Computational Fluid Dynamics
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Lecture – 03
Flow in a rectangular duct: Discretization of flow domain

In lesson two we have seen the basic CFD approach, which enables us to solve a partial differential equation, as a set of algebraic expression. the basic approach is that, we take the fluid domain and put lots of grid points, and these grid points are spread throughout the domain, and it is at these grid points, we would like to have the velocity of velocity or the corresponding variables which is there in the in the equation.

We substitute finite difference approximations which are valid at that grid point into the governing equation, and covert the partial differential equation into an approximate algebraic equation valid at that particular point. And the idea is to solve this algebraic equation, in order to get the variable value. For example, the velocity at point i j, but what we saw is that the variable value at point i j is expressed in terms of the variable values at the neighboring points; that is at i plus 1 j i minus 1 j i j plus 1 and i j minus 1.

So, we have to write down similar approximations at each other grid points, and then we will end up a set of n algebraic equation, for the n grid points at which we want to get the velocities. So, in the end we would have converted one partial differential equation into n set of n algebraic equation. And the idea is that if we solve all this set of n linear list algebraic equation together, we will be able to get the solution which is the w at discrete nodes. So, in this lecture we will see how this particular process goes on.

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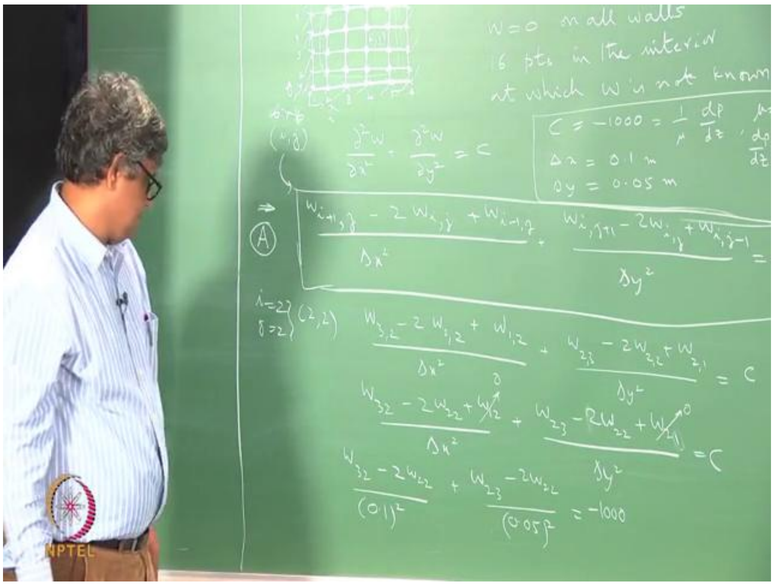

So, we are considering a rectangular duct, we had divided this into five equal intervals in the x direction, and similarly in the y direction. Although we cannot see it from this picture here; delta x is the same and delta y is the same. So, we have x going in this direction, and y going in this direction. And similarly i going in this direction, and j going in this direction. We can identify any grid point with two indices i and j representing the ith grid line, coordinate line in the x direction, and jth coordinate line in the y direction, and the intersection ith coordinate line and the jth coordinate line will give us the grid point.

For example, if this is i coordinate line, and this is j coordinate line. this point here will be i j. and the governing equation which is dou square w by dou x square plus dou square w by dou y square equal to c, was converted into w at i plus 1 comma j minus 2 w i comma j plus w i minus 1 comma j divided by delta x square plus w i j plus 1 minus 2 w i comma j plus w i j minus 1 by delta y square is equal to c.

So, at point i j, this is an approximate form of this equation, and we see that if we solve this for w i j. If we can solve this equation, is for the value w at i comma j. So, if we know these values; the j plus 1 value j minus 1 value, then we will be able to get w i j, but unfortunately at this stage we do not know what the neighboring points are. So, we write, we take this as the template and write down similar equations for all the points that we do not know. All the points at which we would like to know the velocity. So, that we will have a system of equations in which figure all the velocity variables that are unknown and we solve them simultaneously.

So, this is what we are going to look at in today's lecture, with the specific example of this 5 by 5 division of the flow domain. So, in this flow domain we have boundary conditions, and since we have 5 by 5 intervals. So, you have 6 by 6, 6 by 6 points at which we have the grid points. So, there are 36 grid points these are intersection of the coordinate lines here. And out of this we have the boundary condition; that w is equal to zero on all walls. So, that the velocity at this point which is on the boundary, at this point on the boundary, all those things are known.

So, it is only these interior points are unknown. So, we do not know the value at this point, at this point, this point, this point, at these points marked by a circle. So, there are 16 points in the interior at which w are not known. So, we would like to apply this for every grid point at which the value is not known, and then we would like to get a systematic equation.

