D}$, this $\frac{8V}{D}$is a nominal shear rate or apparent shear rate for a non-Newtonian fluids and then it is true shear rate for the case of Newtonian fluids.

So, if you wanted to have the true shear rate for a non-Newtonian fluids especially time independent non-Newtonian fluids then you have to have a correction factor, that correction factor is nothing but $\frac{3n'+1}{4n'}$ right.

Then n' here we found it as nothing but $\frac{dlog\tau_w}{dlog(^{8V}/_{D})}$ that is shear stress versus apparent shear rate when you plot on a log-log scale whatever the curve that you get. So, that slope different τ_w that you can find out that is nothing but n' ok.

Then after that what we have seen? After though we have; though we have a relations to get the true shear stress and true shear rate information for a time independent non-Newtonian fluids, then we what we realized that there are several possible sources of errors existing in this measurements.

Because what we have seen what are the primary assumption that we have taken, we have taken that you know flow is a laminar fully developed steady and then fluid is incompressible. So, then obviously, when the flow is not fully developed. So, then there would be some errors and then if the flow is not laminar region in if the flow is not within the laminar region then again there will be errors like that different errors are possible.

So, we have enlisted all such kind of errors and then we have seen primarily end effects, wall slip effects and deviation from laminar behavior then kinetic energy losses due to the sudden contradiction of cross section; sudden contraction of a cross section area at the connection or at the point where we are connecting the capillary to the barrel right.

Then variable fluid head then viscous dissipation and so on so. We have seen several errors are possible right. So, what we have seen? Some of them may be minimized or we can get rid of some of these errors by making small-small changes in the design or just operating in such a way that some of these errors are negligible or even or completely disappeared that is possible, but what we have seen this, end effects and then wall slip effects are very essential.

So, then accordingly what we have started? We started with the end effects in the previous lecture. What we have seen? Because of the end effects they cause increased pressure drop at the entry; at the entry because of the sudden contraction of the flow area. So, there is an increased pressure drop.

So, then we have to find out how much that increased pressure drops so that we can make adjustment in the rheological measurements and then that may we have we had a method. And then we found how much pressure has increased at the entry that we found and then that pressure we subtracted from the overall experimentally measured pressure.

So, whatever the corrected pressure is there that we have used to obtain the corrected wall shear stress right, because of the entrance effects what happens you know the pressure is increasing if the pressure is changing the change in pressure is occurring. So, then obviously, that will show influence on the shear stress because shear stress is directly proportional to pressure gradient that is what we have seen.

So, then accordingly once we found out how much is that pressure increase that pressure we have subtracted from the measurable or measured pressure drop then we have obtained the corrected wall shear stress corrected for the end effects. So, that is what we have seen and then after seeing that pressure we have also seen an example problem.

Because of which what we realized that after incorporating the end effects the rheological behavior by using different lengths of capillaries you know they are all superimposing onto each other. Before incorporating the correction for the end effects, they were not superimposing onto each other that, is shear stress versus nominal shear rate. They were giving different curves for different L values; different L values of the capillary, but after incorporating the end effects in the shear stress calculations then that corrected shear stress versus a nominal shear rate or apparent shear rate when we plotted, we found that all the curves all the data from different capillary lengths are super imposing that ensuring that the correction for the end effect has been incorporated right.

So, now in this lecture what we will be seeing is the wall slip effect, first we discuss what it is and then we see how to make amendments corrections, because of that such because of such existing walls slip at the wall ok at the capillary tube wall that is what we are going to see now.

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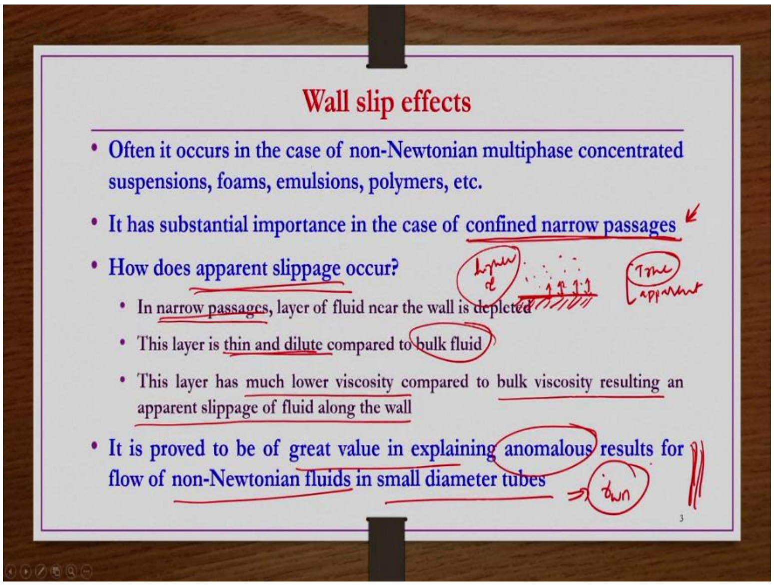

So, wall slip effects are very much essential especially in the case of a multi-phase non-Newtonian concentrated suspensions, foams, emulsions, polymers etcetera, right. It has been found that in the case of a confined narrow passages the role of wall slip becomes more important right, confined narrow passage so that means, diameter. Diameter is playing a role, right.

Let us say capillary your within in the context of capillary if we talk if the confinement increasing or decreasing; that means, you know diameter increasing or decreasing accordingly, right. So, when we it has been found that if you change the diameter that is so, the wall slip whatever is there that is playing you know essential role very important roles, it plays essential role on the you know rheological behavior or overall flow equations or flow data that we are getting ok.

So, it has been founded substantial importance of wall slip effect in the case of confined narrow passage; that means, the passage whatever is there if that decreases the wall slip effects become more and more significant. We started taking the narrow passages because we wanted to have the variation in the shear stress or variation in the shear rate from center to the wall of the capillary should be negligible.

With that purpose, we have started the you know equation development etcetera by taking narrow passages narrow capillary cylinders like that we have taken. But when you take such kind of narrow passages then wall slip will come, will become you know significant and then accordingly corrections should be provided. So, how does it occurs?

The so called apparent slippage how does it occur that we have to understand, actually there are different types of slips are have been you know reported. So, one is the true slip another one is the apparent slip which are relevant as of now. True slips in general occurs in the case of a low viscosity fluids or gases where the attraction between the molecules of the low viscosity fluid or gases and this wall whatever the container wall or you know capillary wall whatever you have taken.

So, there is an attraction force obviously. So, at higher shear stress what happens? These molecules overcome the attraction force whatever is there between the molecules of these gases. And then this solid surface that attraction forces would be overcome by these gas molecules or low viscosity fluid molecules especially at higher shear stress.

And then because of that one there will be a depletion of fluid layer at the surface of the wall and then because of the that depletion the local viscosity at the wall would be much more lower compared to the bulk viscosity of the fluid in the bulk region because of that depleted viscosity or depleted layer at the wall.

So, the velocity of the fluid rather becoming 0 at the wall, it will be having some substantial velocity and then that velocity is the slip velocity. And then such kind of you know slip velocity in the case of low viscosity gases because of the overcoming the attraction force between the molecules and then solid wall surface you know at high shear stress that is called is a true slip in general.

But such kind of slippage would also occur in the case of a high viscosity fluids like non-Newtonian fluids concentrated suspensions etcetera as well. What happens in the case of a high viscosity non-Newtonian multi-phase concentrated suspensions when they are flowing through narrow passages, that is important when they are flowing through narrow passages layer of fluid near the wall is depleted. And this layer is very thin and dilute compared to the bulk fluid its viscosity is low.

And then this layer has much lower viscosity compared to the bulk viscosity resulting an apparent slippage of fluid at the fluid along the wall right. So, now, this causes you know because of this you know depleted layer at the wall you know the local viscosity becomes very less. So, then; obviously, the velocity would be high because the resistance amongst the fluid molecules is less if the viscosity is less. So, then viscosity is decreased that means, the velocity at the wall would increase.

