Process Control and Instrumentation
Prof. A. K. Jana
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 9
Dynamic Behavior of Chemical Processes (Contd.)

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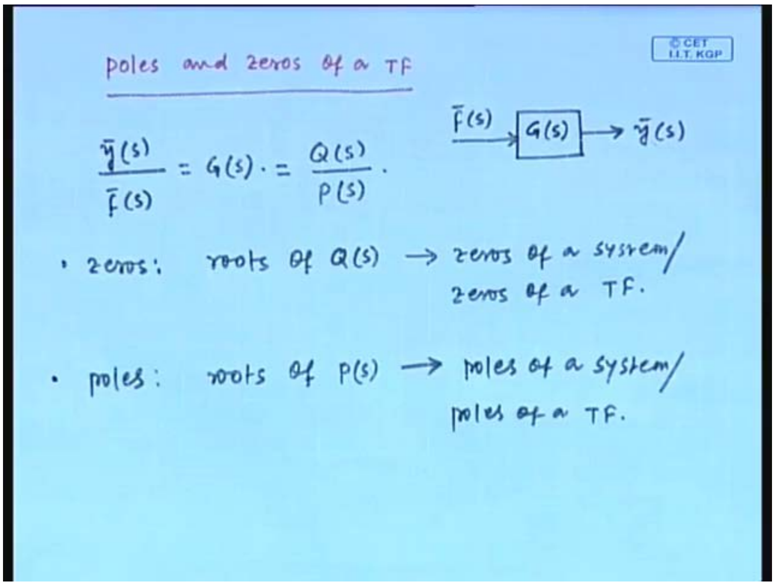

We will start the poles and zeros of a system. So, topic is poles and zeros of a system or you can say transform function. Suppose, this is a blog diagram for if single input, single output system G s input to this process is f bar s and output of this process is y bar s. So, we can write this in this form y bar s divided by f bar s equal to G s. G s is the transform function we know, now the transform function G s may be, the ratio of 2 polynomials, one polynomial is Q s another polynomial is P s.

Now, what is the zeros of a transform function, zeros the roots of the polynomial Q s are the zeros of the transform function or the system. If Q s is the polynomial, then the roots of the Q s are the zeros of the system, roots of the polynomial Q s are called zeros of a system or zeros of a transform function, got it is a very simple Q s is a polynomial now, we need to determined the roots of that polynomial and those roots are zeros of that system or transform function.

In the similar fashion, roots of the transform function P s are called poles of a system or poles of a transform function. Now, we will take one simple example, to explain the concept of poles and zeros.

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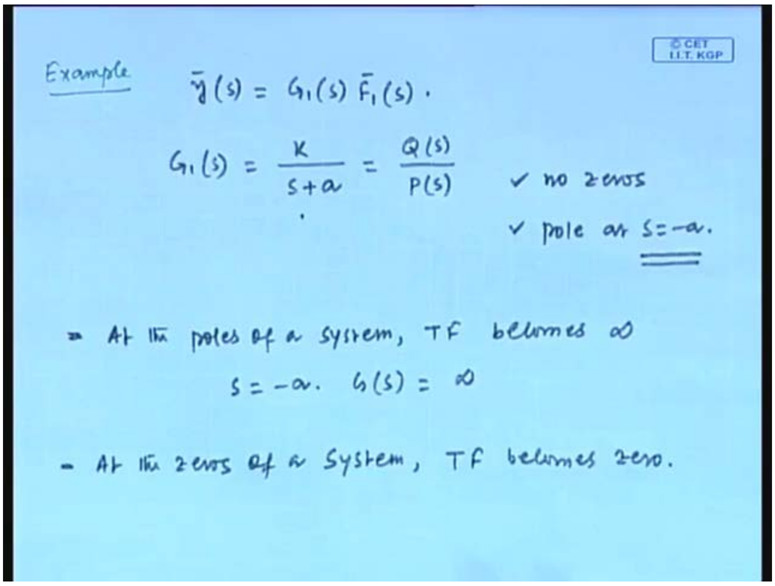

So, will take next one example will consider the equation in s domain that is s a G 1 s f 1 bar s, this is the representation of the model for a SISO system in Laplace domain, Hoyer G 1 is given as k divided by s plus a. You can write this as Q s divided by P s, but the numerator is basically constant. So, that is not the polynomial of s; that means, there is no zeros. So, for this example system, there is no zeros fine and what about P s, P s is s plus a.

So, there is pole at s equals to minus a understood there is no zeros for the example system and there is a pole at s equals to minus a. Now, one point I want to mention that is, at the poles of a system, the transfer function becomes what, at the poles of a system transform function becomes infinity. Our pole, is the pole for the system is at s equals to minus a, if we substitute in this equation s as minus a the transform becomes infinity, lesson it, if we put s equals to minus a then G s becomes infinity, but the similar way we can say at the zeros of a system the transform function becomes 0. So, at the zeros of a system transform function becomes 0. So, these are the concept of poles and zeros, in the next we will discuss with the general form of transform function.

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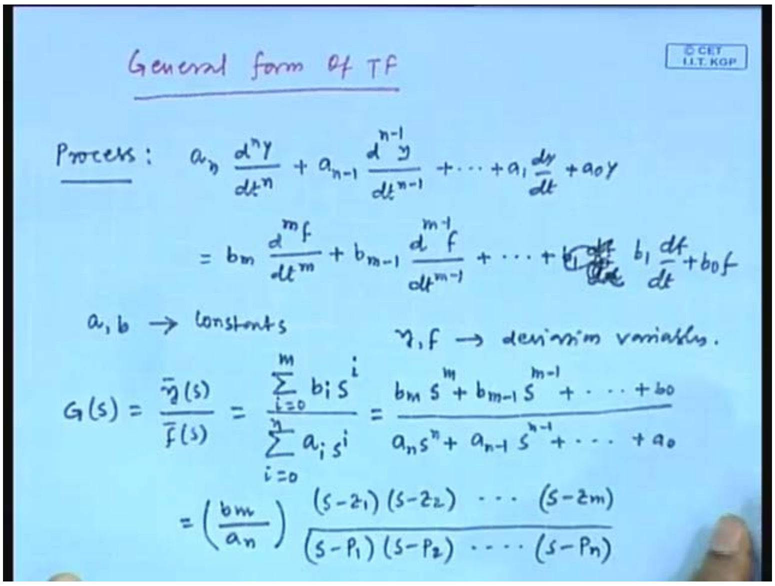

