Fluid Mechanics

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Lecture No. # 25

Welcome to this lecture number 25 of the NPTEL course on fluid mechanics for chemical engineering under graduate students. In the previous lecture, lecture number 24, we had derived the Navier stokes equations; these are the differential balances of momentum for a Newtonian fluid. And we had also pointed out, what are the boundary conditions that are to be used in commonly encountered situations in engineering fluid mechanics.

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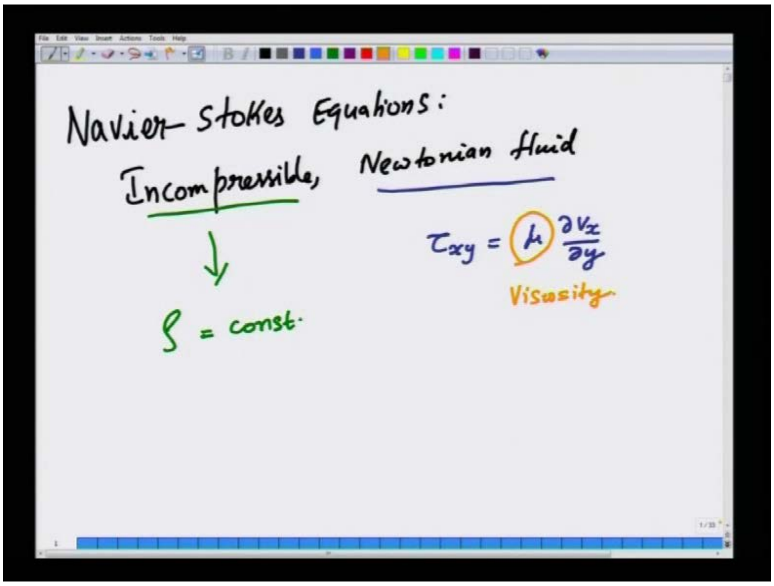

Few important points about the Navier stokes equations - the Navier stokes equations are generally written as follows, these are valid for an incompressible, and Newtonian fluids. An incompressible fluid simply means that density of the fluid is a constant. That is, even if there are pressure changes within the flow, the associated density changes are negligible, so that you can treat density to be a constant.

And a Newtonian fluid means that the stress is directly proportional to the gradient of velocity, the shear stress is directly proportional to gradient of velocity. And it is a linear relation and the constant of proportionality is called the viscosity of the fluid. So these are the two restrictions that we are going to impose in this course. That is the fluid is incompressible, the density is constant and the fluid is Newtonian that is the shear stress is directly proportional to the velocity gradient.

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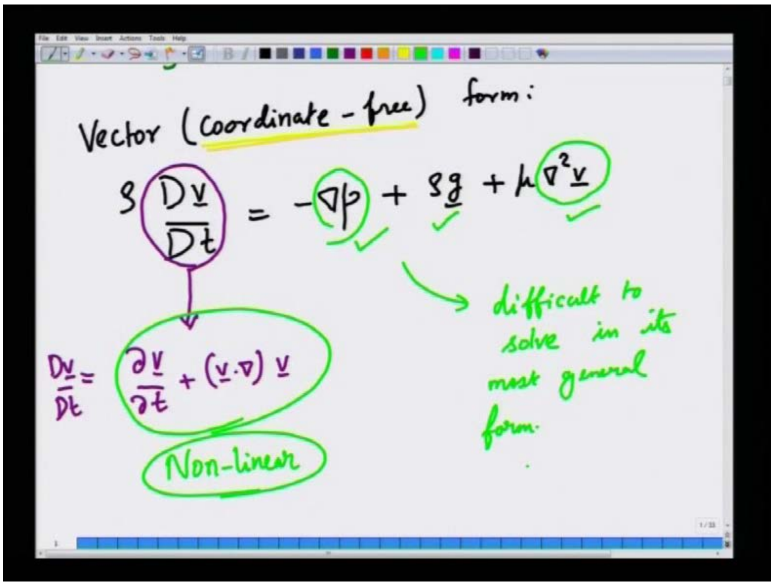

Now having said this, we can write the vector form or coordinate free form of the Navier stokes equation as follows, rho times the substantial derivative of velocity is minus del p plus the body force rho g per unit volume plus mu del squared v. So this is the vector form, this is also called the coordinate free form, because this form is true independent, it is independent of coordinate system. And if you want write it in different coordinate system such as Cartesian, cylindrical or spherical coordinate systems. You must first write down the, you must first understand how to write the various differential operators in these different coordinate systems.

These are normally tabulated in many text books. So, ones you look them up in their respective sections, whether you are requiring a Cartesian coordinate form or cylindrical coordinate form or a spherical form, then you .can write down these equations in those respective coordinate systems. By writing the differential operators in these differentials various coordinate systems.

We are already pointed out that the differential operators in various coordinate systems can be very different and so one cannot readily write down equations in other coordinate systems by analogy with Cartesian coordinate systems. So that is something that is very important and it must be kept in mind while trying to solve problems, in using Navier stokes equation in different coordinate systems.

But, to begin with another important point, before I move on is that this expression this is the substantial derivative of velocity. This is equal to the normal derivative, partial derivative plus v dot del v. So this lends, this renders this equation to be a non-linear equation. every other term in Navier stokes equation is linear, this is linear, this of course a constant, because it is given this is linear. That is by linear, we mean that the unknown occurs only once in a given term it does not occur as a product or as a non-linear function as itself. Whereas here, the unknown occurs as product of the velocity dotted with the gradient of velocity. So this non-linearity renders the Navier stokes equation in general to be very difficult to solve, to solve in its most general form. So, one has to make suitable physically motivated approximations to render the Navier stokes equations in a more simplified form before we are able to solve them. Now, I am going to illustrate that shortly.

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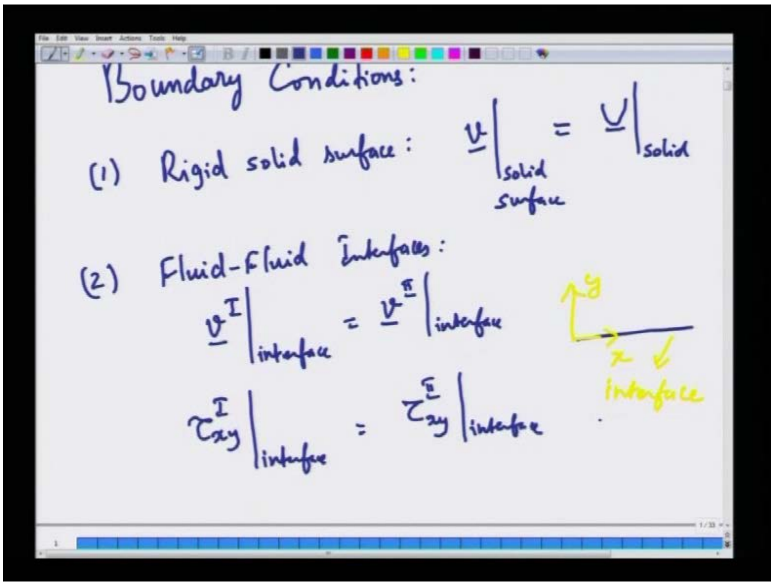

Also, in order to solve the Navier stokes equation; we need boundary conditions that we pointed out in the previous lecture. The boundary conditions are for a rigid surface, rigid solid surface. You have the velocity of the fluid at the solid surface is equal to the known velocity of the solid. If the solid is moving with a constant velocity the fluid velocity at the boundary of the solid will take the same velocity as the solid, this is called the no slip condition. And if the solid surface is stationary, then this fluid will also be stationary. So this is one important condition, at a rigid solid surface. At liquid interfaces, a fluid-fluid interface in general at one has continuity of velocity. Velocity in fluid one at the interface is equal to velocity in fluid two at the interface. And secondly, you also have continuity of stress and so you have continuity of shear stress. Suppose, you have an interface, let us put a coordinate system, we have x then y and this is the interface. Then tau x y in phase one at the interface is tau x y in phase two at the interface this is the continuity of shear stress.

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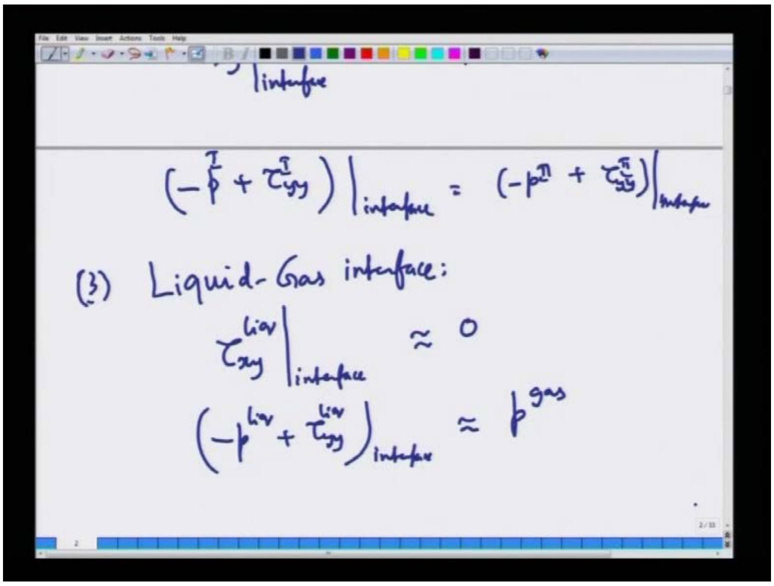

And you also have continuity of normal stress minus p plus tau y y in phase one evaluated at the interface in fluid one is minus p 2 plus tau y y 2 in fluid two evaluated at the interface. This is for a general fluid-fluid interface, but we also pointed out for the special case of a liquid gas interface. It can, this equation can be simplified, because at a liquid gas interface the viscosity of gases are typically very very small of the order of 100 times smaller than that of viscosity of liquids.

