Soil Mechanics
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Lecture – 29
Stress Distribution in soils
Lecture No.2

Students, we have heard one lecture on the topic of stress distribution in soils. Today we are going to have the next lecture, lecture number two on the same topic of stress distribution in soils. If you remember in the last session, we discussed the importance of stress distribution. That is why we should know, how the stresses are distributed in a soil when a load is applied. We also tried to see where it is important under what circumstances. By a large we understood that, understanding the stress distribution in soil is important in order to have a safe foundation and hence a safe super structure. So that is the basis. How do we compute this stress distribution? That is what is going to be discussed in today's lecture.

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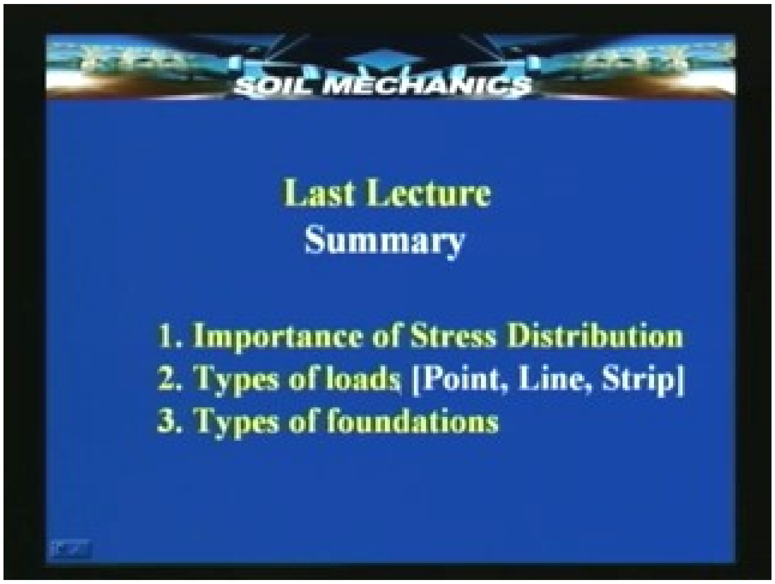

Let us take a quick look at one or two slides before we go further. This slides shows the topics that we discussed in the last lecture, a brief summary of what we discussed in the last lecture may be presented like this. The importance of stress distribution that I mentioned just now because we need to design safe foundations and hence safe super structures. It is also important to understand the different types of loads that are imposed on a soil. We saw that basically there can be three types of loads, the point that is the concentrated load, the line load the kind of load that arises due to a long object, let us say a compound wall or a strip foundation something again similar to a long compound wall foundation or a wall foundation which has got a definite breadth and definite length and hence a definite strip shape.

We also saw that foundations could be of different types. For example we saw that foundation could be rectangular or square, they could be circular, and they could even be of arbitrary shapes. So in order to understand how stress is distributed due to different loads under different types of foundations, we need to have a method and that is what we are planning to discuss today. Let us take a look at these four diagrams. The first one is that of a foundation resting on a soil.

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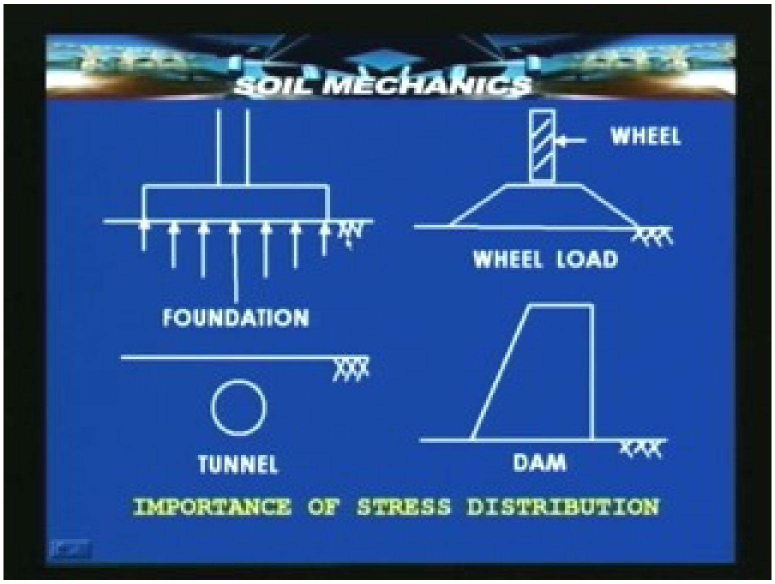

Let us try to discuss this a little bit. Suppose there is a foundation which is subjected to a load from above from the super structure. Strictly speaking, the foundation will distribute the load to the soil in any manner it chooses, depending upon the foundation and the soil. It is very difficult to predict what will be the pattern of distribution below the foundation. Actually it depends upon the relative stiffness of the foundation on the soil and therefore one need to know what is the pattern of distribution before we venture into determining what is the distribution within the depth of the soil.

Another example is that of the wheel load. Let us take a look at this in detail. A wheel load is nothing but the type of load that arises due to traffic. Vehicular traffic when it runs on a pavement with soft flexible or rigid, it imposes a load on the pavement which in turn gets distributed inside the soil. This distribution is of importance because the wheel load varies from vehicle to vehicle. It could not only be static, it could also be dynamic that is moving, and it could be cyclic. And this kind of load induces stresses which vary with time. It is important therefore to have a method to compute these stress distribution under foundation of a typical pavement subjected to traffic or vehicular loans. The third problem is that of a tunnel, all of us are very familiar with tunnels. These tunnels also introduce stresses around them and we need to know how to compute them.

Let us discuss this. In a normal soil medium, initially the stresses which exist are those due to the weight of the soil itself. When you make an opening like that of a tunnel or when you make a trench, when you cut a trench or when you install a pipe. For example suppose this is ground level, you cut a trench and inside that you install a pipeline and then fill it back with soil. There is soil all around and below also. In view of the fact that we have introduced an external object into this soil.

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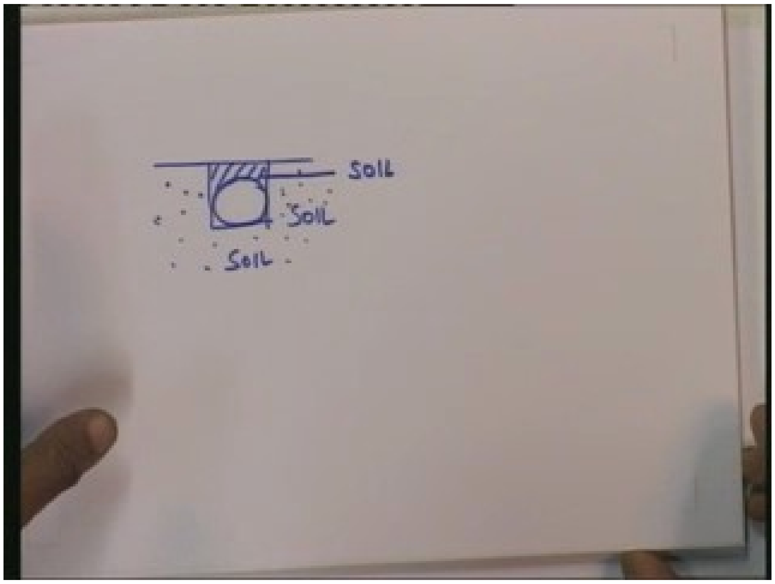

It is going to introduce new stresses all around. The stresses here are no longer the original stresses which were exiting without the tunnel or without the pipeline. So that needs to be a method which will help us to compute the new stresses which are going to arise when either a tunnel is constructed or when an excavation is made or when a pipe is installed. There is also another kind of problem or another instance in practice where stress distribution is important. Take a look at this diagram. The diagram of a dam, this is a typical dam and this imposes stresses on its foundation, the shape of the dam, the water that is stored on one side. All these are going to impose loads on the foundation. The loads are not going to be uniform and we need to know how the stresses are going to be distributed inside this foundation.

