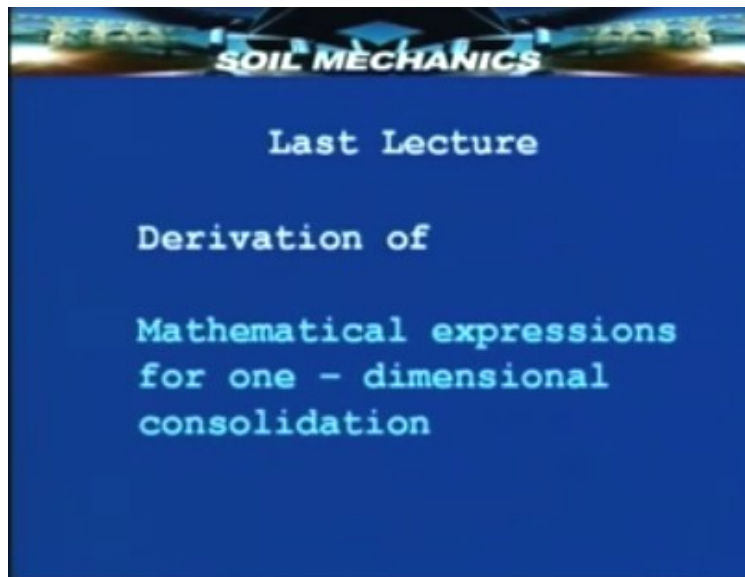


Soil Mechanics
Prof. B.V.S. Viswanathan
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 39
Consolidation and Settlement
Lec No. 6

Students welcome again to one more lecture on the topic of consolidation and settlement. This is going to be sixth in the series. As always I can start with briefly reviewing in whatever we had seen in the past 1 or 2 lectures and then pass on to new material in this lecture. So in keeping with that let me pass on to one slide the starting slide. This shows that in the last lecture primarily we dealt with the derivation of the mathematical expression for one dimensional consolidation.

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Unless we express the mathematical way of the phenomenon of consolidation. There is no way we can possibly predict the amount of consolidation that ever take place, if and when a building or any other structure is erected. That is we must know in advance, we must have a method to calculate in advance what is the likely settlement that a structure is going to undergo and it's in that context that the derivation of the mathematical expression is important. We had already seen in the one of the earlier lectures that we had to make several restrictive assumptions but practices shown that these restrictive assumptions are not really inducing any serious errors.

For example we had idealized the medium; we have idealized the process of consolidation. The process of consolidation was considered to be a one dimensional process. Now based on all these we derived an expression for mathematically representing this phenomenon.

What is this phenomenon basically? This phenomenon is basically one of water flowing out of a soil element under the action of a load and correspondingly the soil element undergoing a certain amount of compression essentially due to the compression of the voids. That is essentially due to reduction in volume of voids. So the clue to the derivation of the mathematical expression lies in equating these two. That is what is the rate at which flow of water takes place from the element and what is the rate at which the pore volume reduces. If you equate the two, actually they have to be equal. They must correspond to each other that gives us a method to see how the flow is taking place and how the dynamics of the hydrodynamic phenomenon is and then we are able to see how the compression phenomenon takes place.

And since the phenomenon of flow is related to time, we in turn are able to relate the process of compression also to time and that's essential because only then we will be able to predict at any given stage what will be the consolidation and what will be the settlement in a structure. This is important because we had already seen that to compute the total amount of settlement or total amount of consolidation that is likely to take place; we do not really need a detailed mathematical derivation. This derivation is necessary in order to find out the so called time rate of settlement.

Let us proceed further. So this slide briefly summarizes the beginning of the derivation for the mathematical expression, rate of outflow from the element is equal to the rate of change of void volume and the volume is $\Delta x \Delta y \Delta z$ and this led us to the derivation of the differential equation for consolidation which is $C_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$.

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Rate of outflow from element = Rate of change of void volume

$$k \frac{\partial^2 h}{\partial z^2} (\Delta x \Delta y \Delta z) = \frac{\partial}{\partial t} \left(\frac{e}{1+e} \Delta x \Delta y \Delta z \right)$$

Thus, the governing equation for consolidation is:

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

Where c_v - Coefficient of Consolidation

This expression shows that u is a function of z as well as a function of time. The constant C_v that has come in this expression is known as the coefficient of consolidation. And we have to now solve this differential equation and find out what is the relationship

between u the excess hydrostatic pressure and the 2 parameters z and t on which it depends. Let us see the next slide. See this slide shows the sequence in solving this basic phenomenon. This being a differential equation as is a normal practice, let us assume the type of solution, the possible pattern of the solution and then try to check that if this has got to be a solution for the given differential equation, what is the way to solve this?

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Taking the solution be of the form :

$$u = F(z)\phi(t)$$

$$c_v \phi(t) F''(z) = F(z) \phi'(t)$$

$$\frac{F''(z)}{F(z)} = \frac{\phi'(t)}{c_v \phi(t)} = \text{const.} = -A^2$$

So we are assuming that the excess hydrostatic pressure is a function of, if you see the slide u is a function of Fz and ϕt where Fz is a function of only depth and ϕt is a function of only time. Since these 2 are independent functions involving independent variables, this is a process known as separation of variables. We can now express these or substitute these in the differential equation involving $C_v \Delta^2 u = \Delta u / \Delta t$ and then we get this constant. Once we have this constant we can in fact formulate two equations, one for z and one for t which if solved will give us the total solution for the differential equation.

So the total equation or the total solution for the differential equation is a product of the solution for Fz and the solution for ϕt . So Fz into ϕt , if we multiply we get u . Now Fz involves a sine and cosine function and there are constants C_1, C_2, A and ϕt also involves a constant C_3 in addition. Now when we multiply these two, some of these constants also get multiplied and therefore we can combine these constants and we get a simpler equation like this $u = C_4 \cos Az + C_5 \sin Az e^{-A^2 t / c_v}$. So now this is the solution in which the constants are unknown. If we determine these constants, basically there are 3 constants C_4, C_5 and A . If we can determine these constants automatically now the solution is ready. To determine these constants always we need conditions.

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Since $u = F(z)\phi(t)$

$$u = (C_4 \cos Az + C_5 \sin Az)e^{-A^2 c_v t}$$

Evaluation of constants C_4 , C_5 and A from Boundary and Initial conditions:

Boundary Conditions

when	$z = 0$	$u = 0$	for $t > 0$
when	$z = 2H$	$u = 0$	for $t > 0$

Initial Condition

when	$t = 0$	$u = u_i$	for $0 < z < 2H$
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We need to know some aspects of the phenomenon which we are trying to study. For example this phenomenon includes or involves z which means the boundaries of the problem, the geometry of the problem. Now if we know the value of u at some values of z that can be used for calculating some constants. Similarly if you know the value of u at some known value of time that can be used for calculating some constants. So that's precisely what we will be doing. We did infact; we defined the so called boundary conditions and the initial condition.

Let us take a look at the boundary conditions and the initial condition. The boundary conditions were at the top of the clay layer and at the bottom of the clay layer. The value of pore pressure excess hydrostatic pressure that is zero for all times and the initial condition is at $t=0$, the pore pressure is uniform throughout the depth from zero to $2H$ its equal to the initial pore pressure u_i . Based on all these we derived essentially expressions for 3 parameters. The 3 parameters for which we derived expressions are one the excess hydrostatic pressure but just knowing the pressure alone is not our objective.

Our ultimate objective is to know what is the amount of consolidation that has taken place at any given point of time. So we need to know the so called degree of consolidation. Why we resort to defining the consolidation in terms of a degree is we know the total consolidation we can always compute the total consolidation without resorting to this mathematical expression by simply knowing the change in void ratio over a change in stress change. Now once we know the total compression that is likely to take place, the compression at any point of time can be expressed as a percentage of that which is what we call as degree of consolidation. There are 2 ways we will express the degree of consolidation, one is U_z that is degree of consolidation at any depth z which is nothing but the amount of consolidation that has taken place at a particular time divided by the total consolidation that is expected there.

And another parameter that we use is U which is basically the average of all U values over the depth z . So we have expression for u , small u we have expression for capital U_z

we have expression for capital U. These are 3 basic expressions that we derived last time which we will be using in our computations. The expressions are let us take a look at these slides, u is equal to this. You can see that it is a function of a parameter capital T which is nothing but C_v small t divided by H square and it is known as the non dimensional time factor.

T is usually written as T_v , v standing for vertical direction that's because a little later we will also be seeing another form of consolidation which is not only vertical but it is also radial in direction. So distinguish T for vertical from other forms of consolidation we have used the letter T_v . And U_z on the assumption of equal excess hydrostatic pressure over the entire depth at T equal to zero from that and if that value is equal to u_0 , we get the degree of consolidation at any depth z in terms of again the parameter capital T, the parameter H.

