**Soil Mechanics**
**Prof. B.V.S. Viswanathan**
**Department of Civil Engineering**
**Indian Institute of Technology, Bombay**
**Lecture – 30**
**Stress Distribution in soils**
**Lecture No.3**

Welcome students we meet once again for a lecture on stress distribution in soils. We have come a long way in the last two lectures. We now have a fairly good idea about what this problem of stress distribution is all about. In the first lecture we had seen the importance of this problem, where this problem is arising, where we need stress distribution in soils, where we are particularly concerned about the ability of the foundation to withstand the stresses from the superstructure without distress. Under all these circumstances we need a method to compute these stresses. So in the last lecture in particular we were trying to take a look at the methods that are being used for computing stresses.

So let us take a look at one of these slides. This slide summarizes briefly what we discussed in the last lecture. Let’s read it. We saw in the last lecture particularly what is the method for computing stresses which is based on the theory of elasticity. If you remember I mentioned that there are major contributions based on the theory of elasticity to the subject of stress distribution from scientist such as J Boussinesq who gave a theory for computing stresses based on theory of elasticity rather a method for computing stresses based on the theory of elasticity. This was followed in the year 1938 almost 40 years later by Westergaard who extended the method of Boussinesq to circular areas. Then came in 1942 a contribution from Newmark for computing stresses beneath any arbitrary area. Then we also saw in the last lecture how to compute the stresses due to a concentrated load.

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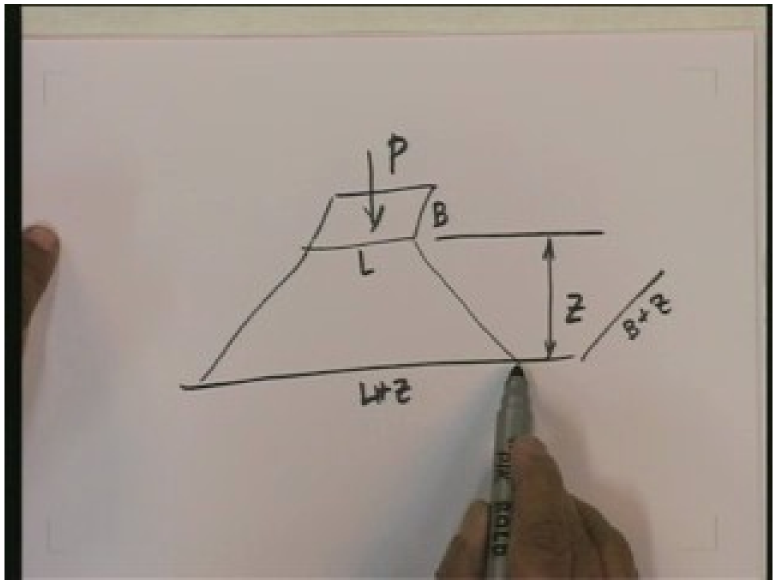


We saw in fact two broad approaches, one which I called the approximate method and another so called Boussinesq solution. If you recollect in the approximate method we just assume that soil is a particulate material. The stress from the foundation gets transmitted to the soil through the grains and in that process it disperses. If this is the foundation, we saw that the soil distributes the stress and at any depth z the area over which the stress gets distributed is no longer length into breadth but it is length plus depth into breadth plus depth.  And so if there is a load P here, its gets uniformly distributed over this area of L plus z into B plus z. This is in approximate method because the actual line which represents the boundaries of the area of distribution is approximate. Now on the other hand, the boussinesq's solution takes a different approach. It is based on theory of elasticity.

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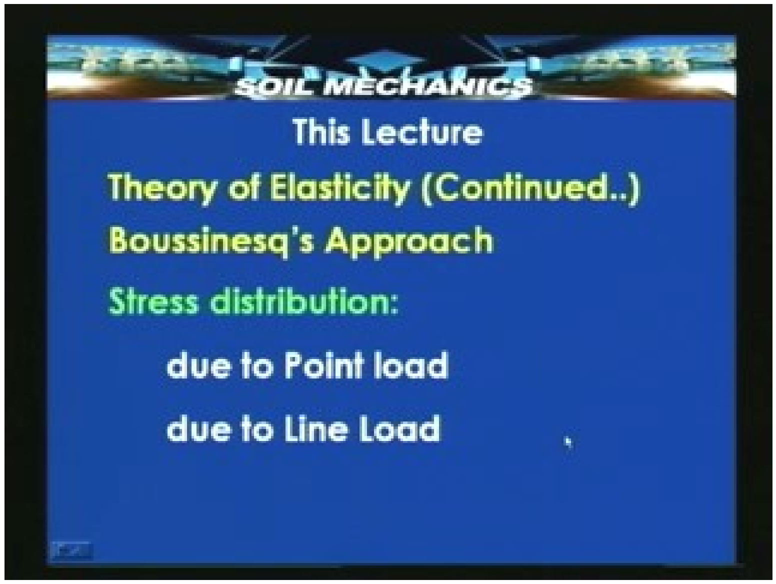


Let us see in today's lecture how to use Boussinesq's approach for computing stress distribution due to a point load and then due to line load. Let us briefly recapitulate what theory of elasticity is all about. It is necessary although we have covered it in the last lecture, to briefly go through this so that we will be able to appreciate Boussinesq theory a little better. If you see this slide, we started with the assumption that soil is a semi infinite medium or half space. It is a continuum meaning there is no break, although the medium is consisting of granular material with distinct boundaries between the grains. We assume this to be a continuum; next we assume the medium to be homogeneous. The advantage is every element of the medium is similar to any other element.

If we understand the behavior of one element then we can extrapolate it to all the other elements and hence to the entire half space. That is the idea of assuming homogeneity, it considerably simplifies the problem, and in nature of course it may be difficult to find perfectly homogeneous material. We will see later on how best non homogeneity or in homogeneity can be included. We also assume that the soil has equal properties in all directions that is isotropic. Most importantly we assume that the soil has no weight which meant that there were no initial stresses and there was no initial deformation.

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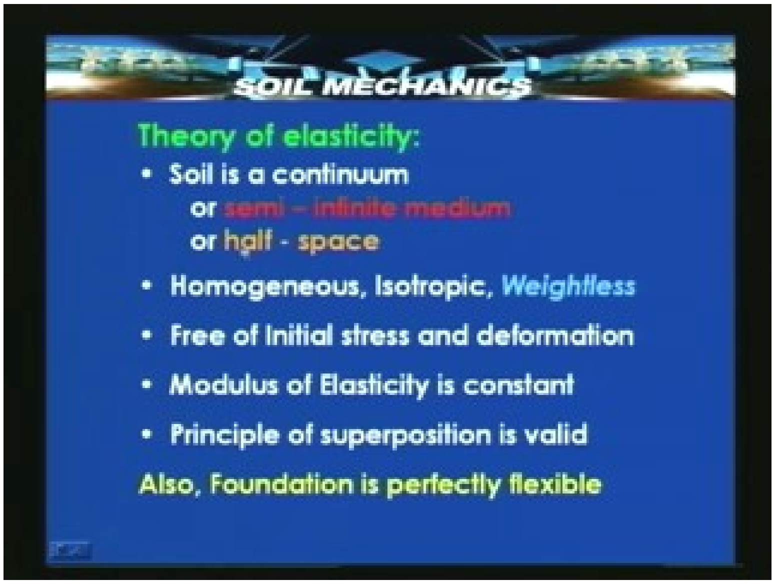


The idea is that we will be in a position to calculate the stress in the medium, purely due to the applied stress due to a foundation. The self weight of the material of the soil which contributes a stress by itself can always be computed, if we know the depth and if we know the unit weight of the soil. And therefore theory of elasticity need not be used for computing the stress due to the weight. Then modulus of elasticity is assumed to be constant, also Poisson’s ratio and other elastic constant and then lastly principle of superposition is valid. That means if for example more than one load is applied to the foundation, the stress due to each load is added to the stress due to the next load and therefore we apply the principle of superposition.