We will discuss the, general form of transform function. So, and n’th order system, we will represent by a linear ordinary deferential equation I mean process will represent by an n’th order ordinary differential equation in linear form. So, that has the form a n d n y d t n plus a n minus 1 d n minus y d t n minus 1, like this a 1 d y d t plus a not y equal to b m d m f d t m plus b m minus 1 differentiation. So, this is d t d 1 d f d t plus b not f this is a general representation of a linear process.

Hoyer all the a and b are constant coefficient and y f both are deviation variables, a and b these are constant coefficient y and f both are deviation variables. Now, if we take Laplace transform and rearrange, we get the transform function and that has the form of G s equal to y bar s by f bar s equal to i equal to 0 to m b suffix i and s to the power i. Similarly, here i equals to 0 to n a suffix i s to the power i and we get b m s to the power m plus b m minus 1 s to the power m minus 1 last term is b not a n s to the power n a n minus 1 s to the power n minus 1 last term is a naught.

Now, if we take common b m divided by a n we get s minus z 1 s minus z 2 s minus z m hole divided by s minus p 1 s minus p 2 s minus p n. This is a general form of transform function, here all z represent the zeros and p represents the poles.

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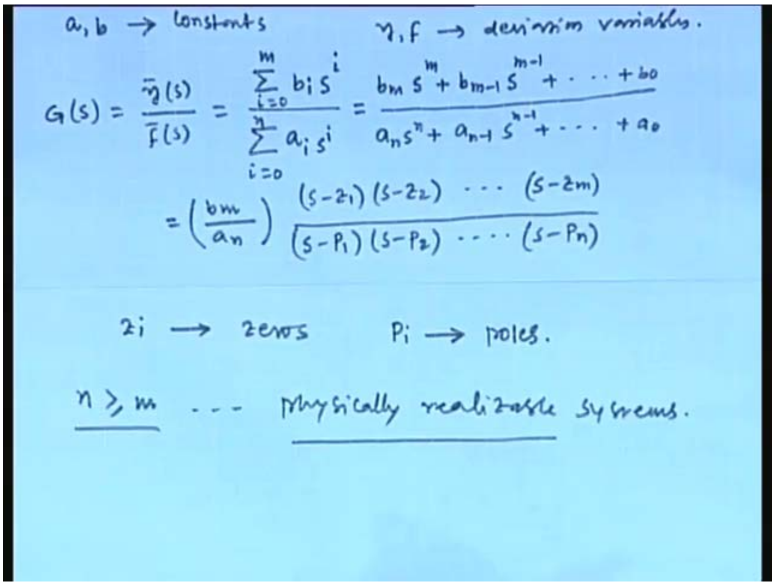

Z i denotes the zeros and p i represents the poles. Now, what about the order n and m, I mean n will be greater than m or equal to n or less than m for all physically meaning full system n should be greater than or equal to m, for all physically mining full system n should be greater than or equal to m, physically realizable systems, this term is used in process control, physically realizable systems.

I mean all physically meaning full system n should be greater than or equals to m you can test it, you consider n less m and you check what type of response, you are getting like for example, a step change in input variable, produces and infinites spike in the output. If you consider suppose n equals to 0 and m equals to 1, then you give a step change in input variable f you will see that, there is an infinites spike in the output, for this reason for any physically meaning full system, we have to consider n greater than or equals to m. So, next we will discuss transform function and it is analysis.

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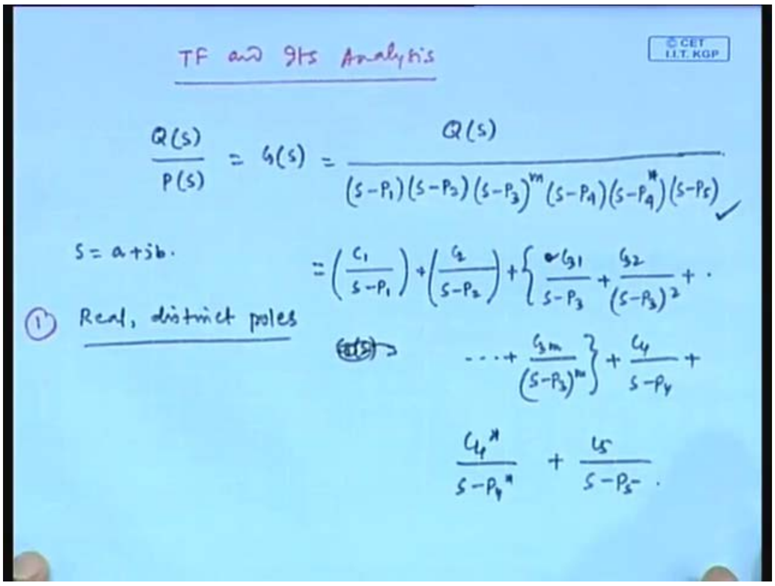

Transform function and it is analysis. One important point I mean the use the transform function we have discussed, that is to observe the transient behavior of a process, we can use the transform function. Another important I mean the use of transform function is the, stability analyses, to know the stability of a system, we can use the transform function that we will discuss in the next.

So, we have mentioned earlier that, this Q s divided by P s this is basically, the representation of a transform function I mean transform function is the ratio of 2 polynomials, 1 is Q s divided by P s. Now, we will take one general form of this transform function, that is represented by this equation Q s divided by s minus p 1 s minus p 2 s minus p 3 hole to the power m s minus p 4 s minus p 4 star s minus p 5, this is the general representation of a transform function.