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Then:

$$u = \sum_{m=0}^{\infty} \frac{2u_0}{M} \left(\sin \frac{Mz}{H} \right) e^{-M^2 T}$$

where

$$c_v t / H^2 = T \quad \text{- NON - DIMENSIONAL TIME FACTOR}$$

T is usually written as T_v [v stands for vertical]

And

$$U_z = 1 - \frac{u}{u_i} = 1 - \frac{u}{u_0}$$

$$= 1 - \sum_{m=0}^{\infty} \frac{2}{M} \left(\sin \frac{Mz}{H} \right) e^{-M^2 T}$$

The parameter H that has come here is known as the longest drainage path because if you recollect, all these equations have been derived on the basis of the time taken for flow to occur. That is the rate of flow of water from out of the element, H is therefore the distance of travel of the water that is the maximum so called drainage path. Then the average consolidation has this expression which again is a function of capital T. Now let us see some details of the actual values of these degrees of consolidation. That's what we will be doing in today's lecture.

In today's lecture we shall try to understand the importance of this consolidation equation and you have seen that in this consolidation equation now, there is still one parameter C_v which is not depended upon the initial and boundary conditions.

This is not a constant which depends upon initial and boundary conditions but it's a material property. Though this C_v needs to be determined by some experimental technique and we already know that for consolidation phenomenon for studying the

phenomenon of consolidation, we have an excellent technique the experiment called the oedometer test. So we shall also see how to use this oedometer test to compute the coefficient of consolidation C_v and then we will follow it up with 1 or 2 examples on the use of these values in computation of settlement.

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Therefore Average Consolidation over entire depth:

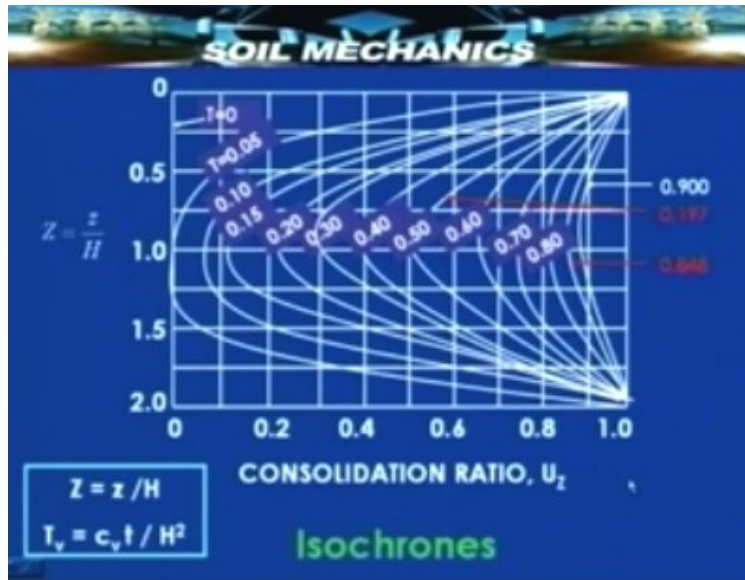
$$U = \frac{1}{2H} \int_0^{2H} u_z dz$$

$$U = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} e^{-M^2 T}$$

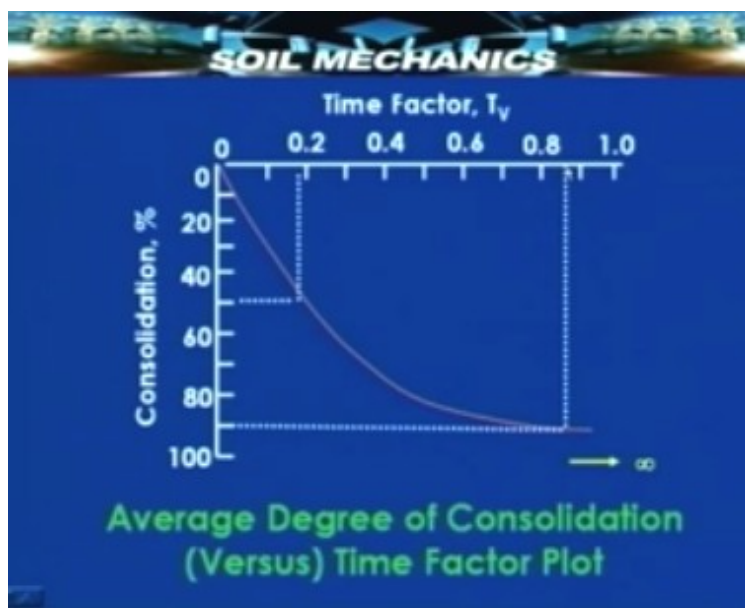
Particularly the time rate of settlement that is how much settlement will take place at any given time. So to begin with let us take a look at what are known as isochrones. We have just now said that as the depth varies excess hydrostatic pressure also varies and at any given depth it varies with time as well. And that is very conveniently represented in terms of curves of equal times, isochrones and we have such a curve which can be in fact derived from the expression for U_z which we saw in one of the previous slides.

Once we know the expression for U_z in that expression we can put different values of time and that time is expressed in fact in terms of the non dimensional time factor T_v by H square. So we shall express time in terms of this non dimensional parameter T_v and substitute that in the expression for U_z then we will know how the variation of the excess hydrostatic pressure is with respect to time and depth. And the isochrones in fact depict this. Along the vertical axis we have the depth again non dimensional form depth divided by the total thickness and along the horizontal axis we have the consolidation ratio U_z that is the degree of consolidation at any depth z . And it has to be zero at the top, zero at the bottom and it has to vary according to the log which is given by the expression for u . And keeping time constant time means the time factor t constant and making it range from zero to 0.9, insteps of 0.05 we can generate a series of curves as shown here.

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These are all curves each one corresponding to a different time but constant value at all points on a given curve. Those are therefore known as isochrones. You will find that we have $t = 0$ and going up to $t = 0.9$ and 90% consolidation at any given depth and at any given time would correspond to a time value of 0.848 where as 50% consolidation will correspond to a value of 0.197. Now these 2 values also have been arrived at by substituting the degree of consolidation, in the expression for degree of consolidation and computing the corresponding t . So either you can substitute capital T and go on finding out how the degree of consolidation varies or for a given degree of consolidation say 90% you can find out the corresponding time.
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And that's shown here for 90% we have 0.848 and for 50%, 0.97. Here is again the average degree of consolidation over the entire depth and once again you find that at time

factor 0.197, the average degree of consolidation over the entire depth is 50% where as 90% consolidation takes place at 0.848 time factor. Now we find that the point of 100% consolidation is rather difficult to locate, it takes theoretically infinite time but in practice it takes several hours. So if you apply an increment of load and wait for a very sufficiently long time then there are chances of the dial gauge reading in the oedometer test becoming virtually constant indicating that the level of 100% consolidation has been reached.

Now we very often need this U T_v relationship in two ways. For a given degree of consolidation or average consolidation, we would like to know what is the time that is going to be taken by a given structure or for a given structure at a given time what's going to be the amount of consolidation that would or would have taken place. So we need these values, we have already seen in the form of graphs, 2 graphs or 2 charts, how u versus t .

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U	T_v
0.1	0.008
0.2	0.031
0.3	0.071
0.4	0.126
0.5	0.197
0.6	0.287
0.7	0.403
0.8	0.567
0.9	0.848

But it's more convenient to use a look up table like the one which is showed in this slide. In this slide you find a look up table U T_v , U varying from 0.1 to 0.9 and T_v varying from 0.008 to 0.848 and for known reasons $u = 1$ has been left out here and 2 values which have been highlighted are those corresponding to 50% consolidation and corresponding to 90% consolidation. These 2 values in particular are very useful as you will be seeing shortly. There are 2 other expressions which are conveniently used, they are slightly simplified forms of the expression for U versus T and they are shown here.