So, we can assume that the shear stress are the liquid side at the liquid guarantee, gas interfaces is approximately 0. And the normal viscous, normal stresses in the liquid at the interface are balance by the pressure in the gas. So these are the two simplified conditions. When you use these two conditions, there is no need to specify any condition on the velocity of the liquid. We merely have to say that the stress at the liquid at the liquid gas interface is 0 and the normal stresses must be equal to the pressure in the gas. So, these are the three commonly encountered conditions that are of relevance to this course. The boundary condition, as I told you in the last lecture comes from the physical nature of the problem. So, one has to look in to the physical nature and then write suitable boundary conditions.

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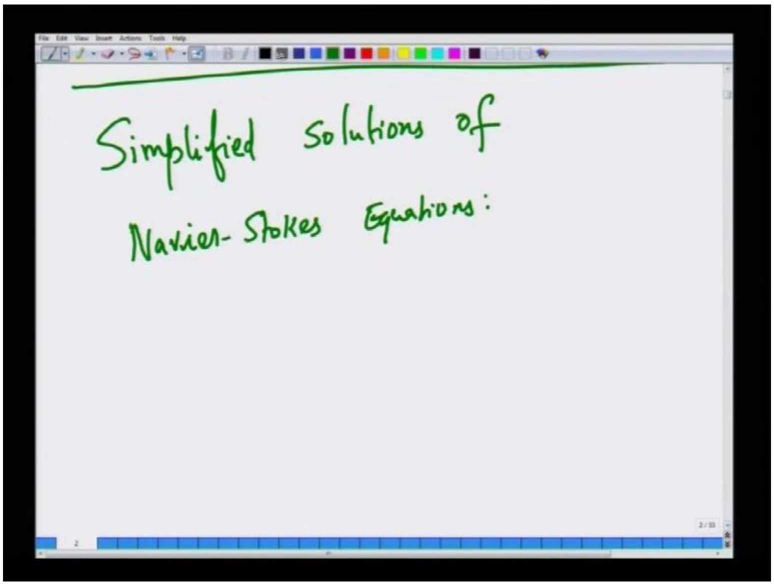

Now, let us now try to solve some problems, simplified solutions of Navier strokes equations. Let us imagine the very first problem that we are going to do is what is called the plane couatte flow. Plane couatte, couatte is the name of the person, who analysed at first, plane is refers to the geometry of the flow. Imagine having two long plates that are separated by a distance H.

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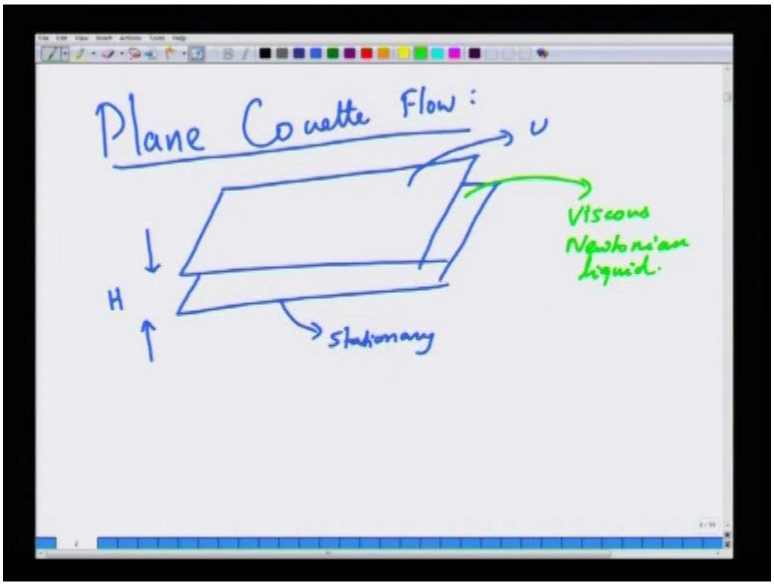

The bottom plate is stationary and the top plate moves with a constant velocity u. And we want to find what is the velocity profile that is present in between the two plates. Now, one question that one can ask is suppose you have, of course this the space in between these two plates is occupied by a viscous Newtonian liquid. The question one can ask is, suppose I have two plates that are separated by distance H and if I place a viscous Newtonian liquid in between them, let say an oil.

What is the force I need to apply in order that the plate moves at a constant velocity? What is the force that must be applied on the top plate, so that up the plate moves with a constant velocity U. In order to answer this question, we will first start off by saying that let us assume that the plate is moving at a constant velocity U. And then find the velocity profile in between the two plates, of the fluid in between the two plates and then calculate the stress. And therefore the force that is the procedure we want to do.

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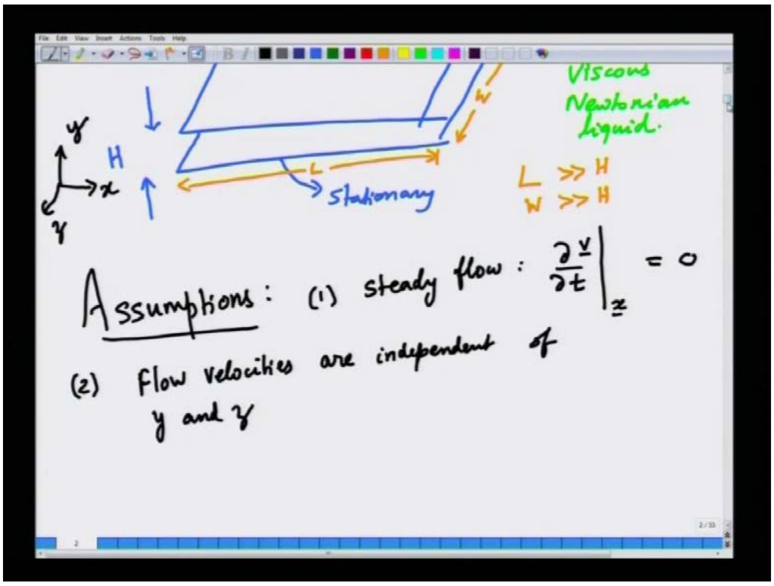

Now, before I solve this, we have to make some assumptions, in order to simplify the Navier stokes equations. The first assumption I will use is that the flow is steady. When the flow is steady, any quantity the time derivative of any quantity such as velocity at a given point in space is 0. If I look at a fixed point in space that velocity is independent of time and so is the pressure and so on. So, we are going to assume steady flow and it is a good approximation, because we are told that the top velocity top plate velocity is a constant, its independent of time. If the top plate itself is moving with a time independent velocity, then there would be no physical grounds for us to expect that the velocity in the fluid is steady.

In fact that can be situations where the top plate can oscillate with the velocity of the top plate can oscillate with time that is it can go up and down like a sinusoidal function. And in such cases of course, there is no reason for us to believe that the fluid velocity inside the two plates be or steady there the fluid velocity will also be time dependent. But given the fact that the top plate is moving with a constant velocity, then there is no reason for us to believe that the fluid inside the velocity the two plates will have a time dependence. So we will assume that the flow is steady. Now, the other assumption that we are going make is the geometric assumption. Suppose, you consider the width of the plates and the length of the plate L.

We are going to consider the limit that L is large compare to H, W is large is compare to H. That is, we have a very long and broad plate and the thickness that gap thickness between the two plates is very small compare to the length of the plate as well as the width of the plate. Now, that will give us the motivation to assume that flow quantities like the velocity are independent of and before I do that let us put a coordinate system. So, I am going to assume the coordinate system where the flow direction is x, the gap normal direction is y and the direction along in to the plane of the board is z of coordinate directions y and z, because they are too long in the other two directions. So and since the plate is moving in the constant x direction, plate is moving in the x direction. There is no reason for us to believe that the velocity inside the bulk of the gap is a function of the other two directions.

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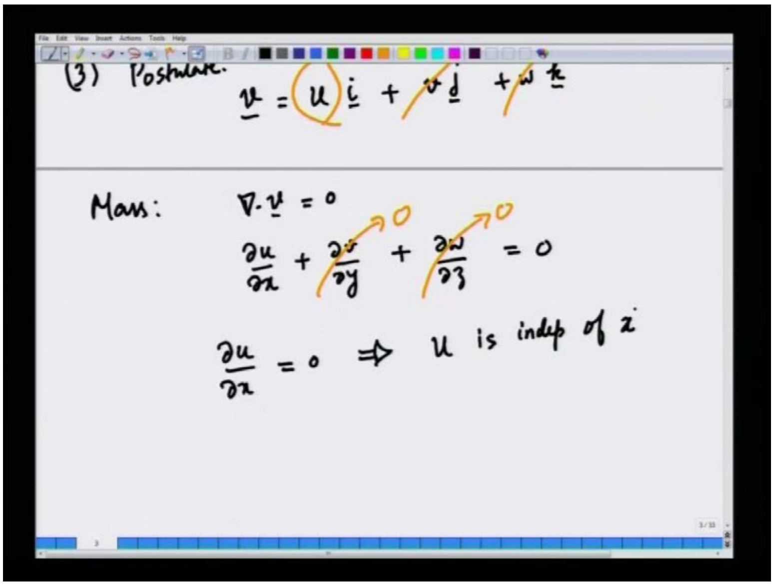

So, we are going to assume that the flow velocities are independent of y and z. So that is an assumption p will make. Now having said this, we will also postulate that the knowing that the, remember that the velocity is a vector, it has component u along the i direction, v along the j direction, plus w along the k direction. We are going to postulate that the only non-zero velocity is u, the other velocities are 0. And that is also that also comes from physical; this is a physical assumption that we are making. And it’s motivated by the fact that the top plate is dragging the fluid in the plus x direction and there is no reason and since there is no other driving forces.

There is no reason for us to believe that there will be flow in the two directions. So this is an assumption. And these can be checked for consistency later. So these are the three main assumptions that we will make. And the fourth assumption, well we will not we need not make that assumption. So let us look at the governing equation. So we will first take the mass conservation equation. Which is del dot v is 0, in Cartesian coordinates you will get partial u by partial x, partial v by partial y, is partial w by partial z is 0. Since we have assumed that v is 0, w is 0. The mass conservation, mass conservation equation is going to tell us that partial u by partial x is 0, which implies I am sorry flow velocities are independent of x and z here. Because the direction along the x is long and direction is z is long.

