Soil Mechanics
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Lecture – 37
Consolidation and Settlement
Lec No. 4

Dear students today will be the fourth lecture in the series on consolidation and settlement. We will recall that in the previous 3 lectures, we have devoted considerable amount of time and discussion to the physical aspects of the phenomenon of consolidation. To put it in a nut shell I had described the phenomenon of consolidation and the resulting settlement as a phenomenon which is predominantly observed in clays and that to saturated clays it is primarily a process in which when a load is applied to a saturated clay layer in the form of say a building or some construction activity then since clays are very poorly pervious, the water requires considerable amount of time to move out of the pores of the soil, this is known as hydro dynamic lag. They take a long time in view of the low permeability of the clay layer that the resulting readjustment of the solid particles and the reorientation and the resulting compression always lag slightly behind the process of hydro dynamics. So this entire phenomenon is known as hydro dynamic lag.

And therefore this compression phenomenon under a load is basically a product of or a result of the gradual movement of water out of the pores of the soil due to a gradual application of a load. The word gradual is very important both from point of view of application of load and from point of view of expulsion of water because we will be probably seeing in some other lecture, in other topic the impact of sudden loading. When there is a sudden load, the phenomenon is not the same as what we are now reviewing under the topic of consolidation. So consolidation and resulting settlement is primarily observed in clays, saturated clays in particular. It is particularly important mainly for saturated clays and it's a result of gradual expulsion of water when a load is applied.

Now this aspect needs to be put in mathematical terms so that we can evolve a theory. This also we had briefly seen, an introduction to this we had discussed in the past few lectures, we referred to it by the name one dimensional consolidation. I had described a theory which we shall be discussing in more detail today and later as the one dimensional consolidation theory. This phenomenon of one dimensional consolidation considerably simplifies the process of consolidation and makes it much easier for us to understand and describe it mathematically and also to make predictions about settlements with a reasonable degree of accuracy from practical point of view. Imagine there is a soil; imagine we construct a foundation and then a super structure on it. The process of construction definitely takes some amount of time. It's not an instantaneous process, this gradual process transfers load to the soil below very gradually and correspondingly in a very slow manner water is expelled out of the soil. And therefore the resulting compression also follows suite very slowly and gradually and this in fact is the cause of concern for us. Although gradual, it is actually a matter of concern. The concern is this compression could take place gradually over a long period of time during the life time of the structure which means that at some point of time during its life this compression may exceed certain acceptable or permissible values which means that up to some point of time the structure may be apparently behaving very satisfactorily performing very satisfactorily but at some point of time, it will start showing distress because the settlement would have probably exceeded some acceptable or permissible value for that combination of the structure and the soil. And this is the cause of concern and this requires that we be able to compute the total settlement that the building is likely to undergo and not only that we also need to know at what rate its going to takes place.

All this we had discussed and we also in the previous lectures how the total settlement can be computed. It so happens that irrespective of how much time it takes the total settlement that a building will undergo depends on the void ratio change, on the pressure total pressure that is applied and the pressure void ratio relationship. It actually does not really depend upon the magnitude of the rate at which the water is expelled. It simply depends upon the total load applied and the total resulting void ratio change or volumetric change in the soil. Therefore the total settlement we could compute very easily, we saw in fact two examples of computation of total settlement in the previous lectures in very great detail. These examples had shown very clearly that computation of total settlement really does not involve any complications and once we know the initial and the final void ratios of the soil corresponding to the initial state of stress and the final state of loading and corresponding stress, we can compute the total settlement.

We saw two examples of this and we concluded by saying that the relationship between e and p can be taken as logarithmic and a very convenient linear relationship can therefore be formulated between e and p and that can used in the computation of the total settlement. Now today we shall proceed further to see the next aspect that is the time rate of consolidation. Fortunately or unfortunately though it is possible to compute the time rate of settlement very well, it is some what involved and it requires a detailed mathematical treatment of the subject. We shall proceed step by step. To begin with we will see some essentials or some fundamentals are prelude to the computation or evolution of a computation procedure for time rate of settlement.

So let us see the next slide to some up what we had seen in the past. A summary of what we had seen in the past is what is shown in this slide, as you can see we discussed in detail first of all the oedometer test. Because it’s the oedometer test which gives us the pressure versus void ratio or void ratio versus pressure relationship and it is this void ratio versus pressure relationship which we get from the oedometer test that we idealize into a straight line in the semi log plot. Once we idealize this into a semi log plot into a straight line, we find that a convenient simple parameter called the compression index can be thought of to relate e with log p and this serves as the basis for computation of the total settlement.

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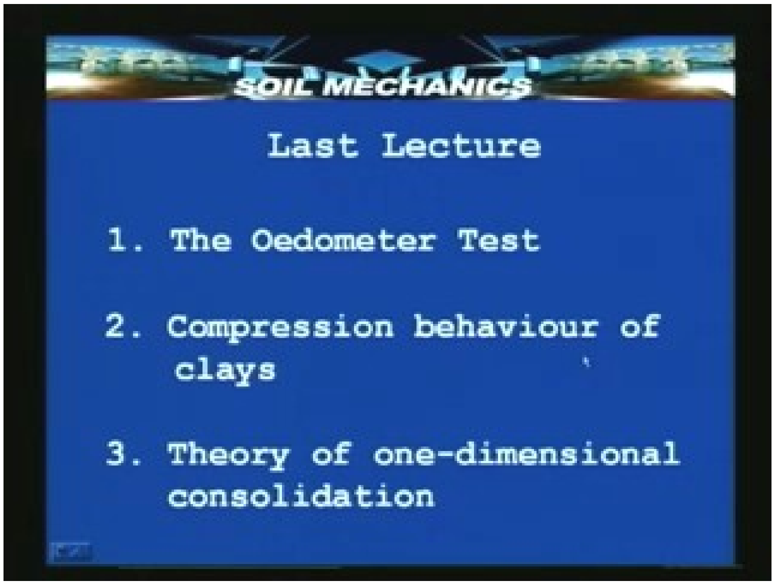

Then we saw compression behavior of the clays in general, in order to lead to the importance of the time rate of settlement. We saw for example that compared to sands in clays under the application of load there is a long duration over which the compression takes place. In case of sand much of the compression takes place, almost instantaneously or during the erection of the structure itself and therefore there are no time dependant compressions which are manifested during the life time of the structures but not so in clays. In clays we always expect, we also we always find time dependant long duration consolidation and settlement. And this was illustrated very nicely by typical oedometer test results, particularly the e log p relationship.

