Rock Mechanics and Tunneling
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Lecture 37
Slopes

Hello everyone, I welcome all of you to a new module. So, we will start today, module 8, and in this module we will discuss about the slopes, mainly the slope stability analysis, then we briefly discuss about different underground excavations, though about tunnel we will spend a good amount of time in subsequent modules.

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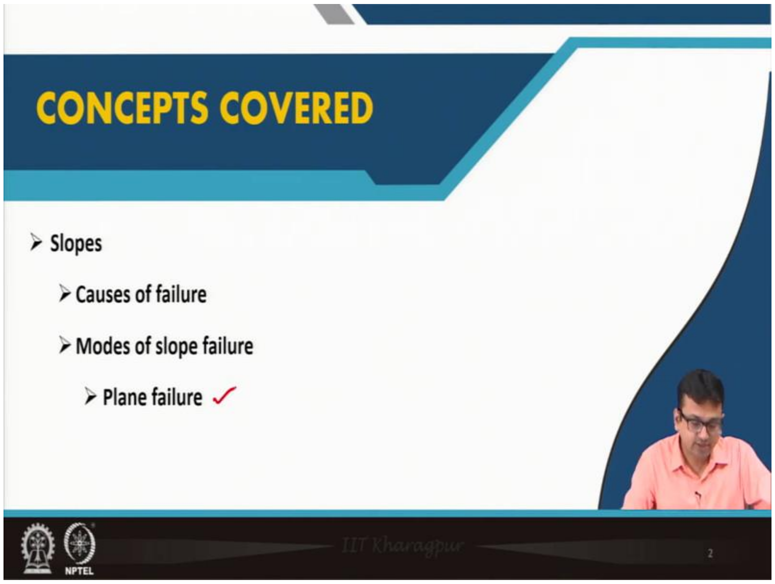

So, today we will start with the slopes, and these are the concepts we will cover today, so basically the causes of failure and then modes of slope failure and we see that there are different types of mode, different modes of slope failures out of that one type that is plane failure we will discuss today.

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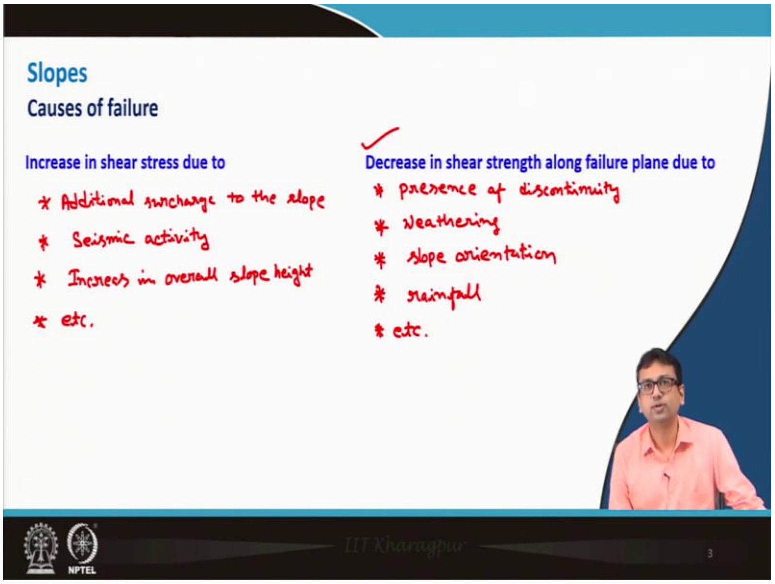

So, generally because of two reasons slope failure happens, one is increasing shear stress due to like additional surcharge if we apply, so let me write down, additional surcharge to the slope. Then obviously, another important reason is due to seismic activities also, the shear stress increases, so seismic activity, then also apart from that, if the overall height of the slope increases then also it increases the shear stress, so increase in overall slope height.

So, likewise, several other reasons are there, for increase in shear stress. So, another important thing is your slope failure can be due to the decrease in the shear strength along failure plane due to, again it may be due to the presence of as we know in case of rock presence of discontinuity planes, plays very important role, so here also if discontinuity plane present there then obviously the shear strength will be less there, so that is why, so presence of discontinuity.