The direction along the y is a small smaller dimension, so that we cannot assume. This is consistent with our assumption that u is independent of x, it is merely conforms our assumption that our assumption is consistent with the mass conservation equation. And this is the only one equation that this is only one information from the mass conservation equation that u is independent of x.

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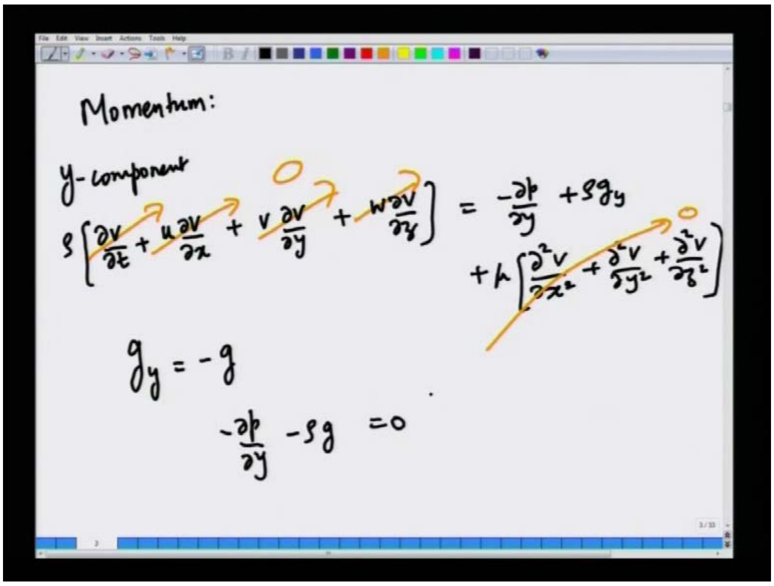

Now we will look at the momentum equation. The momentum equation is a vector equation, it has three components along x, y, and z. And let us first write down the y component. Just to tell you how that there is not much information that one gets. So, the y component of the momentum equation will tell us that this, these are the momentum equations in Cartesian coordinate system, that you can look up from any text book.

Times the laplacian of the y component of velocity. Now, let us look at whether which of these terms are important. Now, we have assumed that there is no velocity in the y direction, so all these terms are 0, so is this term. So the y component of the momentum balance simplifies to and let us assumes that gravity points in this direction. Let us assume that gravity is pointing in the direction of minus y.

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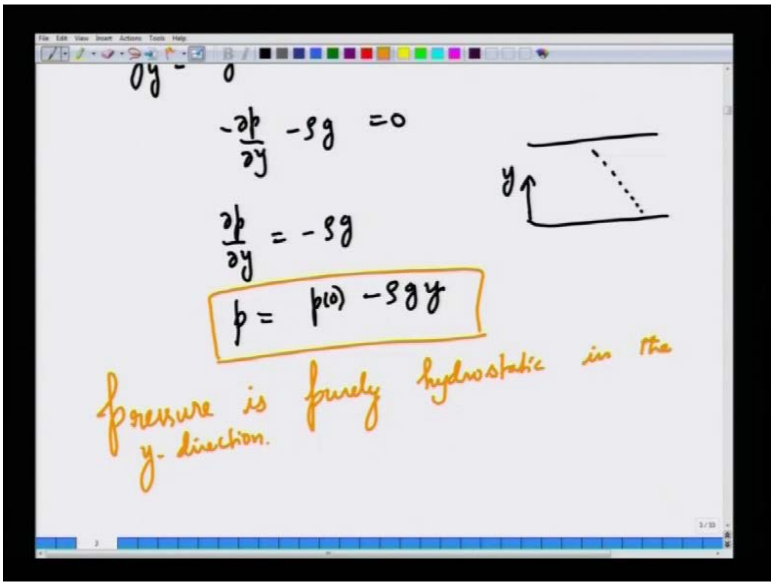

So, g y is minus, so you get minus partial p partial y minus rho g is 0 or partial p partial y is minus rho g or p is some p naught minus rho g y. So, all this essentially is that the pressure distribution in the y direction is purely hydrostatic. So the pressure decreases as you go up in y direction, this is the hydrostatic pressure distribution. Now, we are going to look so there is no other information that we from the y momentum balance except that the pressure distribution. The pressure variation is purely hydro static in the y direction, because there is no flow in the y direction and the only two forces that are acting on the fluid in the y direction or the pressure forces and the gravitational force. So they will balance each other out to give raise to hydro static variation.

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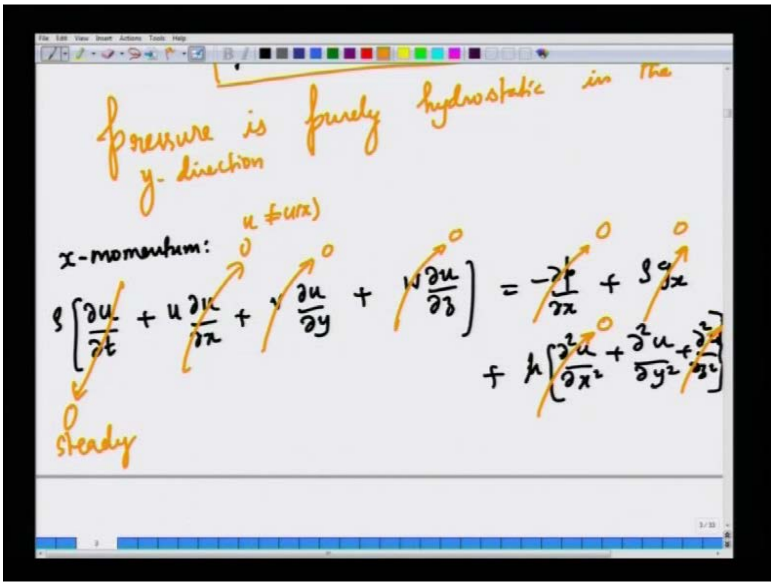

Now, we are going to look at the x momentum equation. Again, we have to look up text book to find out what is the x moment, x component of the momentum equation in Cartesian coordinates, but I am (( )) writing it for you here times the laplacian of the x component of the velocity. Now, again we have to see which terms away. Now this term is 0, because the floe is steady. This term is 0, because u is independent of x, it is not a function of x. This term is 0, because v is 0. This term is 0, because w is 0. This term is 0, because g is purely aligned along the y direction, so there is no component of the gravity in the x direction. Now pressure gradient, since the pressure is purely of hydrostatic, it is a function of y. And since we are not imposing any pressure difference along the x direction this is also 0.

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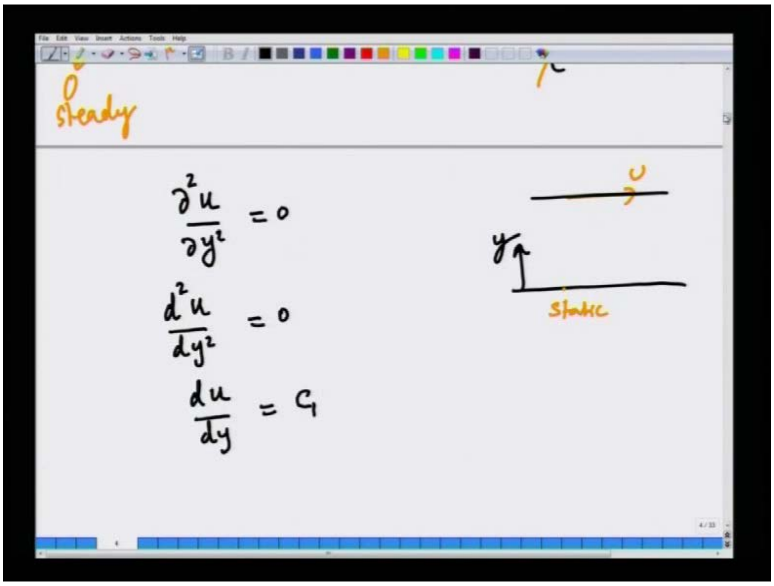

Now u is not a function of x, so this is 0. u is not a function of z, so this is 0. So eventually, the x component of momentum equation simplifies to partial squared u by partial y square is 0. Now this is not 0, because the top plate is moving with a constant velocity u, the bottom plate is stationary. So clearly u within the two plates will be a function of the normal coordinate, which is y direction in our problem, so this cannot be 0. And since use only a function of y, I can change the partial derivative to an ordinary derivative. So, if I integrate this once, I will get d u by dy is some constant c one. If I integrate this once more, u is some constant times y plus some other constant.

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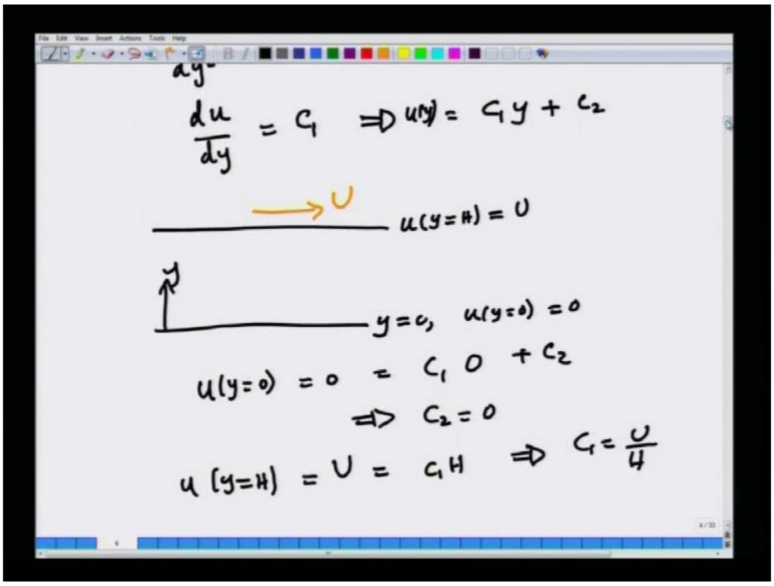

So we have two constants for the velocity profile which is a function of y. These two constants can be fixed by using the boundary conditions. That at y equal to 0 the fluid velocity is 0, because the plate is stationary, this is the no slip condition. At y equals s the fluid velocity is the top plate velocity which is moving with a constant velocity capital U. So, we can use these two conditions, if at y equal to 0 small u is 0, so this means that u at y equal to 0 is 0 which implies C 1 times 0 plus C 2. Which implies C 2 is 0, u at y equals H w, which implies that capital U is C 1 times H or C 1 is u divided by H.