So, because of that one at the wall rather having the no slip velocity or 0 velocity, we will be having a apparent slip velocity that would be having substantial value if the passage area is very narrow; if the fluid passage area is narrow. And then it has been proved that it is of great value in explaining anomalous results for the flow of non Newtonian fluids in small diameter tubes.

We are going to see with an example as well. We do certain kind of simplifications or you know design equations whatever we have developed previously for the case of shear rate nominal shear rate those equations would be corrected and then new equations when we use.

So, then again we can see that you know after incorporating the wall slippage you know the shear stress versus apparent shear rate information you know by using different capillaries you know does not show different curves. All the curves would be super imposing onto each other ok.

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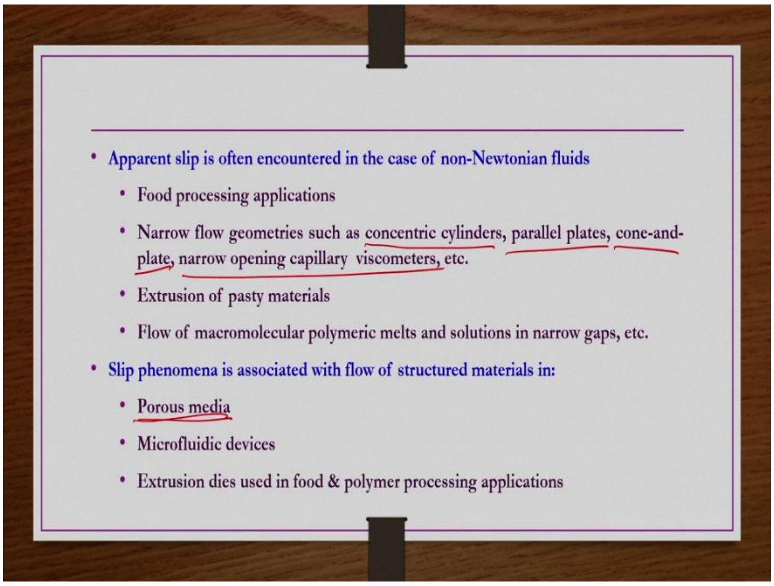

And then this apparent slip is often encountered in the case of non-Newtonian fluids like in food processing applications, narrow flow geometries such as concentric cylindrical rheometers, parallel plate rheometers, cone and plate rheometers, narrow opening capillary viscometers etcetera.

So; that means, in almost all rheology measuring systems or rheometers or viscometers what we have? You know we can we may have you know fluid slippage at the wall because all in all such kind of geometries in general we have narrow passages. Why narrow passages we select? In order to reduce the difference in the shear stress from the wall to the center or to reduce and or reduce the difference in the shear rate from the wall to the center.

So, that region we have taken the narrow flow passages for these kind of rheometers, but you know that is inducing apparent slip. And another application where we can find you know apparent slip in the case of non-Newtonian fluids is extrusion of pasty materials, polymeric solutions etcetera then flow of macromolecular polymeric melts and solutions in narrow gaps and so on so.

Further in the case of structured materials as well slip phenomena may occur some example as in porous media. If you have a porous media even if the fluid is a Newtonian fluid it is a low viscous liquid, then also slippage or slip phenomena would occur. So, that is quite possible. So, slip phenomena may possible even in the case of structured material like in the example of porous media then microfluidic devices and then extrusion dies used in food and polymer processing applications etcetera.

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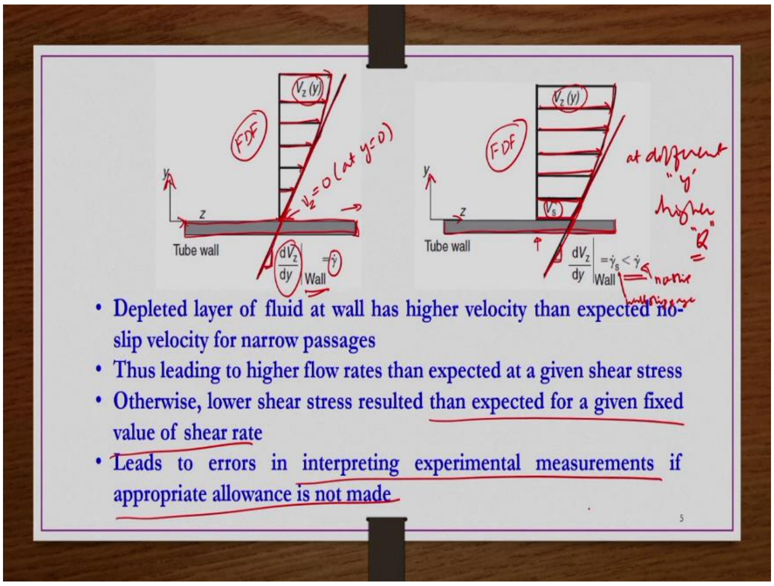

So, how do we realize? So, now, what we do, how do we realize this slippage existing or not that pictorially we can discuss now. So, what we have? We have a tube wall let us say capillary only we can take no problem. We have a tube wall and then let us say coordinate systems you are taking like this. So, this is y direction, this is z direction. So, fluid is moving in the z direction, the velocity whatever is there that is function of y V_z is function of y ok.

So, then at certain z value which is in the fully developed region, so, you know if you draw the velocity profile you can have a curve like this that is at the wall there is no velocity so, 0 velocity and then gradually if you moving up. So, the velocity of the fluid gradually increases. And then it reaches maximum value at the center of the capillary that is what we have seen in the case of no slip, when we take a no slip assumption when the fluid is flowing through capillary. So, this is what we have, right.

Now, if you wanted to know the, if you wanted to know the shear rate at the wall so what you do? At the wall at whatever desired location you consider because it is a fully developed flow. So, then whether you consider any z value it would be fine, right. So, you draw a tangent to the velocity curve.

At the velocity profile at the wall what you do? You draw a tangent like this and then find out the slope of that curve that is nothing but $\frac{dV_z}{dy}$ at the wall that we call wall shear rate $\dot{\gamma}$ ok. This is what you in general have in the case of a no slip you know boundary condition that we use when the fluid is flowing through a capillary or you know confined region something like that ok.

So, now, but if there is a slip what happens? The same situation now we take we have the tube wall and then fluid is moving in the z direction and then velocity is changing in the y direction like this. So, this is z direction this is y direction. So, V_z is function of y here again right. So, now, velocity profile if you draw at any location let us say at this z location if you wanted to draw the velocity profile.

Here again it is fully developed flow. What happens? At the wall now we do not have a v is equals to 0. Here in the case of you know no slip condition what we have seen? V_z is equals to 0 at y is equals to 0, but now here because of the depletion of fluid layer at the wall especially in the case of non-Newtonian concentrated suspensions.

So, depletion of fluid layer at the wall will take place because of that one the velocity of the fluid layer at the wall would be certainly having certain larger value it will not be 0. So, that value is now we are calling is V_s and after that as you move further away what happens again gradually the velocity increases same like in the previous case. But, now the magnitude overall magnitude of the velocity if you see if you see a different y values compared to the previous case, so, what you can see?

The velocity is you know higher at every location. Because of this one you know what will happen? You will be getting higher flow rate, higher flow rate than expected, whatever the flow rate you expect when you take no slip boundary condition as in the previously we have done analysis.

So, whatever the flow rate that you are expecting you do the pre, you have now you have all the equations etcetera. So, then without doing experiments also you can get a certain kind of values. So, that is not that should not be an issue right. So, whatever the pressure drop for a given pressure drop whatever the velocity you expected, compared to that you get higher velocity or higher flow rate you will get if there is a velocity slip at the wall ok.

And then if you wanted to know the shear rate at the wall now at this point you how to draw the tangent again in a similar way right. So, then the tangent slope of the tangent whatever you are drawing at the wall you know to that velocity profile what happens? That will be having the slope smaller than the slope in the case of no slip condition when you are applied.