Then we started discussing the theory of one dimensional consolidation. First of all we said that one dimensional consolidation is an assumption said that it is valid under certain specific circumstances. The major circumstances under which this is valid or one a very large loaded area, a deep seated clay layer these are sufficient to justify that the loading is uniform over a small range of the clay layer at the centre of the clay layer and we are justified in assuming that the entire flow of water, expulsion of water is one dimension and the corresponding is also in one dimension.

Let us see what to discuss further in today's lecture. In today's lecture we will continue with this discussion on the one dimensional consolidation and go towards evolution of a mathematical description or a computation procedure. First let us take an overview of this before we actually take up discussion of the mathematics involved in the problem. In the overview we shall be discussing certain parameters that we will eventually be using to describe the void ratio time pressure relationship. We will also be seeing an analogy which is similar to the consolidation phenomenon but is much easier to comprehend and which helps us to understand the consolidation phenomenon much more easily than any other way.
So let us a take look what are the parameters that are available to us for describing

compressibility. We have already defined in fact one particular parameter and that is the compression index and we saw that the compression index can be determined from a typical e log p curve provided the curve is drawn as a semi log plot.

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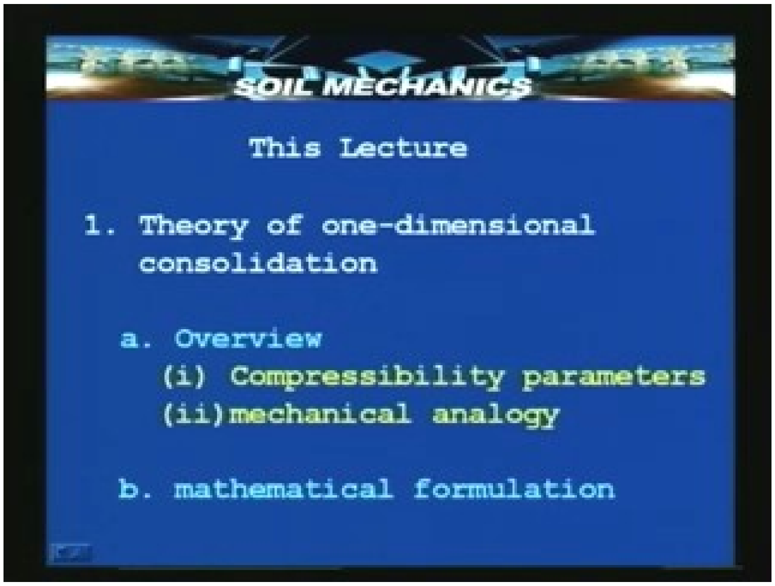

And from an e log p curve we can determine C_c the compression index as the slope of the virgin compression curve. And the expression for the compression index is negative, it is being as negative slope it will be minus of change in void ratio by log of the ratio of the final to the initial effective stress.

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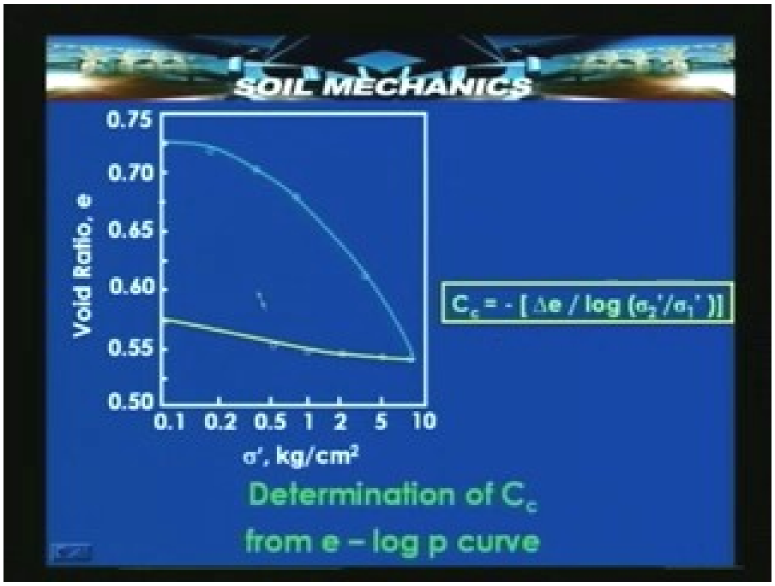

Now we come to a very important parameter which is essentially useful in time rate of settlement calculation. This parameter also defines compression in some ways but it is different from the compression index in the sense here we do not make any assumption regarding straight line relationship in a logarithmic plot. We in fact take a relationship directly from the e p or e sigma dash curve. The e sigma dash p curve or p curve is shown here, need not necessary be linear but any small segment of this if we take and considered to be fairly linear then once again we can effectively describe the change in void ratio as a function of the change in stress and that parameter is what is known as coefficient of compressibility and that's equal to the change in void ratio divided by the change in stress both being taken as positive.

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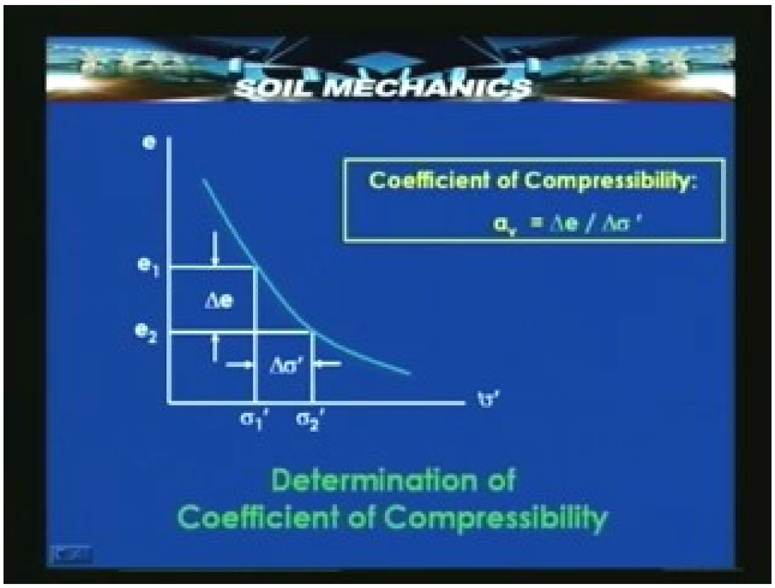

Here is a summary of the parameters that are normally used for describing compressibility, in addition to the 2 parameters which we have just now discussed; I have also added another parameter here. The first one is as I said minus delta e log sigma_2 dash by sigma_1 dash. Now it is not always convenient to use a negative quantity and therefore the change in void ratio is expressed as e_1- e_2 and for convenience the negative parameter is converted into a positive parameter and used. Now the coefficient of compressibility is delta e by delta sigma dash and this helps us in fact to compute the change in void ratio as a function of change in pressure.