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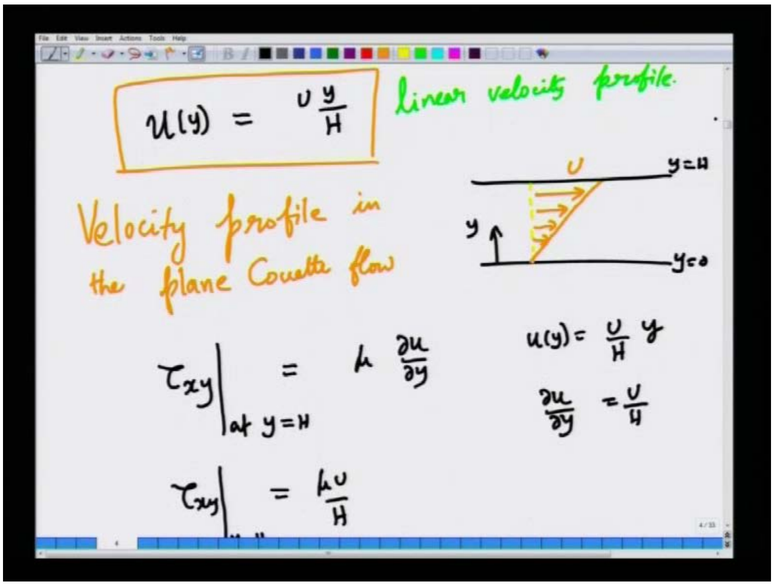

So this gives us that u of y is nothing but, u y u times y by H. So, this is called the velocity profile in the plane couette flow, this is the velocity profile in the plane coquette flow. So, we can plot it qualitatively as follows. Now, at the bottom the velocity is 0 at the top it is u and it varies linearly from 0 to H. So that is implication, this is y equal to 0 is y equals the width of the plate H, width of the gap H. So the velocity profile varies as shown here, it varies linearly; this is a linear variation of velocity, linear velocity profile. Now, once you have the velocity profile, you can find out the stress tau x y is mu partial the shear stress on the top plate at y equals H. because we want to calculate the force the we must exert on the top plate, so that the top plate moves with a velocity u, because there is a viscous fluid in between.

So tau x y is mu partial u partial y. Now, u of y is u by divided by H times y, so partial u partial y is u divided by H. So tau x y is a constant right through the fluid, so it is mu u by H, that is because the velocity profile is linear. So tau at y equals H is mu u by H. Now, comes the matter of interpretation. Therefore, that in order to find what this stress is, we have to see the sign convention.

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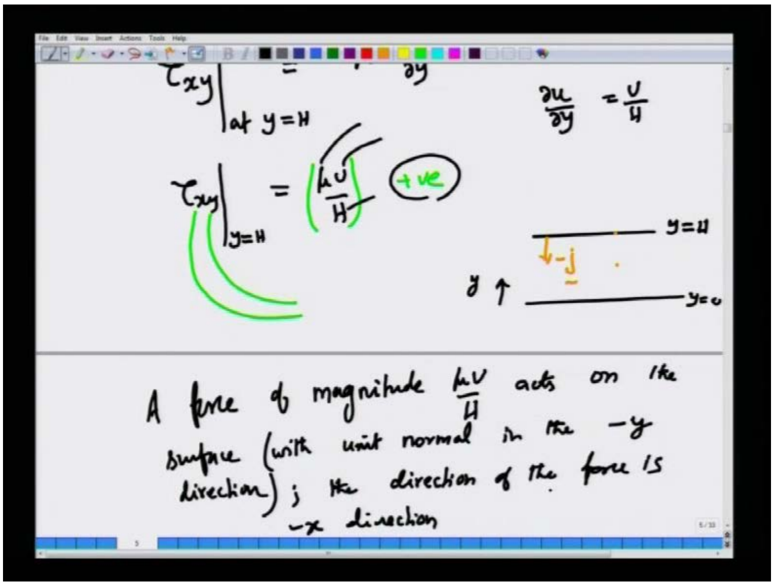

So tau x y is mu U by H, its positive. So that means the direction of the force and the direction of the unit normal both must be in the positive direction or in the negative direction. So, if tau x y is positive, the direction of the force is positive and the direction of the, so which unit normal we are talking about? We are talking about the top plate; we are looking at the top plate. Let us draw the system, you have the top plate at y equals H, this is y equals 0, this is y, the unit outward normal to the top plate points in the minus j direction. So we want the force that is exerted by the fluid on this surface. So, we want the force to that is exerted by the fluid on this surface. Now, that force will act in the negative, so since tau x y is positive. If tau x y is positive, since U is positive number, it is a velocity in the plus x direction it's positive, mu is positive, H is positive, tau x y is positive. If tau x y is positive that means that the force of magnitude mu U by H acts on the surface with unit normal in the minus y direction.

And the direction of the force is the minus x direction is minus x direction, this is the sign convention that we follow. Either both the outward unit normal and the direction of forces are positive or both are negative, that is implication of tau x y being positive.

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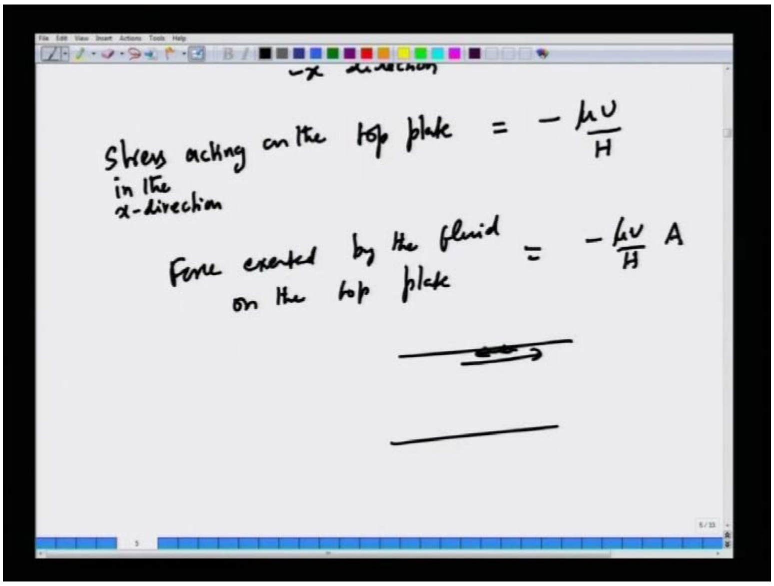

So, the stress acting on the top surface, top plate on the top plate is minus mu U by H. Because we have to take the sign also into account, the direction of the force is minus x direction. So the stress in the x direction is minus that is it acts in the direction of minus x. So the force exerted by the fluid on the top plate is minus mu U divided by H times the area of the top plate. So in order, for this is the force exerted by the fluid on the top plate, so the fluid while when it goes here, it exerts a force in this direction, the force exerted by the fluid on the plate is in this direction, so the force on this plate.

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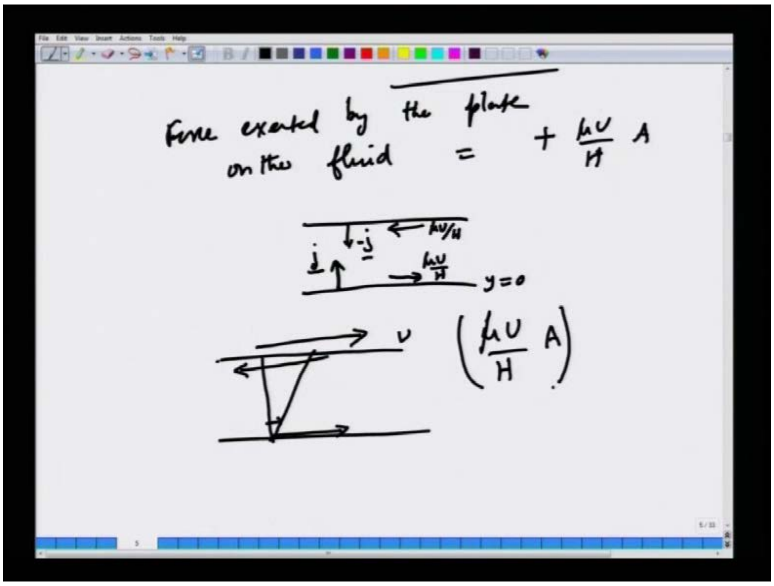

So, the force exerted by the plate on the fluid must be plus mu U divided by H times A. Because the plate is not accelerating it is moving at a constant velocity. So if you do a force balance on the plate the sum of all the forces must be balance out to 0, so it must be equal and opposite. What is important is that? Suppose we were to ask, what is the force exerted by the fluid on the bottom plate which at y equal to 0, the unit outward normal to the bottom plate is in the plus y direction the force will be in the stress will be mu U divided by H in the plus x direction. But, if you go to the top plate the unit outward normal is in the minus j direction, so the fluid will trying to retard this is the viscous drag that is exerted by the fluid on the plate.

That will be in the minus, the direction of the force will be in the minus x direction. The magnitude the direction of the stress will be in the minus of x direction, the magnitude of the stress will be again mu U divided by H; it is only the direction that is different. So the force exerted by this the fluid on the plate, on the bottom plate is in the plus x direction, which is mu U divided by H, whereas the stress on the top plate will be in the minus x direction. That is because the unit normal to the top plate is in the minus y direction whereas, the unit normal to the bottom plate is in the plus j direction.