So, that is $\frac{dV_z}{dy}$ at the wall in the case of wall slippage you will be having $\dot{\gamma}_s$, which is smaller than the $\dot{\gamma}$ and then this $\dot{\gamma}$ for no slip condition and then $\dot{\gamma}_s$ is for the wall slip condition ok. So, depleted layer of fluid at wall has higher velocity than expected no slip velocity for narrow passages. Thus leading to higher flow rates than expected at a given shear stress or otherwise lower shear stress resulted than expected for a given value of the shear rate ok.

So, this leads to errors in interpreting experimental results if appropriate allowances is not made, how much error that is going to be you cannot qualitatively say something and then say that that error is negligible. Because by seeing one example problem you will be realizing that velocity is substantially higher, it is substantially higher that we can see by one example problem that data is also a true experimental data, ok.

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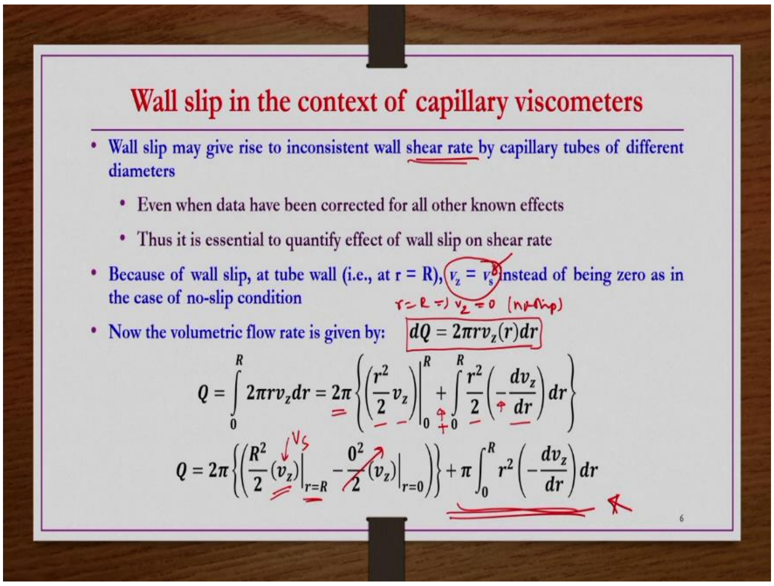

So, wall slip in the context of capillary viscometers; wall slip may give rise to inconsistent wall shear rate because now what we happen what we have seen because of the wall slippage at the fluid slippage at the wall of the capillary you know the flow rate is higher.

So, if the flow rate is higher, then obviously the velocity would be higher, average velocity would also be higher because the cross section area is remaining same.

So, that higher velocity is giving inconsistent or not reliable shear rate at the wall at the wall of the capillary because velocity is related to the shear rate now, right. So, pressure drop is related to the shear stress that we have seen volumetric flow rate or average velocity is related to the shear rate.

So, now there is an inconsistency or inaccuracy in the measurement of velocity or flow rates. So, that inconsistency or inaccuracy whatever is there that will be reflecting in the value of wall shear rate ok. So, we have to make corrections. So, this error may be there even when you incorporate corrections for all other rest of the errors whatever so many possible errors we have seen. You incorporate and make amendments accordingly.

So, that you know all other errors have been taken care. Despite of that one if there is a wall slippage so that is going to give inconsistent or inaccurate results or rheological behavior of an unknown fluid ok. Thus, it is essential to quantify effect of wall slip on shear rate and then that is what we are going to see now.

And then because of a wall slippage the tube wall now $v_z = v_s$. A tube wall in the case of no slip velocity we have taken v_z is equals to 0, but there is a fluid slippage at the wall. So now, we cannot say v_z is 0 at r = R, but it is having certain value v s. What is this v_s, we do not know, as of now we do not know what is this v_s, but we will find. There are a method, there is a method, a reliable method that we can go through and then make correction for this v_s as well.

So, volumetric flow rate for the case of a fluid flowing through a capillary we have seen it as $dQ = 2\pi r v_z(r) dr$, v_z is function of r and then when you do the integration here 2 π is a constant integration of r is $\frac{r^2}{2}$ v_z is as it is then minus then integration of r is $\frac{r^2}{2}$ and then differentiation of v_z is $\frac{dv_z}{dr}$. So, then what I am doing? I am writing $-\frac{dv_z}{dr}$ and then here I am writing plus and then dr. So, this is what we have seen.

Now, if you substitute the limits, here $\frac{R^2}{2}$ v_z at r = R $-\frac{0^2}{2}$ v_z at r = 0. So, since this term is anyway 0 because 0^2 is there. Now, in the previous case when we are, when we were designing equations to measure the shear rate of a fluid using a capillary viscometer previously what we have taken? v_z is 0 at r = R because of this no slip boundary condition.

But now we realized that you know when you use such equations we are getting anomalous results. So, then we realized that there may be certain thing like called wall slippage. So, that slippage at because of the wall slippage the fluid velocity whatever is there at the wall at r = R that is now v_s it is not 0, but it is v_s.

So, then what we have? $2\pi R^2 \frac{v_s}{2}$, that is $\pi R^2 v_s$ plus this term is same as our previous derivation that we have done when we are designing the equations for the capillary viscometer; designing the equations; that means, deriving the equations for shear stress and shear rate when we are using capillary viscometer. So, this term is going to be same and then we have to follow the same thing, but we are not going to do any integrations etcetera all that are not required now.

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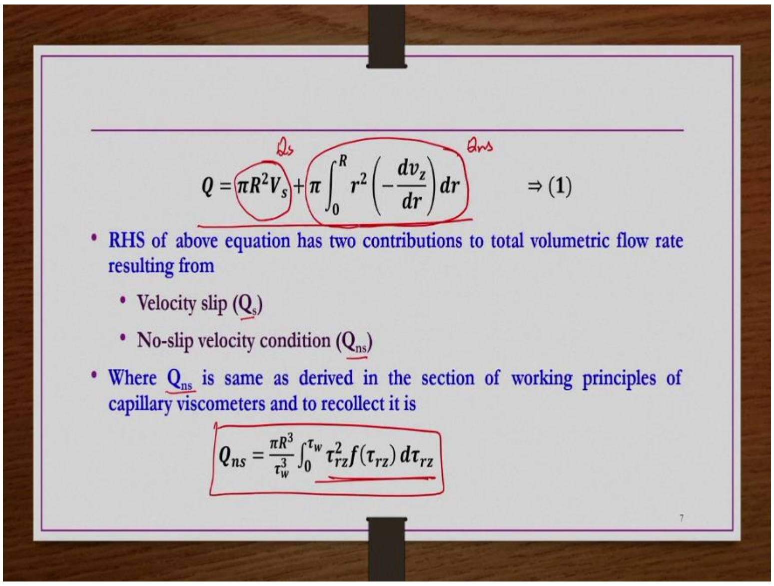

So, after substituting these limits after substituting limits and then simplifying then this is what we have for Q, $Q = \pi R^2 V_s + \pi \int_0^R r^2 \left(-\frac{dv_z}{dr}\right) dr$ right. So, now this volumetric flow rate is having two contributions one contribution because of the fluid slippage another contribution because of the fluid no slip condition Q_s right and both of them are random.

So, RHS of above equation has two contributions to total volumetric flow rate resulting from no slip velocity condition and then slip velocity condition. Because of the slip velocity condition Q_s, let us call it Q_s first term in the RHS and then because of the no slip whatever the volumetric flow rate is that is you know Q_{ns} that is the second term, ok.

So, where Q_{ns} is same as derived in the section of working principles of capillary viscometers and to recollect this is what we got it previously, right. And then we found that this right hand side integration integral part whatever is there you do the differentiation both sides with respect to τ_w and then we get into the Leibniz form and then we have done all subsequent steps you have done.