So, let us. To illustrate this let see how we can apply this template here, to the first point here. So, the first point here, is i equal to 1 2 3 4 5 6 and j is also equal to 1 2 3 4 5 6. So, the first point has i equal to 2 and j equal to 2. So, we will call this as 2 2. So, if you take the template equation a here and apply this to this point here. So, the value of w i plus 1 j comma j this will become w 3,2 minus 2 w 2, 2 plus here w i minus 1 will be 1,2 divided by delta x square plus here this is w 2, 3 minus 2 w 2, 2 plus w 2, 1 divided by delta y square equal to c.

So, at this point where there is no confusion, we can leave out those commas, because we have only single digit indices here. So, we can leave out the commas and just write this as w 3 2 minus 2 w 2 2 plus w 1 2 by delta x square plus w 2 3 minus 2 w 2 2 plus w 2 1 divided by delta y square equal to c, and we can see that, we have got a specific equation with values here in order to make further progress, we can substitute all the variables that are known in this.

For example, c is a given value. So, let us take as a part of problem formulation c to be minus 1000. This actually 1 by mu d p by d z, if we take mu to be 0.001 Pascal second in s i units for water, and d p by d z is negative for the flow to be positive in the, for the w to be positive. So, we will take d p by d z to be minus 1 and that gives us minus 1 by

0.001. So, that is minus thousand. So, this is in s i units.

And we need to have delta x and delta y. So, let us take this in order to get some numerical value, let us take delta x to be 0.1 meter and delta y to be 0.05 meters. So, if you know the total length, and if you divided this into five parts then you can get this. So, this will imply that the total length in the x direction is 0.5 meters and total length in the y direction is 0.25 meters, 25 centimeters and 50 centimeters.

So, these are numerical values, and these come as part of the problem specification. If you want to find the velocities and all that, you need to know what all the physical dimensions of the domain, and from the physical dimensions, and the number of grid points that you want to have you can fix delta x and delta y. the constant here is part of the equation, and correspond to equation you have a certain numerical value. So, you have this, and you also have the boundary condition that w equal to zero on all walls.

Now if you look at this equation here, wherever there is an index of one, either for x or for y; that indicates a boundary point, because one means that this one here. So, x is equal to 1 is this 1. For all this points x is equal to 1, and all these are lie on the boundary, and similarly y equal to 1. So, that is this point this index here, refers to the bottom boundary. Similarly for anything which is 6, which is the extreme location.

So, if x is equal to 6 then that refers to all this boundary points on this boundary, and y equal to 6 is j equal to 6 is refers to all this points. So, at all this points if the, either i index or the j index is either 1 or 6 w equal to 0. We can make use of this to simply this. So, when we specifically looking at this point here, this equation this is 0, and similarly this is 0, and we can substitute the x equal to delta x equal to 0.1 meters, and delta y equal to 0.05 meters, and we can put this as w 3 2 minus 2 w 2 2 divided by 0.1 whole square plus w 2 3 minus 2 w 2 2 divided by 0.05 whole square equal to minus 1000. So, this 0.1 whole square is 0.001. So, this is 100.

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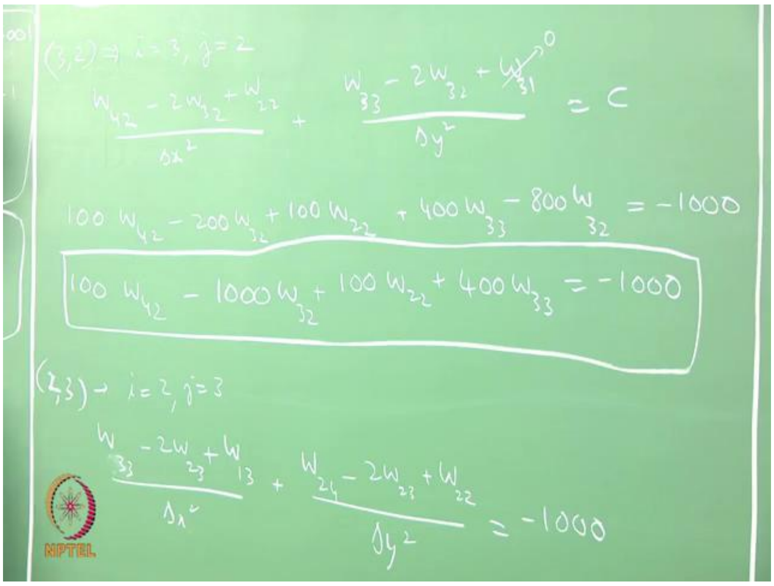

So, this becomes 100 w 3 2 minus 200 w 2 2 this is 400. So, this is plus 400 w 2 3 minus 800 w 2 2 is equal to minus 1000. So, w 2 2 800 and w 2 2 minus 200 will finally, give us 100 w 3 2 minus 1000 w 2 2 plus 400 w 2 3 equal to minus 1000. So, the application of this template at 0.22 has given us this equation and we can see that this equation has w 2 2; that is the w velocity at 0.22 as one of the variables and this is given we cannot solve this equation directly, because we do not know what w 3 2 and w 2 3 are.