So, this so when a fluid is flowing like this, the bottom plate the fluid exerts viscous drag in this direction, on the top plate the fluid exerts viscous drag in this direction. So in order to push the top plate in the plus x direction, you must exert an equal and opposite force, so that the velocity U is achieved. And what is the force you must exert is this simply magnitude of the force is mu U divided by H time of the area of the top plate. So this simple illustration of how very simple solution of the Navier stokes equation. Can be used to find something practically, perhaps practically important that is if you want to push one plate related to another and if the gaps between the two plates are filled by very a viscous liquid.

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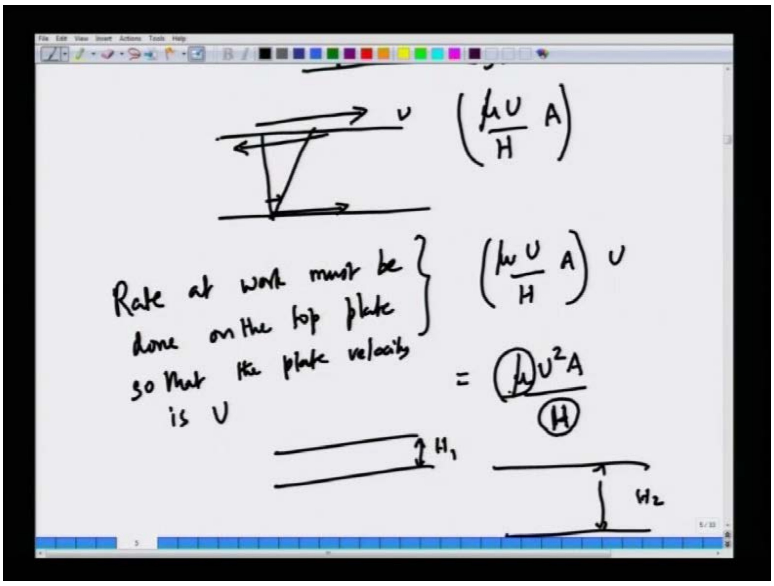

Then you can ask, what is the force that I must exert on that. So you can also ask what is the rate at which I must do work on the top plate be done on the top plate. So that the top plate, because it is continually moving. In order to do that on the top plate, so that the top plate moves with the constant velocity, top plate velocity is u. Well, the answer is the rate at which work is done is force times the velocity, so you will get mu U square a divided by H. So, if you have a viscous very viscous liquid very confined between the two plates, the rate at which you must do work is much more compare to when you have a less viscous liquid. That because, a more viscous liquid resists flow in a in a more stronger way compare to a less viscous liquid.

So this is an illustration of a very simple application of an analytical solution of the Navier stokes equation. Wherein, we made several assumptions that the flow is steady and we also made the assumption that the flow velocity is in the x direction and its independent of the x and z direction and it is only a function of the normal direction y. And have after having done that the Navier stokes equation in the x, momentum the Navier stokes momentum equation in the x direction simplify to a great deal. And we solve it to find that the velocity u varies linearly from the bottom wall to the top wall. And once you find once you found what the velocity profile is, then you can find what is the stress exerted by the fluid on the top plate and that is in the minus x direction. And the magnitude of the stress is mu U divided by H, where mu is the viscosity is the top plate velocity and the H is the gap thickness. Now, once you find what is the stress, then you can find the force by multiplying this stress by area.

If you want to push the plate with the constant velocity, then the sum of forces on the top plate must be 0. So that force that you must exert on the top plate, so that it move with the constant velocity must be mu U, in the plus x direction, the force must act in the plus x direction magnitude of the force the magnitude of the force is mu U divided by H times A. You can also find, since it is continues process you can keep on applying the force, you can also find what is the rate which you can also must do the work on the top plate, so that it happens. That is simply obtained by multiplying the force that you are exerting on the top plate by the velocity of the top plate.

So that is simply mu U square times a divided by H. So this tells you a few things that the rate at which you do work is directly proportional to the viscosity of the fluid. If the viscosity of the fluid is large that means you must do more work, also it also tells you that the same fluid the rate at which you do work decreases with increase in it. That is, if I have a plate thickness of H and A plate thickness of h 1 and h 2, where h 2 is larger than h 1, everything else being constant that the area of the top plate being constant, the liquid being constant, the velocity being constant. Then the rate at which you do work is inversely proportional to the gap thickness.

So you have to do lesser work, in order to push the top plate fluid. And as you decrease the top plate fluid gap thickness, then the rate at which you must you work will increase as one over h. So, these are the salient features of these simple problems called planar couette flow problem. Now, you may ask where is it really applied. Well, it is not just applied whenever you just have only two parallel plates.

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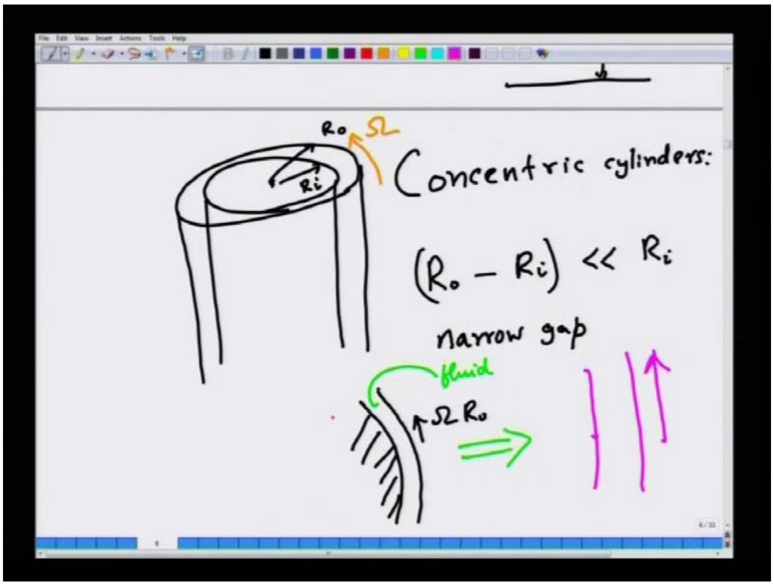

Suppose you have two concentric cylinders, you have one cylinder, inner cylinder surrounded by an outer cylinder, the two concentric cylinders. Now the radius of the inner cylinder is R i, the radius have two concentric cylinders. Now, if the gap between the inner and outer cylinder R o minus R i is very small compare to R i, that is, this is the gap is too narrow. And let us assume that the outer cylinder is rotating with constant angle of velocity omega. So, if the gap is too narrow, so if I draw the top view, if the gap is too narrow and one cylinder is moving with the constant velocity angular velocity so the linear velocity is omega R times naught. The inner cylinder is stationary and the fluid is only present in the gap, so the gap is occupied by fluid.

Now in the narrow gap limit, locally it will appear as if this is like a planar couette flow, with one plate stationary the other plate other plate moving. So, you can use this simple solution. Locally, even in cases where the two plates are not parallel to each other, but if the gap between the two plates is narrow then locally the velocity profile will be just the planar fluid velocipede profile, which we just derived.

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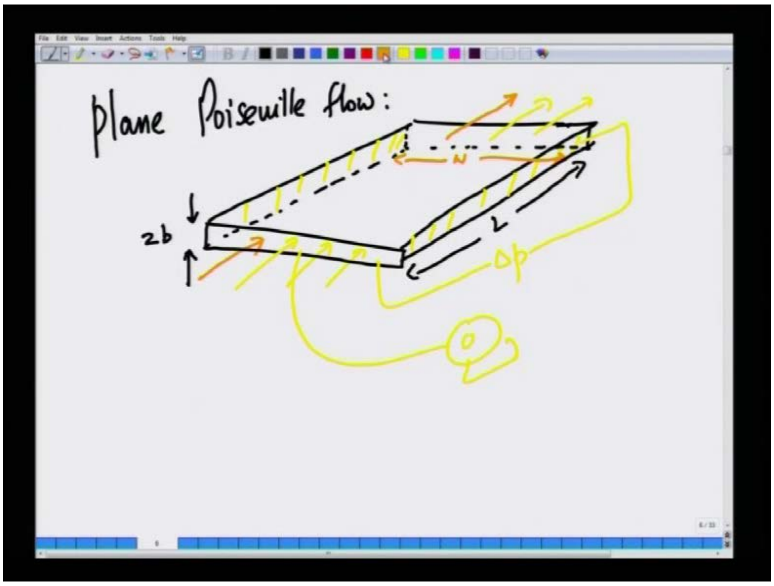

We are going to next move to another application of the solution of Navier stokes equation and this application is called the plane poiseuille flow. Now, the system will appear very similar to what we did before for the plane coquette flow. Except that, what drives the flow is not the motion of the top plate, but instead we are going to look at the effect of a pressure gradient or pressure difference applied across the two ends of the plates. Now, before I simplify the problem or ideally is a problem, I want to tell you that this really what we are essentially going to solve. In practice, is going to be like a slit, flow between, you have a rectangular channel. And the distance between the two plates the gap width is essentially, let us call it, let us call that 2 b. Let the length be L and let the width be w and fluid is flowing in this way. And the reason why the fluid is flowing, this is I mean imagine that the ends of channel are closed; the sides of the channel are closed. That is, they are also rigid balls and the top and the bottom of course, are rigid walls.

And there is a slit there is a gap in which fluid is entering and there is a gap outside, where fluid is leaving. And why is the fluid flowing? The fluid is flowing because you are connected this inside and outside to a pump. There is a pressure difference across the inlet and outlet, so you are pumping fluid and that is the reason why fluid is flowing. So there is a pressure drop delta p that happens between the inlet and outlet. So, this is an example of what is called pressure driven flow.