In all these we have made an assumption also that the foundation is perfectly flexible, so that the contact pressure at the interface between the foundation and the soil is uniform. All these assumptions and their importance have been laid out very clearly, both in the last lecture and I have emphasized it again this time. Are they really applicable, a question automatically arises in the mind. Are we over simplifying the problem or we justified in making these assumption or they really acceptable? To some extend they are acceptable, to a great extend application of this theory in practice has vindicated our stand that it is reasonable to assume the soil to be homogeneous, isotropic and a continuum and also elastic in nature.

You can say that the validity of the method, validity of these assumptions used in the method of computing stresses has been borne out by trails in the field. And therefore we can justifiably say that the method can be used with reasonable confidence in practice. However it must be noted that there can be deviations from the conditions which have been assumed. If there are deviations we shall have some approximate way of accounting for those deviations, using as a basis the expression which we will be deriving on the basis of these assumptions.

That is we will derive a set of conditions, a set of equations or expressions for these ideal situation and any deviation from this will be accounted in some empirical way. Let us take a look at the next slide. This shows the soil element and the coordinate system, this we have seen already in the last lecture two.

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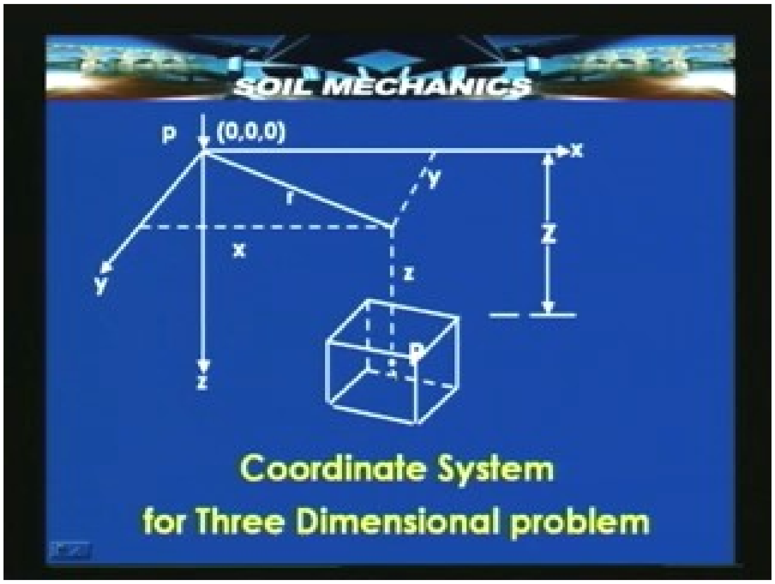


So this is the element which we shall be analyzing to find out what is the stress in the element. And this is the coordinate system that we shall be using and this is the load P which is known as the point are the concentrated load, due to which we are interested in finding out what this stress at point P. We were taking a look at the fundamentals of the theory of elasticity yesterday. The theory of elasticity assumes 4 or 5 major tenants. Number one, it says that the material is in equilibrium.

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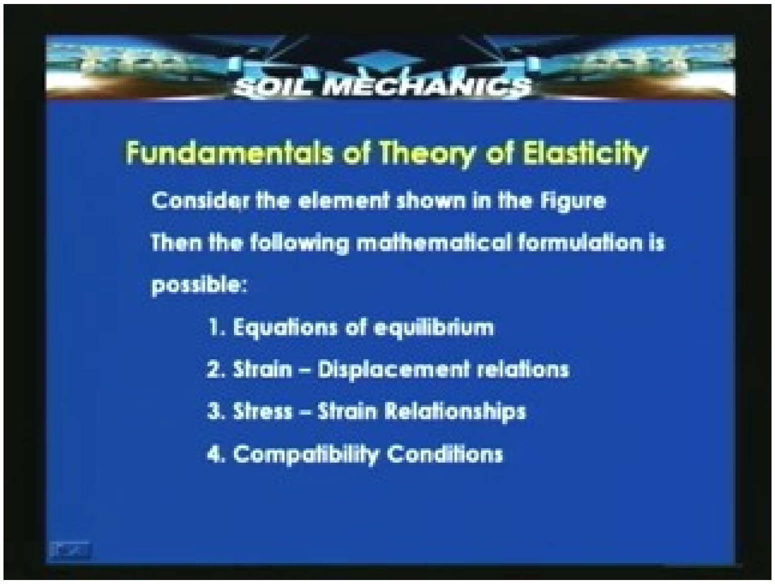


A typical element inside the soil is in equilibrium under the action of the stresses which are acting on it. This is an essential condition and justifiable condition because in practice we want safety. We want the element to be in equilibrium, therefore the foundation will be in equilibrium and hence the superstructure. Therefore it is a perfectly valid, justifiable requirement to state that equilibrium must be satisfied by the element, for the set of forces or stresses which will be acting on it. We have seen in one of the diagrams earlier that every surface of this parallelepiped is subjected to one normal stress and two shear stresses. If we calculate the forces on each one of these planes and write down the well known equations of equilibrium for sigma x is equal to zero, sigma y equal to zero and sigma z equal to zero, this is what we will get.

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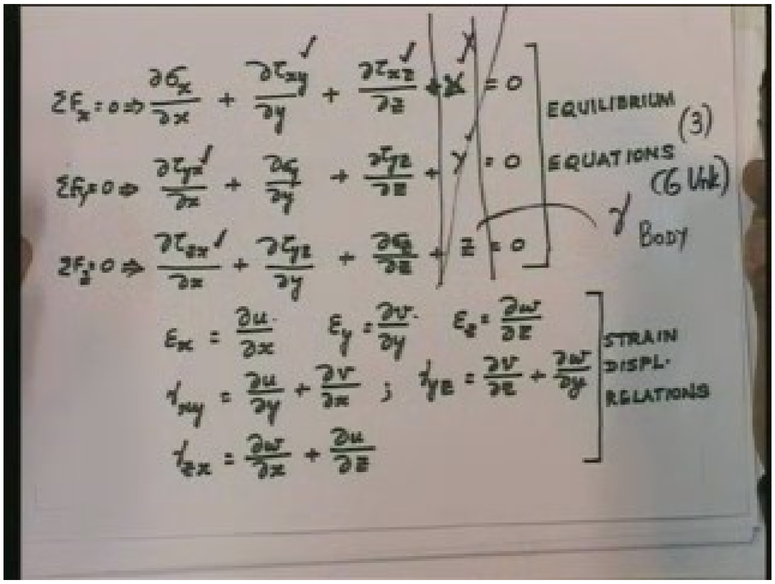


You can see here I have written three equations, one of them says sigma $F_x$ is equal to zero and you find that it has a term which is noting but the rate of change of sigma x in the x direction, then the rate of change of shear stress dow x y in the y direction, rate of change of shear stress x z in the z direction plus so called body force x in the x direction. This is a force, which is force per unit volume which I mentioned yesterday. If we write equations for $F_y$ equal to zero and sigma $F_z$ equal to zero, we will get similar equations. All these three together constitute the so called equilibrium equations of a typical three dimensional problem of stress distribution.

In this if z is the vertical direction, z in fact represents the force in the vertical direction per unit volume of the soil which means that it is nothing but the weight per unit volume of the soil. It is nothing but gamma of the soil. We have just made an assumption that gamma or the medium weight does not exit. The medium is weight less and which means therefore we are ignoring the body stresses. We will also be ignoring the stress x, we will also be ignoring the stress y that means we shall be ignoring all these body stresses. And then the equations of equilibrium will simplify to nearly rate of change of stresses sigma x, sigma y, dow x y, sigma z and so on.
You will see here that there are 3 normal stresses, 6 shear stresses but moment equilibrium equations if we write and solve we will find that dow x y is same as dow y x, dow x z is same dow z x. We will also find that dow y z is same as dow z y which means that effectively we will be having 3 normal stresses and 3 shear stresses which are unknowns. We have 3 equations but we have 6 stresses which are unknowns, so 6 unknowns 3 equations. Obviously equilibrium condition alone will not help us to calculate stress distribution because we have lesser number of equations and greater number of unknown stresses. Therefore we need to make or we need to consider some other condition as well. And that is why we went yesterday also to the second condition which is the strain displacement relations.