As I said in every one of these instances, ultimately we are interested in knowing whether the stresses which come in the soil are within safe limits or not. Having said this it is not only the stresses we should be within safe limits, it’s also important that for a structure to perform satisfactory without the stress, the net deformation or the displacements that the structure undergoes or the foundation undergoes should also be within safe limits. Actually we need to compute not only the stresses but we also need to have a method to compute the displacements that arise due to these stresses and ensure that both stresses and the displacements are within safe limits. Of course the subject matter of this lecture or this topic as a whole is only related to determination of the stress distribution. The computation of displacements we shall not include or we shall not be seeing during the course of this particular lecture, may be some other time or in some other lecture. Let us proceed further.

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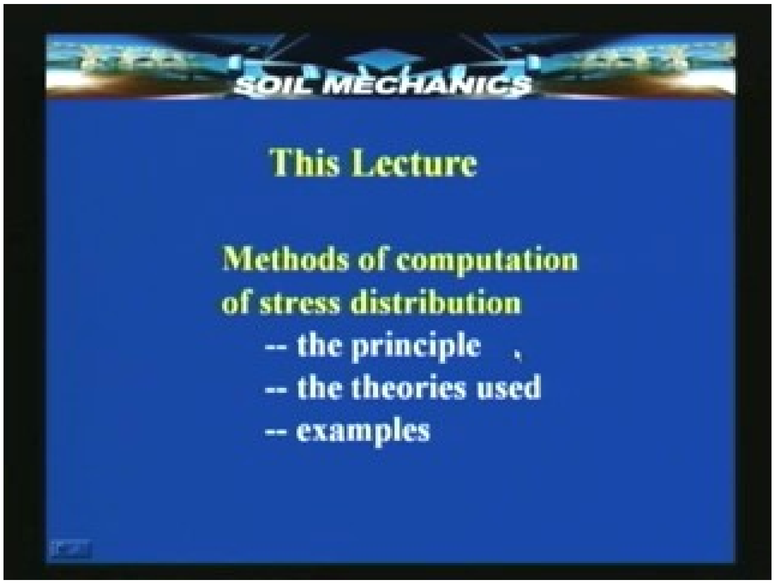

So in this lecture we shall be seeing methods of computation of stress distribution which means we will first see the principle behind the method of computation. Then we will see what are the theories that are available, then we conclude with the few examples. So let us start with the principle. Let us take a look at this ground level subjected to uniform load over a large area.

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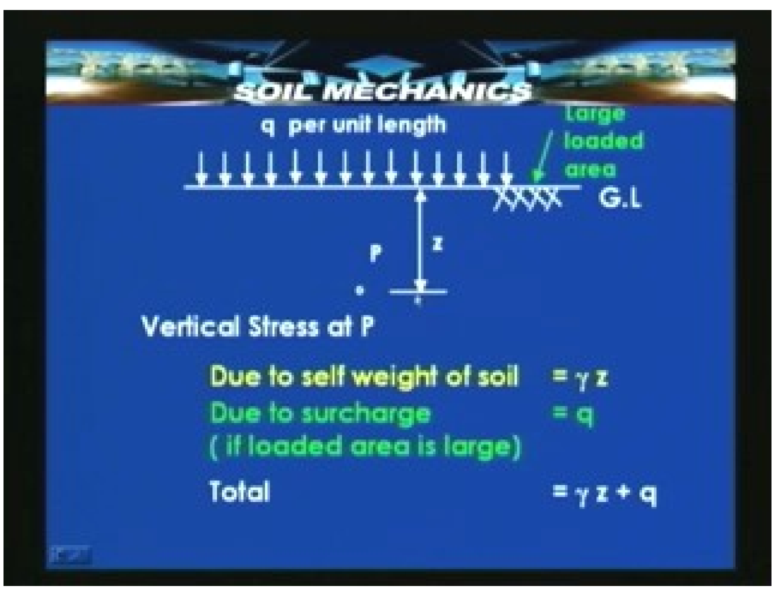

This is very important here that this area is a large loaded area. It is subjected to uniform stress or pressure, load due to some super structure which is tentatively written here as q per unit length of loading. Due to this, at any typical point P at a depth Z there is going to be a stress and what we are interested in is primarily the vertical stress at this point P. Sometime back I mentioned in the beginning of this lecture that these stresses in the soil are primarily due to its own weight. Initially due to the self weight of the soil itself there is going to be a stress at the point P. What is that stress going to be? That stress is going to be the weight per unit volume of the soil into the depth z that per unit area. Let me draw a diagram, effectively we will be considering at depth a point P, we will be considering unit area around the point P which is at a depth z and then consider the volume of soil that is enclosed inside this parallelepiped. The volume of soil that is enclosed in this is going to be the depth z into the area which is one.

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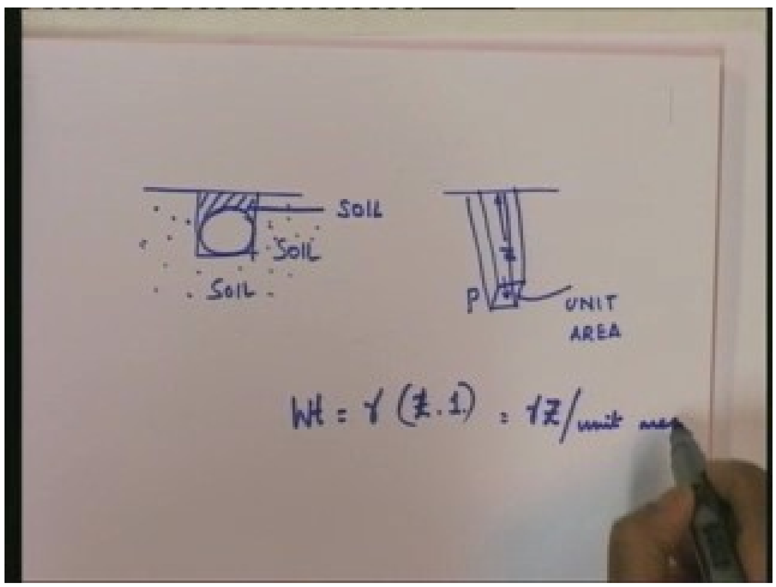

So the weight of this soil is going to be unit weight of the soil into z into one per unit area. Therefore you find that this weight is equal to gamma z and that is the pressure because it acts on unit area. So we have the self weight of the soil which is gamma z and that is per unit area and that is the stress at point P. We are applying a load here in the form of q by constructing some structure. Any activity on the surface be it the construction of a tall tower, a retaining wall, building whatever the structure may be, let us say it imposes a stress q. As long as this loaded area is very large at somewhere near the center of this loaded area, the point P is going to experience a stress which is equal to this q itself. This is actually valid only if the loaded area is sufficiently large compared to the area within which we are trying to determine this stresses. If the loaded area is small, as we go away from the loaded area at a depth z you will find that the effect of the loaded area decreases. And this if you remember, we saw in the last class also when we studied the variation of the vertical stress in the lateral direction.