Now, s is basically if variable, which is defined in the complex plain, in this o a I mean this is the form of s, s equals a plus j b, s is defined in the complex plain by this form. Now, considering this general form of a transform function we will discuss, different types of poles, for example one system which is 2 poles, they both are real, but distinct, in another case we will consider a system which has multiple poles and all the poles are real, what about the stability of that system.

One system which has 2 complex conjugate poles, what about the stability of that system, like this different situations, so we will consider and all these poles are included in this general form. So, first we will consider distinct real poles, so in first case will consider, real, distinct poles. Now, to consider this case we will reduce the general form of the transform function, to this expression G s equal to any way, we can take the partial fraction of this equation, before considering the different cases.

So, we can write this equation in this way. So, c 1 s minus p 1 plus c 2 s minus p 2 plus c 3 1 s minus P 3 plus c 3 2 s minus p 3 hole squire, like this way c 3 m s minus p 3 hole to the power m plus c 4 s minus p 4 plus c 4 star s minus p 4 star plus c 5 divided by s minus p 5. This is the partial fractions of this transform function, general form of the transform function.

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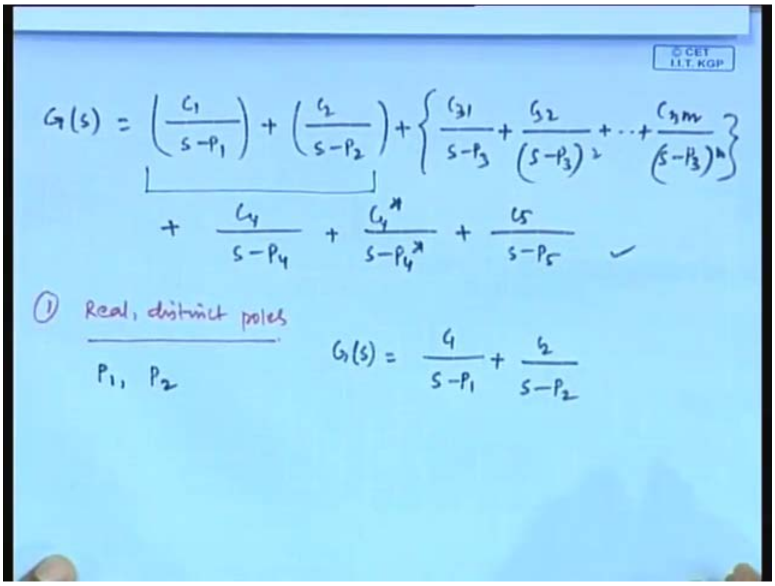

I think I am writing again that G s equals to c 1 s minus p 1 c 2 s minus p 2 plus c 3 1 s minus p 3 c 3 2 s minus p 3 hole squire last term is c 3 m s minus p 3 hole to the power m plus c 4 s minus p 4 c 4 star s minus p 4 star plus c 5 s minus p 5 I have written that general form in this equation. Next we will consider the first case that is real distinct poles. So, in case one we consider real, distinct poles. In this case we will consider just the 2 poles, both are real and both are distinct.

Those 2 poles are one is p 1 another one is p 2. So, the general form of the transform function is reduces to G s equals to c 1 by s minus p 1 plus c 2 by s minus p 2 can I write this, we are presently considering a system which has 2 distinct real poles and this poles are p 1 and p 2. So, we are considering just this part of the transform function, I have mention another thing that the s is define in the complex plane, by s equals to a plus j b.

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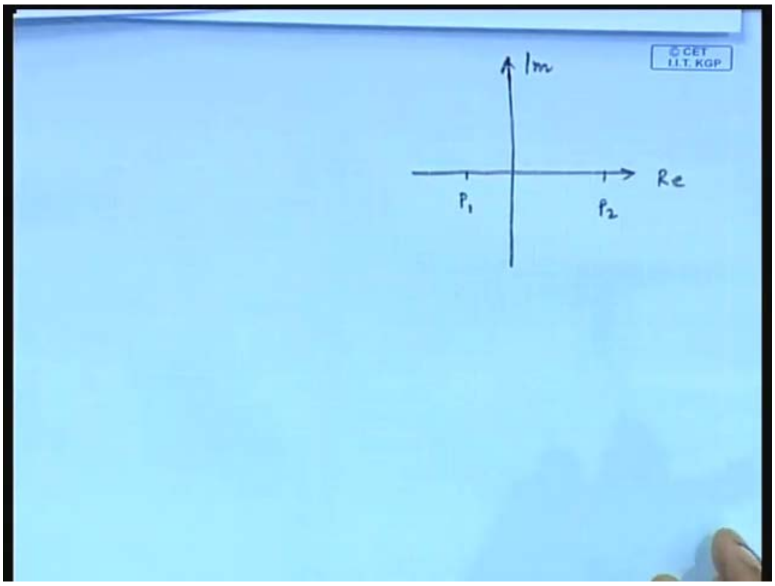

So, we can represent this 2 poles, in the complex plane in this way, suppose this is real axis this is imaginary axis. So, both the poles are real and distinct. So, one pole we will consider in this side, I mean the left hand side of this imaginary axis, another pole we will consider right hand side of this imaginary axis. One pole of in the left of imaginary axis and another pole P 2 is in right of the imaginary axis.

