Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 41
One-dimensional Heat Conduction (Contd.)

Hello, everyone. Welcome back once again in another lecture of Heat Transfer in Chemical Engineering Fluid Dynamics and Heat Transfer.

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We were discussing One-dimensional Heat Conduction.

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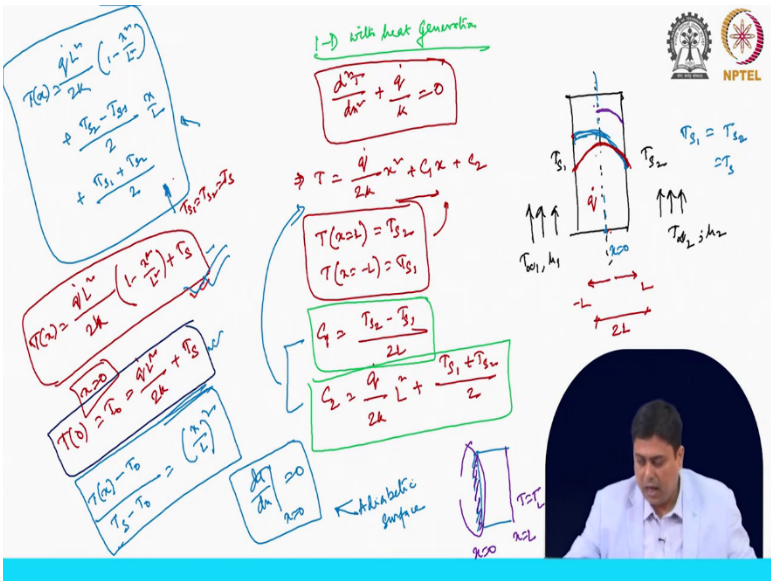

And, we were on the section that where we are dealing with conduction with heat generation or conduction with thermal energy generation. Now, in the last class we have seen this work or this slide particularly where we have derived from the heat equation or the heat diffusion equation for such kind of a thing that where we have a heat or thermal energy generation inside the domain or inside the medium with appropriate boundary condition what would be the temperature profile.

Now, then if we look at this expression that is here that:

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_S$$

the temperature at (x = 0) is given by this expression:

$$T(x = 0) = \frac{\dot{q}L^2}{2k} + T_S$$

Now, in case of thermal energy generation in the domain and when we have a symmetrical domain or symmetrical shape of the domain, then in that case we can see that the maximum temperature that can be in the domain is at the symmetry plane or the plane of symmetry.

Now, So, from if we have this expression then it can also be written in this form:

$$\frac{T(x) - T_0}{T_s - T_0} = \left(\frac{x}{L}\right)^2$$

So, this expression have an another variation of the temperature profile or the expression of temperature profile in this manner.

So, the plane of symmetry is basically; that means, where we have temperature gradient at 0 which means:

$$\left.\frac{dT}{dx}\right|_{x=0} = 0$$

So, which means there is no heat transfer across this plane and this can be represented as an adiabatic surface or the adiabatic condition. So, now the point is that the other scenario can be that means, this plane that we are talking about is say the adiabatic surface.

Now, other scenario can be or say the implication of this expression when it comes to the application. The other thing can be that say for example, we have a surface which is insulated on the one side it is perfectly insulated on one side. So, if now we consider this at (x = 0), if it is perfectly insulated at (x = 0) the same boundary condition would be applied here.

And, say if it is exposed to at $(T = T_L)$ then on this boundary the similar or identical expression we can apply; which means at this plane the temperature is maximum in case of heat generation in the system and then it forms a parabolic path. It is not linear that we have to understand and remember because of the previous discussion that we did on this case.