And this can further be modified into yet another parameter which is known as coefficient of volume decrease or coefficient of volume change or coefficient of volume compressibility. There is a certain difference between the coefficient of compressibility and coefficient of volume compressibility. See here this (Refer Slide Time: 15:52) parameter is known as m_v, this parameter is defined as change in volume per unit volume as a ratio with respect to the change in stress. So earlier it was merely the change in void ratio which in effect is nothing but the change in volume where as now it is change in volume per unit volume with respect to the change in stress.
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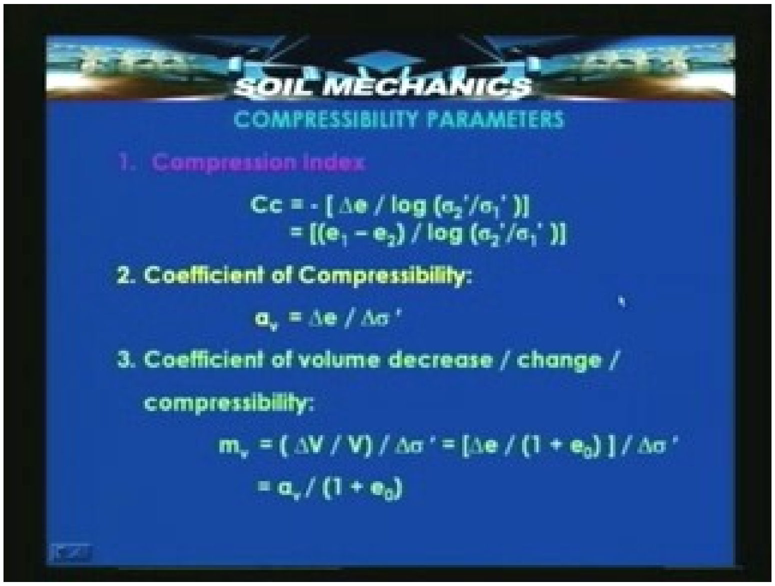

So this will be in terms of void ratio delta e divided by 1+ e_0 divided by delta sigma dash. This if we try to relate it to the coefficient of compressibility which is also an important and convenient parameter then we get that this parameter m_v is nothing but a_v divided upon the original volume of the sample or soil which is 1+ e_0. So we have 2 parameters delta e by delta sigma dash and a_v by 1+ e_0 and these two are based on an e log p curve which is the natural relationship and not an idealized logarithmic linear relationship. And these two are what are found to be useful for determining the rate of change of settlement or rate of change of compression in the soil under the application of the load.

Now how we can possibly relate this with the rate of change. A little bit of thought will tell us that this parameter after all represents the change in stress, it represents in some way the change in the compression as a function of change in stress. Now if we go back to our earlier discussion, we will find that we had expressed the phenomenon of consolidation as a gradual process of expulsion of water and corresponding transfer of stress to the solid grains. The word stress there, very clearly shows that there is something that relates the stress with the gradual process of expulsion of water and the transfer of stress from water to the solid grains. So we can easily see that there is some kind of relationship there, in a very definition of the process between the transfer of stress and the change in stress and the change in void ratio as a result flow of water.

So the rate of flow water can be sequentially related to the volume change and then the stress change. And this is the kind of approach which was thought of by terzaghi in developing the so called one dimensional consolidation theory where he gives an excellent mathematical method for computing the rate of settlement and predict the rate of settlement, when we are given a certain stress change due to a certain activity construction activity in a given soil and when we know what is the permeability of the soil and what would be the rate at which water will be flowing out of that soil.
So given a soil if we know its permeability, we know the rate at which water will move out of this. If we know the rate at which water will move out of this soil that means we

will know the rate at which the volume change will take place. If we know the rate at which the volume change will take place since we have a relationship between the volume change and the stress change, in effect we will be able to find out the relationship between rate of moment of water and rate of stress change. Now once we know the rate at which the stress is changing and since we already have a relationship between void ratio and stress change we will also know the rate at which the void ratio is changing or in other words the rate at which the settlement is occurring. So this link is what is crucial to understand the phenomenon of consolidation, particularly the rate or the speed at which it is taking place.

That means we need a methodology to link the rate of change of stress with the rate of change of void ratio as a function of rate of flow of water and this is what one dimensional consolidation is all about. Today we shall spend some time to see the basic physical aspects of this problem rather than the mathematical aspects of this problem. We shall go into detailed mathematical derivations in one of the subsequent lectures but today we will form the basis or formulate the foundation for the one dimensional consolidation mathematical theory.

Let us see the next slide. Here is a slide where we have a detailed description of a system which is analogous in behavior to the consolidation phenomenon. This is a system which consists of a spring embedded in a cylinder with a piston and containing water or a fluid. We can imagine the soil to be consisting of some solid particles containing water in their void spaces and the entire soil skeleton together with its voids is undergoing compression when water is expelled. So this amounts to having a system in which there will be a facility for water to flow out when load is applied and a facility for compression of the skeleton in proportion to the load.

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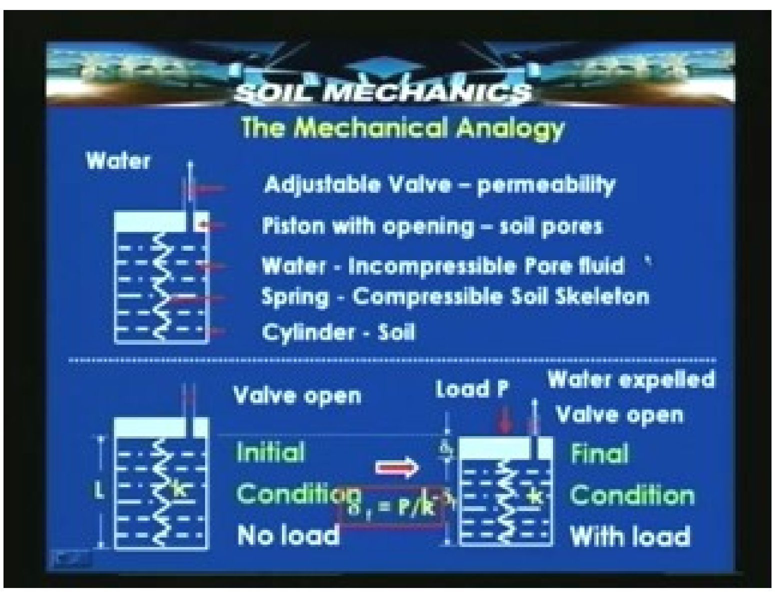

It is very easy to immediately now relate this to a system of a cylinder containing water, where in there is a piston arrangement through which we can apply a load where there is an opening or a valve through which water can expel itself and a spring inside which will allow us to simulate the compression of the soil skeleton. There can be no better analogy than this to understand the phenomenon of consolidation very effectively. Let us go into some depth in and some detail in understanding this mechanical analogy. See for example this first figure. As I just now mentioned the first figure completely simulates an entire soil skeleton with its water in its pore spaces and capable of expelling this water when subjected to a load and undergoing compression.