What are these strain displacement relations? I had explained yesterday again that if any typical surface of the parallelepiped is considered, it elongates as well as distorts. So there will be normal strains as well as shear strain. In a parallelepiped on every surface there will be two normal strains and one shear strains and therefore in all, in a three dimensional problem we will have normal strains epsilon x, normal strain epsilon y, normal strain epsilon z. And also distortion or shear strains which are nothing but variation or deviation in the angle where the original value of the angle was 90 degrees. So the change in angle of an original 90 degree angle is known as shear strain. It can be shown from geometry that the shear strain will be equal to this in the x y plane, in the y z plane this and z x plane this (Refer Slide Time: 14:37).

You will find on the right hand side of all these equations, there are notations such as u, v and w. These are nothing but displacements. So effectively now we have strains and the displacements related to each other. After all strain is nothing but displacement by original length or change in length by original length. Therefore strain displacement relationship can be expressed as, epsilon as a function of u or v or w. And that is what we have; if you see this once again we have 6 equations. And how many unknowns are there? There are of course 6 strain components but the displacements are also unknown and therefore we have 9 unknowns. Though we have 3+6, 9 equations, we have 6 + 9, 15 unknowns. Obviously we are therefore not yet ready to solve this problem completely to obtain the stresses. Not only that, in this second set of equations which we have written down there are no stresses that means there are additional unknowns coming into the picture in the form of strains and displacements.

Actually therefore these 15 are the unknowns of this problem, is not only the stresses but the strains and the displacements are also unknown. Primarily of course in this chapter which we are discussing we are interested in the stresses and therefore our basic unknowns are the stresses. However we need to solve all these equations in order to arrive at finally the stress values. There are obviously inadequate numbers of equations compared to the number of unknowns. Therefore we need to look for some additional conditions but without increasing the number of unknowns. And that is what really theory of elasticity does for us. Let us see the next transparency. Take a look at this, here I have written down these stress strain relationships. Actually in the previous two sets of equations really there was no theory of elasticity coming into picture.

It was all equations pertaining to the equilibrium or the deformation of a typical element in the medium, in a continuum. Theory of elasticity really enters the picture now. If you see, the first equation is nothing but strain on the left hand side, modulus of elasticity and all the stresses on the right hand side, Poisson’s ratio is also included. This equation is extremely simple to understand and derive. Any strain in any direction, we know according to theory of elasticity must be equal to the stress divided by the modulus of elasticity. I am simply assuming that stress is linearly propositional to strain or in other word sigma is equal to some constant which I am calling as the modulus of elasticity into the strain. If you see this epsilon x is the strain in the x direction, it must be therefore obviously getting a component from the normal stress sigma x which is also in the same direction as epsilon x.

So the component contributing to normal strain in the epsilon x in the x direction will be sigma x upon E, but when we are dealing with an element; If sigma x is contributing to a stress or a strain epsilon x due to Poisson effect, due to the Poisson's ratio of the material, this material if it undergoes compression it will undergo in the vertical direction. It will undergo elongation in the lateral or horizontal direction which means that this is also going to affect the strain in the horizontal direction. If this stress sigma x is affecting the strain in the x direction, due to Poisson effect it will also affect these strain in the y direction. Now conversely the stress in the y direction will affect the strain in the x direction through the Poisson's ratio. And that is what we have written here. We have seen that if there is a compression in the vertical direction here, there is an elongation in the horizontal direction. Similarly therefore here if sigma x upon E is a positive contribution to epsilon x, the contribution from sigma y through Poisson's ratio, the contribution from sigma z to epsilon x will both be negative.

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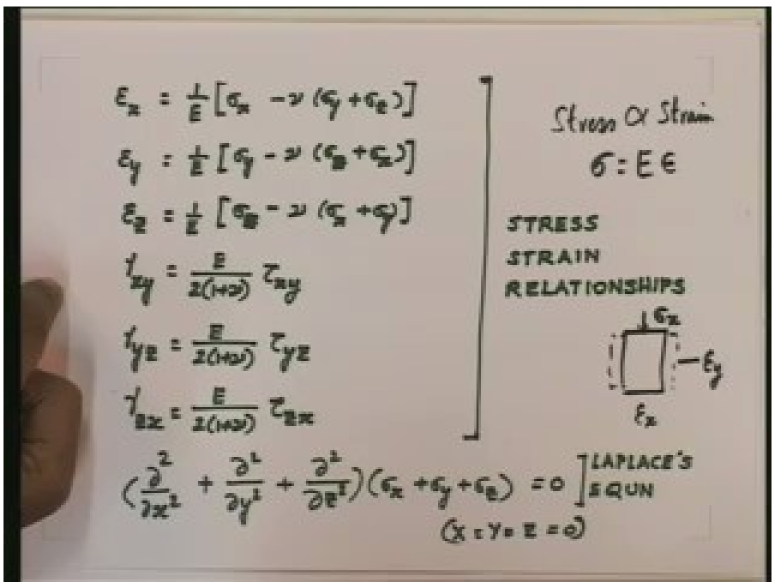


And so the stress strain equation or rather the strain stress equation or relationship for epsilon x in terms of the three unknown stresses will be like this. We have to appreciate the fact that we have succeeded in creating an equation based on theory of elasticity in which we have not introduced any new unknown.

The unknowns are sigma x, sigma y, sigma z and epsilon x which were also there in the previous equations. The constant epsilon and new are material constant and they are supposed to be known and they are constant. We can determine this epsilon and new in the laboratory for the material through certain test and therefore epsilon and new are not unknowns. If we now go to the strain in the y direction, an identical equation can be written in terms of sigma y, sigma z, sigma x also strain in the z direction. Similarly by similar argument and assumption of linear elasticity we can say that the shear strain in the plane x y will be the shear modulus G into dow x y where dow x y is the shear stress. This G also can be expressed in terms of the well known modulus of elasticity in the

Poisson’s ratio as E upon two into one plus new. This can be continued further and we can write similar equations for gamma yz and gamma zx. These constitute the stress strain relationship.

Now we are in a very comfortable position to solve a problem of stress distribution because we have not introduced any additional unknowns here and therefore our number of unknowns remains same as before that is 15. But on the other hand whereas we had 9 equations earlier, we now have 6 additional equations which mean we now have 15 equations and 15 unknowns. These can therefore, theoretically speaking we solved in order to get the stresses. Once we get the stresses, we have after all the stress strain relationship right here in front of us. If we know the right hand side, we can always calculate the left hand side and if we know these strains we can always calculate the displacements. And therefore we can say that this approach can be used to completely solve the problem of not only the stress distribution but also the strains and the displacements which are also unknowns in a typical problem. This is a set of equations which theory of elasticity attempts to solve.

If you try to solve the 15 equations step by step, you can finally arrive at a condition which is known as the Laplace’s equation. I shall not be going into the details of this, you can always obtain the details of this solution process from any book on theory of elasticity. This is an equation which is known as Laplace’s equation, the left hand side is nothing but consisting of second derivatives of the stresses and the right hand side is zero. We have made an assumption, we should not forget that the body stresses x, y and z are zero here. If the body stresses are zero then we have a Laplace’s equation. It may be worthwhile mentioning here, although I shall not go into details that if the body stresses are not assumed to be zero, we can still solve the problem. Then on the right hand side we will have a non zero term and the equation will be then known as Poisson’s equations.