But if the loaded area is large, we can safely take the surcharge or the stress due to the surcharge as q at depth z as well, which means at depth z the total stress is going to be gamma z plus q. But what are we interested in? We as civil engineers are primarily interested in the stress that we are going to additionally impose on the soil at any point by our activity. By constructing an embankment or by constructing a building, what is the additional stress that we are going to impose on the soil? Why is this important? This is important because the soil is after all in equilibrium; the soil is at rest due to its own weight. It has achieved a state of equilibrium probably over a long period of time running to millions of years, geological time and therefore it’s the additional stress that we are imposing by constructing a building which is of primary interest. And that is the vertical stress that we will be talking about throughout the course of this lecture.

When I say, we want to compute vertical stress; I actually mean the vertical stress that is arising due to the load that is applied. This vertical stress that is arising due to the load applied, some times I will be calling this as the vertical stress, sometimes I may be calling this as the increment in stress. Both mean the same; effectively what I imply is the stress that is arising at any point due to the externally applied load which in this case is a uniformly distributed load. But in general the stress need not be uniform. Let us proceed further.

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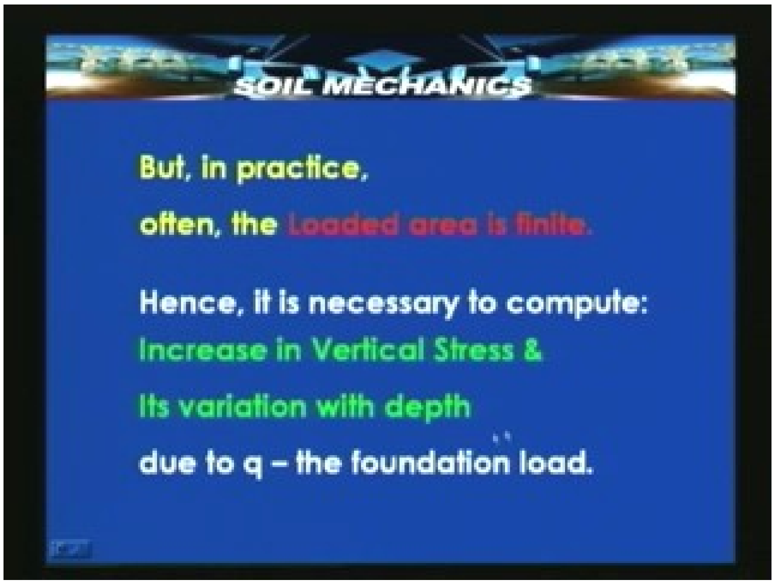

So as I mentioned, in practice very often this loaded area is finite, it is not very large. Therefore we often need to compute the increase in the vertical stress and its variation with depth due to the surcharge q which is nothing but the load that the foundation is transmitting. This is merely emphasizing what I told, that is it is the increase in the vertical stress that we are interested in. Further as I said, one component to the stress is the self weight but then it can always be easily computed if we want by knowing gamma and the depth z.

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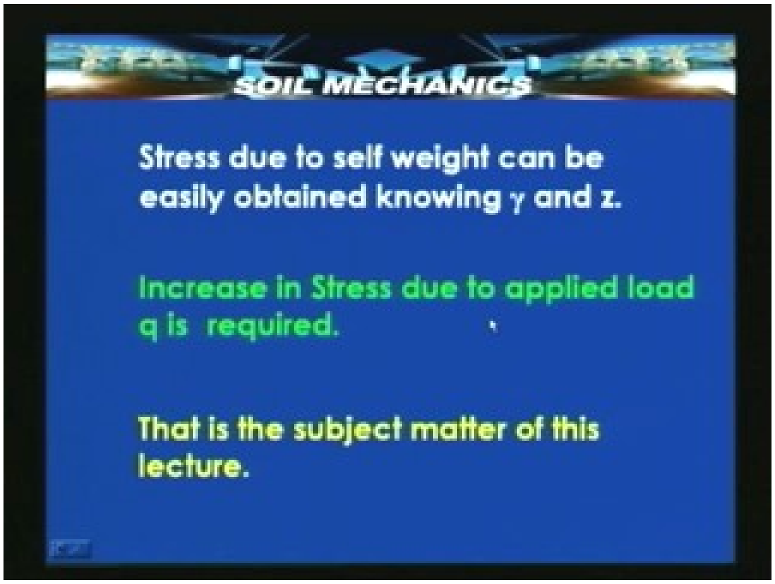

Because we have just seen that the vertical stress due to the weight is gamma z. So it is only the increase in stress due to the applied load which we are interested in and that is what we are going to discuss in today’s lecture. For understanding how these vertical stresses is computed we need to understand what is contact pressure. When a foundation is constructed and it is in contact with the soil, if this is the soil and we put a foundation on the soil or below this. It is in contact with soil below and it is at this surface of contact that the foundation is actually transmitting its stress. So if the foundation is imposing a load on the soil, in turn the soil is going to impose a resistance which is what is depicted here in the form of a pressure distribution under the foundation. This is what is known as the contact pressure.

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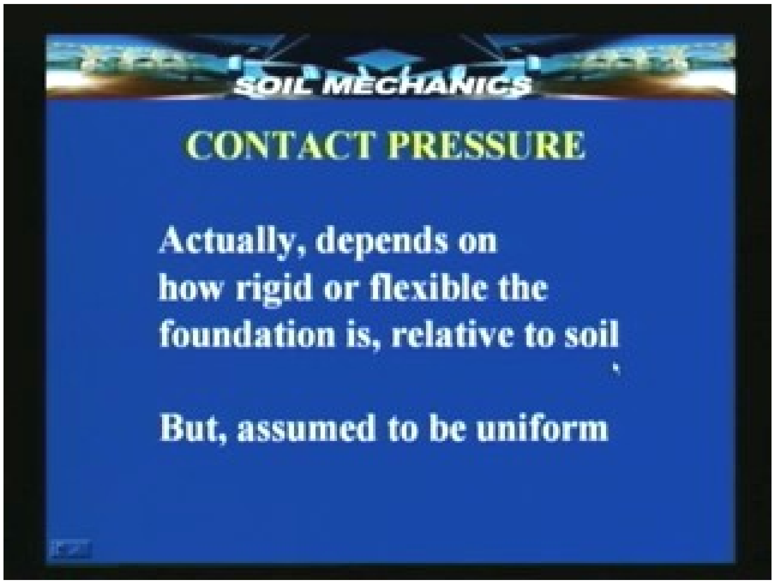

This contact pressure which is arising below the foundation is responsible for creating stresses at several points inside the medium. Let us see next. This contact pressure actually depends on the rigidity or flexibility of the foundation relative to the soil. Imagine suppose I have a rigid foundation, this foundation is so rigid that when I apply a load on this it is going to settle uniformly because it is a rigid object, it will simply go through. If on the other hand this foundation is a flexible object then when I apply a load this is likely to bend like this which means that it will impose or it will experience uniform test from below. Whereas in rigid case it may experience very large stresses at the center but lesser stresses at the edge and it may have a stress distribution like this.