So here to begin with, this is a cylinder which in fact represents the total soil. Suppose we take a finite amount of volume of soil then this cylinder represents that finite amount of volume of soil. Then in that we have water which represents the saturated clay that we have been talking about. So if clay is represented by this entire cylinder containing water then presence of water contributes to the saturation effect. Then there is a spring which is nothing but the entire skeleton, the over all skeleton which includes the particles, the void spaces and it is capable of compressing when the water goes out. So here is a spring which is compressible and represents the skeleton. So this is the over all soil, this is the skeleton and this is the water and these constitute the system as a whole which is analogous to the system that undergoes consolidation and compression in nature (Refer Slide Time: 24:57). Then this water which is here as I said, not only represents the equivalent of saturated clay but it is also capable or helping us to simulate the incompressible part of the skeleton that is the pore fluid.

Then we have this piston which is nothing but an arrangement through which load can be applied to this soil and there is a hole in the piston which in fact represents the soil pores through which water can go out, this arrow (Refer Slide Time: 25:47) but there is also an adjustable valve here which is capable of controlling the flow of water out of this cylinder when a load is applied. And this in fact is very crucial and it is this which represents the permeability or the coefficient of permeability of the soil and this is what is responsible for inducing into this time dependency. It is this which controls the flow of water and therefore this is what is controlling the rate of transfer of load from water to the solid grains and contributing in turn to the compression of the skeleton. So this system consisting of all these components is a very nice illustration and equivalent of the compression phenomenon.

Now let us see what really happens during the process of consolidation. There are several stages that the phenomenon goes through that is suppose we have a soil over which we are building a structure then there are several stages through which the soil goes through and ultimately it ends up in a consolidated state. To begin with when there is no structure, when there is soil it is saturated, there may be pores but it is saturated but it is under hydro static state and there will be no flow of water and it will be in full equilibrium. What I mean by that is suppose there is this ground level, this is this clay layer and it is completely saturated, when there is no load, then there is no load this means that this entire thing is in equilibrium and there is no flow which is occurring from the clay layer due to the application of this load.

There is no load and therefore there is no expulsion of water. On the other hand the moment we apply a load since expulsion does not take place immediately, it implies the load is transferred to the water first because the water is incompressible and until it deforms or goes out, it cannot transfer the load to the solid grains. Therefore initially when the load is applied it goes straight into the water and then the water experiences pressures which are more than the hydrostatic pressures which it was experiencing before the load was applied and this is the fundamental idea that helps us to compute how and at what rate the settlement should take place.

Now see we have two states, one is without load another is with load. When there is no load there is no pressure in the water other than the hydrostatic pressure. The hydrostatic pressure corresponds merely to the depth of water at any point inside the clay layer here for example, if the height of water above that point let us say this is the point. The height of water above this if this is d, then the water pressure here is merely the hydrostatic pressure equal to gamma$_{wd}$ where as when you apply load since the water cannot escape and since it is not deformable it has got to resist, it has got to take up entire load that is being applied on the soil and because of this water immediately goes into a mode where it experiences what is known as excess hydrostatic pressure. That is a pressure which is over and above the normal hydrostatic pressure which it is expected to experience.

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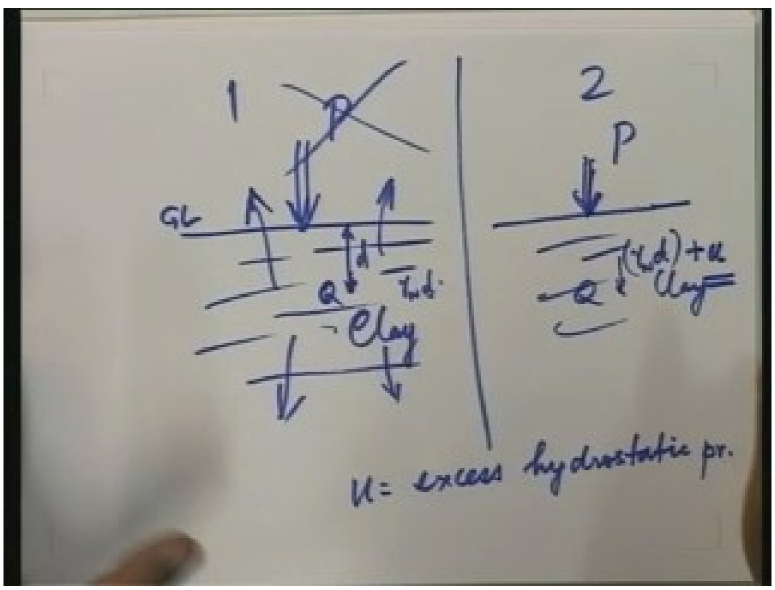

So if this is the second stage, this is the first stage then now the water or any point inside the clay layer at the same depth is undergoing or experiencing a load corresponding to what is applied here and that load is in the form of an excess hydrostatic pressure that means the pressure here is not just gamma d but it is gamma$_{wd}$ plus a parameter which we call as which we denote as u. This u is what is known as the excess hydrostatic pressure and this excess hydrostatic pressure is in the water as soon as the load is applied. But then water cannot remain their if we are talking about a porous material like a soil.
Since there are pores which are not sealed, the water has to find out the way out and as the water moves out the pressure gets released and the rate at which water moves out decides the rate at which this pressure u is released. The rate at which this pressure u is

released decides the rate at which this is getting transferred to the solid grains. After all ultimately for equilibrium any load that is applied at the surface must eventually been bourn by something. So if water is responsible for transferring the stress to the solid grains then the solid grains are ultimately responsible for carrying the load that is applied on the surface. So now if we go further we will find here, the first stage is a stage where the valve is opened that means the pores are opened but there is no flow of water, water does not go out because it is in complete equilibrium under the hydro static stress that is acting on it due to its own depth.