And then we found that this is a constant after doing the integration and substituting the limits you will get a constant or in terms of $\tau_w$you will get and τ_w is the constant for a given pressure drop ok. So, but anyway we do not need to do anything now. So, now, the equation number 1 that is whatever the $Q = \pi R^2 V_s + \pi \int_0^R r^2 \left(-\frac{dv_z}{dr}\right) dr$, dr is there this equation.

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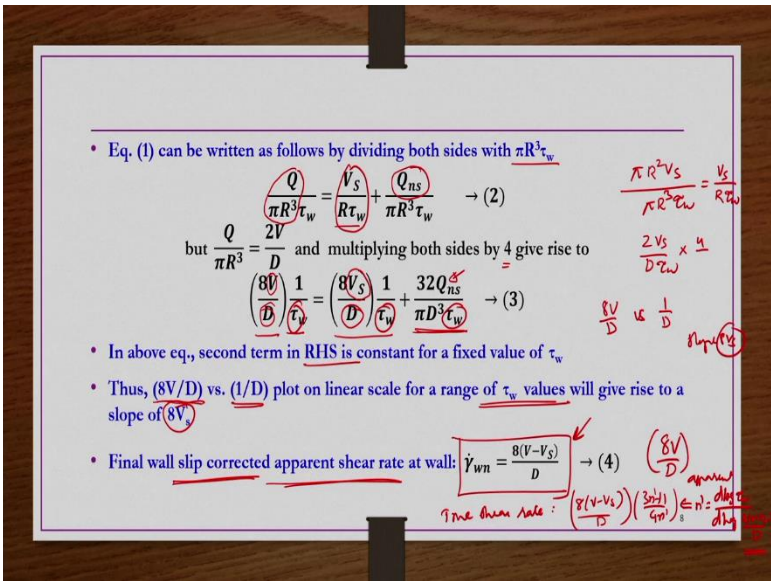

We are dividing both sides by $\pi R^3 \tau_w$. So, that $\frac{Q}{\pi R^3 \tau_w}$ in the left hand side, right hand side you will be having $\frac{V_s}{R\tau_w}$ + whatever the integral part that is there we are not worrying; we are not doing it just you know we are giving notation Q_{ns} to the second term in the RHS of equation 1.

So, $\frac{Q_{ns}}{\pi R^3 \tau_w}$ ok. So, first term in the RHS is nothing but $\pi R^2 V_s$ if you divide it by $\pi R^3 \tau_w$ then you will be having $\frac{V_s}{R\tau_w}$ right. Now, $\frac{Q}{\pi R^3}$ is nothing but $\frac{2V}{D}$. So, in place of $\frac{Q}{\pi R^3}$ we will be writing $\frac{2V}{D}$ and then both sides we will be multiplying by 4. The first time in the RHS whatever is there.

So, what we have? $\frac{V_s}{R\tau_w}$. So, then in terms of D we can have $\frac{2V_s}{D\tau_w}$. So, we multiply and then multiply by 4 both sides. So, then you will be having the first term in the right hand side as $\frac{8V_s}{D\tau_w}$. So, left hand side $\left(\frac{8V}{D}\right)\frac{1}{\tau_w} = \left(\frac{8V_s}{D}\right)\frac{1}{\tau_w}$ is the first term in the RHS and then this term you know in terms of D if you write $\frac{32Q_{ns}}{\pi D^3 \tau_w}$.

And then Q_{ns} we find it is a constant in terms of equations also it is coming in terms of tau w that is what we have seen. So, then altogether what we understand? This equation you know equation number 3, second term in the RHS is constant. First term in the RHS is having two values V_s and the two parameters V_s and D are there, both are changing, if you D if you are changing D. So, then V_s is also changing if they are existing a wall slippage.

So, we are taking the cases where there are wall slippages are there. Remember if you have a large diameter capillary, so, then wall slippages maybe very small you may avoid it, but when you take the large diameter capillaries then shear stress and shear rate both will change from center to wall.

So, then you cannot assign one single value of shear stress or shear rate ok. So, in the left hand side you know what we have? The average velocity V and then D. So, what we do from this equation? If you plot $\frac{8V}{D}$ versus $\frac{1}{D}$ then what will happen? It is possible; it is possible that you know you can get a straight line of slope $8V_s$ alright. So, that is one.

Second term in RHS is constant for a fixed value of τ_w and then for a range of τ_w values, you get data $\frac{8V}{D}$ versus $\frac{1}{D}$ from the experimental results. And then you plot it $\frac{8V}{D}$ versus $\frac{1}{D}$ then, you will get a straight line with slope V_s, with slope V_s right. So, you may be thinking why τ_w because it may be cancelled out now, because what we wanted to get we wanted to get a relation for different shear stress.

Because now, what after incorporating all other corrections shear stress is true shear stress that is reliable, because all other we assume that all other errors you know whatever are there corresponding corrections have been incorporated; such as end effects etcetera. So, whatever the shear stress that we get that is a reliable, right.

So, that is the reason for different values of shear stress we are trying to do you know this information so that we get some value for the velocity slip ok. So, now, final wall slip corrected apparent shear rate would be nothing but $\frac{8(V-V_S)}{D}$ actually $\frac{8V}{D}$ is nothing but apparent or a nominal wall shear rate, is not it.

Now, after making after realizing there is a fluid slippage at the wall that is fluid is having certain velocity at the wall. So, then that velocity we have to find from and then subtract from the average velocity and then multiply by 8 and divide by D to get the wall slip corrected apparent shear rate, remember this is also apparent shear rate only. It is not the true shear rate if it is a time independent non-Newtonian fluid right.

It is still apparent only, but apparent shear rate also having some errors some anomalous because of the change in diameter. If you change the diameter of the capillary different shear rate values you are getting because of the wall slippages. So, then how much was slippage is there or what is the velocity at the wall because of the wall slip effects that you find out and then you are subtracting that one from the average velocity V and then getting this corrected wall slip corrected apparent shear rate ok.

So, then true shear rate if you wanted to get that again $\frac{3n'+1}{4n'}$calculation you have to do and then multiply it ok. True shear rate after making wall slip corrections we will be having $\left(\frac{8(V-V_S)}{D}\right)\left(\frac{3n'+1}{4n'}\right)$.

That $n' = \frac{dlog\tau_w}{dlog\left(\frac{8(V-V_S)}{D}\right)}$ now. Not $dlog\left(\frac{8V}{D}\right)$ like previous case, $dlog\left(\frac{8(V-V_S)}{D}\right)$, because now $\frac{8(V-V_S)}{D}$ is now the reliable apparent shear rate or wall slip corrected apparent shear rate ok. We will take an example problem also now to understand about this one.

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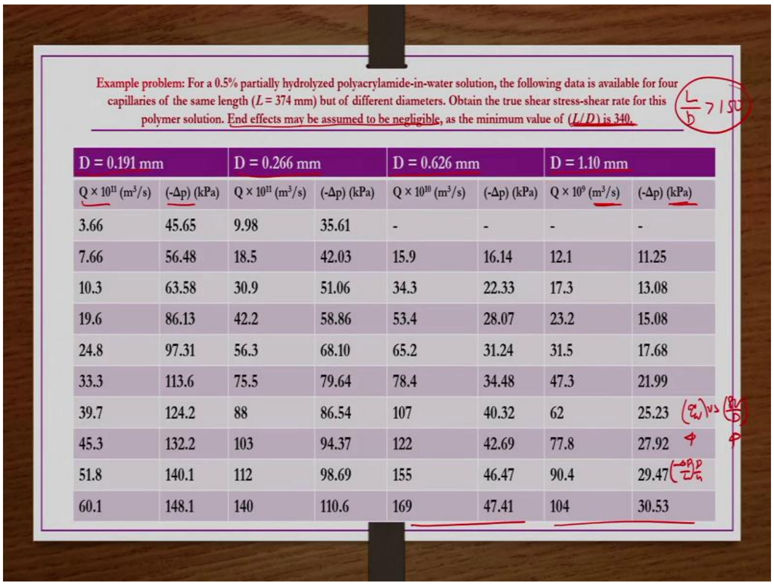

Example problem: For a 0.5% partially hydrolyzed polyacrylamide-in-water solution, the following data is available for four capillaries of the same length (L = 374 mm) but of different diameters. Obtain the true shear stress-shear rate for this polymer solution. End effects may be assumed to be negligible, as the minimum value of (L/D) is 340.