This depends not only on simply the rigidity of the foundation but it depends on the rigidity of the soil as well, because even if the foundation is rigid, if the soil is also rigid then it is the relative rigidity which controls the pressure distribution. So it is a relative rigidity or flexibility of the foundation with respect to the soil which controls this pressure distribution and in general this pressure distribution could be non-uniform. It need not necessarily be uniform. However we shall be making a simplistic assumption, we shall be making an idealization that is, this foundation contact pressure is actually uniform. This considerably simplifies the computation of stresses inside the soil without incurring huge loss of accuracy or large errors.

This is the stress distribution which we want to determine, the one which is arising due to the contact pressure which we shall be taking as uniform. There are two ways of computing the stress distribution; one is an approximate method and empirical method. Another is a more rigid rigorous method which is based on the theory of elasticity. The relative importance or the relative advantages of these two methods is more than obvious.

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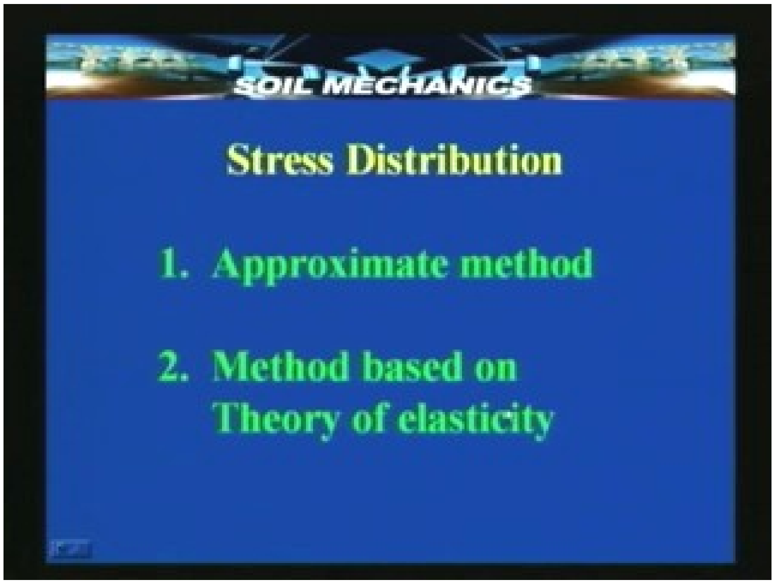

The approximate method as the name implies makes a few gross assumptions, makes a few approximations and gets an estimate of the stress at any point. This estimate is often useful in practice where we may not always need a rigorous method to compute the stresses. Whereas in very important situation where a very rigorous calculation of stress is necessary. This approximate method may be in error by such a large margin that we may not be able to use that. So there could be a situation where we only need an approximate estimate of the vertical stress at any point in the medium. And the approximate solution is good enough and this is what is used very frequently in practice where we only need some preliminary estimate of the stresses.

Whereas when it comes to an actual design of the foundation we always want to do it in a rigorous and it very strict manner. Therefore we like to use a sound theory which is not very approximate, which of course may also make assumptions but will be far more rigorous compared to the approximate method. Let us see what this approximate method is. As you all know that soil is a particulate material, I have represented this soil grains here in the form of circles. This diagram shows the stress transmission from grain to grain in a soil. Take this grain suppose it is subjected to a load P. This is in turn imposing a load on two supporting grains at their contact. This is a point where the first grain is in contact with this lower grain. Similarly this is a point of contact between the left hand side grain and the upper grain. This transmits a part of the load P here and another part of the load P here. Assuming ideal and symmetrical conditions, this will experience P/2 and this will experience P/2.

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But in general, a soil particle need not be circular, need not be spherical, it need not be regular in shape. It may be highly irregular and there may be more than one contact points. The lower soil grains may also not regular in shape but to understand the basic principle, let us take idealize situation.

This is good enough to convey the importance of the grain structure, to convey how the stress gets dispersed inside the soil because the soil is a granular material. The stress gets divided between these two. These two particles in turn are supported by three more particles like this (Refer Slide Time: 22:46). So this transmits stresses here, as well as here, as well as here and this goes on and on. Ultimately this load P gradually gets dispersed over larger and larger areas as we go deeper and deeper. That is a very important finding, this shows that this load P acts on larger and larger areas as we go deeper, which means the stresses per unit area go on decreasing with depth.

This is a very useful thing because the stresses must decay with depth, otherwise at infinite depth we will have infinite stress and the soil will not be in stable condition. This structure which has shown here is a singe grain structure. You must have already studied that there are other forms of soil structures and therefore there are other forms of distribution of stresses inside the soil. However we shall make a gross assumption that all soil particles are spherical, uniform, identical and this stress distribution is symmetrical. Let us see the next slide, take an area like this. Let us say this is a foundation, this is being subjected to a load P.

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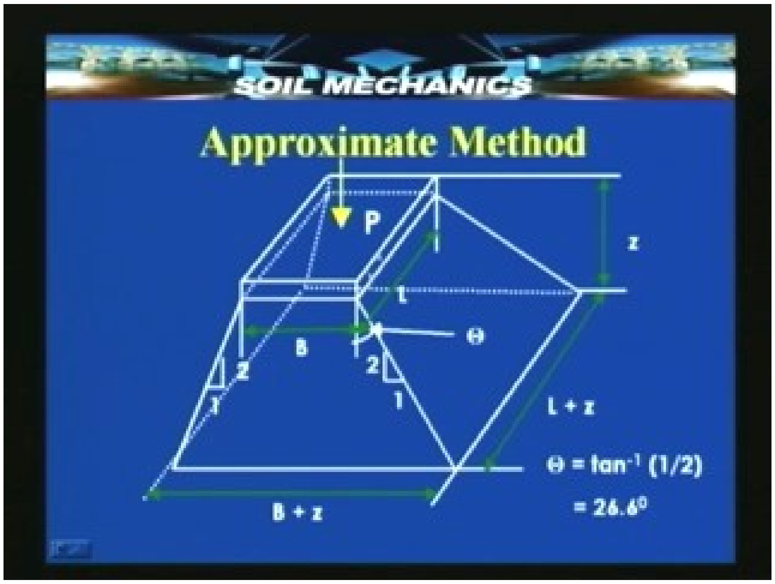

This is known as a concentrated load, this is a single vertical load of magnitude P acting vertically down at the center of this. How will this load be distributed inside the soil? We have just now seen how the particulate nature of the soil helps in the distribution. Suppose this is the foundation on which I am applying a load P because there are a number of particles below this. Gradually this load is going to get distributed like this and like this and in the third dimension as well it is going to get distributed along lines approximately as drawn here. A better diagram is there in this slide. In the slide we can find that the stress gets distributed latterly like this and also in this direction. Here also there is a distribution line which means if the area of this foundation is length into breadth that is L into B.