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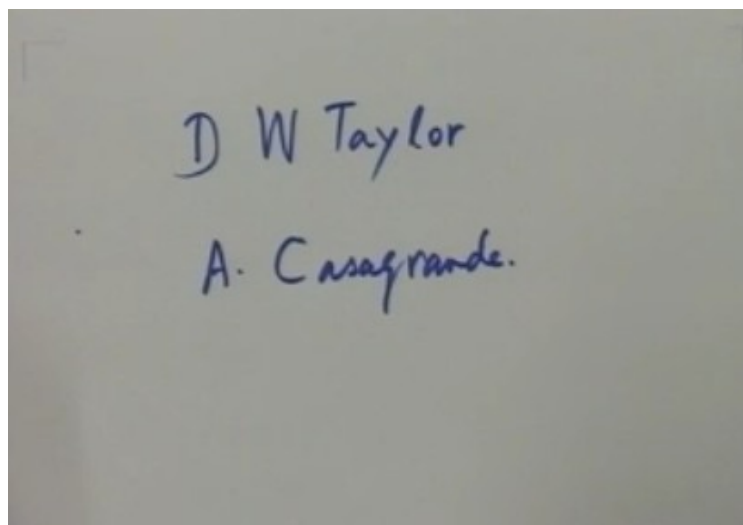
Approximate Expressions for Degree of Settlement
SOIL MECHANICS

$$U = \sqrt{\frac{4T_v}{\pi}} \quad (T_v \leq 0.6)$$

$$U = 1 - \frac{8}{\pi^2} e^{-\pi^2 T_v / 4} \quad (T_v > 0.6)$$

You find U is approximately equal to root of 4 T_v by pi as long as T_v is less than or equal to 60% or 0.6 rather and U is equal to this as long as T_v is greater than 0.6 actually I would like to make a correction at this point. This should have been 0.2 as long as T_v is less than or equal to 0.2, you will find that U is proportional to this or equal to this and as long as T_v is greater than 0.2, u is equal to this. The most important issue now is the determination of the parameter coefficient of consolidation. Coefficient of consolidation can be conveniently determined from the oedometer test. There are 2 methods for this purpose, one is devised by DW Taylor, a well known soil mechanics engineer and another by A Casagrande.

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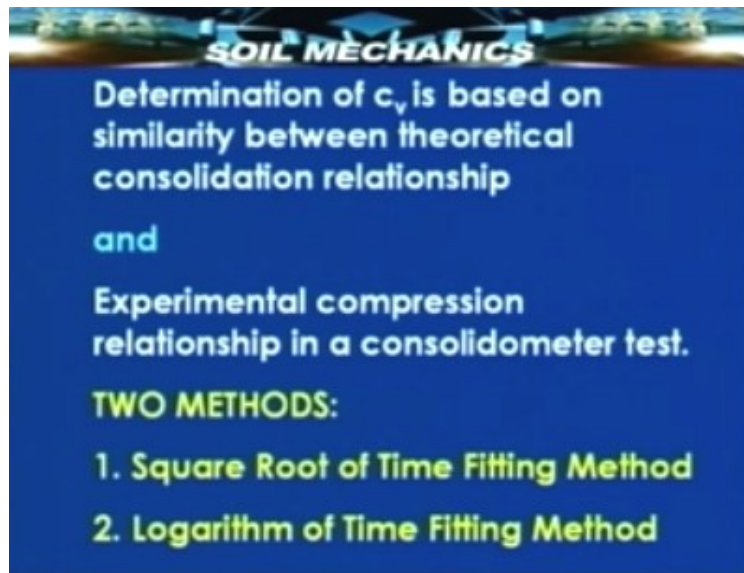


What is special about these two methods is they are both curve fitting methods. That is we get an experimental curve of compression versus time and we do some curve fitting on these in order to get certain data on the basis of which we can compute the value of

C_v . C_v is basically dependant upon small t and therefore if we can get 2 values of t and the corresponding degree of compression in an experiment, we will be in a position to compute the value of C_v and that's what we try to do. There are 2 different approaches; one is that of Taylor which is known as the square root of the time fitting method and another Casagrande's which is known as the logarithm of time fitting method.

Let us take a look at the square root of the time fitting method. As I mentioned both these methods are dependant on the fact that there is a similarity between the theoretical consolidation relationship and the experimental compression relationship in a consolidometer test. The meaning is in a theoretical consolidation equation, we know the précised relationship between U and T . We have seen it in great detail in several of the slides just now. U versus T , now if you look at it a little bit carefully, this capital U is nothing but the representation of the compression of the voids in some terms and capital T is nothing but a representation of the time elapse.

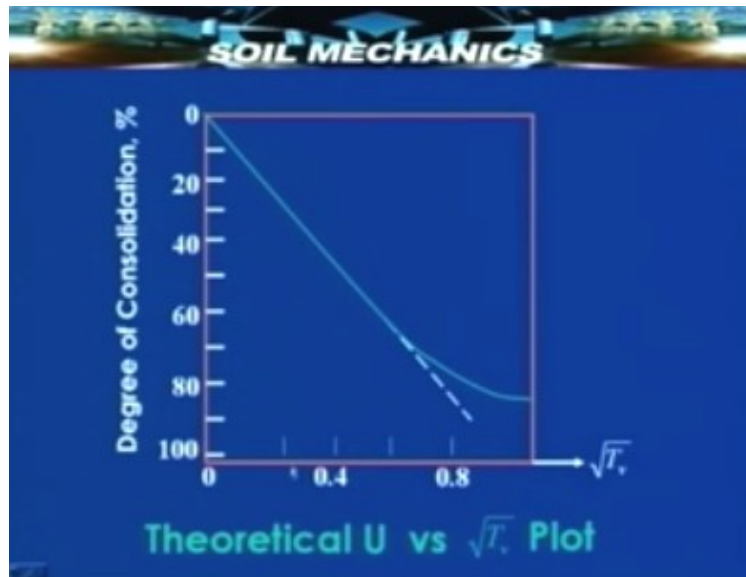
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So the relationship between capital U and T has to have some similarity to the plot of compression that is dial gauge reading variation with respect to time. It is based on this similarity that the methods of curve fitting have been evolved for determining C_v . What Taylor has done is he has used dial gauge reading versus root of time and considered to be similar to U versus root of capital T , where as Casagrande has taken dial gauge reading versus logarithm of time as similar to degree of consolidation versus logarithm of capital T . Let us take the square root of time fitting method and see how that operates. Being a curve fitting method essentially it's a graphical procedure.

So the theoretical U versus root of T_v if you take degree of consolidation is plotted here in the vertical direction and root of time is plotted in the horizontal direction. We had seen earlier that this U versus T curve was a curve going some what like this.

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Where as now we find that once it is plotted against root of time, it has become almost a straight line. Now this straight line is the basis for comparing this curve now with a curve obtained from typical oedometer test, a curve of dial gauge reading versus root of time in the method. So the initial part of the theoretical U versus root of T_v curve is found to be linear, is linear up to 60% consolidation.

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The Square Root of Time Fitting Method

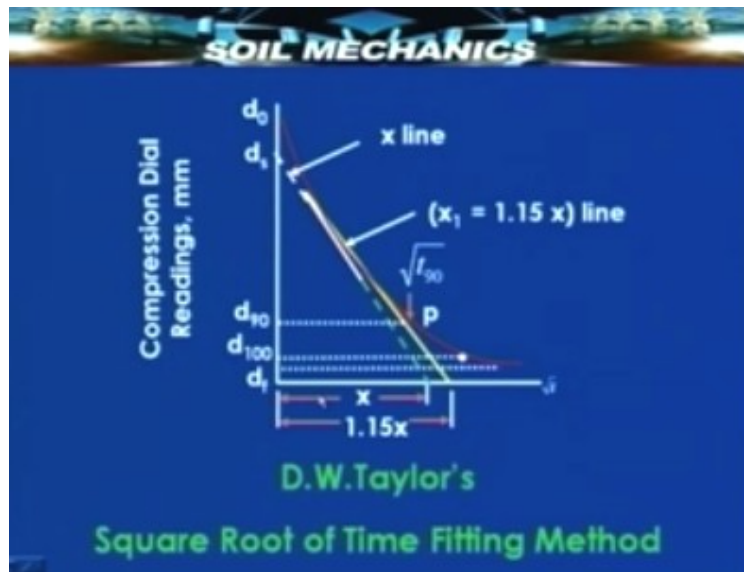
Basis: The following observations by D.W.Taylor

- The initial part of the theoretical U vs $\sqrt{T_v}$ curve is linear upto about U =60 %;
- On the theoretical U vs $\sqrt{T_v}$ curve, the point corresponding to $\sqrt{T_{90}}$ has an abscissa equal to 1.15 times that of extension of the initial linear part.

The point corresponding to root of T_{90} is found to have an abscissa of 1.15 times that corresponding to the extension of the initial linear part. Let us take a look at the curve again. The point corresponding to 90% consolidation here is found to have an abscissa which is 1.15 times that of a corresponding time on the line extended here. And that's a basis for determining C_v that is we take an similar looking curve that is the curve of dial gauge reading versus root of small t and in this we try to locate in fact that point which

has got an ordinate which is 1.15 times that on the straight line.