However we are interested now in the problem of stress distribution using the method of Boussinesq and we shall confine ourselves therefore to that main assumption which he has made and the corresponding equation which we have, that the main assumption if you remember is the medium is weightless and therefore the equation that we shall apply now in our conditions is the laplace’s equation. How do we solve this Laplace’s equation to get the unknown stresses? As I said once we solve this and get the unknown stresses, we can always go to the stress relationship, get the strains and then the displacements. So let us see how this is solved. Once again I may not be in a position or it may not be required to go into the elaborate details of solving the Laplace’s equation.

I will just mention that the Laplace’s equation is solved by defining a so called stress function. This function phi here is known as Airy’s stress function. This Airy’s stress function is a function which tells you what the stresses are in a medium and how they are distributed in terms of an equation like this. This equation has been derived by Boussinesq for those 15 equation’s which we had considered earlier. And the special property of this function phi is del to the power of 4 into phi =0. If you remember, let us take a look once again at the Laplace’s equation. This Laplace’s equation can also be written as del square (sigma x plus sigma y plus sigma z) =0 where this del square stands for a sum of the partial derivatives, del square by del x square, del square by del y square and del square by del z square. But this is of the second order. On the other hand this stress function phi satisfies the condition, del to the power 4 into phi, this is del to the power 4 into phi that means the fourth order derivatives come into picture. Fourth order derivatives of this function phi equal to zero.

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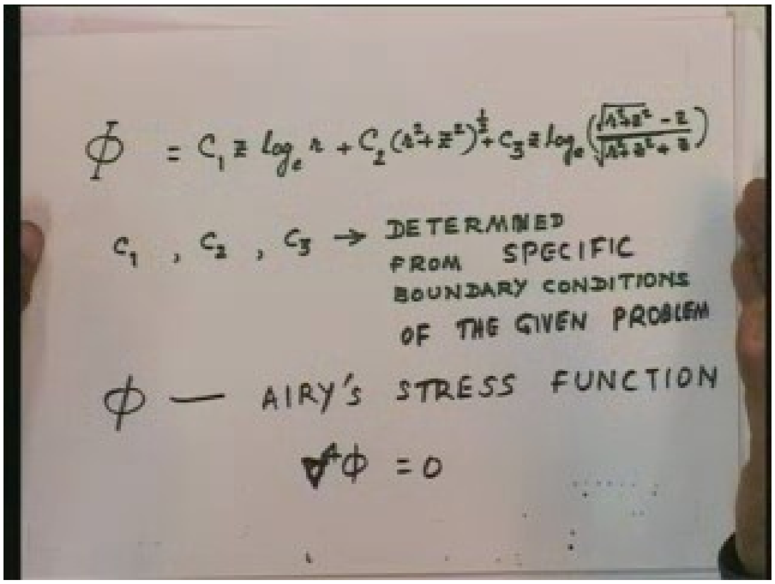


This has been shown in the theory of elasticity that this will satisfy that Laplace's equation. This will serve as a function, that is a function of position, a function of the coordinates r, z of this form which will satisfy the condition del to the power 4 into phi equal to zero, will also satisfies the Laplace's condition which we saw earlier, which means that this is a valid solution and this will give us the stresses. From here the stresses have been obtained by Boussinesq. If you see here there are constants $c_1$, $c_2$, and c3. The solution which we have derived or evolved so far has not considered any specific problem. Although I have been saying that we are interested in this stress distribution problem, up till now we have not specified what the stress distribution problem which we are considering is. We have not stated whether it is a problem of a wheel load on an embankment or a dam sitting on a rock foundation. We have not stipulated whether it is a foundation problem or a tunnel problem. This means that up till now whatever solution we have evolved is a very general solution and therefore it consist of certain constants and these constants will vary from problem to problem.

And thus this same solution will apply to different problems, however in each problem there will be a different value of the constants $c_1$, $c_2$ and $c_3$ and these $c_1$, $c_2$ and $c_3$ are all determined from the specific boundary conditions of that particular problem. They will all be determined from the specific boundary conditions of the given problem. I am not again, going into the details of the all the boundary conditions. But just to illustrate one boundary condition which we can easily appreciate is on the surface of the medium, there is no vertical stress. That means at z equal to zero where z is the depth and at z is equal to zero on the surface there are no stresses, sigma z is zero. So that is a boundary condition. That is an example of a boundary condition, there will be other boundary conditions as well. And all these are taken into account and Boussinesq has arrived at a solution for the stress distribution problem in general of this shape.

Lastly if you see this next slide, I have mentioned something known as compatibility condition. Let us take a look at what these compatibility conditions are. I have just now stated that this is a general solution valid for all problems of stress distribution irrespective of whether it is a tunnel or retaining wall or a dam. Then one condition which must be inherently satisfied is that when under these stresses, strains and deformations or displacements takes place, our basic assumption of continuum should not be violated. What it means is, suppose I take this surface of the soil. Suppose I consider this element actually it is a three dimensional element like this. I consider also the adjacent element, actually these are so called contiguous elements meaning they are adjacent, touching each other. However for convenience in imagination, I have drawn them as to separate elements.

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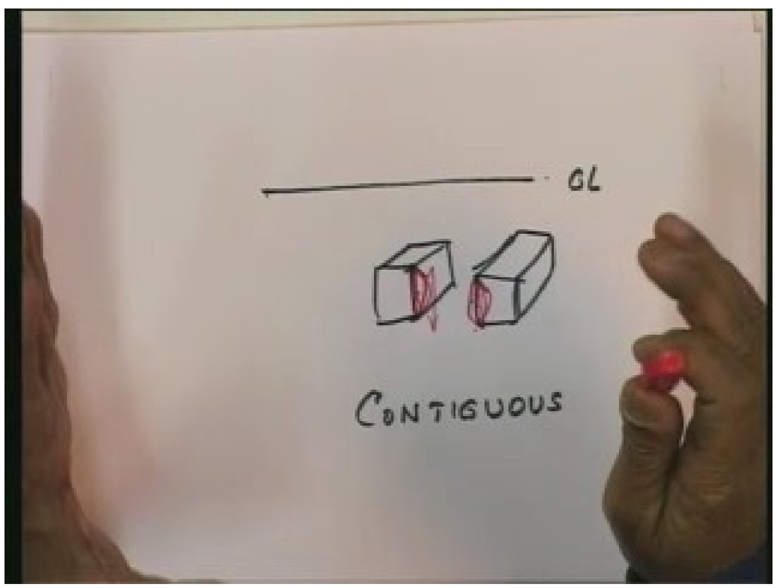


You see when these two elements are adjacent elements, when each element undergoes some strain or deformation. If this material has to remain as a continuum then there must be some relationship that must exit between the strains on the deformations of the two elements. Otherwise what will happen is for example, suppose this undergoes a strain and takes a shape like this. And if this takes a shape like this (Refer Slide Time: 30:35) obviously there is a loss of contact between the two elements.

That means these two strains are not compatible. You can extend this concept at three levels that is along this surface and along this surface. At this corner and at this corner and at all points the displacements must be similar or equal, so that the two surfaces remain in contact. Then from this point to this point, if there is a variation in displacement here that means if there is a strain then the strains must be same in both. On the other hand it is not enough if only strains are same, if this now undergoes a deformed shape like this, then the slope of the deformed shape at every point must be also equal to the slope of the deformed shape of the other corresponding surface. And also the curvature of this surface must be same as the curvature of this surface. All these are stated mathematically and a set of conditions known as the compatibility conditions have been derived in the theory of elasticity. I am not going now into the details of all those equations and how they have been derived, but suffice it to understand that the complete compatibility between adjacent elements in terms of strains and displacements and curvatures has to be maintained and that is possible provided we satisfy the compatibility conditions.