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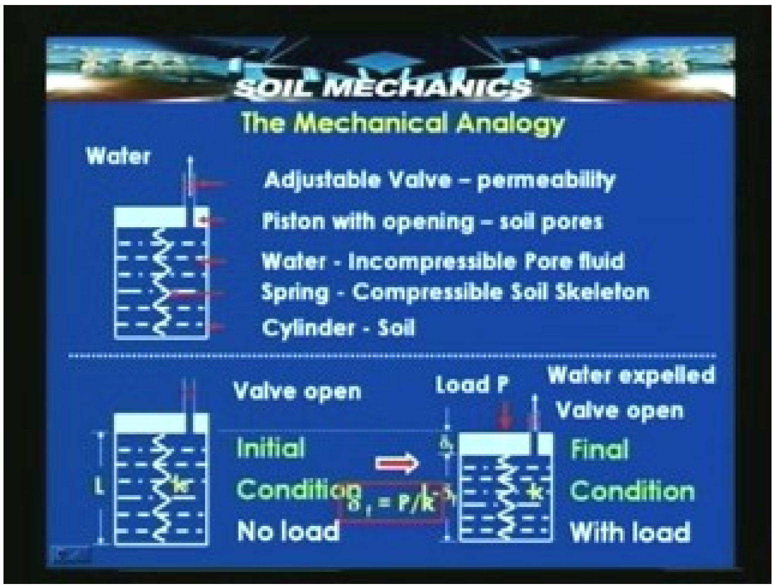

So this is an initial condition where there is no load, the valve is open, the soil is saturated there is a spring and the spring remains in its uncompressed state. There is no compression in this spring. This is the starting level, starting stage and now suppose we apply a load let us say a load p is applied to this system on the piston. When a load p is applied on to this system then as long as this valve is closed nothing will happen. The piston will with stand the load since water is incompressible the piston will remain its original position, there will be no movement.

But the moment we open this valve and allow the water to get expelled then we will find that the piston gradually moves down and in that process the load gradually gets transferred to the spring and the spring starts compressing. If you see here, this phenomenon if it continues let us say for a very long period of time, theoretically infinity then the final stage will be reached. And the final we will find that the spring would have compressed to the maximum possible extent corresponding to this load capital P and the final condition with load would correspond to a compression of the spring by an amount delta f meaning delta final.

That means the piston would have moved down by an amount equal to delta f then the spring would have compressed and it would have taken up a length equal to L minus delta f where L was the original length and compressed length and delta f is the compression that has been induced by the movement of the piston. Now if k is the spring constant let us assume a linear spring in which load is directly proportional to the deformation. So p is k delta or in other words this delta final that takes place in this system where the spring is being compressed, the final compression must therefore be equal to the total load applied and divided by the spring constant k. This is the final state but we must understand that this final state is not reached over night, there is a long duration over which this final stage is reached. Now we find that our spring piston cylinder system has an initial state in which the spring is not at all compressed and has a final stage in which the spring is totally compressed to an extent called delta f corresponding to the load p that has been applied over the piston.

Now between these two there are several stages and gradually the spring compresses as the water goes out and gradually the load p gets transferred little by little to the spring in a manner proportional to the corresponding compression of the spring. And this goes on until the final stage is reached. So between these two extremes at any given instant what is the amount of compression that has resulted in the spring and what is the corresponding load that has been transferred to the spring is what is of interest to us and that's what is helping us to compute the rate at which the settlement takes place. So at any given intermediate instant, how much would be the compression? If that can be computed then we have a method by which we will be able to predict not only the total settlement but also at any given point of time how much settlement would have occurred and not only that how much more settlement is likely to take place and over what period of time. So this is a complete answer in fact to the entire problem of consolidation.

So let us proceed further, what ever I have explained very briefly now is illustrated in the slide. For example stage one, no load that is the initial stage, no load and the valve is open but there is no flow and the water is in equilibrium and only hydrostatic pressure acts, so there is no load that means the applied load capital P is zero. That means the load transferred to the water is also zero because no water has been expelled and the corresponding load transferred to the solid grains is also zero. That means external load has not been applied and so there is a completely free system with no load acting. Now comes the second stage where we apply a load p, this simulates the construction of a structure. Now the moment a load p is applied then we start the counting the time for the compression phenomenon.

The compression phenomenon really gets initiated only after the load is applied, so the moment the load is completely applied is the time t =0 at which the compression phenomenon may be considered to have got initiated. Now if I take the initial state at time t =0, we have just now discussed that at time t =0 the load that is applied is carried completely by the water, only it gets transferred when the valve is opened. So at this stage where now the valve is closed, load is applied the load simply is carried out by the water.

So the load carried by the water is equal to the total load p applied and the load carried by the soil solids is now zero as before. This is the starting point and now in order to simulate this consolidation phenomenon, we will open the valve. Once the valve is opened, water will start flowing out and then we will see how the load gets transferred to the soil. So this was the second stage where the load p was applied and at t =0 water was carrying the full load and solids were carrying no load because the valve was closed and there was no flow.

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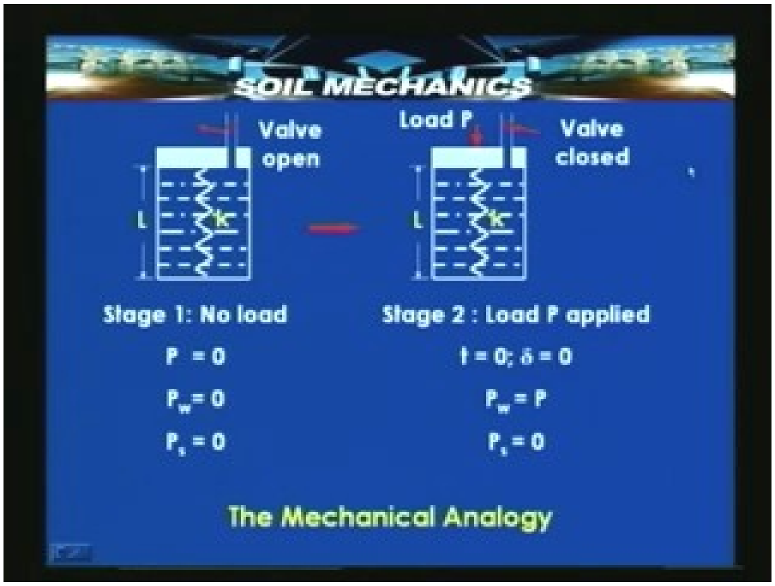

Now comes the second stage where this is an intermediate stage, this is the third stage. This is an intermediate stage where we find that water has been partially expelled out from the voids, the valve is open, water flow is still taking place and at any time t_i an intermediate instant the compression that has taken place is delta i and the load that has been taken up by water is therefore the total load p which it was originally carrying minus the load now which has been transferred to the spring because the spring has undergone a compression equal to delta i.