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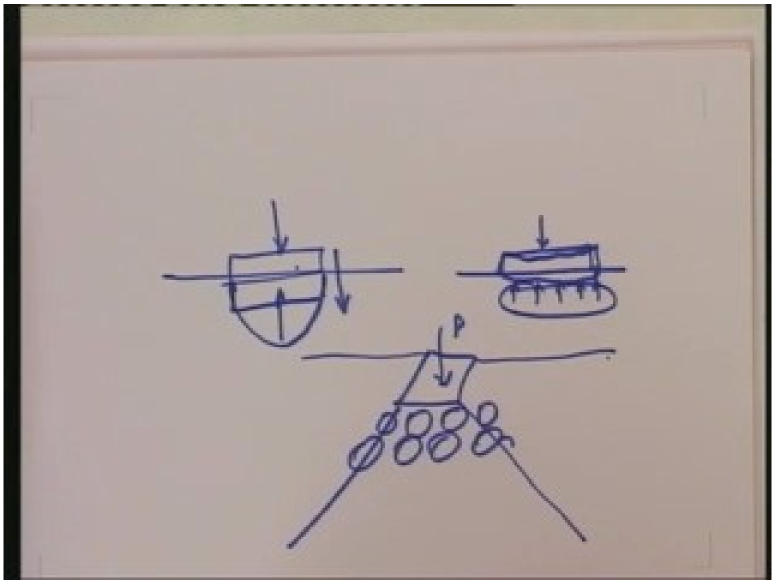

And if we assume for all practical purposes, a distribution given by two boundaries or four boundaries to be more precise, which are all sloping at an angle theta which has a tangent of ½ or rather we say that the distribution is along a slope of magnitude 2/1. If we take this as the distribution line, then what happens is at any depth z below the foundation from the top to the bottom, because of this distribution this length becomes L + z and this length becomes B + z. And therefore at depth z the vertical stress due to the load P will be simply P divided by this area which is L + z into B + z. This angle corresponding to a tangent of one upon two is nothing but 26.6 degrees which is approximately 30 degrees.

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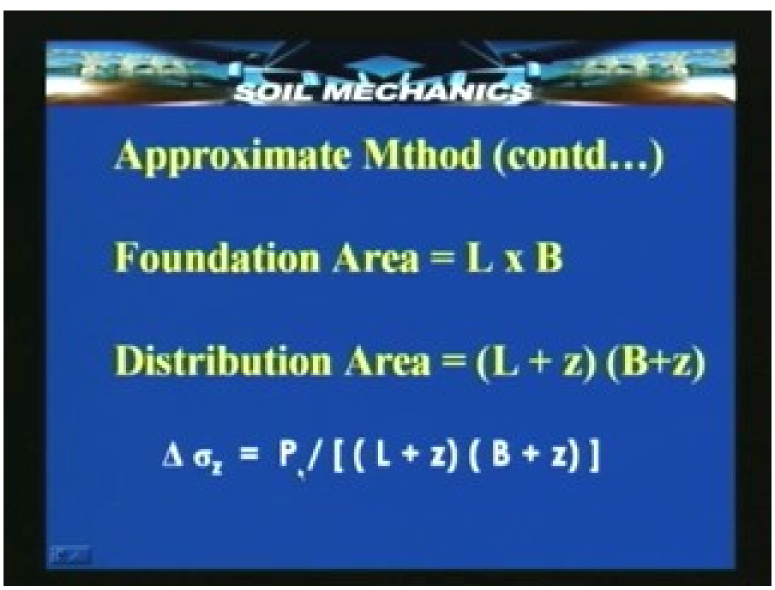

So in practice we often assume a distribution of approximately 30 degrees. So continuing this approximate calculation, we find that the foundation area if it is L into B, the area over which the load is actually distributed at depth z is L+ z into B + z. And therefore any vertical stress or so called incremental stress, delta sigma z will be equal to the applied load divided by the area at depth z which is L + z into B + z. Having understood the principles of dispersion of stresses inside the soil, which has been very nicely brought out by this approximate method. Let us see how to compute the stresses by a more rigorous method that is the theory of elasticity.

If you see this slide you will find that three names are mentioned. The first name is J Boussinesq a French mathematician, the other two names are of Westergaard and Newmark. They have all contributed at certain interval of time to the computation of stress distribution below a foundation using theory of elasticity. There are many others who have contributed subsequently, but some of the major or important contributions, conceptually different contributions have come from Boussinesq, Westergaard and Newmark.

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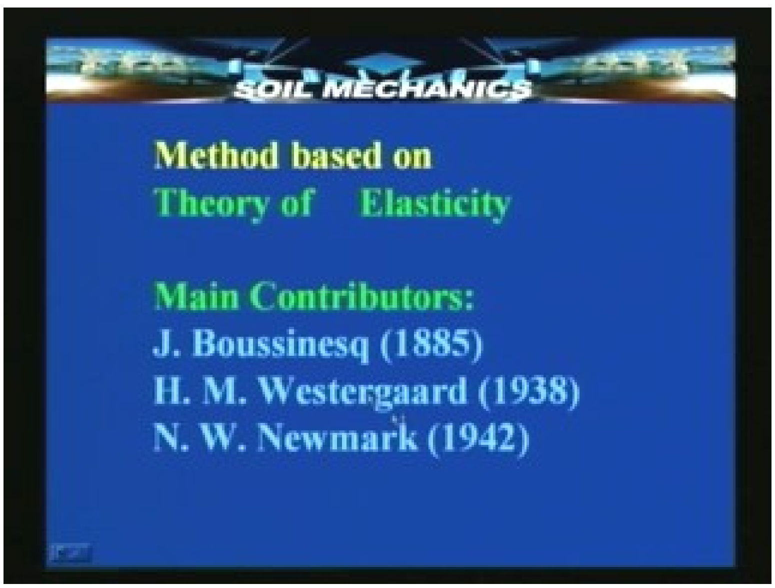

We shall see now Boussinesq theory of stress distribution which is nothing but essentially application of the principles of theory of elasticity to the problem of stress distribution below a foundation resting on a soil. Let us basically try to see what this theory of elasticity is all about, before we see how it has been used by Boussinesq in computational stresses. Theory of elasticity is a theory, the very name suggest that it is based on concept of elasticity. What is this concept of elasticity? What is an elastic material? We know an elastic material is one in which the stress is propositional to strain. If we say that the stress is linearly propositional to strain, directly propositional to strain then the well known Hook’s law which says sigma equal to kz applies. Actually soil is a particulate granular material but if you want to apply theory of elasticity, we have got to assume that it is an elastic material. So what are the importance assumptions that will be making, before applying this theory is that soil is an elastic material. But there is also other assumption which is needed to be made in order to apply this theory. So let us take a look at all those assumptions which we will have to make before we proceed with the basics of the theory itself. First of all we have to define the soil medium. The soil medium is defined as a continuum, it is sometimes called a semi infinite medium, sometimes half space. All of them mean one and the same thing.