Other than that like the case, suppose the weathering is there, so because of that also strength reduces, also overall this slope orientation that is also important, and because of that also shear strength may reduce and other than that like the presence of water, specifically if the rainfall is there then obviously the water pressure means that will increase as a result of that stress will decrease and because of that also or the slope may fails. So, likewise several other reasons maybe there some of the important points I have discussed over here. Mostly I hope you know these basic things because already you must have done the soil mechanics course. So, basic are same.

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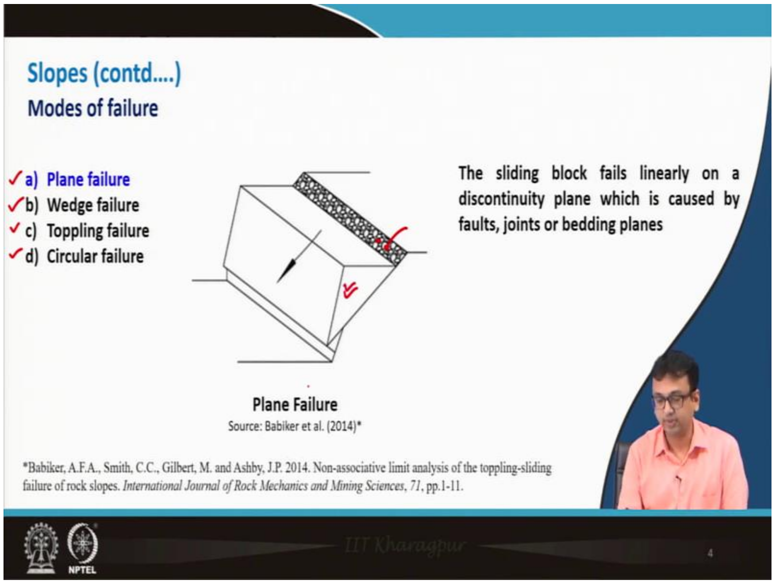

But the thing what you will find new here is most of failure in case of rock there are generally four modes of failure we consider one is plane failure, one is wedge failure, then toppling failure and circular failure. Circular failure is again something what we also have seen in case of soil, so when you are similar to that only but plane failure, wedge failure, toppling failure they are something new what we will see over here. So, we will discuss about everything one by one.

So, this is you see plane failure, you see this is a failure plane you see and so let us see what it says, the sliding block wall, so this is your sliding block, sliding block fails linearly on a discontinuity plane which is caused by faults, joints or bedding plane. So, this may be this discontinuity plane maybe due to the joints or faults or bedding planes, and this is the sliding block. So, this way it fails simply.

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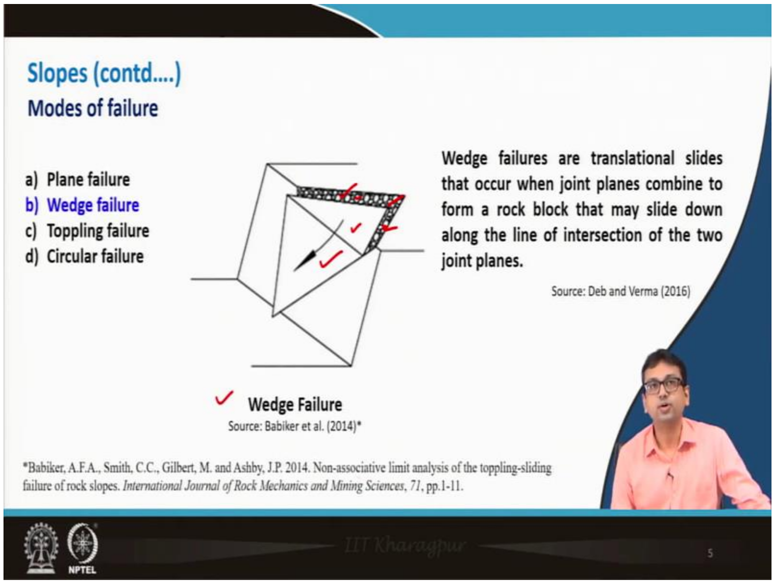

So, now second is wedge failure. So, in wedge failure you see a wedge like this actually fails, so what happens there? So, wedge failures are translational slides that occur when joint planes combined to form a rock block that may slide down along the line of intersection of the two planes. So, you see you can assume this is as one plane, this as one plane and this is suppose your line of intersection of these two planes.