So corresponding to compression delta equal to delta i some intermediate level of compression, the load carried by the spring would be k into delta i where k is its stiffness. So that load carried by water is total load minus k delta i, this is an intermediate stage and there will be such several intermediate stages corresponding to several levels of delta i and its this delta i which we want to compute corresponding to any time t_i and that is where the mathematical formulation of one dimensional consolidation helps. Now let us see from stage 3, we will go on to several such stages and finally reach the last stage where the spring is totally compressed with final value delta f and the load carried by water therefore now is zero and the entire load is carried by the solids. So the water has now been expelled to the extent that it was required and the excess hydrostatic pressure which was there in the water due to the load p is no longer there because water is not carrying any load and the pressure in the water is once again hydrostatic pressure.
The excess pressure in the form u has been completely released; this is what the mechanical analogy and let us see how it translates into what is really happening in the field. Let us see the application of this analogy to a real situation in the field. The first figure represents the real situation in the field in which there is a compressible medium

which is sandwiched between 2 permeable layers. That is stage one, now there is no load and that means talking in the terms of stresses, the excess hydrostatic pressure u is zero and the effective stress sigma dash is also zero because there is no stress applied and the soil is in equilibrium under what ever stress is there due to its own weight.

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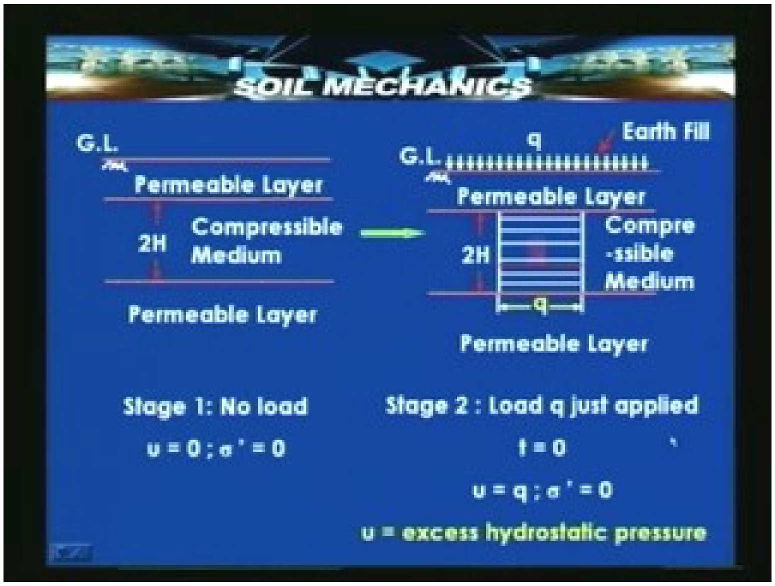

Now in the second stage where q has just been applied, we started counting the time for compression phenomenon t =0 and the entire load is carried by water and therefore the excess hydrostatic pressure u in water is equal to the load applied that is q and the corresponding effective stress is zero and the excess hydrostatic pressure is now in its full or in its maximum value that is equal to q. Then from this second stage where u =q and sigma dash is equal to zero at t =0, we move on to any intermediate stage where t =t_i and u will correspond to some remnant or remaining value of excess hydrostatic pressure which we shall call as u_i and the corresponding effective stress that has already been transferred to the solids is called as sigma i.

This sigma i prime or sigma i dash is the net effective stress that the solids are now experiencing. They will experience the full stress due to the applied load once this excess hydrostatic pressure is completely dissipated. At this point of time it is only dissipated to an extent to equal to q –u_i. Now if we proceed further we find that we reach gradually the very last stage where at t= t final entire excess hydrostatic pressure is totally dissipated and the effective stress is equal to the applied load q that means the solids have now taken over the complete load from the surface.

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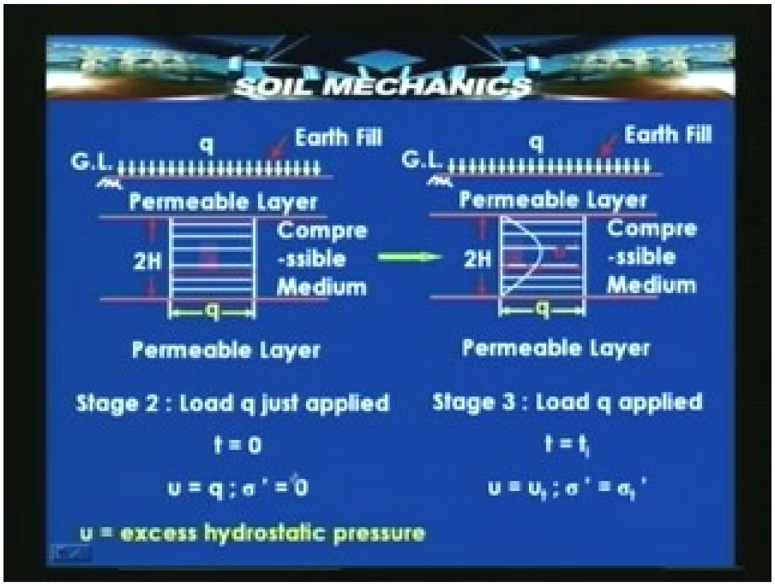

Let us go back a little bit, you have to look at the pressure distributions diagrams as well. See here when there is no load, there is no pressure in the compressible medium. The moment we apply the load that is we are applying q in the form of a surcharge then the pore pressure distribution in the soil is uniform with depth and it is equal to u. So that's why we have written u is equal and sigma dash is equal to zero. Now see here in stage 3, u has decreased a little bit because some expulsion of water has taken place and some dissipation of excess hydrostatic pressure has taken place and this any intermediate stage at this depth for example, this is the excess hydrostatic pressure and this is the effective stress that has already been transferred.

Now what is surprising is that this u is no longer uniform with depth, until now u was uniform with depth at t =0 but here it is no longer uniform. The reason is for stress to be transferred water has to flow out, flowing out of water depends on the availability of pervious medium. Suppose we have 2 permeable layers one above and one below then at the contact between these permeable layers, the excess hydrostatic pressure will get dissipated immediately. At the very next instant after application of load, the excess hydrostatic pressure at the permeable boundary will immediately get dissipated but as we go deeper into the layer either from top or from bottom, that is from the draining layers if we go inside towards the centre of the medium that is the compressible medium, then we find that since water has to travel from the centre up to this pervious medium either this way or this way.

The excess hydrostatic pressure would not get dissipated immediately as it happens at the two ends which means that the excess hydrostatic pressure distribution at any instant of time will not be uniform but it will be very varying with depth. The least pressure, least excess hydrostatic pressure corresponding to a value of zero will be occurring at the top and the bottom if both the layers are draining and the maximum will occur at the centre.
It is usual to consider a parabolic variation of this and in fact in terzaghi's consolidation theory, the so called one dimensional consolidation theory that we are going to see, this excess hydrostatic pressure distribution with depth will be in fact taken as a parabolic

distribution. And it is found that very well simulates the distribution in nature. So you can see that as this phenomenon goes on, in the very final stage once again the pressure distribution becomes uniform but now this entire pressure is sigma dash and there is no u, u is zero.