D = 0.191 mm		D = 0.266 mm		D = 0.626 mm		D = 1.10 mm	
$Q \times 10^{11}$ (m^3/s)	($-\Delta p$) (kPa)	$Q \times 10^{11}$ (m^3/s)	($-\Delta p$) (kPa)	$Q \times 10^{10}$ (m^3/s)	($-\Delta p$) (kPa)	$Q \times 10^{9}$ (m^3/s)	($-\Delta p$) (kPa)
3.66	45.65	9.98	35.61	-	-	-	-
7.66	56.48	18.5	42.03	15.9	16.14	12.1	11.25
10.3	63.58	30.9	51.06	34.3	22.33	17.3	13.08
19.6	86.13	42.2	58.86	53.4	28.07	23.2	15.08
24.8	97.31	56.3	68.10	65.2	31.24	31.5	17.68
33.3	113.6	75.5	79.64	78.4	34.48	47.3	21.99
39.7	124.2	88	86.54	107	40.32	62	25.23
45.3	132.2	103	94.37	122	42.69	77.8	27.92
51.8	140.1	112	98.69	155	46.47	90.4	29.47
60.1	148.1	140	110.6	169	47.41	104	30.53

So, example problem to realize how much important to establish wall slippage effect in the case of capillary viscometer, and then how to make the correction for the shear rate in apparent shear rate by using that slip velocity at the wall. And then finding out the true rheological parameter of the fluid that is what we are going to see through this example problem ok.

For a 0.5 percent partially hydrolyzed polyacrylamide in water solution the following data is available for four capillaries of the same length, length is same 374 mm, but of different diameters obtain the true shear stress true shear rate for this polymer solution ok τ_w $\dot{\gamma}$ we have to find out, end effects may be assumed to be negligible because the minimum value of $\frac{L}{D}$ is 340.

For time independent fluids $\frac{L}{D} > 150$ itself is sufficient to avoid or to say the end effects are negligible, but now in the case minimum value of $\frac{L}{D}$is 340, why minimum because there are four different values of D, L is constant right. So, now, when D is 0.191 mm Q versus Δp information is given here the first two columns, third and fourth column for D is equals to 0.266 mm.

And then fifth and sixth column for D is equals to 0.626 mm and then last two column when D is equals to 1.1 mm the Q versus $-\Delta$ p information is given. Pressure drop is given in kilopascal volumetric flow rate is given in meter cube per second. So, then Q multiplied by 10 power something is there 9 10 11.

So, then whatever the values are there they should be multiplied by 10 power minus of this whatever the 9 10 11 etcetera ok. So, now, first what we have to see? Wall slip effect is required or not that is what we have seen; we have to see, because what we have seen in the previous class, we have established how to make correction for the end effects.

But now, it is saying that you know in the problem statement itself says that you know neglect the end effects because $\frac{L}{D}$is 340 minimum $\frac{L}{D}$is 340 alright. $\frac{374}{1.1}$ if you do, so, then you get approximately 340 right. $\frac{374}{0.191}$ mm if you do it you may be getting much higher value of $\frac{L}{D}$.

So, that is more than sufficient to say that the end effects are negligible. So, now, the only other effect that we are discussing is wall slip effect, right. So, in order to know whether the wall slip effect is required or not this information first we have to convert in terms of τ_w versus $\frac{8V}{D}$ apparent shear rate wall shear stress and then apparent wall shear rate we have to get by using this data.

Very simple because τ_w you can get $\left(\frac{-\Delta p}{L}\right)\frac{D}{4}$ you can do and then Q is given. So, then V average you can get, $V_{avg} = \frac{Q}{\pi r^2}$ that you can get. So, then this nominal shear rate also you can get for each data set right. Then you plot them on a graph τ_w versus $\frac{8V}{D}$.

If they are super imposing onto each other for all four cases of capillary diameters then we say that you know no other correction is required in the case of whatever this apparent shear rate is there and then we can directly find out its rheological parameters by finding out the n' etcetera those kind of things, fine.

So, that is what first we are going to see. If they are not super imposing; that means, wall slip is playing role because diameter when you are changing; that means, the passage cross section of the passage you are changing right. So, if you have the cross section narrow cross section passage then wall slip would be more important. So, then accordingly you know you have to make corrections that is what we see by solving this problem.

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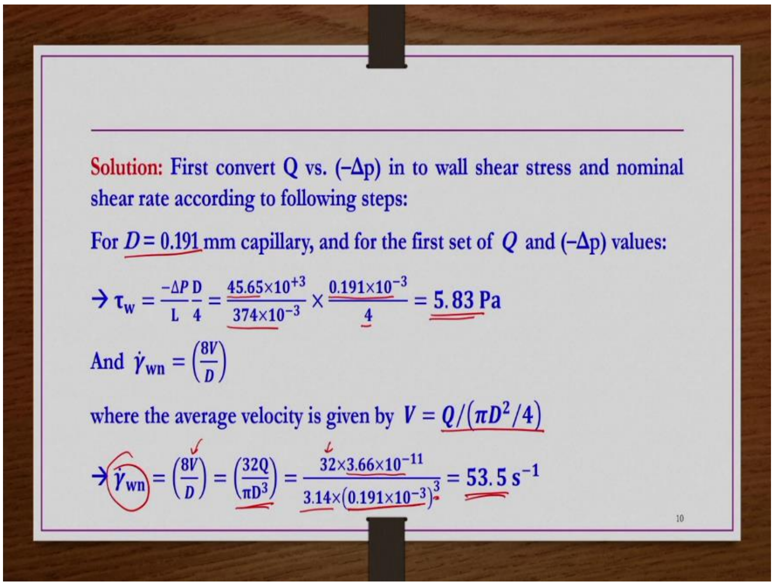

So, let us say for the first data point when D = 0.191 mm, Δp is given as 45.65 kilopascal that is $\frac{45.65x10^{+3}}{374x10^{-3}}$ and then D for the first capillary of D 0.191 mm. So, $\frac{0.191x10^{-3}}{4}$ when you do it you will get τ_w = 5.83 pascals.

Now, similarly $\dot{\gamma}_{wn}$apparent shear rate or nominal shear rate $\frac{8V}{D}$ we have to get. $\frac{8V}{D}$, in terms of Q if you write because V = $Q/(\pi D^2/4)$. So, then that if you make use here in place of this V then you get $\dot{\gamma}_{wn}$ is nothing but $\frac{32Q}{\pi D^3}$, so, 32 constant. First data point when D = 0.191 and when $-\Delta$p is 45.65 kilopascal, volumetric flow rate is 3.66 x 10^{-11} m^3/s.

So, that is what we have written, π is 3.14 and D is first capillary 0.191 x $10^{-3}$and then whole cube. When you simplify it you get roughly 53.5 second inverse as apparent shear rate for the first data point when D = 0.191. So, you got τ_w you got shear rate also $\dot{\gamma}_{wn}$ also as well. So, then what you can do? So, now, you when you get these τ_w and $\dot{\gamma}_{wn}$ for all the capillaries for each capillary of different diameter for each data set.