So a typical theory of elasticity solution will consider equilibrium equations, will consider strain displacement relationship, will consider compatibility conditions and then solve and get the value of stresses through the Airy's stress function. So if you look at the slide you find that our basic attempt is to derive the Laplace's equation and then to use the stress function approach to get the stresses. Though this is precisely what has been done by Boussinesq.

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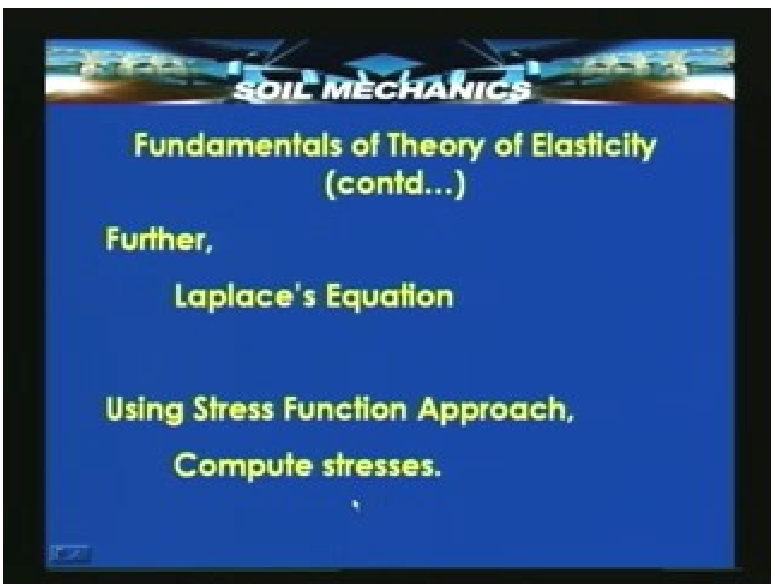


You take a look at all these stresses, first one is a stress sigma x, next one is the stress sigma y, third one is the stress sigma z. What are these stresses? Let us just go back for a moment. You see here this is the element, sigma x, sigma y, sigma z are the stresses which are acting in the respective directions on this elements. So these are the stresses we are talking about. You see here, the stresses are sigma x, sigma y and sigma z. And what is of greatest interest to us is the vertical stress sigma z.

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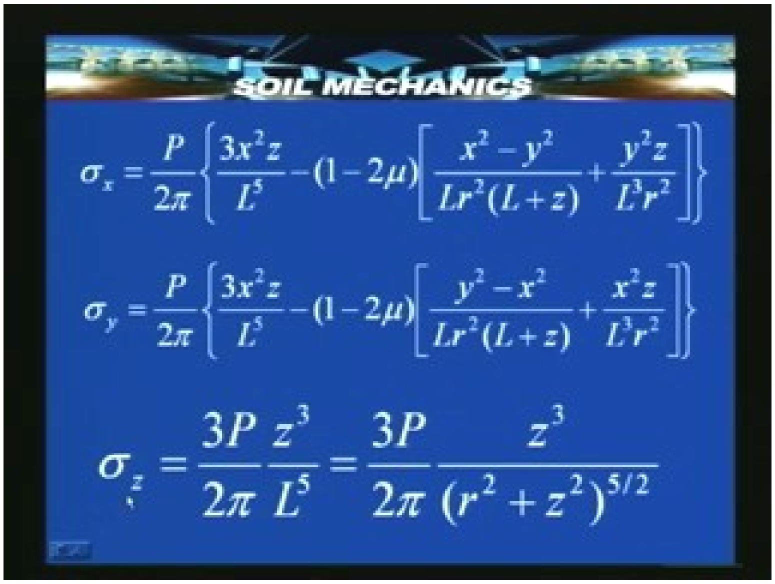


Luckily for us the expression for sigma z is a very simple and compact expression. It is 3 P z cube upon 2 phi L to the power of 5 or 3 P by 2 phi into z cube by r square plus z square to the power of 5/2. That is 3 P by 2 phi into z cube by r plus z square to the power 5/2 where r is noting but square root of x square plus y square and L is nothing but square root of x square plus y square plus z square or square root of r square plus z square. What do these mean? Here you can see this is x, this is y, this is z and r is nothing but this step (Refer Slide Time: 34:06). This figure shows the cylindrical coordinates system and in the case of cylindrical coordinate system rather than having stresses sigma x, sigma y, sigma z, we will be having stresses sigma r, sigma theta and dow r z. These can also be computed by the same business approach and the corresponding expressions that we shall be getting if we solve, will be like this.

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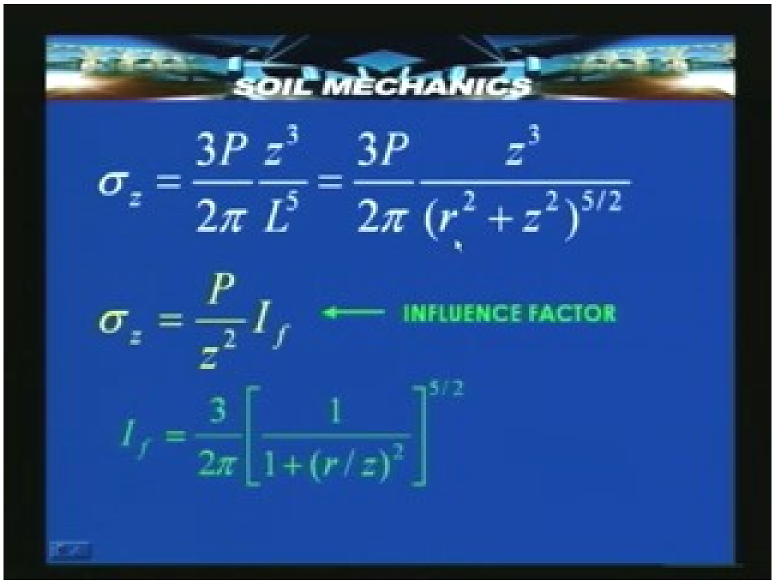


Sigma r will be given by an expression like this, sigma theta by an expression like this and the shear stress dow r z by an expression like this. The parameter mew that comes here is nothing but the Poisson's ratio. Let us see the next slide. This slide is of importance, this talk about the vertical stress sigma z and if you look at the value or the expression for sigma z, you find that capital P is the load applied. This is a constant that is 3/2 is a constant, r and z are the coordinates of any point at which we want the stress or at which sigma z is equal to this. Therefore we can express this sigma z in terms of some constant quantities and in terms of some quantities like r and z which vary in the medium depending upon the point of interest. So if you rewrite this equation in the form sigma z equal to P upon z square into a parameter called $I_f$ or influence factor. That influence factor, if you compare these two will turn out to be equal to $I_f$ equal to three upon two phi into one upon r by z whole square raised to the power of 5/2. This influence factor is nothing but a factor which includes the constant term 3 and 2 phi and the ratio of the coordinates r and z. And if we can evaluate this $I_f$ or the influence factor for different ratios of r and z, then we have a way of generalizing the stress computation problem.

The vertical stress sigma z and the expression can be generalized to a great extend, if we write it in the form P by z square into the influence factor. Because the influence factor can be computed irrespective of this problem and kept ready for different values of r by z. And that is what you will be seeing in the next slide. You see here a table of influence values for vertical stresses due to a point load P on the surface of the medium. A concentrated load P will cause different vertical stresses at different r by z ratios and the corresponding to each r by z ratio, we will have an influence factor $I_f$ which was given by the expression which we saw in the previous slide.