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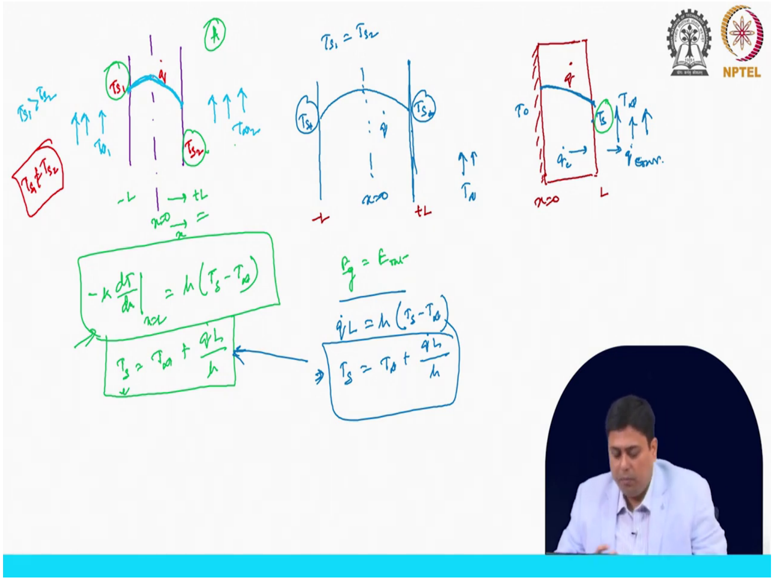

Now, most of the time what happens say for example, I have this surface. Say let me draw a couple of surfaces. So, this is the plane of symmetry in this case. So, and here we have fluid flowing on both the sides. We are having two different temperature $T_{\infty 1}$ and $T_{\infty 2}$. So, accordingly our surfaces attain two different temperature T_{S1} and T_{S2} and then there is a heat generation in the medium and this is a plane of symmetry.

Now, in case of $T_{S1} \neq T_{S2}$ and if this is my plane of symmetry that I have already mentioned that it is $x = 0$, this is $x = +L$ and this is $x = -L$, this is the x-direction. So, in this case what will happen? The profile that would look like based on this discussion that we have seen is something that it if again the condition if I say that $T_{S1} > T_{S2}$, then from a temperature on the wall that is T_{S1} it goes to a higher temperature at the middle and then it drops to T_{S2}.

So, the temperature profile would look like something like this. So, there will be a parabolic profile from here which reaches maximum at the midpoint and then again drops to a value T_{S2}. If the $T_{S1} \neq T_{S2}$ and $T_{S1} > T_{S2}$. If these two are identical or say the T s 1 is equals to T s 2, then we have seen a profile which is symmetric in nature where this is the $x = 0$.

These are the two planes or the two sides where this is T_{S1} and this is also T_{S1} or T_S if we consider. So, the maximum temperature is attain at the midpoint. So, this is the plane or the adiabatic surface as per our previous discussion. In case of perfectly insulated surface (x = 0), (x = L). So, this is (+L), this is (-L). Now, in this case with heat generation, this temperature is T_0 and this is again T_∞ because of the flowing fluid.

If T_S is a temperature or T_∞ is a temperature that is lesser than this domain temperature which means domain is or the medium is being cooled by flowing some fluid outside its outer surface, then in that case the profile that would look like that would start from a highest value here and then it would be like half of it of the parabolic curve where the temperature flowing the heat q conduction is in this direction. This is by q convection.

So, these are the scenarios that can happen. So, the now here the point is that as I mentioned that when we try to calculate this temperature profile say that what would be exactly the temperature profile from the generic expression or for the particular point which (x = 0). We see that to find out this temperature we need the surface temperature which means we will need this surface temperature values.

Now, these are often difficult or not given. So, in that case or in those cases what we should do we should find a relation between T_{S1} and T_∞ or say T_{S2} and $T_{\infty 2}$; that means, usually the flowing fluid temperature is given which is outside the domain or outside the surface.

But it is not that whatever the fluid temperature outside the surface temperature attains the same temperature because it has the convection resistances. So, in those case we have to write appropriate boundary condition at (x = +L) or (x = -L) depending on how the domain is given.

So, in those case the usual energy balance that we can immediately write is since it is having through a common surface say for example, here it is if I mention that the surface area is A then:

$$k\left.\frac{dT}{dx}\right|_{x=L} = h(T_s - T_\infty)$$

So, at x is equals to L we write this energy balance. The amount of heat lost by the conduction the amount of heat gain by convection or while the convection is happening.

Now, we have a expression of Ts here that we have derived we find the $\frac{dT}{dx}$ expression and put (x = L) and if we substitute that thing here the thing that we will get is:

$$T_s = T_\infty + \frac{\dot{q}L}{h}$$

The expression that we get to compute the value of Ts or the surface temperature from the given information of fluid temperature that is flowing outside, the amount of heat generation or the thermal energy generation, the thickness of the material or the characteristics length of the material the domain and h is the convective or the convection heat transfer coefficient.