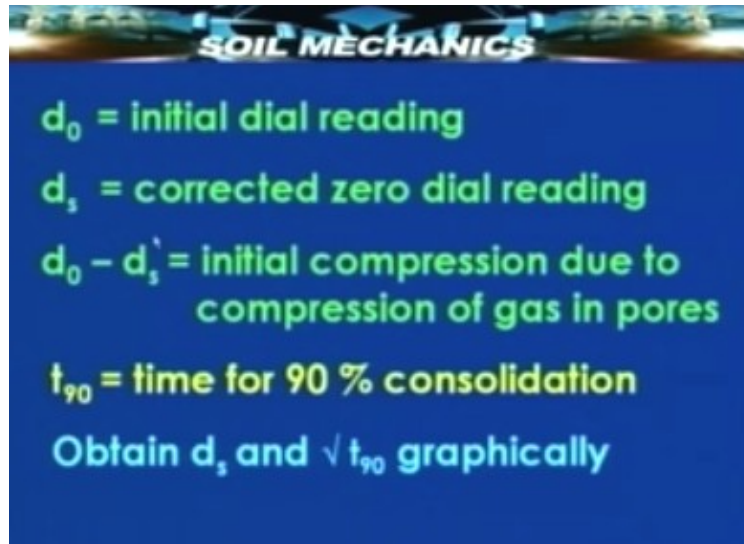
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So suppose this red line represents the curve of dial gauge reading versus root of time. What we do is we find out the starting point of this curve, actual starting point of this curve because original dial gauge reading may be d_0 but there is a certain amount of seating requirement that is when you apply the load, initially the pore gases get compressed and expelled and therefore the actual time at which the hydro dynamic phenomenon starts can be taken as corresponding to the dial gauge reading d_s . So d_s for all practical purposes is the initial dial gauge reading and that is the straight line part of the U versus root T curve. So the straight line starting from d_s and tangential to the red line is considered to be similar or analogous to the straight line part of the capital U versus root of capital T curve which we saw in the previous slide.

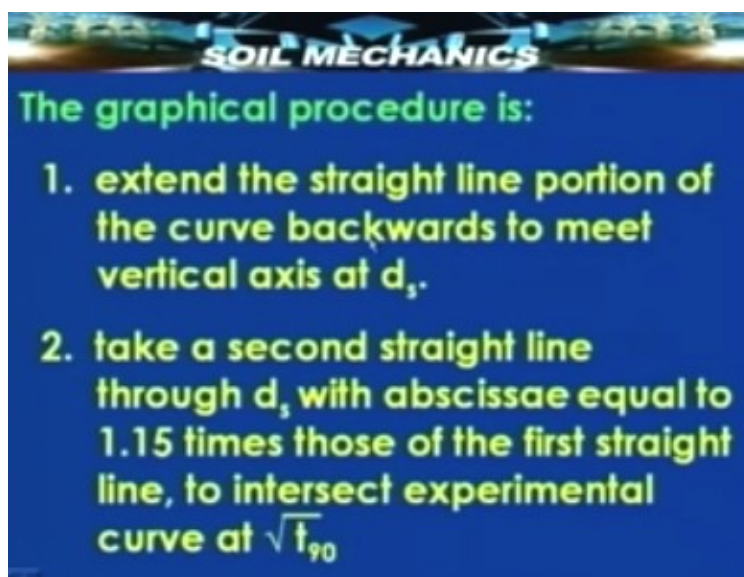
So all that we need to do is to draw another line that is this yellow line which will have abscissa which will be 1.15 times the abscissa of this line starting from d_s , this tangent starting from d_s and that's precisely what is shown here. If this tangent straight line can be considered as the line which may be denoted as x line then this yellow line is nothing but x_1 line which is having abscissa 1.15 times those of the x line and where this line cuts the red curve actually this should have been cutting here that's the point corresponding to 90% consolidation. That's in fact you can see that it's cutting some where here, this yellow line is cutting the red line some where here and that's the 90% consolidation point.

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And once we know d_{90} we will know T_{90} or root of T_{90} for that matter, once we know root of T_{90} we already know there is a relationship between small t and C_v . We know the sample thickness in the oedometer test; we know that we are providing double drainage, drainage on both sides in the oedometer test. We know what is the maximum drainage path? It will be nothing but half the thickness of the sample, that's the maximum distance that water from the centre of the sample has to travel in order to be out of the soil element. So root of T_v corresponding to 90% can be determined from this graph. Once we know that, since we already know the capital H and we also know that the capital T value is 0.848 for 90% consolidation, we can use all these information for calculating C_v .

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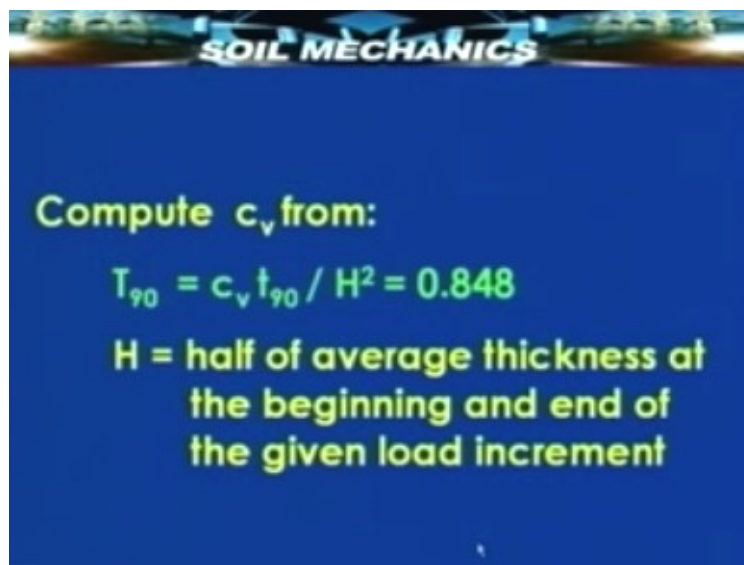


So if you have the initial dial gauge reading d_0 , first you have to work out the corrected zero dial reading after accounting for gas expansion and so on. Because that's the point where the straight line part of the curve will cut the y axis. So $d_0 - d_s$ is the initial

compression of the pores due to compression of gas. Now once we obtain d_s and root of T_{90} graphically as shown earlier, we can easily determine C_v . The graphical procedure as I mentioned earlier is going to consist of 2 parts, one is drawing the straight line portion so that it needs the vertical axis at d_s and then extending it and then taking a second straight line abscissa equal to 1.15 times those of the first straight line and finding out where to intercepts the red curve and that's the point corresponding to root of T_{90} .

Finally once we know capital T_{90} is equal to 0.848, we have a method to calculate C_v since we know small t_{90} and capital H which is nothing but half of the average thickness of the soil sample in oedometer test.

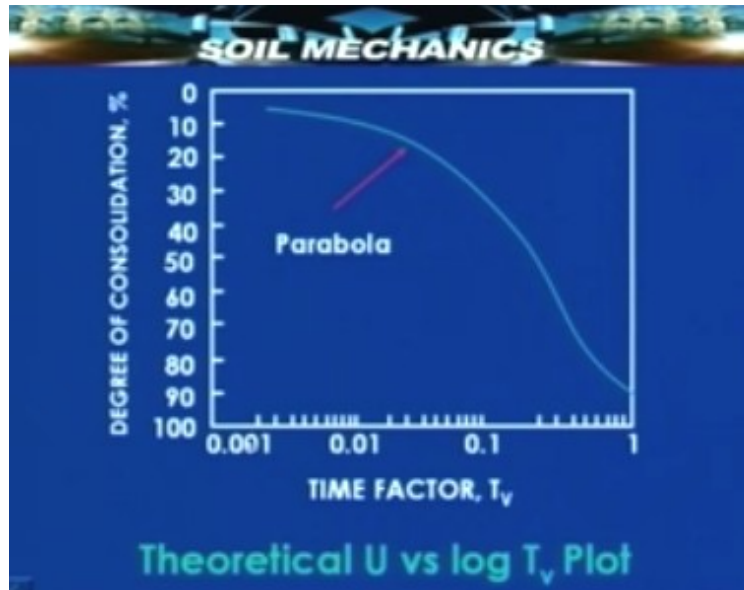
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Now the explanation for the logarithmic of fitting time becomes much simpler. Here degree of consolidation is plotted against log of T_v that is against T_v to a log scale. and this found to be very close to a parabola and its found that this is similar to the experimental curve where we plot dial gauge reading with time though we will now be seeing the logarithmic method of Casagrande in which U versus log T is used and it is seen that the U versus log T curve is similar to the dial gauge reading versus with log time.

The basis as you can see in this slide is that U versus log T_v curve is similar to the curve for dial gauge reading versus log of time. In this method instead of T_{90} we locate T_{50} . So we need to locate starting the point corresponding to d_s and we need to locate the point d_{100} . So once we know the dial gauge reading corresponding to zero and to 100 we can interpolate and find out dial gauge reading corresponding to 50 and corresponding time factor capital T_{50} and that's the method.