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D = 0.191 mm		D = 0.266 mm		D = 0.626 mm		D = 1.10 mm	
τ_w (Pa)	(8V/D) (s^{-1})	τ_w (Pa)	(8V/D) (s^{-1})	τ_w (Pa)	(8V/D) (s^{-1})	τ_w (Pa)	(8V/D) (s^{-1})
5.83	53.5	6.33	54	-	-	-	-
7.21	111.1	7.47	100.1	6.75	66.02	8.27	92.6
8.12	150.6	9.08	167.2	9.34	142.4	9.62	132.4
11	286.5	10.46	228.4	11.74	221.7	11.10	177.5
12.42	362.5	12.10	304.7	13.07	270.7	13.00	241.1
14.50	486.8	14.16	408.6	14.42	325.5	16.17	362
15.86	580.3	15.39	476.3	16.87	444.3	18.55	474.5
16.88	662.2	16.78	557.4	17.86	506.6	20.53	595.4
17.89	757.2	17.55	606.1	19.44	643.6	21.67	691.8
18.90	878.6	19.66	757.7	19.83	701.7	22.45	796

And then you converted data whatever is there that you write you make a tabular column like this. So, now, τ_w versus $\frac{8V}{D}$, see first case when D = 0.191 mm τ_w we got 5.83 and then $\frac{8V}{D}$ we got 53.5 second inverse. Similarly remaining points you know we got here for the same D = 0.191 mm.

Similarly, for other diameter capillaries also we got this τ_w versus $\frac{8V}{D}$ information. Now, for these four capillaries of different diameters τ_w versus $\frac{8V}{D}$you already got or you know whatever the Q versus Δ p information was there you converted in terms of τ_w versus $\frac{8V}{D}$. And then now you plot them.

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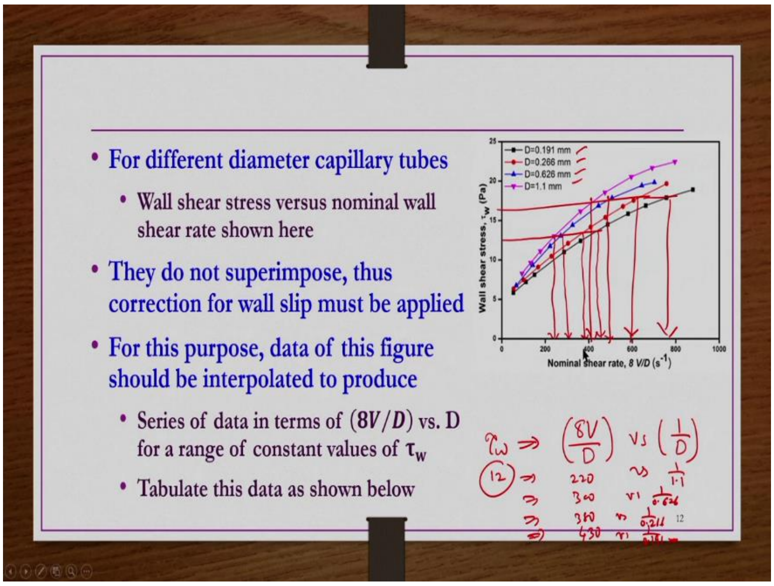

So, what you see? When you are plotting this τ_w versus $\frac{8V}{D}$for different diameters you are getting different curves. So, this is for a smaller diameter, this is for the next diameter and then this is for the other diameter and this is for the larger diameter 1.1 mm. So, they are not super imposing onto each other they are not super imposing onto each other.

That means, even after incorporating the end effects the rheological measurements of that particular fluid by obtained by capillaries of the different diameter is not reliable; that means, diameter is really playing a role you have already incorporated the end effects. So, then obviously, you know if no other errors are existing so, then all these even if you change the diameter, so, then all they should be super imposing τ_w versus $\frac{8V}{D}$ curve should be a super imposing.

If they are not super imposing; that means, something is happening because of changing the diameter. And that something is nothing but the fluid slippage or slip velocity at the wall of the capillary that is what we have to find out as we discussed a few slides before, ok. So, now, what we do, how to get that information? What we have done in the couple of slides before? We have to plot $\frac{8V}{D}$ versus $\frac{1}{D}$ for different τ_w values.

So, now what you do here? You select different values of τ_w maybe 10, 15, 20 like this right. So, let us say when you take τ_w of something around 12 value here and then draw a straight line. And then and for each curve you draw a vertical line like this wherever they are intersecting with the curves of this τ_w versus $\frac{8V}{D}$ curves wherever they are intersecting.

So, corresponding $\frac{8V}{D}$ values you have to find out let us say when it is 12. So, $\frac{8V}{D}$ for the you know 1.1 mm diameter is roughly let us say 220 $\frac{1}{1.1}$ and then similarly next maybe 300 when D = 0.626 mm. When D = 0.266 mm it is roughly 380.

And then when it is 0.191 mm, it is roughly 430 right, this is what you get for one τ_w. Now, you take another τ_w maybe 17 draw a line like this and then corresponding values you find out similar way and then you tabulate them and then you tabulate them as here shown here.

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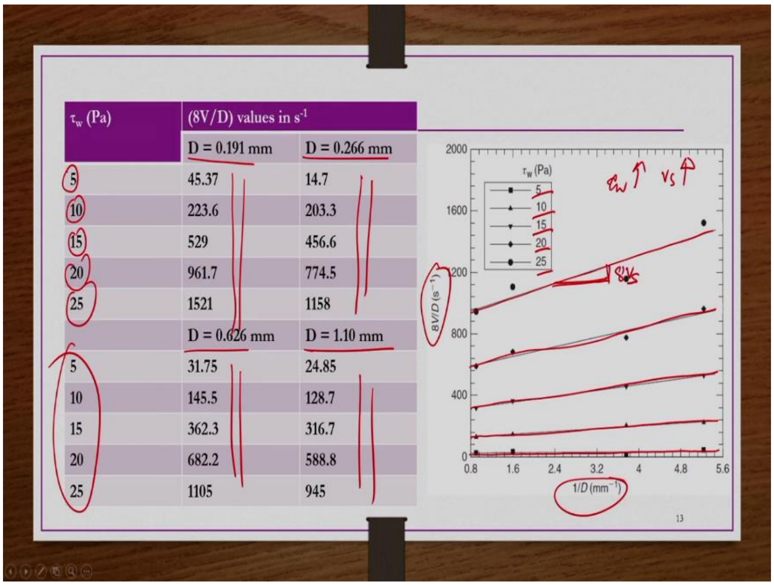

So, I have taken τ_w 5, 10, 15, 20, 25. So, for each diameter of a capillary corresponding $\frac{8V}{D}$ values are tabulated here for different τ_w values right. Now, we are going to plot this $\frac{8V}{D}$versus $\frac{1}{D}$ here for different τ_w values. So, then you can see when tau w is very small, so, then slip wall it is almost like horizontal line that is slope is almost like 0 it is not 0 exactly, but almost like ok.

So, then as you increase the τ_w value the slope of the line is increasing the slope of the line is increasing; what does it mean? When τ_w is increasing the wall slip is increasing.

So, V_s magnitude of V_s is going to be increasing that we can see. Now, for each of these curves you find out what is the slope so, that is nothing but $8V_s$ right.

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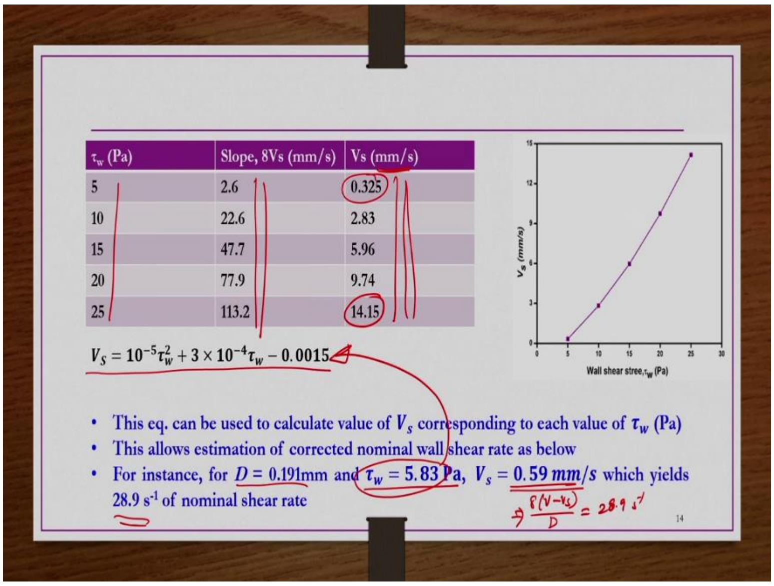

So, that slope value for each of this τ_w I plotted here shown here tabulated here. So, corresponding V_s you can see. See now, in mm per second the velocity slip velocity can be as much as 14, 15 mm per second. It is not 0, you cannot say 14 mm per second velocity is 0 velocity even you cannot say 0.3 to 5 mm per second velocity is a 0 velocity ok.