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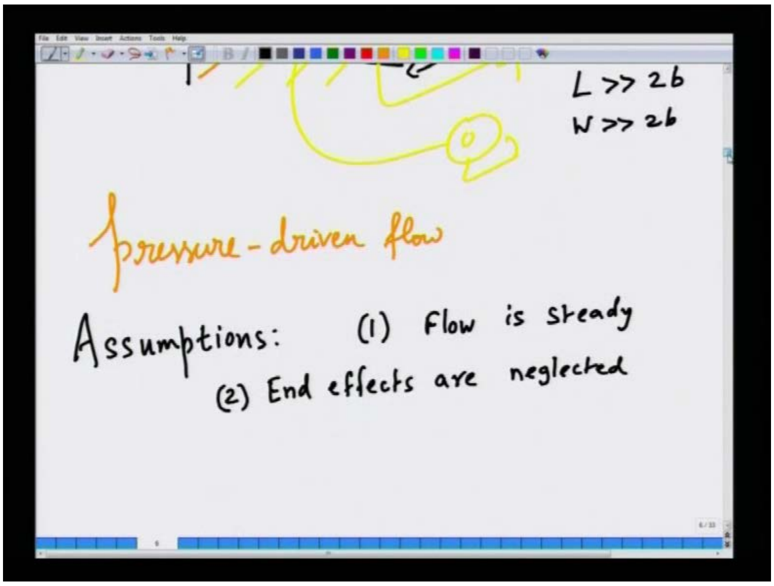

So this example of what is called a pressure driven flow. Now, we are going to make assumptions which will make the problem tractable. The assumptions are namely that the flow is steady. Now, this assumption is again a physical assumption, just as before that when you are applied pressure difference is independent of time, then you would expect reasonably that the flow inside this channel is also independent of time. The flow is steady, end effects are neglected. That is, you have these two side walls and the fluid is entering and the fluid is leaving. If L is very large compare to 2 b and w is large comparative to 2 b that is the gap thickness is very very small compared to the length, as well as width of the plates.

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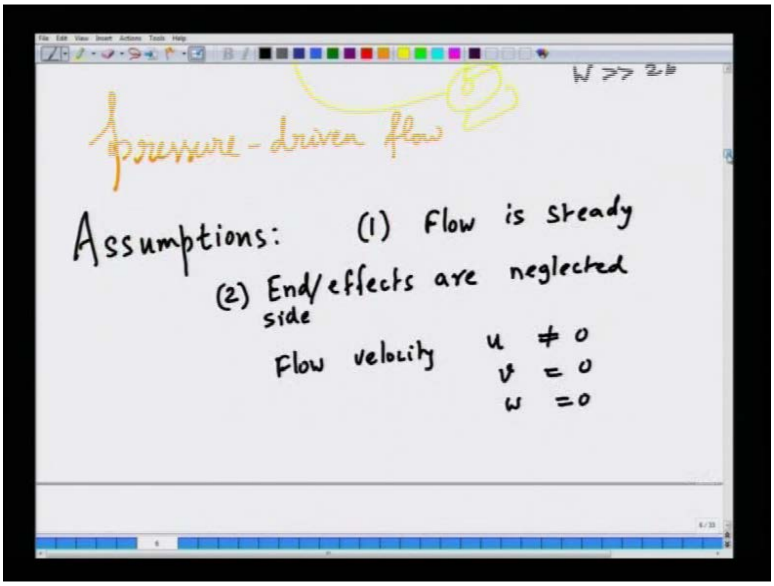

Then you can hope that the end or side effects are neglected. So the flow velocity is, the only non-zero velocity flow is the x component, so we have to place a coordinate system. So this is the direction of the flow x, the normal component is y and the third component is z, as usual. So the flow velocity u, only u is non-zero, but v is 0 and w 0. v 0, because there is no driving force in the other direction the y direction and same for w, the z component of the velocity.

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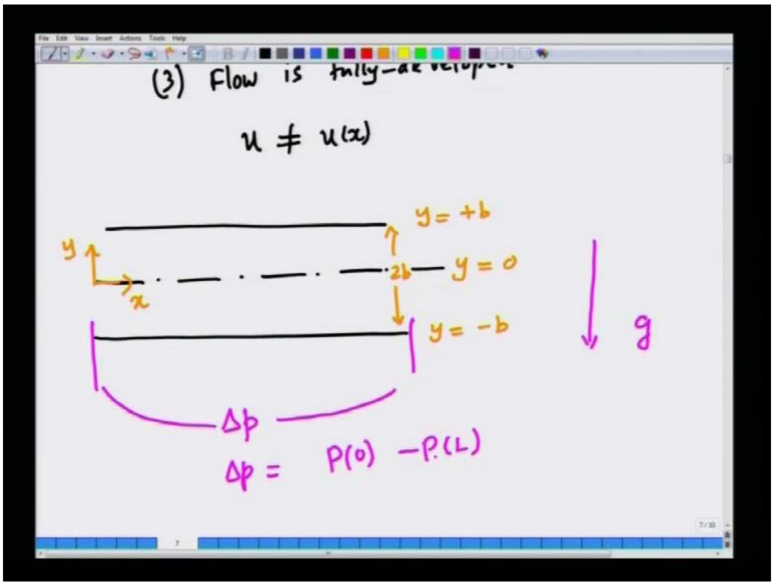

And we are also going to assume that flow is fully developed. When I say flow is fully developed, I mean that u is not a function of x. You can imagine that u will be independent of w, when you are far away from two side walls. Because they are near very close to the side walls there will be some dependence of u on the z direction, z coordinate. But in the middle in the, since we are considering a very very very wide channel. In the middle of the channel, the flow will be reasonably independent of z. Similarly, when flow enters from a reservoir, it will initially be a function of the x direction. But, when you are sufficiently far away from entry or exit, we say that the flow is fully developed, if the velocity of the flow u is independent of the direction of in which it is flowing.

The flow is steady, the flow is fully developed, u is independent of x and we also assume that the only non-zero components of the flow are, is just u the other components v and w are 0, identically 0. So, the flow is only in one direction. Now therefore, we can simplify our geometry as flows. So you have two plates, this is the centre line. Now, I am going to place the coordinates as follows, I am going to place the y equal to 0 at the centre line. So the top plate is y equals plus b, the bottom plate y equals minus b, because the width of the channel is 2 b, the gap thickness width of the channel is 2 b.

Now, we can have without loss of generality gravity acting in the minus y direction as before. The reason why fluid is flowing is because of a pressure difference. There is a pressure drop delta p is p pressure at 0 minus pressure at L, that is a pressure drop. And L is in general the length of the channel, as you have pointed out in this figure; L is the length of the channel, this length. So this is the simplified problem that is you consider flow in the x y plane, because things are independent of z. Because the diagonal width of the channel is very large compare to the gap thickness.

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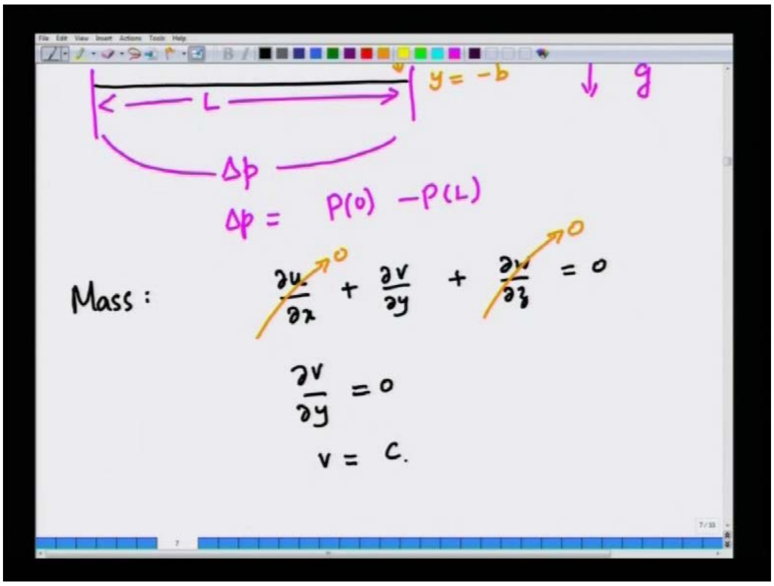

Now, so we are going to assume, we have to first write down what happens to the mass conservation equation. In Cartesian coordinates, because that is convenient to analyse this problem. First of all, we can find that we can say that w is 0 and since u is fully developed, this is 0, so the mass conservation equation will tell you that partial v by partial y 0 or v is some constant that is independent of y. So, since things are independent of x, it just a constant.

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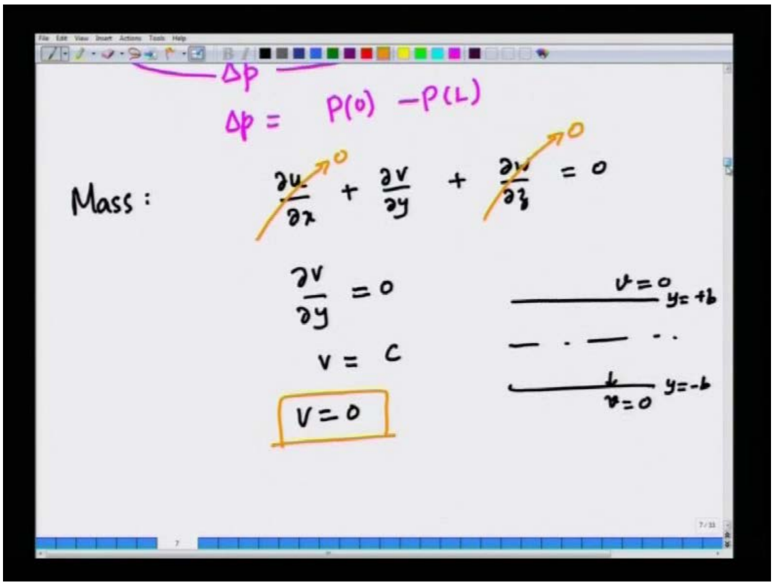

Now, the boundary conditions at the top and bottom wall at y equal to minus b, as well as y equal to plus b is that the velocity v is 0. Because there is no normal velocity, velocity v is 0 y equal minus b and y equals plus b. So, if it is 0 at one of the walls, and if it is the velocity is a constant, the constant has to be 0 everywhere. So, the normal component of the velocity is 0 everywhere in the flow. You do not have to make any independent assumption that normal component velocity is 0. It comes out naturally as fall out of the continuity equation or the mass conservation equation. Or you can say that the mass conservation equation is consistent with our assumption that the normal component of velocity is 0.