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So here is the explanation for the method in pictorial form. If this yellow line represents the dial gauge reading versus logarithm of time. Then first step is to take this as a parabola because it is found that in the theoretical U versus $\log T$ curve the first portion, this part is a parabola. So assuming this to be a parabola, we will try to find out the corrected starting point d_s that's the point corresponding to 0% consolidation. We will also then find out the point correspond to 100% consolidation and 50% consolidation will be in the middle of this range and from that we will be able to get small t_{50} corresponding to capital U_{50} .

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Logarithm of Time Fitting Method

Basis: The following observations by A. Casagrande

The initial part of the theoretical U vs $\log T_v$ curve is a parabola and d_s can be located as shown in next slide;

On the theoretical U vs $\log T_v$ curve, the point corresponding to T_{100} is located at the intersection of the Tangent and the Asymptote.

From d_s and d_{100} , d_{50} & T_{50} can be located.

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SOIL MECHANICS

Logarithm of Time Fitting Method

Basis: The following observations by A. Casagrande

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On the theoretical U vs $\log T_v$ curve, the point corresponding to T_{100} is located at the intersection of the Tangent and the Asymptote.

From d_s and d_{100} , d_{50} & T_{50} can be located.

Once we know small t_{50} again we use the same formula involving C_v and get the value of C_v . So the method here is since this is a parabola if I take 2 points A and B with ordinates equal or rather abscissa equal to t_1 and $4t_1$ respectively then since this is a parabola from the starting point to this point whatever variation in the dial gauge reading is there that is whatever variation is there in the y coordinate must also be equal to the variation in the y coordinate between this point and this point. Let us see that this $d_s - d_1$ must be equal to $d_1 - d_2$ and both equal to say y . Let us see the reason. The reason is we have taken the first part to be a parabola that is time must be proportional to or equal to αd square.

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Assuming initial part of the curve to be a parabola: $t = \alpha d^2$

Or, $t_s = \alpha d_s^2$; $t_1 = \alpha d_1^2$; $t_2 = \alpha d_2^2$;

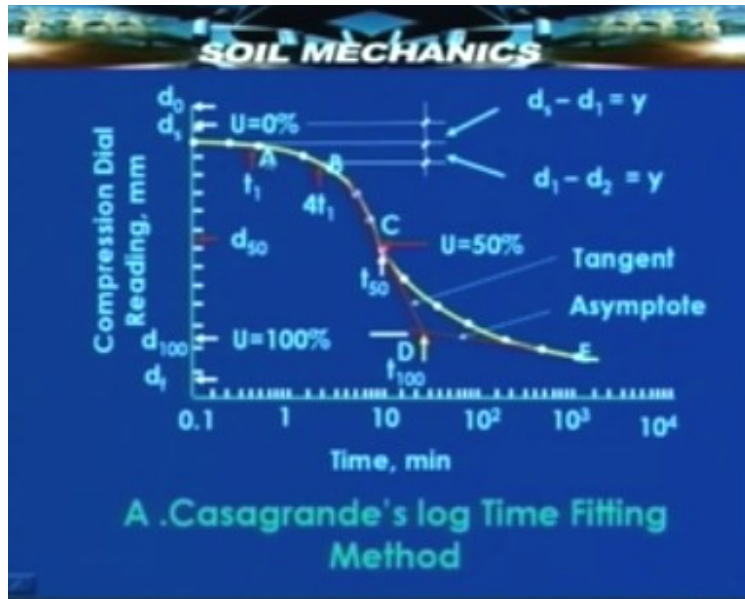
For $t_2 / t_1 = 4$

$\log t_s = 2 \alpha d_s$; $\log t_1 = 2 \alpha d_1$; $\log t_2 = 2 \alpha d_2$

$d_1 - d_s = [\log t_1 / t_s] / 2 \alpha$

$d_2 - d_1 = [\log t_2 / t_1] / 2 \alpha$

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At all times on the parabola, time must be proportional to d square because the parabola has an equation of the type y is equal to ax square. This means that at the starting point t_s must be equal to αd_s square, at point a it should be t_1 equal to αd_1 square and at t_2 it should be αd_2 square and since t_2 by t_1 is 4. We find that on the logarithmic scale $d_1 - d_s$ and $d_2 - d_1$ are both equal absolutely similar and that's the basis for computing this distance and arriving at the correct starting point d_s . So we select any 2 points A and B this having time t_1 , this having time $4t_1$ and using the difference between this as a starting point. We choose a point d_s which will have the same difference in ordinate as shown here. So that means d_s is located. Now what remains is to locate d_{100} and d_{100} is taken as the intersection point of the tangent here and the tangent at the last portion of the curve. (Refer Slide Time 36:18)

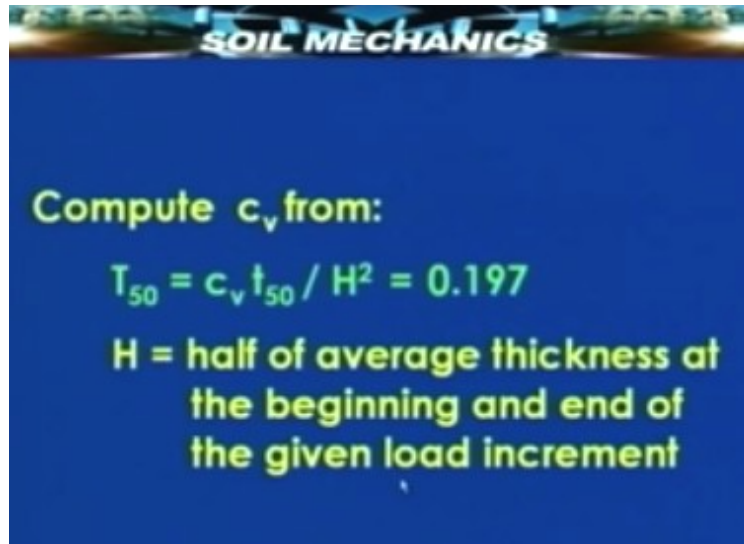
SOIL MECHANICS

By making

$$(d_1 - d_s) = (d_2 - d_1)$$

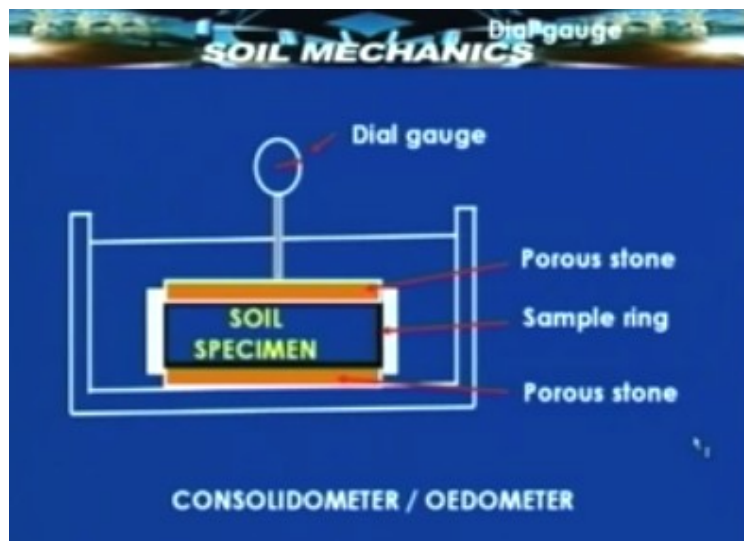
dial reading d_s corresponding to $U = 0\%$ can be approximately calculated.

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These 2 intersect at a point here corresponding to which the time is t_{100} , the degree of consolidation u is 100% and the dial gauge reading is d_{100} . Between these two some where in the middle lies d_{50} and t_{50} on basis of which we calculate the value of C_v . So by making $d_1 - d_s$ equal to $d_2 - d_1$ we are in a position to calculate the starting point of the curve. Once we know T_{50} since it is equal to 0.197 we can substitute that and get the value of C_v . Once again capital H corresponds to half thickness of the sample and since the sample thickness varies during the experiment, we take half the average thickness of the sample for computation of C_v .

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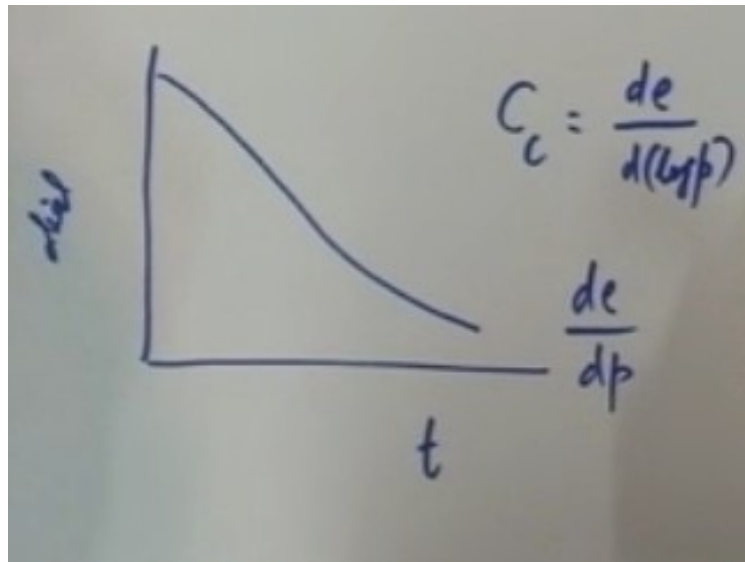


Now I had mentioned in one of my earlier lectures which I am now briefly reproducing that there are 4 different compressibility parameters C_c , A_v , M_v , C_v .