If this information are known we can easily find out the value of Ts for the surface temperature. So, once we have this, we can find we can solve several problems, but this derivation can also be thought of a in a different way for example, the amount of heat being generated if we consider as a overall system instead of looking at the temperature profile here and how it looks like if we think of this scenario as like this that we have a heat generation in a medium, now, that heat has to be dissipated and that dissipation is happening by convection only.

So, that means, there is no radiation ok we are neglecting the influence of radiation here and in this case what happens what we can write is that:

$$E_g = E_{out}$$

Now, if we try to put the expressions on this case what will happen? The amount of energy being generated that we usually mention as $\dot{q}$ which is per unit volume so, in this case considering it is a unit surface area we have $\dot{q}L$ is the amount of heat being generated and the amount of heat being dissipated that is happening only by convection through area A, which is omitted from both the sides is $h(T_s - T_\infty)$, if this is the surface temperature T_s and the flowing fluid temperature is T_∞, in case of T_{S1} if I write that simply T_s in both the cases.

Now, this expression is also giving the same result as of this which means here you can if you rearrange what you would get is:

$$T_s = T_\infty + \frac{\dot{q}L}{h}$$

the same expression we can arrive. So, what are the applications or how do we apply this understanding for problem.

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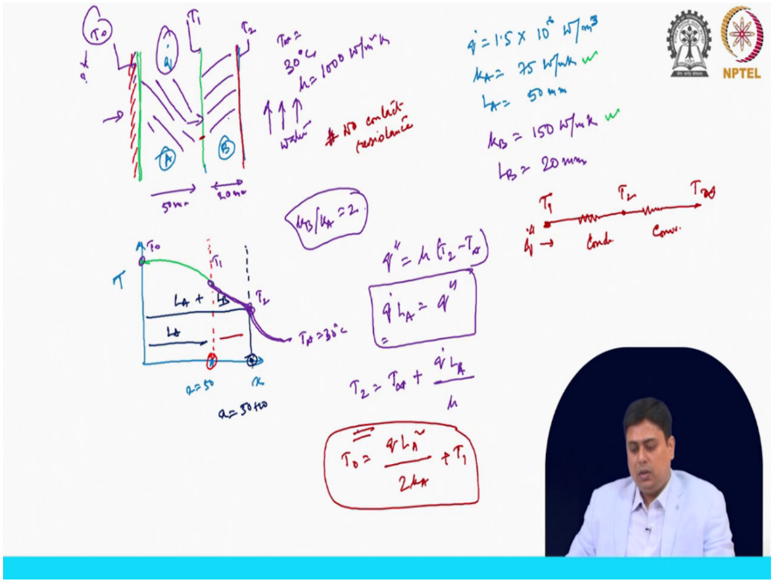

Now, we solve a simple problem where it is mentioned that we have a plane wall of two material. So, let us at first draw the schematic that we have a wall that is having two different material. This is material one and say this is another material. So, this is material B and this is material A.

Now, in this case the wall of material A has uniform heat generation which is $\dot{q}$ and a certain value is known $\dot{q} = 1.5 \times 10^6$ W/m^3. Thermal conductivity of kA = 75 W/mK, width or the thickness of material A is 50 mm.

This wall that is wall B, the wall that is B does not contain any thermal energy generation term or part. So, there is thermal energy generation in part A only or inside the material A and material B does not contain or does not have any thermal energy generation inside that thickness.

Now, that also k_B value or the thermal conductivity value for material B is 150 W/mK. The thickness of B is 20 mm. The inner surface of this material A is which is this part the inner surface of A is well insulated and the outer surface of B is cooled by air a fluid that is flowing at say 30 ^oC in which h = 1000 W/m^{2}K, and let water is flowing outside this.

So, this thickness is 50 mm, this thickness is 20 mm, this is having a temperature that is T_0, this surface is having a temperature say T_1, this is surface temperature T_2 this surface, $T_\infty = 30°C$. So, what we have to do here? We have to sketch not to scale the temperature profile throughout the domain considering this is an one-dimensional heat conduction. And also we have to determine the temperature T_0 that is insulated.

So, to solve such problem at first there are few things we have to remember that we consider or the assumption is that this is a steady state heat conduction happening. This is a one-dimensional problem. We assume this is a one-dimensional steady state heat conduction is happening.