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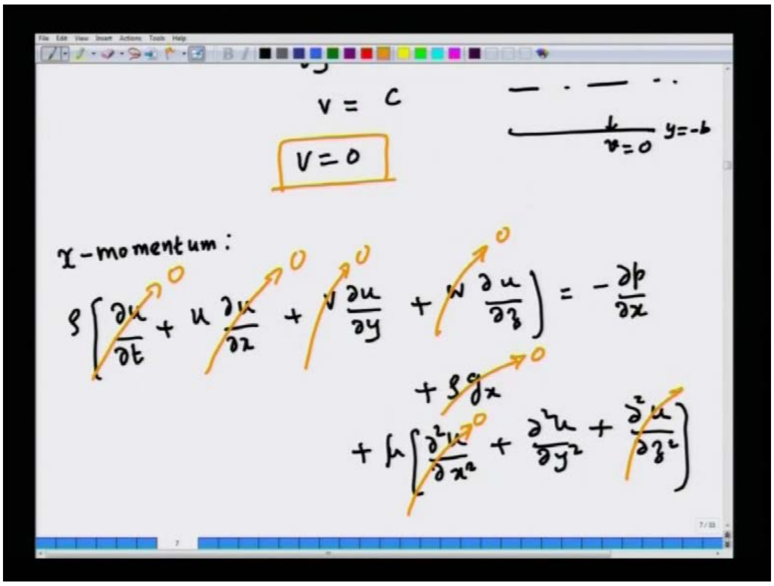

Second, we will go to the y momentum balance or let us just go to the x momentum balance as before. The x momentum balance becomes rho; this is the same as the previous problem. So, let me write this down quickly as the couette flow problem, mu times the laplacian of the x component of velocity. Now, this is the x component, so let us simplify this by making the by using all the assumptions. By invoking the assumptions, the flow steady 0, the flow is fully developed, this is 0. There is no y component of the velocity, this is 0. There is no z component of the velocity, this is 0. There is no acceleration due to gravity in the x direction, this is 0. Flow is independent of the x direction, this is 0. Flow is independent of the z direction, this is 0.

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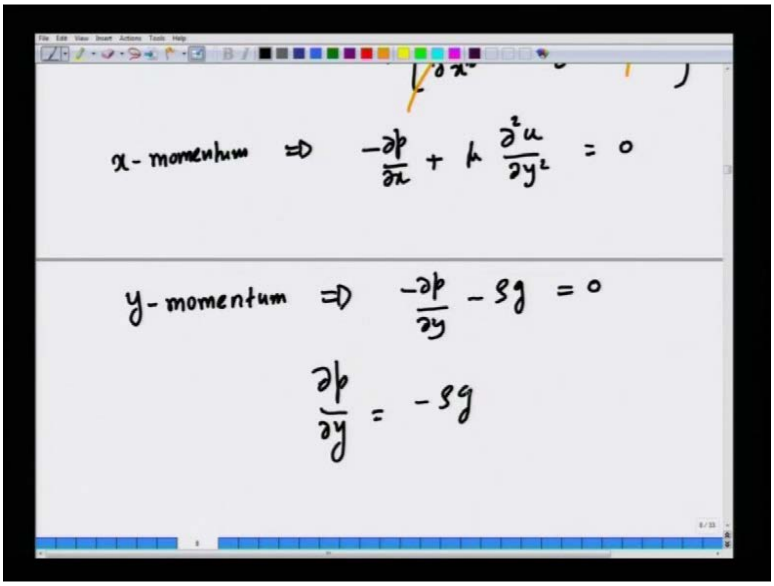

So the x component of the momentum equations, momentum Navier stokes equation simply tells us that minus p partial p partial x plus mu partial square u by partial y square is 0. Now, the y momentum or y component of the momentum balance of the Navier stokes equation is very similar to the previous problem, so I would not repeat it. It is the same as in the couette flow problem; partially p partial y minus rho g is 0. Because, we have pointed the acceleration due to gravity vector in the minus y direction that is why you have a minus sign here. Now, this implies that p is minus rho g y partial p, let us write this down slowly, partial p partial y is minus rho g.

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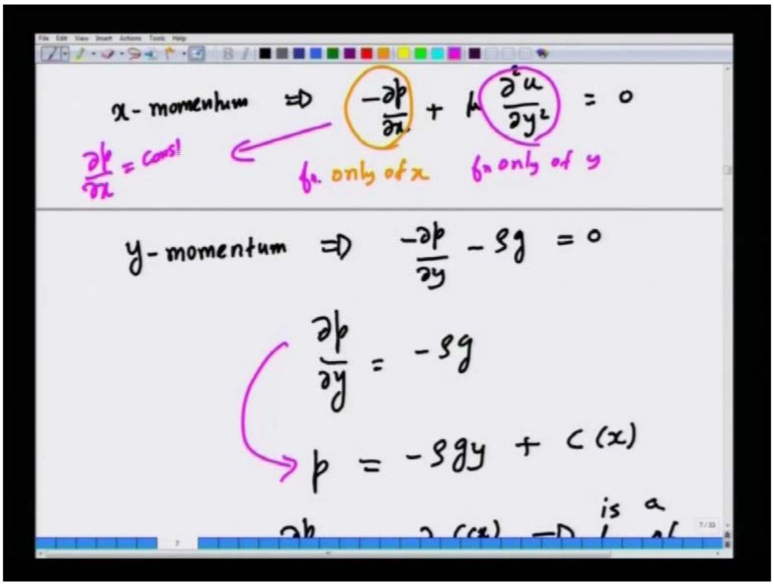

So this is a partial differential equation, so if I integrate partially with respect to y, I will get minus rho g y plus a constant, which is in general function of x, because, this is consistent with this equation. If I take the partial derivative with respect to y, the partial derivative of this constant will becomes 0, because there is a partial derivative, so you will get minus rho g. So if I partially integrate this, I will get a constant that is in principle a function of x. Now, if we differentiate this with respect to x, this becomes 0, because it is partial derivative. So what this means that, this is partial c of x by partial x. So partial p partial x is a function only of x, x only

It is a function of x y, it is not a function of y. So, you look at the x momentum balance, partial p partial x is a function only a function only of x. While u is an only of function y, so partial square u by partial y squared is a function only of y, it is a function only of y. So, that means at each has to be equal to a constant or partial p partial x is a constant.

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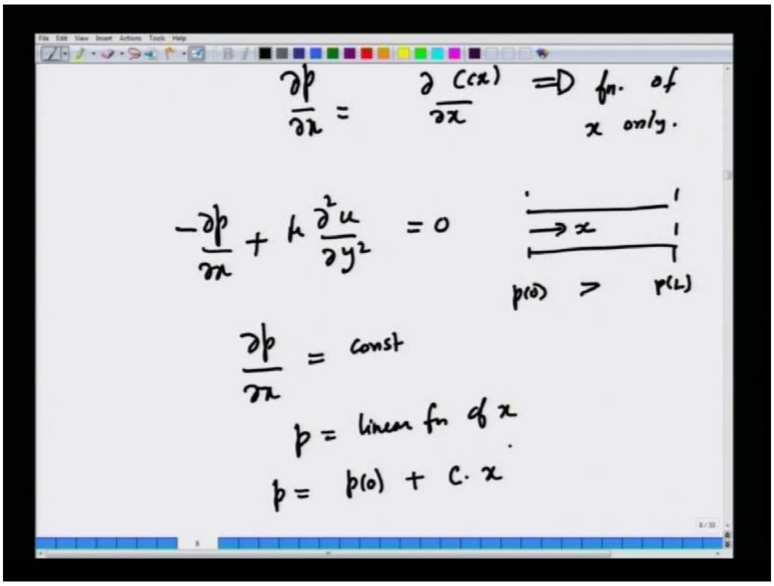

So let me write down here, minus partial p partial x plus mu partial square u by partial y square is 0, but partial p partial x is a constant. Now, what that constant is comes from physics of the flow, the flow is in the plus x direction, that means that the pressure at 0 must be greater than the pressure at L. So, partial p partial x is usually, if it is a constant, then p is then partial p partial x, if it is a constant, that means p is a linear function of x. So, if I integrate this, I will get p is some p naught plus some constant times x that is what it means.

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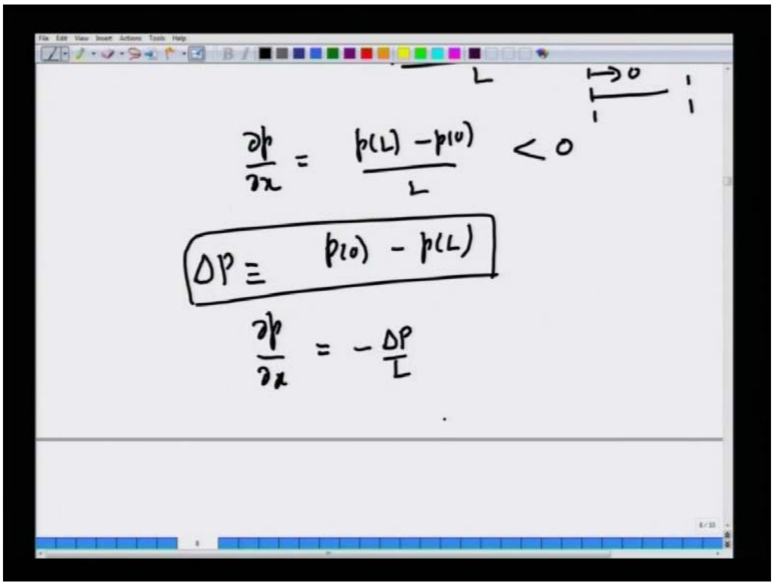

So I can write, p is a constant linear function of x. So I can write, what is this constant. Let us say at x equals to L p is p L, so p at 0 plus c times L. So the constant is p at L minus p at 0 divided by L. So, partial p partial x is nothing but, the constant c, which is p at l minus p at 0 divided by L. if the flow in the plus x direction then d p dx has to be negative, because the pressure at 0 must be greater than the pressure at L. So, if I call del p as p at 0 minus p at L, then partial p partial x will become, if I define del p as this then partial p partial x will become minus del p by L. So, in the x component of the momentum equation, instead of minus partial p partial x, I will get minus of minus del p by L, which will be plus del p by L.