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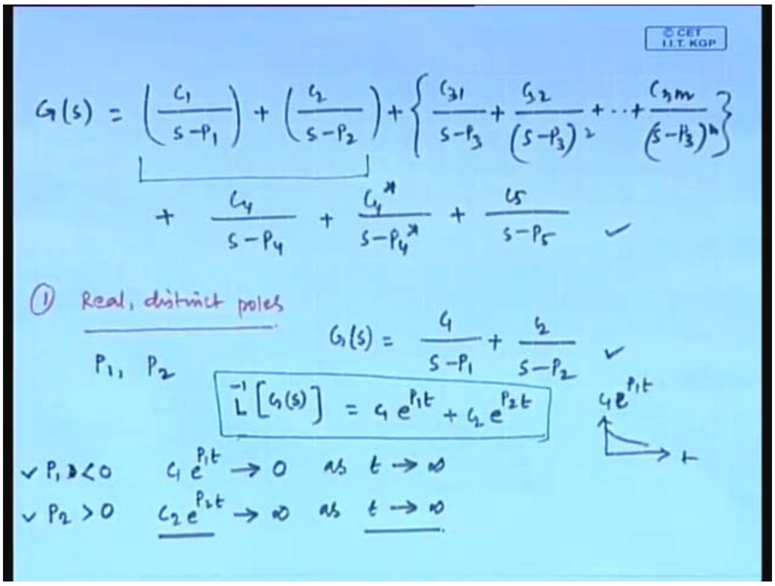

And the transform function is represented by this equation G s equals to c 1 by s minus p 1 plus c 2 by s minus p 2. Now, if we take the inverse of Laplace transform what we get, inverse of Laplace transform we get c 1 exponential of p 1 t plus c 2 exponential of p 2 t, if we take the inverse of Laplace transform, we get this expression. Now, if p 1 is greater than 0, p 1 we have consider in this complex plane sorry this p 1 is less than 0 because, that is in the left of imaginary axis.

So, p 1 is less than 0. If that is the case, than c 1 e to the power p 1 t I mean exponential of p 1 t tens to 0 as time tens to infinity, can we write this, we have consider p 1 less than 0. So, exponential of p 1 t multiplied by c 1 tens to 0 as time goes to infinity, we can represent this graphically also, it is like this going to 0 this is t this is c 1 exponential of p 1 t. In another case, we have considered p 2 that is greater than 0 then c 2 exponential of p 2 t tens to infinity as t tens to infinity agree, if p 2 is greater than zero then c 2 exponential p 2 t goes to I mean grows 2 infinity as time tens to infinity.

So, we can say that if a system has the pole in the left of imaginary axis, then the system is decaying to the 0; that means, the system is stable and if the pole is greater than, 0 in that case it is growing exponentially towards infinity; that means, the system is becoming unstable. In the second we will consider, the multiple real poles.

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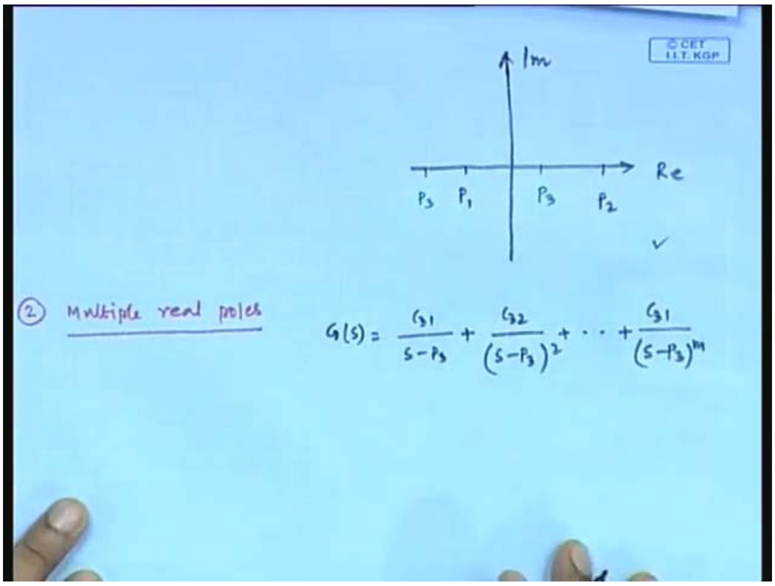

In the second case we will considered multiple real poles, that pole is here p 3 and that is repeated m times. I mean we will consider multiple poles represented by p 3. So, first we will locate that p 3 in this axis, complex plane. So, suppose 1 p 3 is here, this is 1 p 3 another p 3 is suppose, here like this way p 3 is repeated m times. And for this multiple real pole system we will consider the transform function represented by c 3 1 s minus p 3 c 3 2 s minus p 3 hole squire, like this last term is c 3 1 s minus p 3 hole to the power m.

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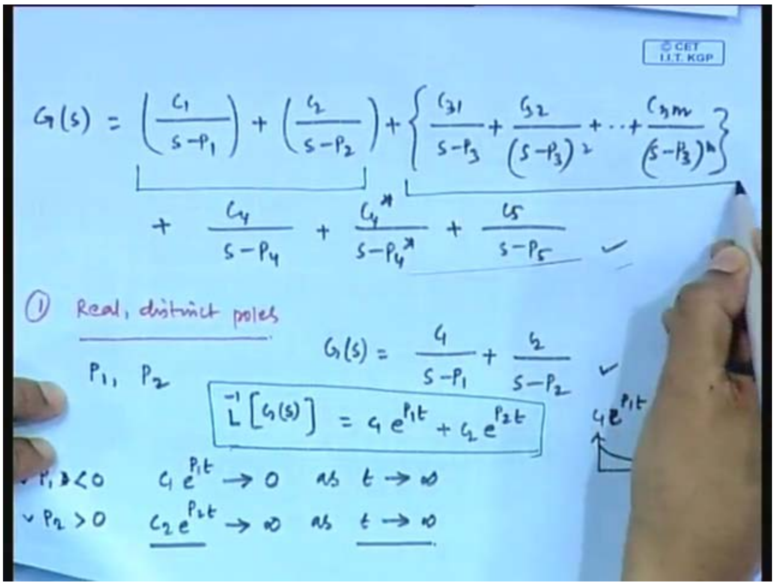

If you see the general expression of the transform function that is this, this is the general expression of the transform function. Now, to consider the multiple real roots we are considering this part, as the transform function that I have return there.