To sum up all these, what we can say is the total stress is equal to q at time t =0 and it remains constant at all times because total stress is equal to the pore pressure plus the effective stress. The pore pressure itself will be equal to q at t =0 but with time it will gradually get dissipated and the ultimately it will become zero at t equal to infinity. And correspondingly the effective stress will be zero initially and it will gradually increase and at t =infinity it will be equal to the applied stress q and the entire applied stress would have got transferred as effective stress to the solids. And this means that this is the progression that takes place and ultimately this sequence of events leads to a settlement where the settlement also starts from zero and goes on increasing and reaches a terminal maximum value at theoretically time equal to infinity. So now we have very nicely seen a mechanical analogy and how it simulates an actual condition in the field and very well understood really what takes place when the phenomenon of consolidation is progressing.

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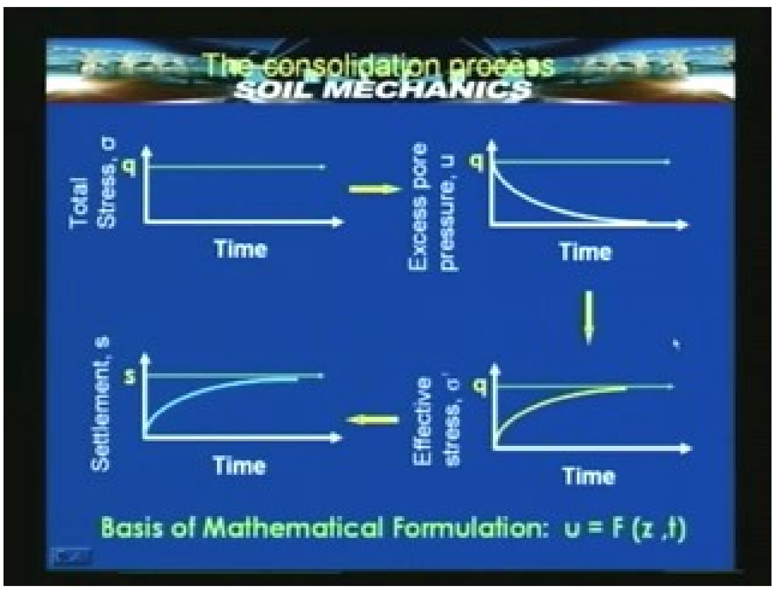

In order to evolve a mathematical formulation, in order to derive some expressions for this what we shall do is we shall derive a series or define a series of relationships. To begin with we have already seen that the coefficient of compressibility is $a_v = e_1-e_2$ upon $p_2- p_1$. This is the relationship which we have already seen. And it can be expressed in differential form as de upon dp with a negative sign to indicate that the change in void ratio is actually a decrease.

Then we can also define a parameter called the degree of consolidation because we have just seen that the compression at any intermediate time during the entire process of consolidation is proportional to the amount of volume change that has taken placed until then, which as a result of the water going out. Therefore the degree of consolidation usually expressed as a percent can be expressed as the change in void ratio at any time t at any instant t divided by the total change in void ratio. At the moment this degree of consolidation is expressed only in terms of void ratios but we know here from one that void ratio change can be related to the stress change through a_v. So now if we go to the next concept that is the total stress always remains constant then we find that the stress change is equal to the excess hydrostatic pressure change.

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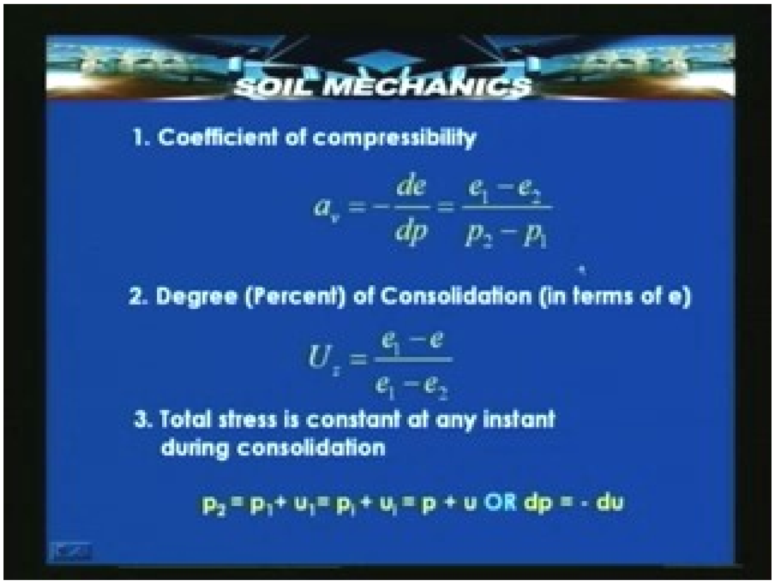

So dp is equal to minus du, since we have a relationship between de and dp and since degree of consolidation is related to de, we find that this degree of consolidation that is nothing but the consolidation that has taken place at any point of time, any instant of time. So this can be related to through 1 and through 3 to du. So here the void ratio change that's gets related to the pressure change and the pressure change gets related to the excess hydrostatic pressure change.

So the degree of consolidation can be rewritten as e_1- e upon e_1-e_2 equal to p- p_1 upon p_2-p_1 equal to initial excess hydrostatic pressure minus excess hydrostatic pressure at any time t divided by the excess hydrostatic pressure that was existing in the beginning, that is nothing but 1- u upon u_i and the void ratio change can therefore be expressed in terms of either the pressure change or the excess hydrostatic pressure change through a_v as shown in this last equation, de is minus a_v dp as we have already seen but its also now equal to a_v du. And this forms the basis of the mathematical derivation for computation of the total settlement.