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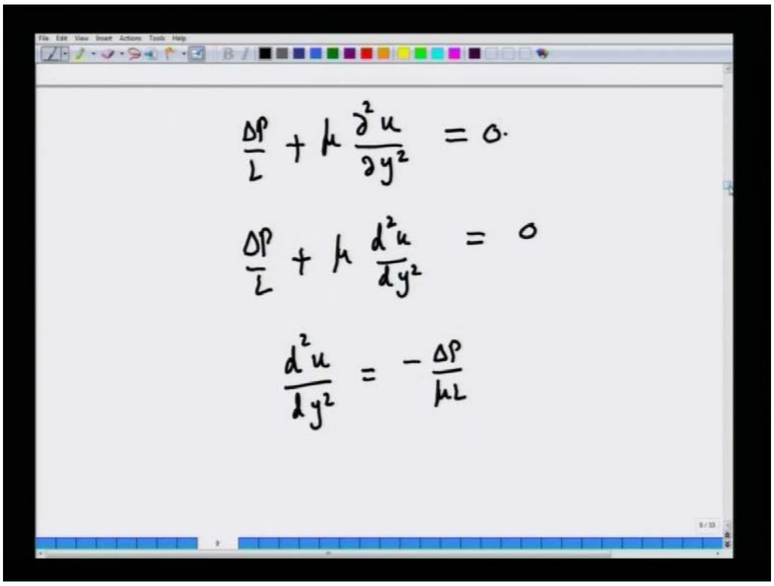

So the x momentum equation becomes, del p by L plus mu partial square u by partial y square is 0. So, we can solve this in the following way. Now, I can of course, without loss of generality write as partial differential with respect to y is ordinary differential equation, because u is function only of y.

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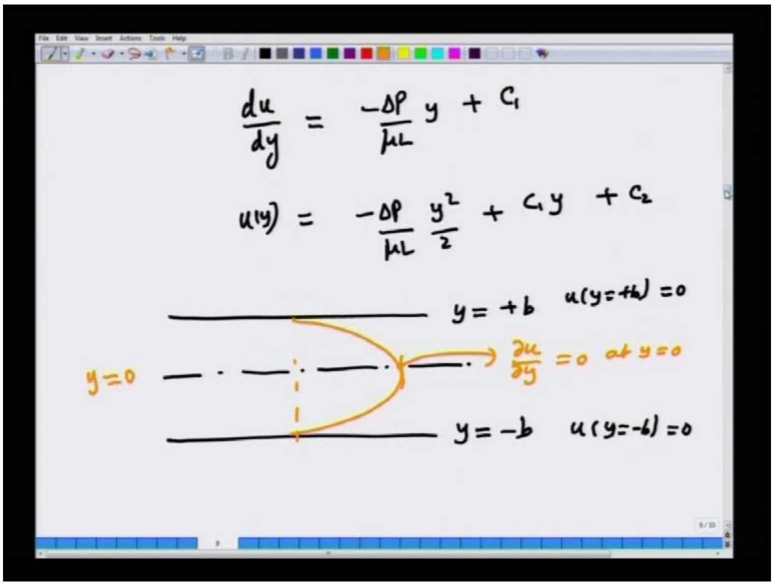

So I can solve this by taking this to the other side, is minus del p by mu L. So, I have to integrate this twice with respect to y, if I integrate this once with respect to y by mu L times y plus c 1 or u is minus del p by mu L times y square by 2 plus c 1 y plus c 2. Now, this means this is u as a function of y. How are you going to fix the two constants? The two constants are can be fixed in the following way. You have a boundary at y equals to minus b; you also have a boundary at y equals to plus b. You can assume that you can use the no slip condition that u at y equals to minus b is 0 and u at y equals plus b is 0 to find two constants. Or you can also use what is called a symmetry condition. This y equal to 0 is the symmetry line, because the velocity profile has to be symmetric about this. Because, everything is it is a mirror reflection of whatever is happening, so the profile has to be symmetric.

That means, d u dy must b 0 at y equals to 0 at the centre line. Because it is the line of symmetry, things will look the same both the in the upper half as in the lower half, so it is symmetric. The velocity profile has to be symmetric about y equal to 0, therefore d u dy has to be 0 at y equal to 0. So we will use this root, because it is simpler. We know what is d u d y. d u dy is this, if I evaluate this at y equals to 0, d u dy at y equal to 0 is 0, this implies that at y equals to 0 this become 0. This gives you the simple condition that c 1 is 0. The symmetric condition straight away gives you that c 1 is 0.

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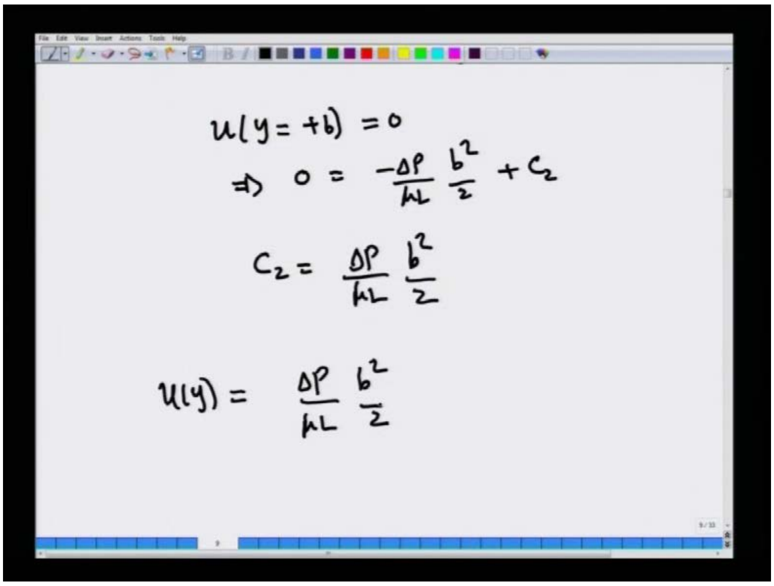

So c 1 is not there. The other condition, the other constant can be found by using u at y equals plus b is 0. This implies, 0 is minus delta p by mu L b square by 2 plus c 2 or c 2 is delta p by mu L b square by 2.

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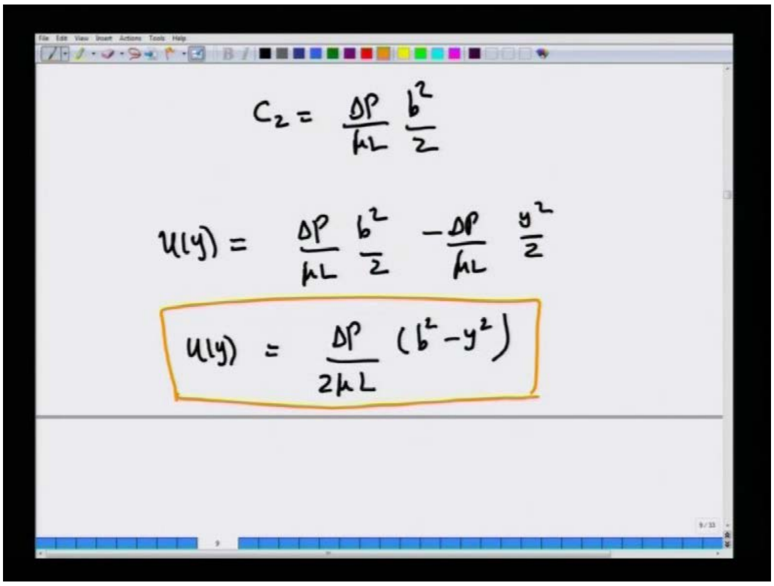

Now, I can substitute this back in this expression for u instead of c 2, I will put this. So, u of y is delta p by mu L b square by 2 minus delta p by mu 1 y square by 2 or u of y is delta p by 2 mu L times b square minus y square. This is the velocity profile for plane poiseuille flow that is flow between two parallel plates driven by pressure difference.

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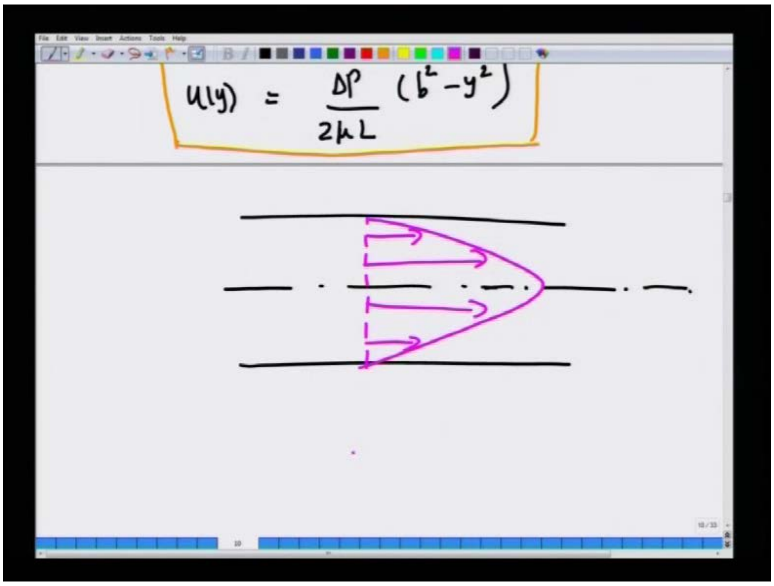

Now this has a parabolic form, so if I had to draw the two plates, and then draw the velocity profile, it will be a parabola centred about with maximum at the centre. The maximum velocity of this parabola will be at the centre. We will stop at this point, and the we will continue in the next lecture.