These are the units and it's possible to determine all these from these well known consolidometer test. As a starting point for determining these, we need to know the void

ratios. We have dial gauge readings which we are able to gauge from the experiment, the so called oedometer experiment and we have the time or root of time or logarithm of time as the objective may be. This is a curve some what like this and ultimately we are interested in knowing the change in the void ratio because we are finally interested in for determining C_c the expression de by $d \log p$ and for determining for $A_v M_v$ we need de by dp .

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This means that void ratio versus pressure curve is a must, its unavoidable it needs to be drawn. This means that its not always sufficient to just plot the dial gauge reading versus time that's okay for **compute addition** of the coefficient of consolidation C_v but in order to compute the void ratio changes, we need to make a little bit of a computation based on the dial gauge reading variations and void ratio computation can be made again by 2 methods, one is known as the height of solids methods, the other is know as the change in void ratio method.

Let us take a look. In a consolidation test typically we know the diameter of the specimen therefore we know the area of cross section which is π by $4 d$ square and we know the initial height 25 mm. It's all standardized, it's all fixed the equipment is internationally stipulated standardized equipments and therefore these dimensions are known. In the 2 different methods we tried to relate the dial gauge readings through the compression of the soil sample to the void ratio.

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SOIL MECHANICS

Determination of Void Ratios

A typical consolidometer specimen has the following dimensions:

Diameter	: 75 mm
Area of Cross-section	: 44.18 cm ²
Initial Height (h_0)	: 25 mm

1. Height of Solids Method
2. Change in Void Ratio Method

So in the height of solids method essentially we are looking for the height of the soils specimen and how it varies. So if we know the dry weight of the specimen, we can calculate the unchanging height of the solids h_s by this formula. This is a very well known fundamental formula where W_s by V_s which is nothing but the density of the solids when divided by γ_w gives us the so called specific gravity called G . So we can find out that the volume of solids V_s in any given case will be this $G \gamma_w$ divided by W_s or the other way round W_s by $G \gamma_w$. So the height of solids can be found out as W_s by $G \gamma_w$ and actually it's a volume and therefore height can be obtained by dividing this volume by area of cross section of the specimen.

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SOIL MECHANICS

Height of Solids Method

If Dry weight of specimen = W_s

We know $h_s = W_s / (G \gamma_w A)$ [const.]

Hence $h_{v0} = h_0 - h_s$

At any stage $h_{vi} = h_{i-1} - (\Delta h)_i = h_{i-1} - \Delta d_i$

and $e_i = h_{vi} / h_s$

Now this means that the height of the voids alone at the starting point will be the initial height of the sample h_0 minus the height of the solids. So at the starting point if this is the

height of solids and this is the initial height of voids can be obtained from the initial height of the sample h or h_0 minus the height of solids. Because the height of solids remains same throughout the experiment which means that at any instant of time during the experiment, the height of the voids which will go on changing will be equal to the previous height minus the change in height that takes place or change in dial gauge reading that takes place.

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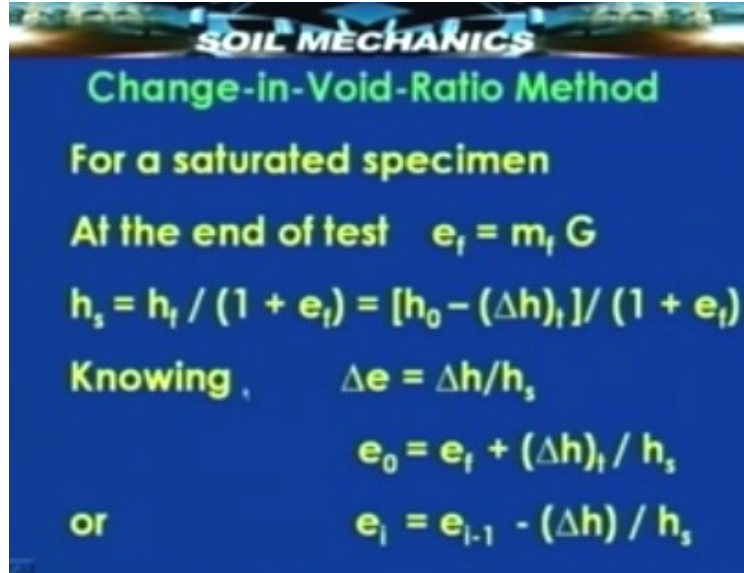
The image shows handwritten notes and a diagram. At the top, it states $V_s = \frac{W_s}{G\gamma_w}$. Below this, it shows $G = \frac{W_s}{V_s \gamma_w}$. To the right, there is a diagram of a soil specimen with a total height h_0 and a height of solids h_s . Below that, there is a diagram of a dial gauge showing a change in height Δh .

Once we know this we can calculate the void ratio as height of voids by height of solids. So the attempt in this method is to determine the change in height of the specimen and relate it to the change in volume of the voids. So e is taken rather as height of voids by height of solids than volume of voids by volume of solids. So at any stage h_{vi} that is volume of height of the voids at time i , instant i will be equal to height of the specimen in the previous stage minus the change in height and the change in height is nothing but the change in dial gauge reading. A similar approach is there in change in void ratio method. Here we take advantage of the fact that the soil sample is saturated and remains saturated throughout the experiment.

Actually sometimes it may not but for all practical purposes we assume that the soil sample remains saturated throughout the experiment. Since it is saturated throughout the experiment, we know that the void ratio for a saturated specimen must be equal to the moisture content into the specific gravity again. So we know the height of the solids can be found out from the final height divided by the volume of solids that is 1 upon $1 + e_f$. If we have several stages of loading then the initial height minus the total change in height divided by $1 + e_f$ will always be equal to the height of solids which remains constant. Because of this we can say that change in volume or change in void ratio must be equal to change in height divided by volume of or rather the height of solids or in other words $e_0 - e_f$ must be equal to total height change divided by height of solids.

Or it can also be said that e at any instant i minus e at any previous instant should be equal to Δh divided by h_s , $e_{i-1} - (\Delta h) / h_s$.

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SOIL MECHANICS

Change-in-Void-Ratio Method

For a saturated specimen

At the end of test $e_t = m_t G$

$h_s = h_t / (1 + e_t) = [h_0 - (\Delta h)_t] / (1 + e_t)$

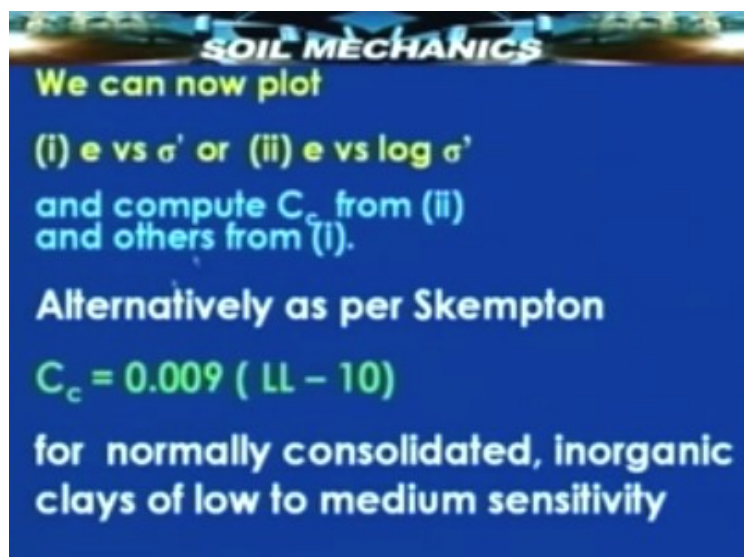
Knowing $\Delta e = \Delta h / h_s$

$e_0 = e_t + (\Delta h)_t / h_s$

or $e_t = e_{t-1} - (\Delta h) / h_s$

So this is the way we can successively compute the void ratio at any stage from the previous void ratio knowing the change in height and the height of solids which we determined previously. We can therefore now plot either e versus σ dash curve or e versus $\log \sigma$ dash curve. If we have the e versus $\log \sigma$ dash curve we determine C_c from that, if we use e versus σ dash curve then we will be determining other parameters.