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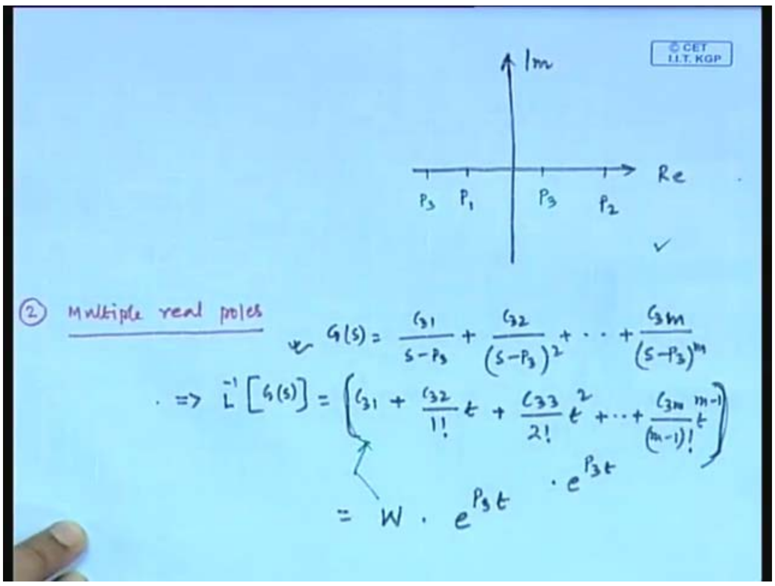

So, G s equal to c 3 1 by s minus p 3 c 3 2 by s minus p 3 hole squire, like this way this is c 3 m divided by s minus p 3 hole to the power m. In the similar way we will take the inverse of Laplace transform G s. So, what will get c 3 1 plus c 3 2 divided by factorial 1 multiplied by t plus c 3 3 divided by factorial 2 multiplied by t squire, like this way c 3 m divided by factorial m minus 1 multiplied by t to the power m minus 1 this will be multiplied with exponential of p 3 t.

If take the Laplace inverse of Laplace transform of this transform function, we get this. Suppose, I am writing this as W multiplied by e to the power p 3 t I mean this hole part this 1, we are representing by W.

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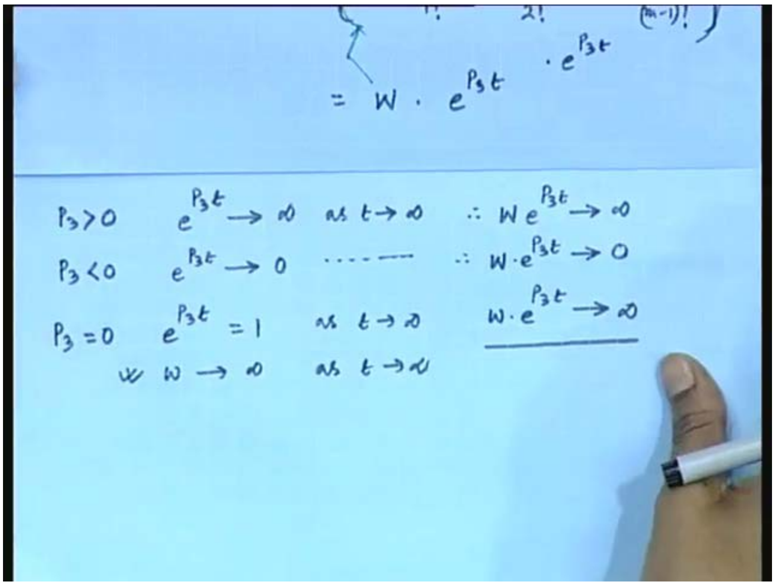

So, next we will consider p 3 greater than 0 then exponential of p 3 t grows to infinity agree, as t tens to infinity. So, what about W multiplied by p to the power p 3 t tens to infinity; that means, again in stability is there because, the system response is going to infinity, in another case we will consider p 3 less than 0, then exponential of p 3 t tens to 0 as t tens to infinity. So, what about this W multiplied by exponential of p 3 t it will d k to 0.

And in the third case, if we consider p 3 equal to 0 exponential of p 3 t becomes unity, exponential of p 3 t becomes 1 as t tens to infinity. So, what about this W multiplied by exponential of p 3 t grows to infinity, due to this W grows to infinity as t tens to infinity. So, due to this term, we are getting this W multiplied by exponential of p 3 t becomes infinity. So, these are basically the different real roots, associated with a process and the stability of that process. Now, in the next case we will consider the complex conjugate poles, that is the third case.

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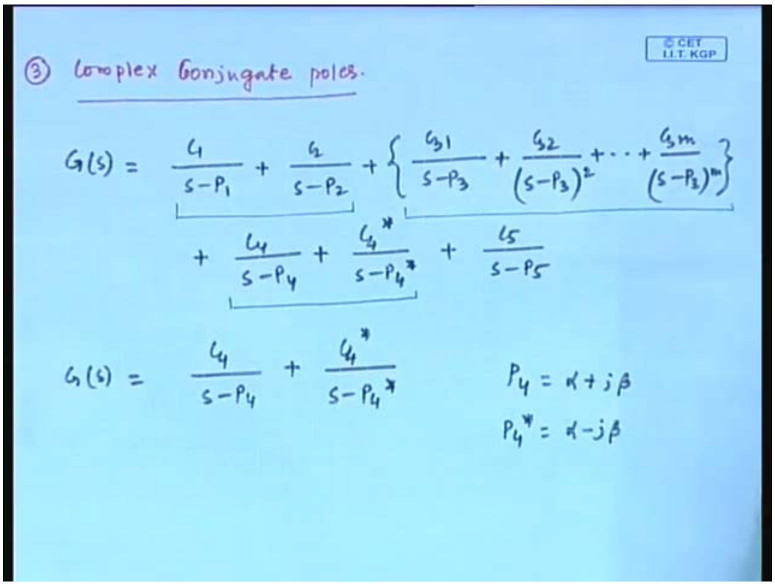

In the third case we will consider, complex conjugate poles. So, for the complex conjugate poles, will consider the third portion of the general form of transform function. The general form of transform function I am again writing that c 1 s minus p 1 plus c 2 s minus p 2 this 2 written terms we have considered for the 2 distinct real poles. Next term we have consider, for the multiple real poles hole squire 3 m s minus p 3 hole to the power m .