So, we need to write equations for these two things, and when we write those two equations we will find new values. So, let us just write down the equation for this point in order to get w 3 2. You need to write the equation at 0. 32 here and 0.32 means that i is equal to 3 and j is equal to 2. So, let us come back to equation a and then substitute i equal to 3 and j equal to 2 what do we get we get w 4 4 2 minus 2 w 3 2 plus w 2 2 divided by delta x square plus, and here its w 3 3 minus 2 w 3 2 plus w 3 1 divided by delta y square equal to c here.

And here again we try to substitute known values of delta x and delta y c and this point which is on the boundary, and that becomes zero. And once you substitute that delta x whole square is 0.001, 0.0. So, this is 100 w 4 2 minus 200 w 3 2 plus 100 w 2 2 plus 400 w 3 3 minus 800 w 3 2 equal to minus 1000. Again we can add these two to get 100 w 4 2 minus 1000 w 3 2 plus 100 w 2 2 plus 400 w 3 3 equal to minus 1000.

Now, if you look at these two equations, we would like to get w 2 2 from this, but in order to do that we need to know what this is, and when we write this equation we have w 4 2 this 1 w 2 2 and this. So, we can say from this we can get w 2 2, and we have one more equation. So, we have 2 equations to get this one, but still w 2 3 is not known, and w 4 2 is not known, w 3 3 is not known.

So, we need to write those equations also. So, let us just a for the sake illustration, let us try to write an equation for w 2 3, which means that i equal to 2 and j equal to 3, if we substitute again in this, and let us just substitute. So, w 2 3 3 minus 2 w 2 3 plus w 1 3 by delta x square plus w 2 4 minus 2 w 2 2 3 plus w 2 2 divided by delta y square equal to minus 1000. So, we substitute the delta x square and delta y square and this is again zero, and we should be able to get 100 w 3 3 minus 1000 w 2 3 plus 400 w 2 4 plus 400 w 2 2 equal to minus 1000.

So, this is one more equation, and this is the equation for 2 3, and this introduces 3 3 2 4, and 2 2 its already known. So, we can see that every time write an equation we have new values. So, let us just try to map this out in this to see where we are. So, when look at 0.22. So, that is this one here. We have value here, and then 3 2 is this point, and 2 3 is this point here . So, you have, these are the. So, this is linked to these two.

Now when we write 3 2; that is this point here, we have 4 2; that is this one, and we have 3 2 here, and we have 2 2, and then we have 3 3. So, that is this point. So, we are bringing in these points, similarly, when we write for 2 3, which is this point here. We have 2 3, and then we should have 2 2, and we have 2 4 which is this point here, and we have 3 3 which is this. So, we have this thing.

So, we can see that at every point, we are involving the immediate neighboring points in our equation, where the immediate points are on the wall, we know the velocity. So, we can substitute them and they are not appearing in the equation, but if they are not along the wall then they are included in the equation. So, this is the template, and for the general point i comma j will have the four neighboring values; which will be coming into this. So, you have a computation molecule, which consists of this.

So, when we write an equation i comma j we can expect to see four neighbors plus this five grid points linked together by this computation molecule. So, this is the characteristic feature of computation field dynamic. This computational molecule depends on the equation that we are trying to solve, and the different approximation that we are trying to make. So, if we make use of another approximation, then the computational molecule may change. And if you have additional terms in this, again the computational molecule it will change, and the coefficients for each of this points again one two one like that that also depends on the difference formula that we use.

So, depending on the type of approximation that we make, and depending on the type of equation that we are trying to solve. We will have a computational molecule, and that computational molecule is encapsulated, in this template equation which we choose to write. And once we develop this template, then we see that this template will work only for that point, and that point alone is not sufficient, we will have some neighboring points for this type of equation.

And we have to put the template at every point, and then we will get a set of equations. So, if we write, if we apply this template for all the 16 points then we will have 16 equations, and all the 16 equations will be such that when you put them together no new variables are required for us to get a solution, will have a set of 16 equations, which together make up sufficient number of equations to solve for all the 16 variables. So, they uniquely determine the solution.

Since we are doing a hand calculation, and since we need to write 16 equations it becomes a big difficulty. So, in the next lecture, we are going to simplify it and then we are going to make it into a four by four matrix. So, we will have fewer number of equations, and will take it down as a tutorial, and try to derive all the equations for a four by four systems, and see that for this four by four system we have enough number of equations that when we put all the simultaneous equation, we will have a close system of equations, and it is just a question of solving this algebraic equation.

So, that will be the part of the next lecture in which it is lecture comes tutorial. We will go through this exercise, exercise of for the same equation, for the same domain, but fewer numbers of points. We derive all the algebraic equations and prove to ourselves that these together make up one complete set of equations, which can be solved. Then we go through a specific method of solving this algebraic equation which is characteristic of CFD and then we go through the solution there. So, that will be part of the next lecture.