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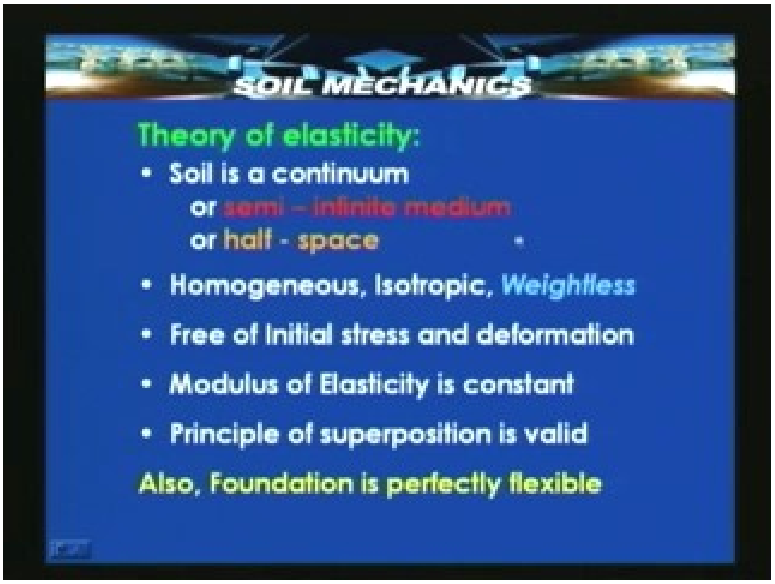

Suppose this is the boundary of the soil medium, as I said in the last class we often represent the ground soil surface by horizontal line and write G L. Ideally or theoretically this soil extends to infinity in the downward direction. That is it can go deep into the earth crust. Above this if we consider the entire universe, we have again infinity. If this is the space, half the space is what we have below this horizontal line and any line on the ground surface may be considered to be locally horizontal, although the earth is round. Although the earth is a globe, over a small area locally we can take the ground surface to be horizontal and the space that is below is the space occupied by the soil may be called as half space.

Another way to call this is by the name semi-infinite medium because it is half of an infinite medium. It is extending to infinity in the downward direction in the two lateral directions. But what is more important is, this medium although it is consisting of particulate materials, we shall be assuming that it is a continuum. It is a continuous medium unintersected by any gaps in between the soil particles. Why is this required? This is required because we are going to make further assumptions. The assumptions that we are going to make further are homogeneous isotropic material. This semi-infinite continuum material is going to be considered as homogeneous and isotropic. The importance of the word homogeneous is that any element that is taken out of this continuum, like the one which we considered last time, if you remember we considered an element like this. Any element inside the soil will be considered to be representative of the whole medium. So that is the meaning of homogeneity.

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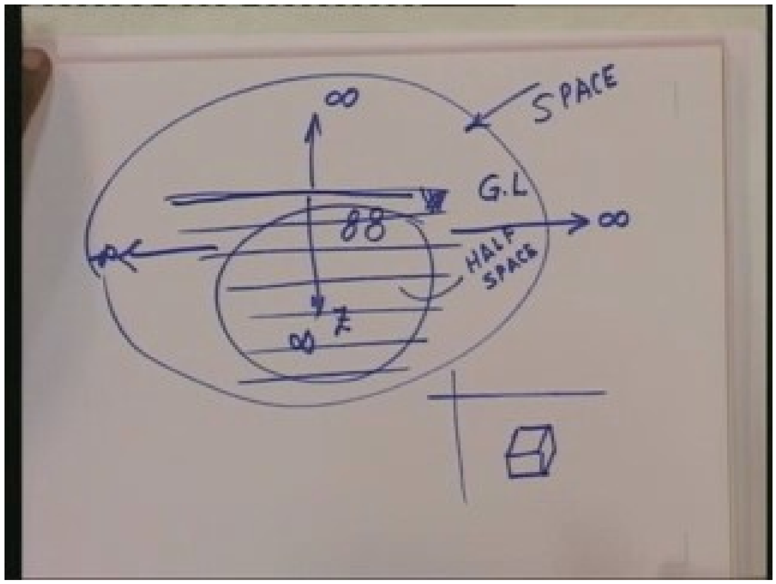

So when we say the soil is homogenous, we mean that any element of this continuum is capable of fully representing the behavior of the continuum as a whole, which implies that if we can analyze the behavior of one element from point of view of stress distribution, we can easily extrapolate it to every other element in the continuum and therefore understand the stress distribution in the continuum as a whole. That is the importance of the assumption of homogeneity. The next assumption is that of the medium being isotropic. In the word isotropic means that all the properties of the medium are same in all the three coordinate directions x y z. That is again important because any stress computation will depend upon its variation in the three dimensions and it becomes much simpler to compute this stress variation, if we do not introduce any other complexity into the calculations such as the variation of the material itself.

So it’s convenient to assume the material to be having constant properties and only the stresses to be varying. And that is what we shall be doing although with advancement in the recent pass, there is no need to restrict ourselves with these assumptions of isotropicity. It is possible to compute stress distribution even in medium which are not isotropic. But then for simplicity we shall be considering only the problem of a medium which is homogeneous and isotropic. One more important assumption we are making about the medium is that it is weightless. Let us see the importance of this.

Why is this assumption of weightlessness important? It is important because we are interested in the incremental stress that is being cast by the load that is applied externally on the surface of the ground. The weight of the medium which is introducing a stress at any point given by gamma z can be easily calculated if you know the depth z. Therefore it is not essential to go for any complex or any complicated or rigorous theory to compute the stress. What we really need to compute is only the additional stress that comes due to the surface loading. And therefore it makes our job much easier, if we restrict to a problem where the way to the medium is ignored and we only calculate the stress due to the external load. The stress due to the weight of the medium can always be added or super post by simply computing it as gamma times the depth. At any points z therefore we will have a vertical stress computed by a rigorous theory for the externally applied load and to that we will add the stress that is arising due to the weight of the medium.

In addition to these assumptions we shall also be assuming that the medium is free of all initial stresses and deformation. Actually this follows from this simple assumption of weightlessness because if there is no weight there is no initial stress and no initial deformation. We have already defined the word isotropic, from this it follows the modulus of elasticity and other parameters such as Poisson's ratio or the shear modulus which define the *elastic nature of the soil are all constant with direction. And then finally we will assume that the principle of superposition is valid. That means if by chance there are two loads applied over the soil, the stress due to the first load and the stress due to the second load can both be added in order to get the total stress due to both the loads. This we had in fact discussed indirectly a while ago.*

For example if we want to know the total stress which comprises the stress due to the externally applied load and the stress due to the self weight. Then we assume that the two can be super imposed in order to get that total stress. But finally a very important assumption that we shall be making is that this foundation is perfectly flexible. The importance of this has already been stated, that is when we have a flexible foundation we have uniform contact pressure and all the derivation, all the theory, all the methods that we shall be seeing now are all valid for uniform contact pressure at the foundation soil contact.

In order to understand the theory of elasticity, we need to have the basic definitions sketch of the problem. So this is the basic definition, there is a load capital P which is applied on the surface of the soil. The surface of the soil is defined by the x y coordinate or the plane x y. The depth is defined by a coordinate z positive downwards and any three dimensional elements such as the one which is shown here will have a coordinates at the center which will be given by x y z or r and z, where r is nothing but root of x square plus y square in this case. We therefore are looking for a method to compute the vertical stress at this point P due to a load which is being a load P that being applied at the origin.