So, now, we will consider this part c 4 divided by s minus p 4 and c 4 star divided by s minus p 4 star and in the 4'th case we will consider c 5 divided by s minus p 5 this we have already considered for the distinct to real poles, this part we have considered for multiple real poles now, we will consider this part I mean the transform function, reduces to c 4 s minus p 4 plus c 4 star s minus p 4 star. Now, the 2 complex conjugate poles are p 4 and p 4 star.

So, will consider p 4 equals to alpha plus j beta and p 4 star will consider that is alpha minus j beta. These are the 2 complex poles. Now, we have to represent them in the complex plane.

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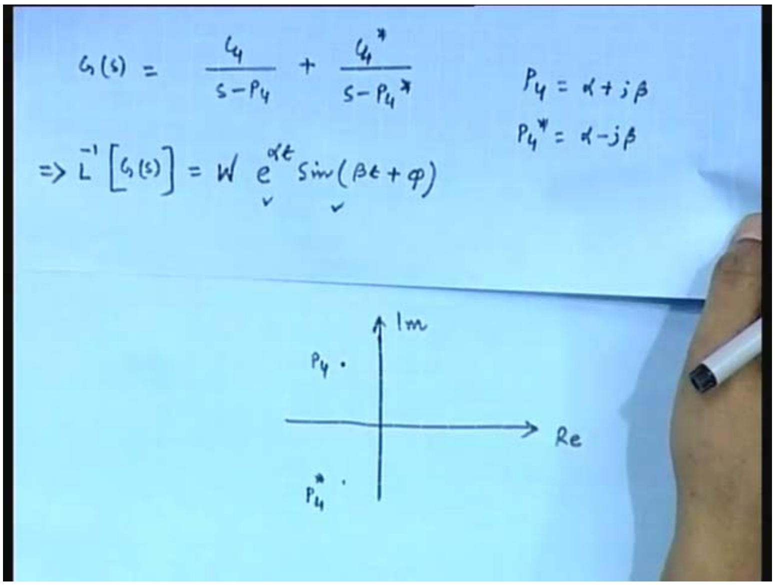

This is the complex plane, this is real axis, this is imaginary axis suppose, one pole that is p 4 and another pole that is here p 4 star. So, this is alpha plus j beta and another one is alpha minus j beta. Now, as usual we need to take the inverse of Laplace transform. If we take the inverse of Laplace transform, the right hand term includes one exponential term exponential of alpha t and one sin function, that is beta t plus 5. The right hand term includes one exponential function, exponential of alpha t and another one is sin sign function. Now, this right term may be multiplied with some value suppose W. So, next we will consider, the different situations I mean different values of alpha, then what will be the response of the process having 2 complex conjugate poles.

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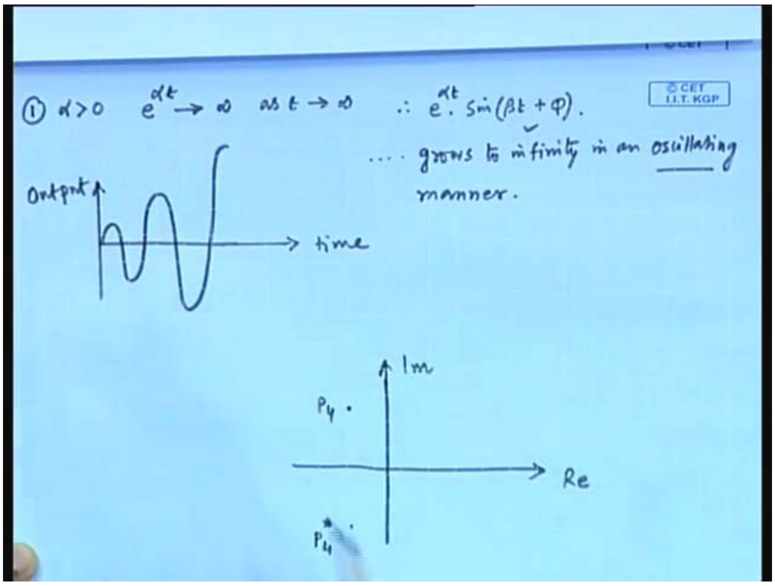

So, in the first case we will consider alpha greater than 0, if alpha greater than 0 then exponential of alpha t tens to infinity as time tens to infinity agree, then what will be exponential of alpha t multiplied by sign beta t plus phi. It grows to infinity, in oscillating manner, it grows to infinity in an oscillating manner, it is going to infinity in oscillating manner because, of this semi schedule function, so how we can represent the system response graphically.

So, we represent in this form suppose, this is time and this is output function, this is output initially like this. So, initially the process was at steady state. Now, we have introduce some input change, then the output response in the manner, it is going to wards infinity. So, it is obvious that if alpha is greater than 0 the system has instability problem. Similarly, we will consider alpha less than 0.