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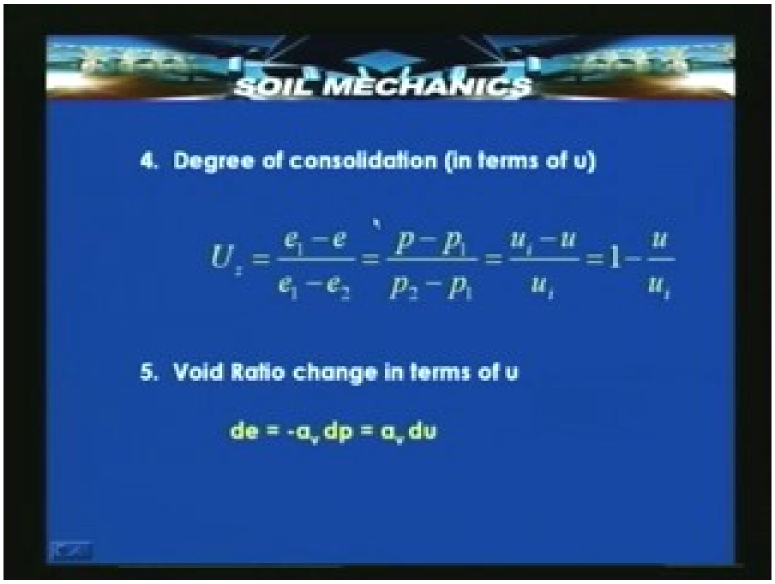

Now the mathematical derivation follows or is based on a number of assumptions, many of which are already familiar to us. We have already assumed that the soil is homogenous, soil is saturated, soil and water are incompressible, small elements are similar to the entire layer because the soil is homogenous. We have already defined that one dimensional compression is valid; one dimensional flow takes place when load is applied. We are now finally also adding another assumption that is properties of the soil particularly permeability remains constant with stress. As the stress change the permeability does not change. This is very much valid in practice normally although it is not possible to totally rule out the effect of stress on permeability and other properties.

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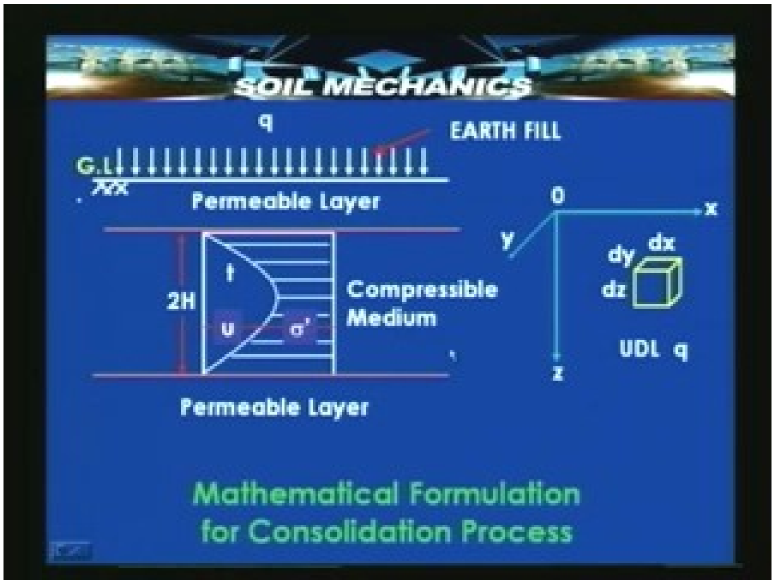

But for all practical purposes we can take it as constant and that's what we do in the one dimensional consolidation theory. And then of course we have already seen that there is an idealized linear ep relationship. Now based on all these by considering a compressible layer bounded between 2 permeable layers and any typical element from out of this layer, we can evolve a relationship between the degree of consolidation U_z and the depth and the time. Because we have seen that the degree of consolidation varies with depth and varies with time. Only thing is here we have plotted the degree of consolidation as a function of a non dimensional parameter z which is the depth divided by the total thickness of the clay layer.

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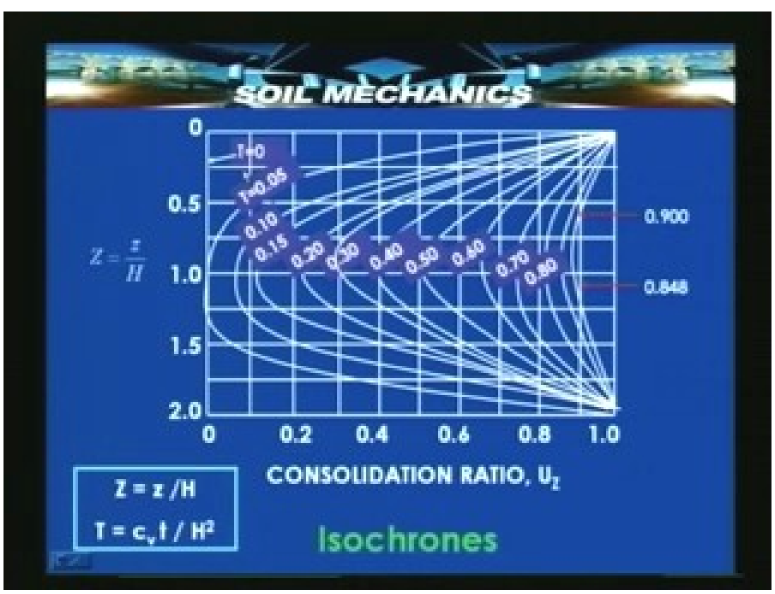

That is the relative depth and a non dimensional parameter t which is defined as $c_v t$ upon

H square where I am yet to define this parameter c_v which I shall be doing in the next lecture. Now what you should pay attention in this picture is that as time varies from roughly zero to its maximum possible value, the pore pressure variation with depth which is parabolic becomes less and less in magnitude and less and less in curvature and ultimately it becomes zero and this is what consolidation phenomenon is.

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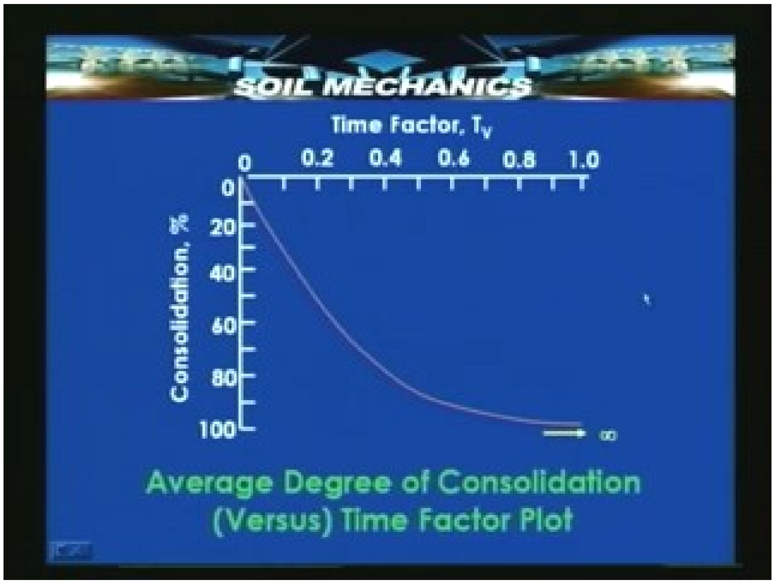

In the next lecture we will see how this t varies and how percentage of consolidation depends upon t and we shall once again briefly review what we have done in today's lecture and then go on deriving the mathematical expressions for consolidation.
Thank you.