So, that means, because of this slippage slip velocity the tau w versus $\frac{8V}{D}$ curves were not super imposing by changing the diameter of the capillary ok. So, now, this information V_s versus τ_w if you plot you can get a line like this. It is not necessary that you should get always like this, you may get straight line and different non-linear behavior also you may get.

So, why do we need is because our true values of τ_w are not exactly like 5, 10, 15, 20, 25 like that. So, by because we cannot say now four capillary diameters each case there are 10 to 12 data points are there. So, many data points you are there. So, for each and every point you cannot do the finding out the τ_w value by you know especially when you are doing graphically.

So, then what you do? You find out the relation V_s versus τ_w. Now, once you know this relation V_s versus τ_w then you can substitute required value of τ_w here in this equation and then find out the corresponding V_s ok. So, that we do. Let us say for the first data point when D is equals to 0.191 mm we found τ_w is equal to 5.83 Pascals. So, when you substitute this 5.83 Pascals here in this equation then you will get V_s = 0.59 mm per second.

So, then you get $\frac{8(V-V_S)}{D}$, we get V already you got it. So, then the $\frac{8(V-V_S)}{D}$ you will be getting 28.9 second inverse this one. Like that for all the values of τ_w corresponding V_s you have to find out and then corresponding $\frac{8(V-V_S)}{D}$you have to find out for all capillary of different diameters.

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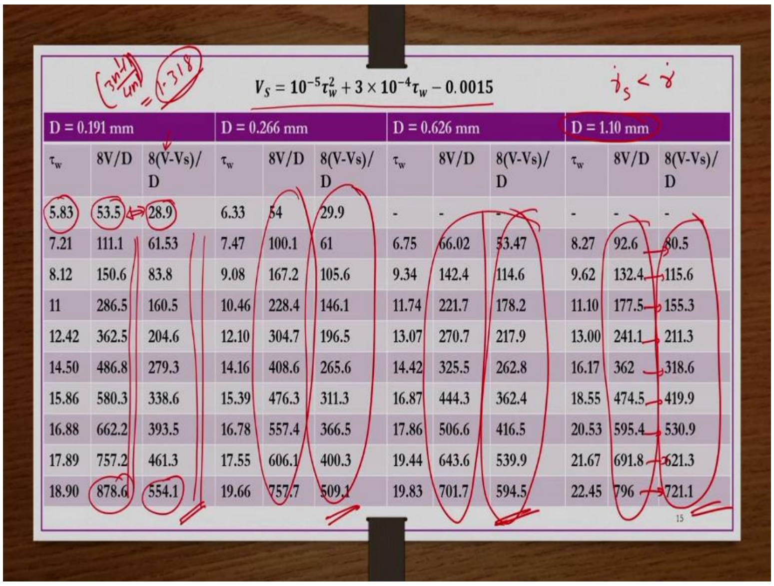

$V_S = 10^{-5}\tau_w^2 + 3 \times 10^{-4}\tau_w - 0.0015$

D = 0.191 mm			D = 0.266 mm			D = 0.626 mm			D = 1.10 mm		
τ_w	8V/D	8(V-Vs)/D	τ_w	8V/D	8(V-Vs)/D	τ_w	8V/D	8(V-Vs)/D	τ_w	8V/D	8(V-Vs)/D
5.83	53.5	28.9	6.33	54	29.9	-	-	-	-	-	-
7.21	111.1	61.53	7.47	100.1	61	6.75	66.02	53.47	8.27	92.6	80.5
8.12	150.6	83.8	9.08	167.2	105.6	9.34	142.4	114.6	9.62	132.4	115.6
11	286.5	160.5	10.46	228.4	146.1	11.74	221.7	178.2	11.10	177.5	155.3
12.42	362.5	204.6	12.10	304.7	196.5	13.07	270.7	217.9	13.00	241.1	211.3
14.50	486.8	279.3	14.16	408.6	265.6	14.42	325.5	262.8	16.17	362	318.6
15.86	580.3	338.6	15.39	476.3	311.3	16.87	444.3	362.4	18.55	474.5	419.9
16.88	662.2	393.5	16.78	557.4	366.5	17.86	506.6	416.5	20.53	595.4	530.9
17.89	757.2	461.3	17.55	606.1	400.3	19.44	643.6	539.9	21.67	691.8	621.3
18.90	878.6	554.1	19.66	757.7	509.1	19.83	701.7	594.5	22.45	796	721.1

So, that is what we get now here. So, then we have this relation V_s versus τ_w relation is there. So, when τ_w is 5.83, we got $\frac{8V}{D}$ is 53.5. Before incorporating the wall slip effects nominal shear rate was 53.5, after incorporating the wall slip effect it decreases to the almost to half 28.5. And then likewise you can see for all data points you can see substantial difference between the incorrected $\frac{8V}{D}$ and then corrected $\frac{8V}{D}$.

After incorporating the wall slip the shear rate has decreased substantially that is what in theory also we discussed $\dot{\gamma}_s < \dot{\gamma}$ right and then, that how much less cannot be neglected, you can see it is very large for each case here also, second case here also you can see, third case here also you can see, fourth case here also you can see, some cases maybe the difference may be less.

As we discussed this wall slip is more important in narrow passages; that means, if you are increasing the diameter of capillary then wall slip effect may be decreasing. That means whatever the $\frac{8V}{D}$ and then $\frac{8(V-V_S)}{D}$ in details are there values are there they are they will not be very different from each other that we can see here, when for the larger diameter capillary still there is a effect, but the difference is less. For larger capillary diameter the difference between $\frac{8V}{D}$ and $\frac{8(V-V_S)}{D}$ is very less compared to this smaller capillary diameter.

Here in the case of 0.191 mm almost 50 percent is reduced, but here in the case of 1.1 mm capillary diameter, 1.1 mm diameter capillaries it decrease very less from 92 to 80 from roughly 800 to you know 720 ok. So, when you increase the diameter the wall slip effect may be reduced, but if you go larger values of capillaries then shear stress may not be constant from wall to the center of the capillary.

Or the difference of in shear stress or shear rate from wall to the center maybe larger if you take the larger capillary diameter so, because of that one we have to go for the narrow capillary and then accordingly we have to make a correction for the wall slip effect.