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SOIL MECHANICS

INFLUENCE FACTOR VALUES

FOR VERTICAL STRESSES DUE TO POINT LOAD

| r/z | $I_y$ | r/z | $I_y$ | r/z | $I_y$ |
|---|---|---|---|---|---|
| 0.00 | 0.478 | 1.00 | 0.0844 | 2.00 | 0.0085 |
| 0.10 | 0.466 | 1.10 | 0.0658 | 2.10 | 0.0070 |
| 0.20 | 0.433 | 1.20 | 0.0513 | 2.20 | 0.0058 |
| 0.30 | 0.385 | 1.30 | 0.0402 | 2.30 | 0.0048 |
| 0.40 | 0.329 | 1.40 | 0.0317 | 2.40 | 0.0040 |
| 0.50 | 0.273 | 1.50 | 0.0251 | 2.50 | 0.0034 |
| 0.60 | 0.221 | 1.60 | 0.0200 | 2.60 | 0.0029 |
| 0.70 | 0.176 | 1.70 | 0.0160 | 2.70 | 0.0024 |
| 0.80 | 0.139 | 1.80 | 0.0129 | 2.80 | 0.0021 |
| 0.90 | 0.108 | 1.90 | 0.0105 | 2.90 | 0.0018 |

And you find here r by z varies from 0 to 0.9, 1 to 1.9, 2 to 2.9, we can go on adding but very often in practice we confine ourselves or restrict ourselves to this range of r by z values. If required we can always go back to the general expression for sigma z and substitute the appropriate value of r by z and go ahead with the computation of sigma z, however if we want to simplify and make use of this table.

This table can be used for all values of r by z which range from 0 to 2.9. You see the values of the influence coefficients at the top, near the surface where r by z is 0, we have maximum influence coefficient 0.478. That means maximum stress occurs directly below the load and as we go away, either in the radial direction or with respect to depth z, we find that the influence factor gradually goes on decreasing to a lower value as 0.0018 at an r by z of 2.90 which means if we go either radially or downwards from the load then the influence factor goes on decreasing and therefore the stress goes on decreasing. In addition if you go back to that equation, you find that it is not only the influence factor which goes on decreasing with r by z and therefore brings down sigma z. Sigma z is a fraction involving P and z square which means that it is inversely proportional to z square, which means as depth increases sigma z goes on decreasing as a function of the inverse of this square of distance. That means it decays very rapidly, although we go down by a distance z, depth wise, the stress decreases by a magnitude of z square. That means there is an inverse square formula in operation and the sigma z therefore both laterally and with depth will go on decreasing. But you will also find that up to certain distance initially it will increase and then it will decrease, which we shall see now.

This is a table of the r by z values and influence factor. This table conveys a lot of important information. For example if we put r equal to constant, how does the stress vary or if you put z equal to constant in this, how does the stress vary or which are the points irrespective of different r by z which will experience the same stress which will have the same influence factor or same stress at least? This we can analyze as follows. If you take a close look at the table of vertical stresses as I mentioned, we can analyze the effect of the variation of z, the variation of r and how the stress itself varies. So coming back to this slide, a close look at the vertical stress expression will tell us, at constant depth z what happens to the stress values, at constant radial distance r what happens to the stress values and what constant stress values will occur where? Take this table, I have given here once again the influence factor and the expression for sigma z in terms of the influence factor and now I am going to consider, at r equal to zero the effect of z.

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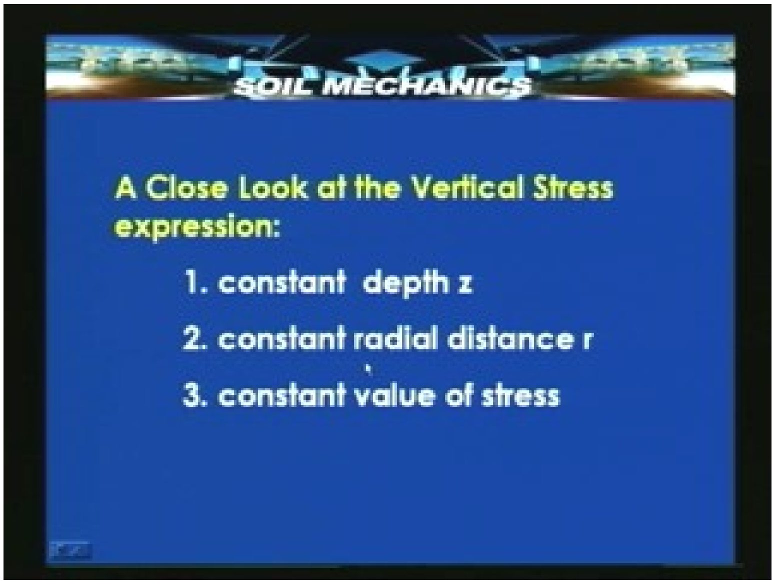


I am taking a constant value of z and at a constant value of z as r increases, how the influence factor varies and how the stress varies? Suppose we start with r equal to zero, at r equal to zero the influence factor can be computed from here to be equal to 0.4775. And as I mentioned while showing the previous slide at r = 0, the maximum stress occurs. So here this shows that r = 0, the influence factor is 0.4775 and the maximum stress that occurs at r = 0 is P upon z square into 0.4775.

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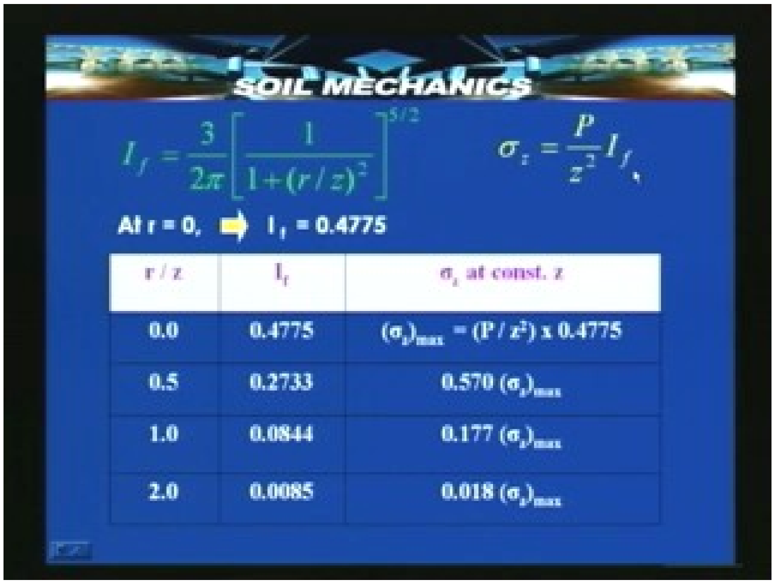


Whereas at the same depth z, as the value of r changes from zero, as it goes to a higher and higher value. We find that at constant z and varying r, at r by z = 0.5, influence factor reduces to almost half this value and the stress reduces to almost little more than half of the maximum. And when r/z = 1 that is r and z are equal, the influence factor sharply drops down to a very low value of 0.0844 and the stress also reduces to a very low value of 0.177 of the maximum value which is occurring at the surface.

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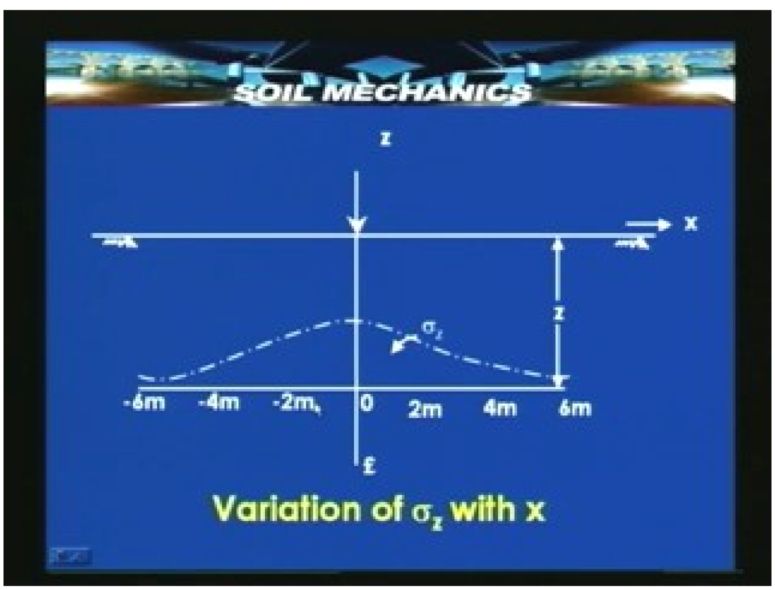