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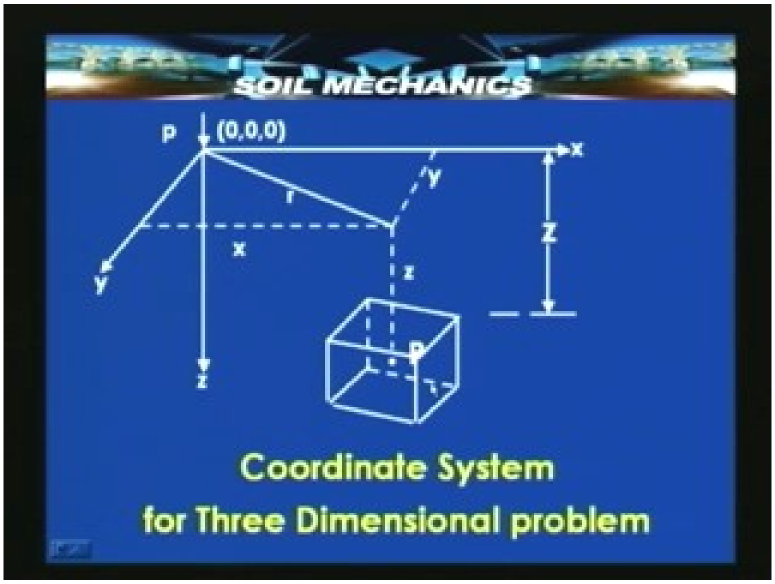

So this is a three dimensional coordinate system and element combination that we shall be using for further work. So let us quickly go through the fundamental of the theory of elasticity. We have already seen the assumptions, now we shall see what are various steps in the use of theory of elasticity in arriving at the stress distribution. The fundamentals of the theory of elasticity all apply to one of the elements which we just saw in the previous slide. Typical element which is shown in the previous slide is similar to any other element because we have assumed homogeneity. If I take any one element we can make a few mathematical formulations on the basis of theory of elasticity, with the help of these formulations we shall be in a position to compute the stresses in the medium.

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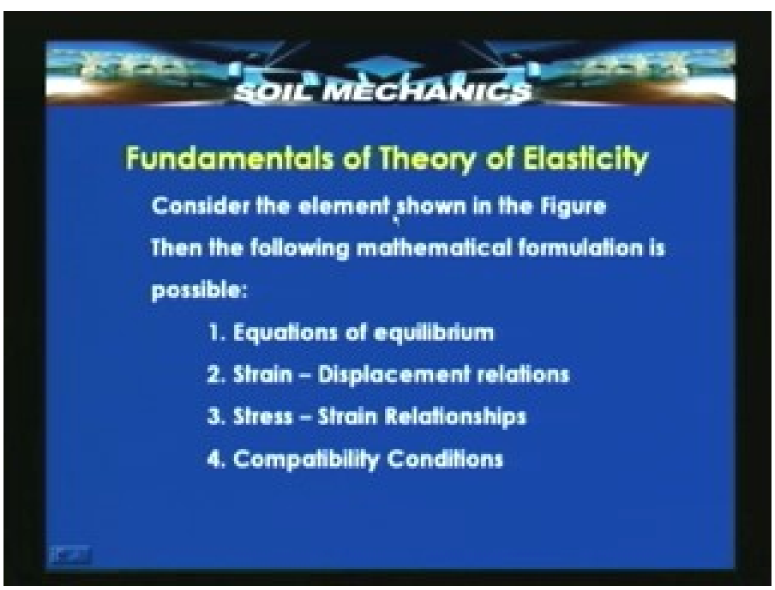

What are the formulations? One is equation of equilibrium, the second one strain displacement relations. We shall be formulating also stress strain relationship and then lastly compatibility conditions. Let us take the equations of equilibrium and try to understand in some detail. If I take the equation of equilibrium, the importance of this lies in the fact that in practice we want the soil medium to be in equilibrium. We want every element of the soil to be in equilibrium, it's only when every element of the soil is in equilibrium that the foundation which is resting on the soil will be in equilibrium and therefore the structure will be safe.

So equilibrium is very easily understandable, it is an important prerequisite and therefore lets us start with the assumption that whatever be the stresses acting on an element the element must be in equilibrium. Suppose this is the element, in the last lecture we saw that every surface or every phase of this element is going to be subjected to a normal stress and two shear stresses, one in each coordinate direction. This is a normal stress, this is a shear stress in the x direction and this is a shear stress in the y direction (Refer Slide Time: 41:14). There will be three stresses on each one of these surfaces. All these stresses we shall assume to be uniformly distributed over this small parallelepiped.

The element is considered to be very small, it is infinitesimally small. Infinitesimally small element implies that the element is so small, its dimensions are so small that the stress that is acting on this that is sigma z, dow z x, dow z y are all uniformly distributed over these, which means that if you want to get the three forces, 1 vertical and 2 horizontal on this surface, you just need to multiply this stress by the area of the surface. Because the stresses are assumed to be uniformly distributed. For example if this dimension is going to be in the y direction that is delta y, if this dimension in this x direction is going to be delta x then the force that is arising due to sigma z will be sigma z into delta x into delta y. Similarly there will be forces acting on each one of these six surfaces of this element. It is very important that this element be kept in equilibrium by these forces. So it could be better than simply writing equations of equilibrium for all those forces which are acting on the various surfaces of this element.

What could be the equations of equilibrium for a three dimensional problem? There could be six equations of equilibrium, three for force equilibrium and three for moment equilibrium. So the equations of equilibrium are going to be sigma F_x that is sigma of all the forces in the x direction is equal to zero, sigma of all the forces in the y direction equal to zero, sigma of all the forces in the z direction equal to zero. Similarly the moments about the x axis must be equal to zero, the moment about the y axis must be equal to zero and the moment about the z axis equal to zero. We shall not be going into great detail of how each force on each face is computed. How these equations are written and how they are simplified to get finally the conditions which depict equilibrium. I shall simply write one final answer that is one typical equation representing equilibrium in the x direction. I shall be giving you the equation which represents the state of stress that keeps the element in equilibrium in the x direction.

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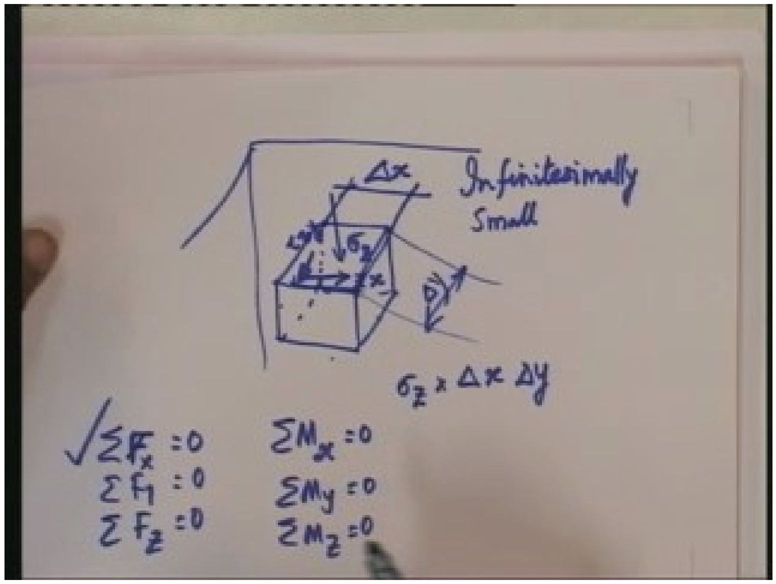

This equation will look somewhat like this. The sigma x upon delta x + delta dow x y upon delta y + delta dow x z divided by delta z + x =0. There will be two more equations, this equation is for sigma F_x =0. There will be two more equations, one for sigma F_y =0 another for sigma F_z =0. We have one extra term here, this is nothing but the force that is arising due to the weight of the material and this is known as the body stress. This is expressed as stress per unit volume. This body stress can be computed just as we can compute the volume and weight of a parallelepiped. The body stress per unit volume is therefore something which can be computed straight away, which can be determined straight away without the use of even theory of elasticity. And therefore this is a known quantity. What are not known are, this stress sigma x, this stress dow x y and this dow x z.