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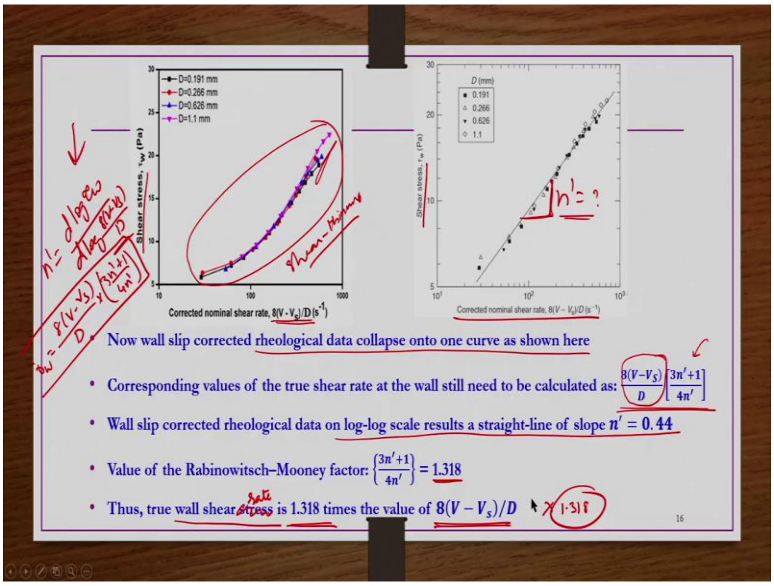

So, now this if you plot shear stress τ_w versus $\frac{8(V-V_S)}{D}$ not $\frac{8V}{D}$. $\frac{8(V-V_S)}{D}$which is nothing but corrected nominal shear rate, still it is not true shear rate it is a corrected one, but after making the wall slip correction you can see for the change in diameter of capillary is not showing any significant effect.

The change in capillary diameter is not showing any significant effects lightly instead is there that is those things may be coming out because of the interpolations etcetera right. So, now, this one what you can do? You can find out n' by plotting d log of τ_w versus d log of $\frac{8(V-V_s)}{D}$.

We cannot take d log of $\frac{8V}{D}$, we have to take the corrected nominal shear rate that is $\frac{8(V-V_s)}{D}$. And then find out the n' and then you can find out the corrected or the true shear rate not the corrected, true shear rate you can find it out as $\frac{3n'+1}{4n'}$ this is how you can get it.

True shear rate right that way you can do as we have done one example problem in the previous class, but what we do? Since we already from this graph we can realize this is you know concave upward curve and then it is a shear thinning fluid; shear thinning fluid. And then for shear thinning fluids if you apply the so called power law curve fitting and then plot log of you know log of τ_w versus log of $\frac{8V}{D}$ then you get a straight line.

So, shear stress versus shear rate corrected nominal shear rate $\frac{8(V-V_s)}{D}$ on a log-log curve if you do you may get curve like this and then you can find out the n' value also, n' you can find it from here right. This way also you can find. This way if you find out, you have to do n' finding out for each value of τ_w.

So, but know from the curve nature of the curve you already realized it. So, then you can apply the power law behavior whatever the so called you know power law fitting you can do and then find out this n' ok. So, now wall slip corrected rheological data collapse onto the one curve as shown above here and then corresponding values of the true shear rate at the wall still need to be calculated as this one.

Because $\frac{8(V-V_s)}{D}$ is nothing but corrected nominal shear rate, but the true shear rate you will get only when you multiply this one by this factor $\frac{3n'+1}{4n'}$. So, wall slip corrected rheological data on log-log scale results a straight line of slope n' 0.44.

That means $\frac{3n'+1}{4n'}$ is coming to be 1.318, so; that means, your true shear rate would be 1.318 times the values of $\frac{8(V-V_s)}{D}$ whatever we have tabulated in the previous slide. So,

$\frac{3n'+1}{4n'}$ is nothing but 1.318. So, then all these values whatever the third, sixth, ninth and twelfth column whatever the $\frac{8(V-V_s)}{D}$ values are there that you have to multiply by 1.318 to get the true shear rate information ok, to get the true shear rate information.

Whatever the $\frac{8(V-V_s)}{D}$values are there in the previously tabulated results, so, that $\frac{8(V-V_s)}{D}$ columns should be multiplied by 1.318 to get the true shear rate.

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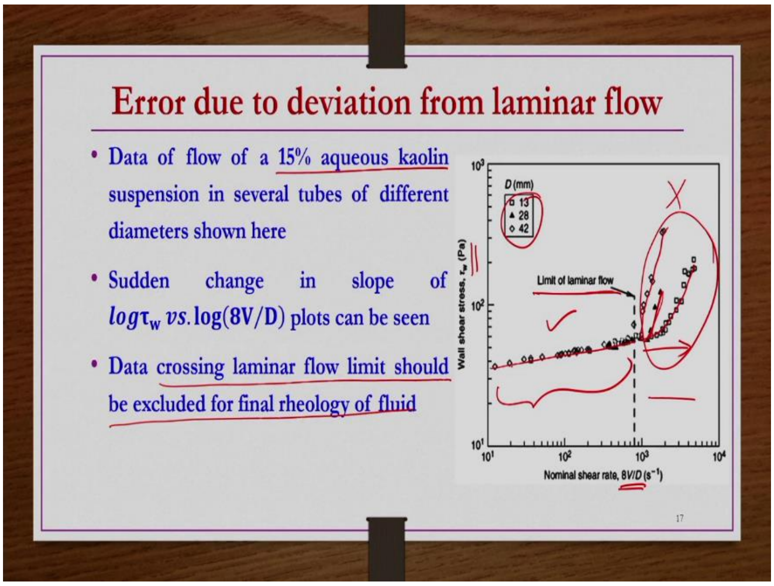

So, the last error: error due to deviation from laminar flow how to find it out ok. So, how do you find? So, whatever the shear stress τ_w versus nominal shear rate $\frac{8V}{D}$ when you plot it on a log-log graph in general for a laminar flow you get a straight line like this right. When you deviate when the flow is deviating from the laminar curve what happens? There is a sudden change in slope whatever the diameter of capillary you take. For different capillary diameters the information is shown here.

So, there is a sudden change in slope that you can see if the flow is crossing the laminar limit. If the flow is crossing the laminar limit, then wall shear stress versus nominal shear rate curve on a log-log scale would be experiencing sudden change in the slope right. So, what you have to do? You have to from your data you how to strike off you have to remove this data because all this analysis whatever we have done for the capillary viscometer that is true for the fully developed laminar steady flow only.

So, this data you should not take into the consideration, simple. So, this is true experimental data for 15 percent aqueous kaolin suspension in several tubes of different diameters. So, log τ_w versus log $\frac{8V}{D}$ plot is seen and then data crossing laminar flow limit should be excluded for final rheology of the fluid.

You have to take consideration of these data points only and then you can see by changing the diameter the all the points are superimposing onto each other. That means, there is no wall slip effect up to this point, after this point only wall slip effect is there and then wall slip effect in general is higher for the higher shear rates.

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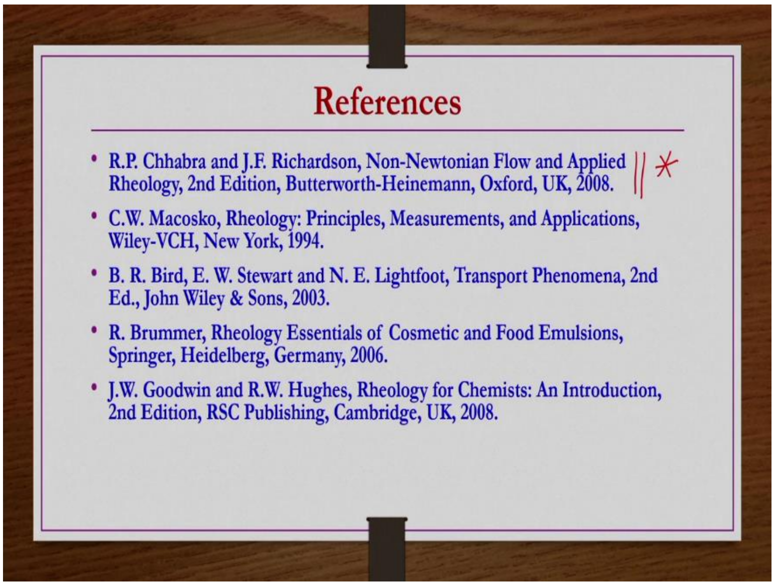

References for this lecture: the entire lecture is prepared from this reference by Chhabra and Richardson including the example problem, including the example problem has been discussed from this reference book by Chhabra and Richardson. Similarly other references books may also be useful.

Thank you.