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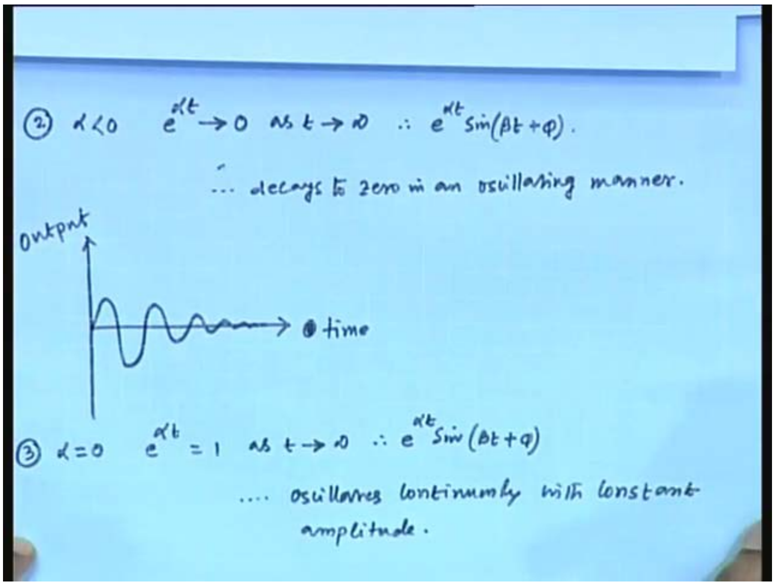

In the next case we will consider alpha less than 0, then e to the exponential of alpha t tens to 0 as time tens to infinity. So, exponential of alpha t sin beta t plus phi decays to 0 in an oscillating manner. So, it decays to 0 in an oscillating manner, the graphical representation is like this, this is the time, this is output. So, it is gradually stabilizing I mean the process is coming back to the original state.

And next we will consider alpha equal to 0, if alpha equal to 0 then e to the exponential of alpha t equal to 1 as time tens to infinity, so x exponential of alpha t multiplied by sin beta t plus phi, what will be the response.

Student: ((Refer Time: 42:51))

Sin a shodel with yes sin a shodel I mean in constant amplitude. So, we can write oscillates continuously with constant amplitude.

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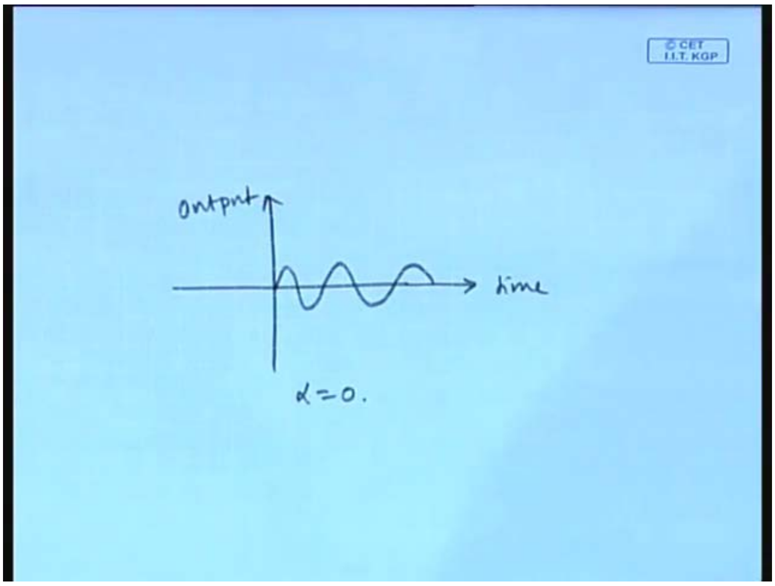

This is sometimes called marginally stable and for this case graphical representation is, after this is time, this is output like this, this is a case for alpha equal to 0. So, we have considered 3 different cases now, another case I mean the last case in which we will consider.

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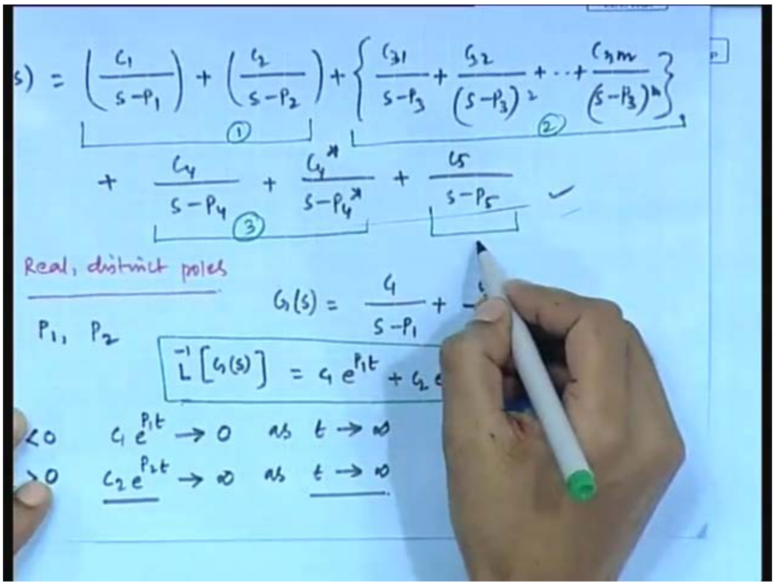

This transform function. This is a first case, second case, this is a third case complex conjugate poles and this is a 4’th case, in the 4’th case we will consider the pole is at the origin.

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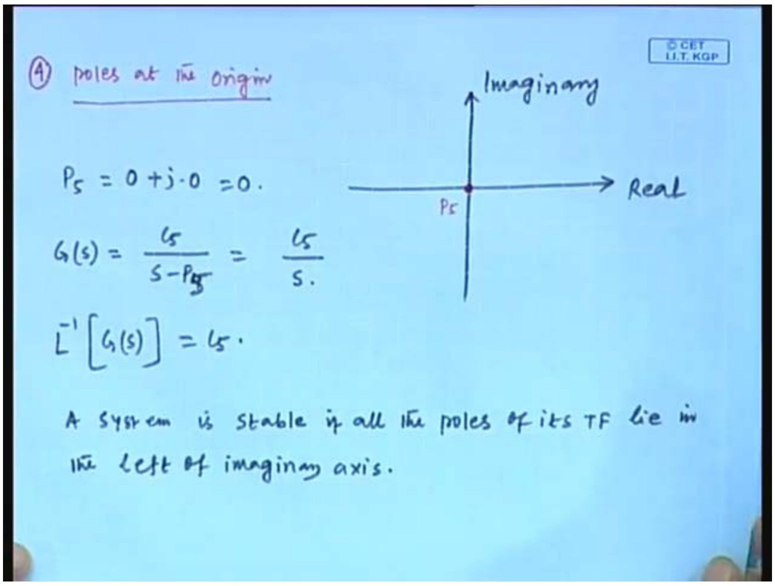