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SOIL MECHANICS

We can now plot

(i) e vs σ' or (ii) e vs $\log \sigma'$

and compute C_c from (ii)

and others from (i).

Alternatively as per Skempton

$C_c = 0.009 (LL - 10)$

for normally consolidated, inorganic

clays of low to medium sensitivity

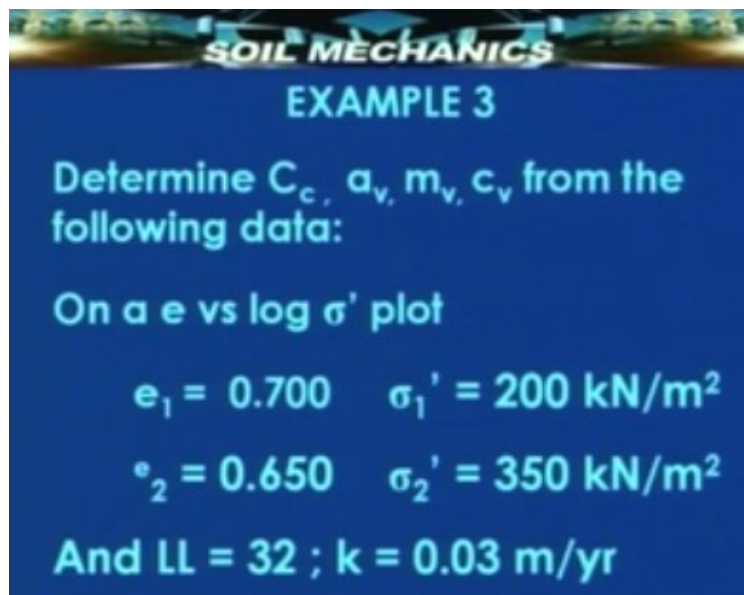
Now incidentally for C_c , for determining the coefficient of compression that is the compression index, we have another expression an approximate expression given by skempton based on a large number studies by him on normally consolidated inorganic

clays of low to medium sensitivity. That equation is one which links C_c with liquid limit. Now liquid limit is such a basic parameter for a clay that any relationship such as this where the compression index is related to the liquid limit is a very useful method for computing the compression index. Because liquid limit is anyway determined and it is a very basic fundamental parameter for a clay and this equation is $0.009(LL - 10)$.

Now let us see an example. To begin with let us see how to determine the various soil parameters from a typical test. I shall not be going into the details of data from a test, let us assume that we have the data. Given the data of dial gauge readings versus the time we will be in a position to have the basic data required for computing the void ratio as one of the methods as per one of the methods which I described just now and once we know the void ratios we will be in a position to compute these various parameters. So suppose e_1 and e_2 are the void ratios corresponding to 2 stress values, σ_1 dash equal to 200 and σ_2 dash equal to 350.

Then this is the change in stress, this is the change in void ratio. If we use logarithm of change in stress we will be able to calculate C_c , if we use directly the change in stress we will be able to calculate the other parameters a_v , m_v , c_v . Incidentally the parameter C_c can also be calculated from LL the liquid limit and the parameter m_v requires the value of a_v and the parameter c_v requires the value of coefficient of permeability.

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SOIL MECHANICS
EXAMPLE 3

Determine C_c , a_v , m_v , c_v from the following data:

On a e vs $\log \sigma'$ plot

$e_1 = 0.700$	$\sigma_1' = 200 \text{ kN/m}^2$
$e_2 = 0.650$	$\sigma_2' = 350 \text{ kN/m}^2$

And $LL = 32$; $k = 0.03 \text{ m/yr}$

So answers are c_c will be equal to the change in void ratio divided by logarithm of the ratio of stress that is 0.206.

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SOIL MECHANICS
EXAMPLE 3

Answer:

$$C_c = (0.700 - 0.650) / \log (350/ 200)$$

$$= 0.206$$

By Skempton's equation:

$$C_c = 0.009 (LL - 10)$$

$$= 0.009 (32 - 10)$$

$$= 0.198$$

We get by skempton's equation a value very close to this 0.198. Now for this load increment σ_1 dash minus σ_2 will be rather σ_2 dash minus σ_1 dash will be 350-200 and a_v the coefficient of the volume compressibility will be nothing but de by dp and that will be 0.0003 and the units of a_v are meter square by kilo Newton, m_v is nothing but a_v upon $1+e$. Since we know the starting void ratio that's the initial volume of the solids 0.0003 by 1.700 or 1.765 into 10 raised to the power of minus 4 again meter square by kN is the value of m_v , c_v on the other hand requires the value of coefficient of permeability and this is the expression for it and if you substitute the known values of k and a_v and e you get C_v equal to 17 meter square per year.

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SOIL MECHANICS
EXAMPLE 3

Answer:

FOR GIVEN LOAD INCREMENT

$$a_v = (0.700 - 0.650) / (350 - 200)$$

$$= 0.0003 \text{ m}^2/\text{kN}$$

$$m_v = a_v / (1+e) = 0.0003 / 1.700$$

$$= 1.765 \times 10^{-4} \text{ m}^2 / \text{kN}$$

$$c_v = k (1+e) / a_v \gamma_w$$

$$= 0.03 \times 1.7 / (0.0003 \times 10)$$

$$= 17 \text{ m}^2 / \text{yr.}$$

Let us take another example. A 6 meter thick clay layer is subjected to a load of 60 kilo Newton per meter square. One year after loading the average consolidation u is found to

be 50%. So the time period small t is given, the stress change is given, the degree of consolidation is also given, average degree of consolidation.

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SOIL MECHANICS

EXAMPLE 4

A 6m thick clay layer is subjected to a load of 60 kN/m². One year after loading, average consolidation is 50 % complete.

What is the coefficient of consolidation if the clay layer drains vertically in both directions? If $k=0.03$ m /yr , what is the settlement at the end of 1 year?

What is asked is the coefficient consolidation assuming one dimensional consolidation and if $k=0.03$ what is the settlement at the end of one year? The settlement that will really take place here will depend upon the final void ratio. So at this point of time, interested in the settlement corresponding to a particular time rather than the total settlement. And that's why we need the coefficient of consolidation here.

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SOIL MECHANICS

EXAMPLE 4

SOLUTION

$$c_v = T_{50} H^2 / t = 0.197 \times 3^2 / 1$$

$$= 1.773 \text{ m}^2 / \text{yr.}$$

$$m_v = k / c_v \gamma_w = 0.03 / 1.773 \times 10$$

$$= 1.692 \times 10^{-3} \text{ m}^2 / \text{kN}$$

$$s (U=1) = m_v \Delta p H = 1.692 \times 10^{-3} \times 60 \times 6$$

$$= 0.609 \text{ m}$$

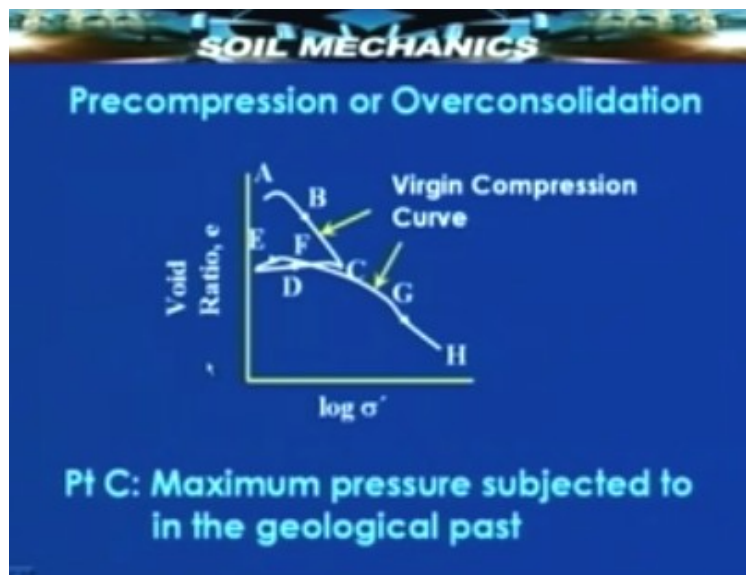
$$s (U=0.5) = 0.304 \text{ m}$$

The solution will be c_v is $T_{50} H$ square by t because we know the time corresponding to 50% consolidation, so we need capital T_{50} which we know is 0.197 from this we get c_v .

Obviously once we know c_v since we know k we can calculate m_v and its known that settlement corresponding to 100% consolidation that is a total settlement in terms of m_v can be expressed like this. Although I have given you the expression for total settlement in terms of c_c and in terms of $e_1 - e_2$ by \log of $p_2 - p_1$. It is possible to express that in terms of m_v as well and m_v into Δp into H will give you the total settlement. We know the total settlement in this case, from these values of m_v Δp and H because Δp is known as 60 kilo Newton's per meter square and this gives us 0.609 meters as the total settlement.