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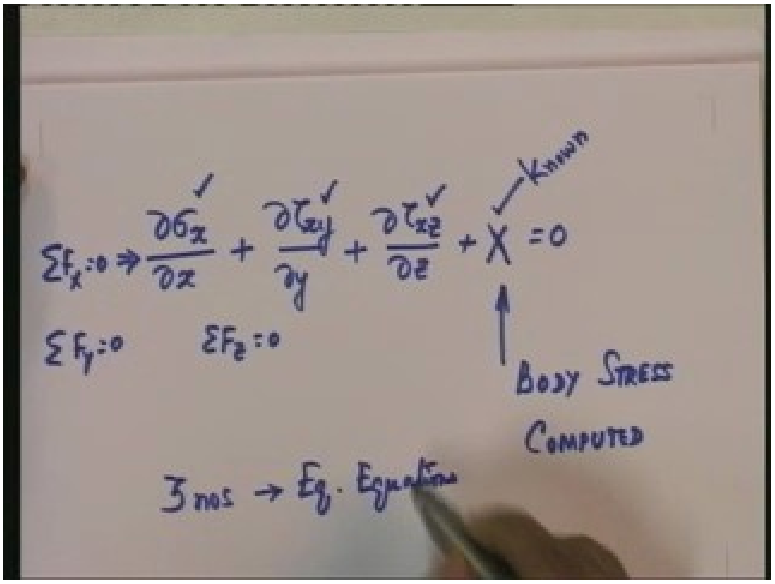

Similarly the other stresses sigma y, sigma z and other shear stresses. One set of equations that is three numbers for equilibrium equations are the first set of equations that we shall be considering in theory of elasticity. The second set of equations corresponds to strain displacement relationship. If I again consider any one phase of this parallelepiped, let us say this is x, this is y (Refer Slide Time: 46:50). When a stress is acting on this, this is going to get distorted some what like this due to the presence of this stresses which are acting on this, which means that any typical side gets extended. Similarly the shape also gets distorted which means there are normal strains and also rotational or shear strains. We need to therefore have some equations to represent these strains. We all know the strain is nothing but change in length divided by the original length.

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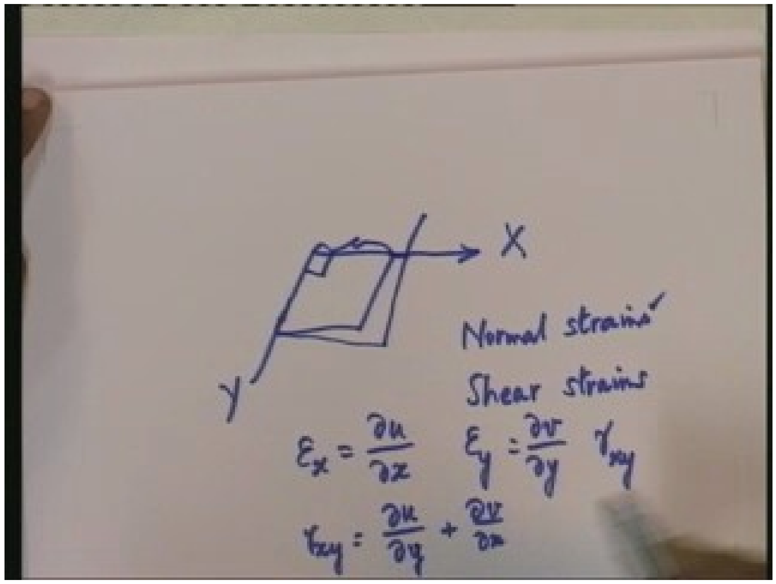

So if we apply that concept and similarly shear strain is defined as change in an angle which is originally a right angle. If we apply these concepts we can formulate equations for the normal strain and the shear strain. For example, for one surface x y, for one plane x y the normal strains will be one in the x direction that is epsilon x equal to del u by del x, another in the y direction that is del v by del y and third one is the shear strain. The shear strain x y is del u by del y plus del v by del x. It is not really essential for our purposes at this point of time to go into the details of derivation of these expressions. It is suffice to understand that if we know the displacements u in the x direction and v in the y direction, we will be in a position to compute the strains epsilon x, epsilon y and gamma x y. Similarly this can be extrapolated to three dimensions and we will be in a position to compute if we know the displacement w in the z direction. We will be in a position to compute the shear strains in the other directions and also the normal strain epsilon z.

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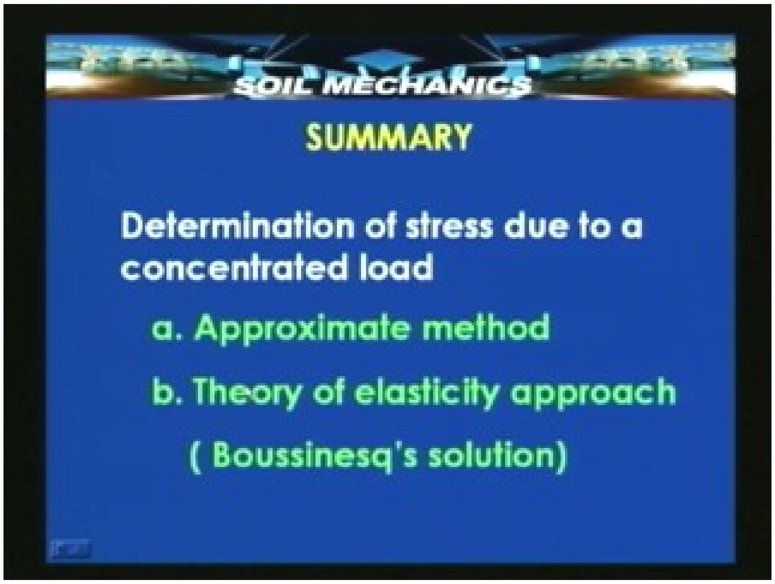

Similarly we will also be seeing in the next class how to compute stress strain relationship and how to define a set of equations or set of conditions known as the compatibility conditions. So now we shall conclude by talking a brief look at summary. Today we have seen the approximate method of computing the stress due to a concentrated load on the surface of a soil medium. We have also seen the approach which is based on the theory of elasticity. We shall see further details of the equations that constitute the basics of the theory of elasticity and how they are solved in order to get the stresses in the medium and this we shall do in the next lecture. So in the next lecture we shall go into the details of how these stresses can be computed due to a line load, a strip load. How stresses vary below a rectangular foundation, below circular foundation and below a foundation of arbitrary shape. And all these we shall be computing or calculating using the theory of elasticity. So we conclude with this.

Thank you.