And, the other thing that we must remember or must explicitly say that there is no contact resistance between the surfaces. So, which means there is no contact resistance at this position where two materials are pasted or attached the joining interface. So, based on this, ok and assuming that we have constant properties of material A and B that is not varying with space or with temperature, we consider those are the constant values.

So, how this sketch would look like? The sketch if I try to now draw say in this direction, we have x this is say the temperature profile. So, what we will have? One sided this side is insulated of material A having heat or thermal energy generation term. So, that part would have a parabolic profile of the temperature. So, if this is my x is equals to L point or x is equals to 50, if this is x is equals to 50 so, from here what we will have? A parabolic profile that is like this. It goes to a temperature which is T_1.

Now, there will be a zero slope at the insulation. So, at the insulation there will not have any slope just immediate that is near the wall scenario because that part is insulated. Then in material B, we will have a linear profile because there is no heat generation. There will not have any slope generation, slope other than the linear profile. So, there what we will have and since there is no contact resistance that we have considered so, there is that we do not consider there would be a temperature drop on this case.

Now, at the interface what we will have there will be a slope change drastic change in the slope. Because here we have a k_A value which is 75 and here we have a slope which is 150 k_B that is the thermal conductivity of material B which is 2 times higher than the thermal conductivity of material A, which means it the conduction resistance in material B is much lesser it 2 times lesser than the thermal resistance for conduction in material A.

So, the degree of slope that would be here if I consider till this point is my so, from here it is (x = 50 + 20) this point. Because this is the 20 mm is the thickness of material B. A is the L_A and this is $(L_A + L_B)$. So, from here to point T_2, so, this is my point T_1, this is point T_0 and this is T_2. From here to here there would be a linear profile. The slope would be $(k_B/k_A) = 2$.

So, after this linear profile we have convection here. So, for convection again there will be a temperature drop till it is $T_\infty = 30$ °C. So, profile may look like this kind of a temperature drop. So, this is the conceptual part that how the temperature profile would be there. This is a schematic of it exact again it is not to not as per the scale. But this helps us to understand how the temperature distribution would be inside the medium.

Now, once we have this what we understand is that q'' for the amount of heat flux that is going out of the system is eventually: $q'' = h(T_2 - T_\infty)$. So, whatever heat is being generated and being transferred till the surface of outer surface of material B that would be taken away by the convective fluid for the fluid that is in motion on the surface.

Now, the point is that this heat flux, this q double prime we can determine by using another say energy balance equation for the control volume A. But, here at particularly what we will see that since at (x = 0) is adiabatic and there is no flow of heat from this side. So, what will happen, the energy that is being generated here that is the only energy that would come out of this boundary.

So, which means $(\dot{q}L_A)$ is essentially q''. The amount of heat being generated inside material A with its thickness is the heat flux in this case. And we also know that T_2, the outer surface temperature if we try to make this related with the fluid temperature then we have seen this expression earlier. This is the material that we have:

$$T_2 = T_\infty + \frac{\dot{q}L_A}{h}$$

So, from here we can find the temperature T_2 from this expression. And again, because once we this is the simple balance that we can write or if you are not able to understand this let me rewrite the initial part. So, here what we have the amount of heat being lost from material B to the outside fluid temperature is basically that what we are doing is the simple this balance that we have written here.

So, from this expression we find the value of T_2. And, we have also seen the expression that would happen if we have one side insulation or adiabatic surface for the temperature profile. So, that means, in this case:

$$T_0 = \frac{\dot{q} L_A^2}{2K_A} + T_1$$

Again, the T_1 in this expression is unknown. But, this T_1 what you can find from a equivalent thermal circuit diagram where the cube double prime is going from this insulated end from this the interface of material A and B particularly here in this case because the cube flux is coming out only from material A. Then this is the T_1 we have a resistance for material B and another resistance that takes us to the T_2 here and this is the T_∞. So, this resistance is for the conduction and this for the convection.

So, we will continue this one in the next class. We will solve this problem for calculation of T_0 from the same expressions that we are having. Meanwhile, you try to understand what I have done here what are the relations of these expressions that we have directly written because these equations we have already derived and discussed previously. So, till then practice this and we will see the solution in the next class.

Thank you.