So the settlement corresponding to 50% consolidation which takes place in one year's time will be 0.304 because 100% consolidation corresponds to 0.609 meters therefore 50% consolidation would correspond to half the settlement that is 0.304 meters. However the time taken for 50% consolidation which is one year need not necessarily be and in fact will not be half of time taken for the total 100% consolidation for obvious reasons. Now we come to an important phenomenon known as the pre compression phenomenon. There are many locations in the world where there have been glaciations these glaciations have come down melted and come down over great distances and covered vast areas and at some point of time for certain unknown reasons they have retreated. But in this process of coming forward and going back they had been responsible for applying enormous amount of loads on soils particularly what is of relevance to us or clayey soils. There the clayey soils which have experience this have undergone consolidation under the load of these glaciations. And after the glaciations have retreated they still remain consolidated but they are more consolidated than one would expect under the present over burden pressure. So such soils are known as over consolidated soils. We need to know in such soils, what is the degree of over consolidation or what's the compression pressure to which it has been subjected to in the past.

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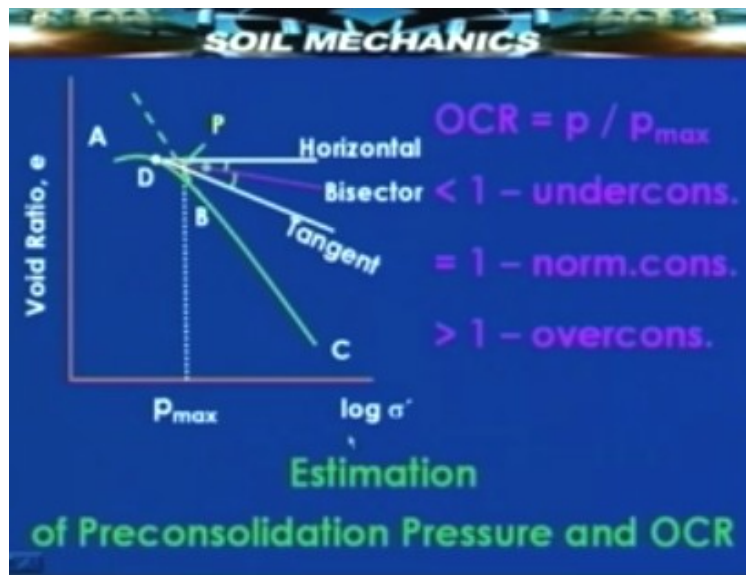


This is important suppose you see this graph which I am showing here. If I apply load on a sample and observe this void ratio change, I know that I will have that so called virgin

compression curve up to some stress. And now if I unload and reload the soil traces a totally different path until it reaches the same pressure at which we started the unloading and beyond that once again it passes through almost straight line curve which again is a virgin compression curve because this soil is undergoing this degree of compression under the stress for the first time. Whereas any soil which starts at this point would have already undergone compression to this level and would have that means experienced a peak compression pressure corresponding to point C. This means that from this graph of loading, unloading it is possible to find out what is the pressure to which a soil has been consolidated in the past.

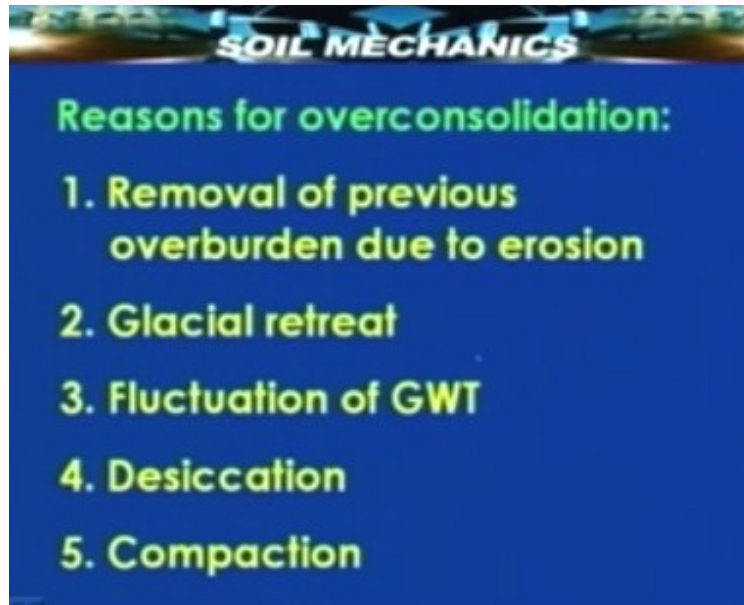
They raise a graphical procedure and this is a very simple graphical procedure. You take the void ratio versus pressure curve plotted to log scale. Draw a tangent at a point corresponding to the maximum curvature. Then also draw the horizontal at that point then the bisector of this angle and where this bisector meets straight line extension of this curve BC that is this point P that gives you the maximum pressure to which the given soil has been subjected to in the past. Now this soil being over consolidated, we can find out what is this value of p_{max} compared to the pressure p that it is now experiencing.

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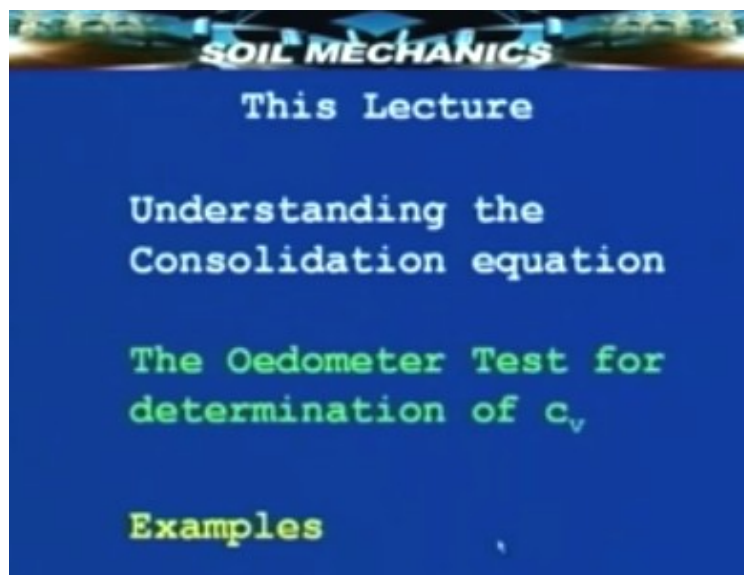
Suppose this pressure p which is experiencing is less than p_{max} then the soil is under consolidated, if this is equal to p_{max} then the soil is normally consolidated, if it is more than p_{max} it is over consolidated or to put it in other words if the maximum pressure to which a soil has been subjected earlier is more than the present over burden pressure then the soil is over consolidated. There are many reasons for over consolidation, I have explain the particularly the glacial retreat as the main reason, there are other reasons as well.

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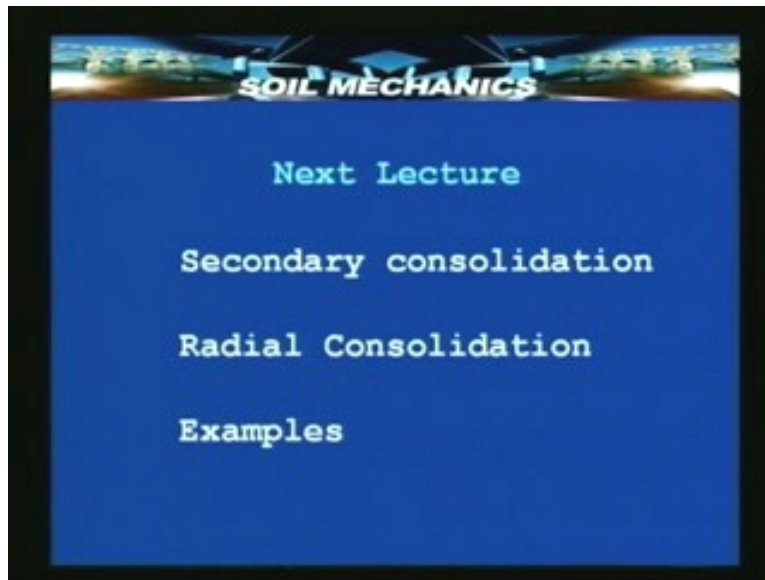


So in this lecture we have understood the importance of the consolidation equation. We have seen how the oedometer test can be used for determination of the coefficient of consolidation and also the void ratio changes and then we have seen a few examples.

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We have closed the lecture with a discussion on precompression and we shall now pass on to some other topics in the next lecture. The topics will be so called secondary consolidation and the radial consolidation which also again we will follow up with a few examples.