At z equal to two or rather at r/z =2, the influence factor further drops down and the stress becomes 0.18 times the maximum stress here. From this table whatever we are able to figure out, can be understood better from a graph. Take this graph, this load is acting at the center here, these are the x and z coordinate. I am taking a constant z and different values of r. At different values of r, I have just shown you that the influence factor goes on decreasing as you go away from this point and therefore the stress goes on decaying and this is what is a typical stress distribution at a constant value of z. This is important or very useful. It shows that the effect of the load fortunately for us does not go on extending to very long distances, it starts decaying after a reasonable distance and becomes very close to a value of almost zero at a reasonably short distance. What this means is if two foundations are kept side by side at a reasonable distance then the influence of one foundation may not affect the stresses due to the other foundation. This is an important idea. This shows that if two foundations can be spaced in such a way that one does not affect the stresses due to the other, then the soil will not be subjected to unduly excessive stresses. Just imagine I have one more foundation and one more concentrated load just very close to this. Then that will also have a distribution like this, which means that there is an overlap between the two distributions due to the two concentrated loads which means that there is a zone which will experience stresses due to both the loads. And therefore the total stress that zone will experience will be almost double the value of the stress due to one of the foundations. In practice sometimes it is unavoidable to keep two foundations close to each other and therefore in such instances we have got to take the relative influence of each foundation on the other.

Let us see the next slide. We shall try to understand what happens to the influence factor and the stress at constant r but varying z. As before this is the expression for the vertical stress where this is the influence factor. This 3/2 phi into this (Refer Slide Time: 45:24) represents the influence factor. If r is constant here, sigma z will vary with z. How does it vary?

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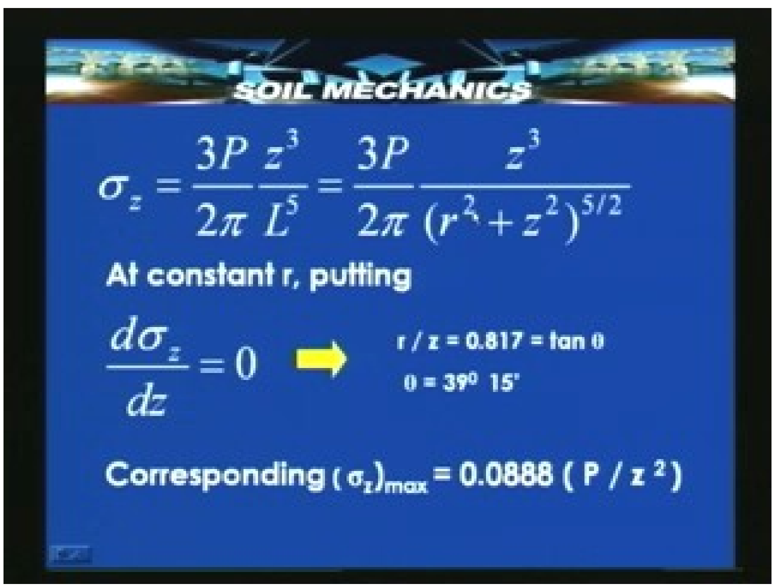

We can see that as it varies, somewhere it has got to be maximum or minimum. In order to find out where sigma z reaches its maximum value with respect to z, we can differentiate this expression with respect to z. The d sigma z by d z can be put equal to zero and if you differentiate this with respect to z and put equal to zero and solve for r upon z, you will find that r upon z is nothing but 0.817. That means sigma z becomes maximum as we go down at an r by z value which is given by 0.817 or by a certain line which depicts r by z which will make an angle theta equal to 39 degrees 15 minutes. At this value of r by z, the corresponding maximum value of stress will be sigma z max equal to 0.0888(P/ z square). That means this is the influence factor, which means that the vertical stress sigma z reaches a maximum value at some depth. This implies that it is not the maximum value at other depths and this is interesting as once I have pointed out in one of my earlier lectures.

The vertical stress first goes on increasing with depth, it reaches a maximum value and then again drops down, luckily for us and therefore at very great depths the stresses will be decaying and they will have a less than maximum value. Let us see the next slide. This slide show how the vertical stress varies with depth at constant r. So let us say these are the two axes, take a distance r this is z, this is r. If I draw a line of r by z, this is what that line we look like.

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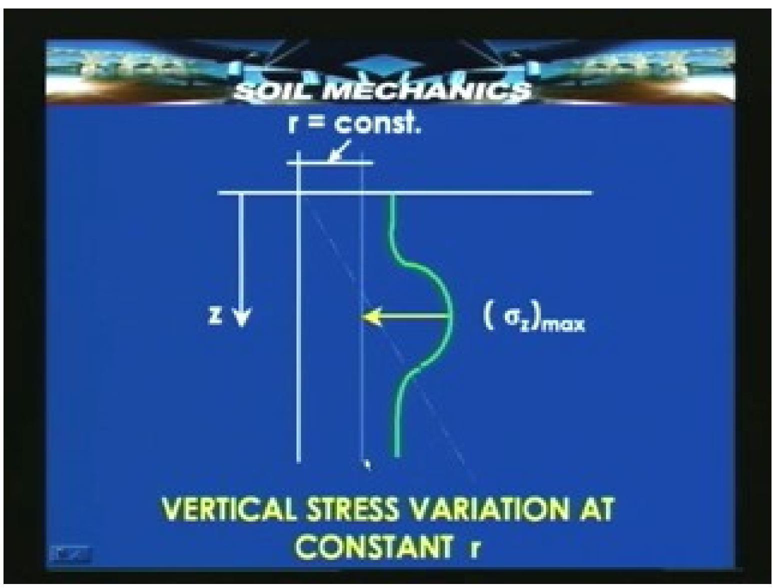


If at a constant r given by this line, I try to plot the value of sigma z computed by the previous expression then I will find that sigma z is some value at z equal to zero and will gradually go on increasing, will reach a maximum value at some value of z and then again it will decrease and will become almost steady as we reach a very high value of the depth. And this line which gives effectively the ratio r upon z will have an inclination theta given by the expression in the previous slide that is 39 degrees 15 minutes. So in effect you will find that the maximum value of stress will occur at a point here which when joined to the origin will make an angle of approximately 39 or 40 degrees with respect to the vertical. Lastly let us take a look at how the stress varies in the medium which are the points of equal stress. The same equation for sigma z can be again used and if we try to calculate at different values of r by z, the value of z and find out all those r by z values corresponding to h, we get the same sigma z and join all of them, we will get what are known as isobar or pressure bulbs.

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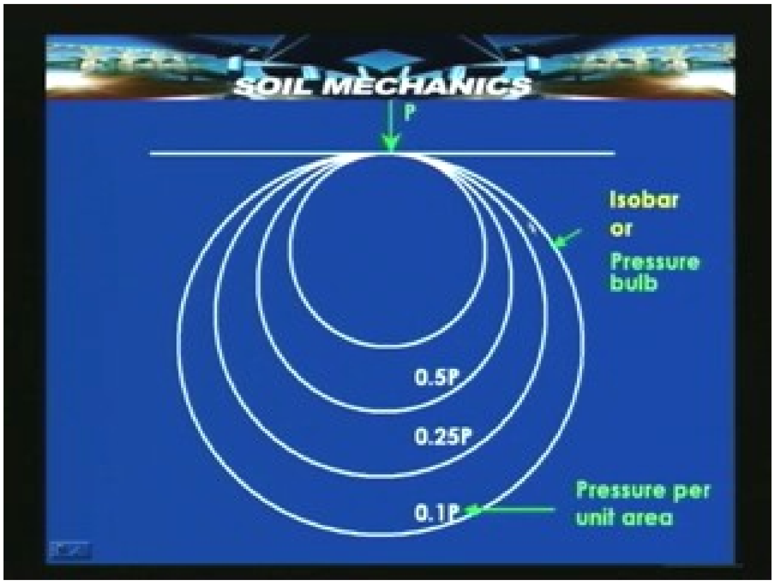


Because this represents pressure, this is in the form of a bulb we call it a pressure bulb and since it represents lousy of all points which have the same pressure. Rather since this bulb represents a line of equal pressure this is also known as isobar. This figure actually represents that if I apply a load P, as I go down 0.5 times the magnitude of P will be the pressure at some depth. As I go further down it could reduce to 0.25 times the magnitude of P and then at some depth it can become 0.1 times the magnitude of P or I can get a pressure per unit area here or the stress per unit area here which will be roughly 10 % of the magnitude of the applied load.