So, in the 4'th case, we will consider poles at the origin I mean if we draw the complex plane this is real and this is imaginary the pole p 5 is present here, this is p 5; that means, p 5 equal to 0 or you can write in this form 0 then transform function G s reduces to c 5 s minus p 5 this is p 5 equal to c 5 divided by s. Now, we will usually take the inverse of the Laplace transform, then we get c 5 1 constant term.

So, these are the 4 different cases which we have considered hear and we have conclude, we can conclude based on this 4 observations I mean considering the 4 cases that, a system is set to be stable, if all the poles lie in the left of imaginary axis, this is our conclusion based on the 4 observations. A system is stable, if all the poles of it is transform function lie in the left of imaginary axis. And in the subsequent chapters, we will study more about the stability, considering different techniques. Now, we will take one simple example, one simple transform function which we have considered earlier.

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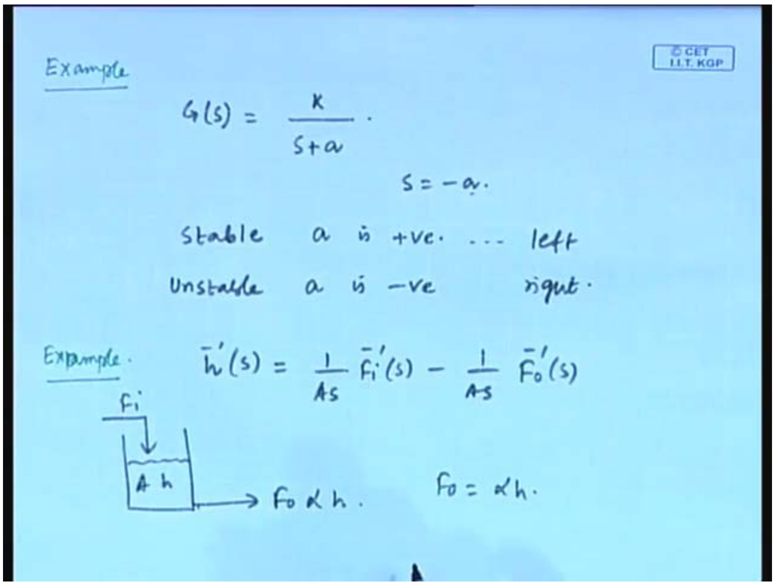

One simple transform function we will consider, that we have discussed earlier that is G s equals to k divided by s plus a. So, this is a stable system or unstable system, stable system why this is a stable system.

Student: ((Refer Time: 48:37))

Yes because, s is equal to minus a. So, if a is negative, if a is negative.

Student: Not stable.

Not stable. So, here one pole exists at s equals to minus a. So, we can say the system having the transform function of G s is stable only if a is positive. If a is negative this is unstable because, in this case the pole lie in the left side of the imaginary axis and in this case, the pole lie in the right side of the imaginary axis. Another example we can consider that is the, liquid level system for the case of liquid level system, we got the transform function that is 1 divided by A s f i bar prime s minus 1 divided by f s f naught bar prime s.

And this model we got considering, this system f i and f naught. A is the cross section gradient, h is the height. Now, we can do one thing, we can consider f is pronominal to h. We are just trying to modify this example. Now, f naught equals to suppose alpha multiplied by h.

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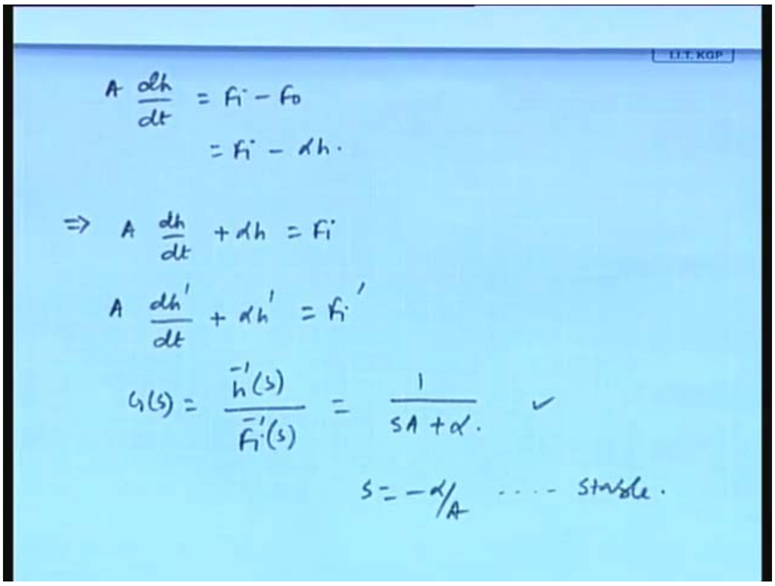

Now, if we substitute this in the model equation then what will get A d h d t equal t o f i minus f naught equal to f i minus alpha into h. So, if A d h d t plus alpha h equal to f i. If we write in terms of deviation variables we will get alpha h prime equals to f i prime. So, what will be the transform function for this, h bar prime s divided by f i bar prime s equal to 1 divided by s A plus alpha, can I write this 1 divided by s a plus alpha agree or not agree.

So, what is the root I mean what is the pole of this equation.

Student: ((Refer Time: 52:41))

Minus alpha divided by A. So, the system is stable or unstable, the system is stable, area is always positive. So, if we consider alpha is positive then the system is stable.

Thank you.