What this signifies is that beyond this, the stress that is imposed by P is not much. It is almost in significant because if we take 10 % to be significant anything less than 10 % to be not so important or not very significant which is a very valid assumption in practice. Then any pressure bulb which is beyond this which corresponds to a stress level of less than 0.1 times P may be unimportant. From the point of view of influence of this P, may be very well assumed to be restricted to a pressure bulb defined by 0.1 P.

Let us take a look at the next slide. This is the statement of a small example problem. Determine the vertical stress on a horizontal plane at a depth of 12 meters corresponding to a concentrate a load of 800 kilo Newtons applied on a semi infinite media.

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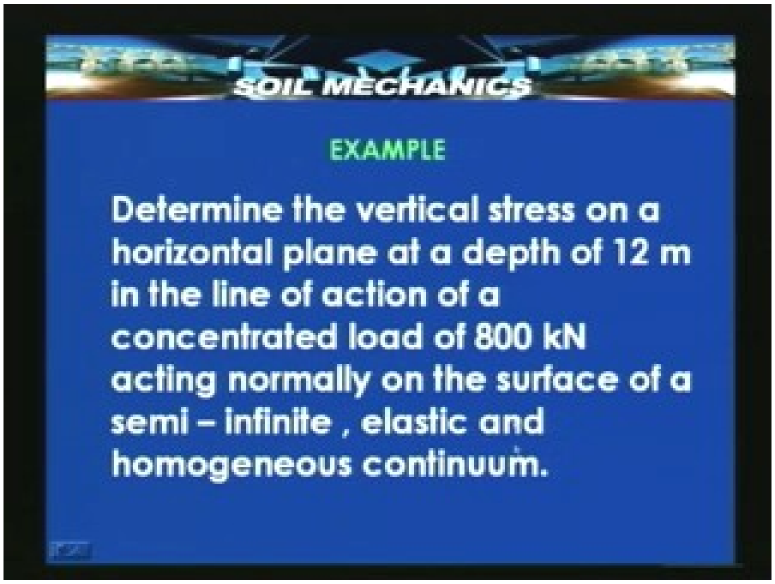


Applying our expression for sigma z, which is P by z square into the influence coefficient. At r/z =0 we find that influence coefficient is 0.47745 which means that the stress sigma z is nothing but 2.653 kilo Newton per meter square. You see how simple the problem of stress computation has become. This has helped us to find out the stress at r/z =0, that is at r =0 and z =0, at that top surface. This is the maximum value of the stress due to the applied load in this particular problem.

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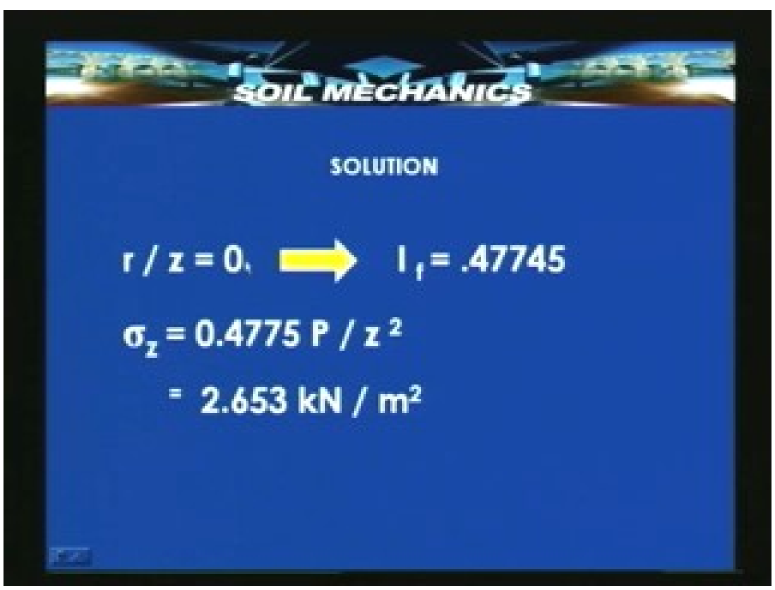

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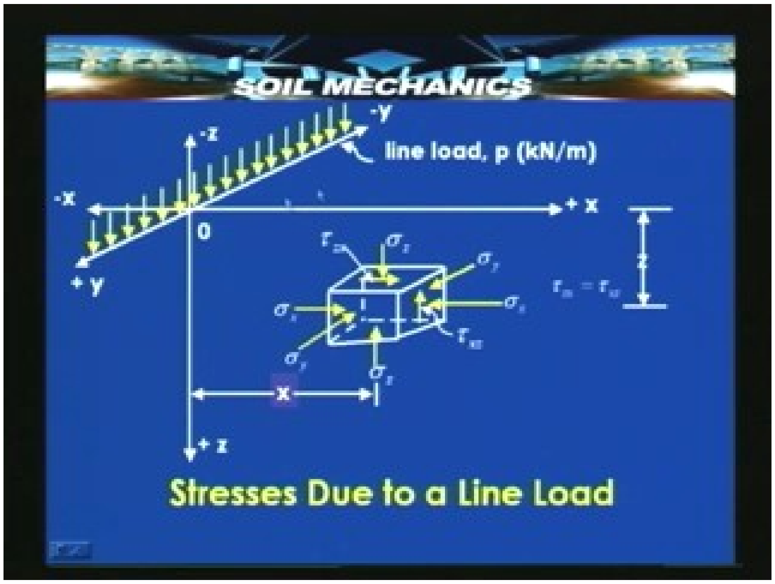


And by taking different values of r by z and different values of corresponding $I_f$ we can go on computing stress at different locations. This can be extended also to line loads. Line loads are nothing but uniformly distributed loads. Suppose we consider a line load of P kilo Newton per meter, we can compute these stresses due to this line load by the same expression. Only thing is that expression will now be integrated for all the points along the line load. And if we do that the Boussinesq expression for sigma z for a line load will turn out to be 2 P by phi into z cube by x square plus z square whole square maneuverability.

So now if there is a line load of 400 kilo Newton per meter, at x = 5 meters and z = 5 meters we will get a value of sigma z from these expression equal to 12.73 kilo Newton per meter square. See now how simple the computation of stresses has become, although we used the theory of elasticity which involved solving a number of equations. We find that luckily for us all those equations can be solved in a very general manner and we can arrive at simple expressions. So we need not solve those equations again and again, luckily for us those expressions and the corresponding influence coefficients are more than sufficient to compute the stress. And by taking different values of x and z or r and z we can go on calculating the stress distribution in the entire medium.

Coming to the end of this lecture, I can say that we have seen the basics of theory of elasticity, the Boussinesq problem and how it is used for computing stress distribution both due to a point load and due to a line load. We have also seen two simple examples of application of these expressions. Tomorrow or in the next lecture we shall be seeing details of how to compute stress distribution below a rectangular area, below a circular loaded area and below an area of any arbitrary shape.
Thank you.