So, now if we read it once again, the wedge failures are translational slides, so it will just simply slide along this line, so wedge failures are translational slides that occur when joint planes combined to form a rock block that may slide down along the line of intersection of the two joint planes. So, this is the line of intersection of the two joint planes, let us see, suppose it is a and b plane and this is the line of intersection, and this is the rock block that is failing, this is our wedge failure. Now, next one is toppling.

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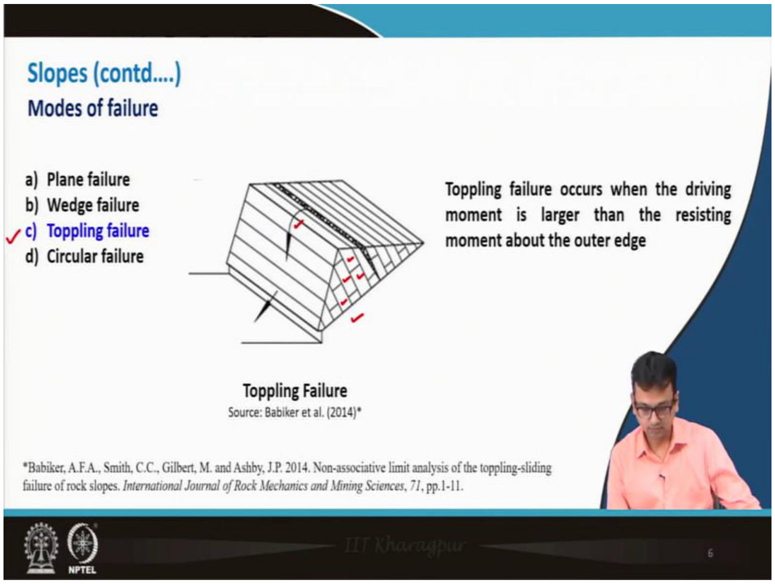

So, this is our toppling failure you see the diagram from here you can get some idea small blocks are there you see. So, toppling failure occurs when the driving moment is larger than the resisting moment about the outer edge. So, you see these are you see this if you look at this arrow you can understand it is this top link, so this is another type of failure we can observe in case of rock slope.

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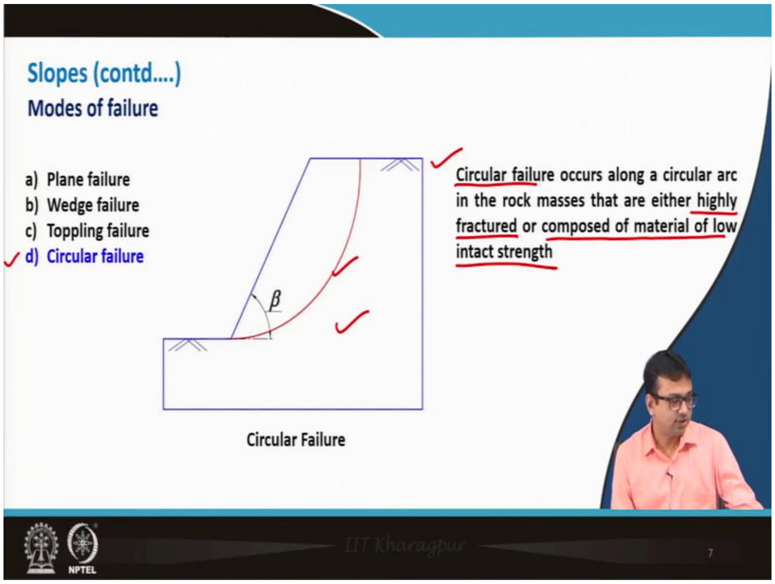

So, now this is something what you have learned earlier also circular failure, so this is quite common in case of soil slope also, so circular failures this occurs circular failure occurs along a circular arc in the rock masses that are either highly fractured or composed of material of low intact strength, so then only means if it is highly fractured or compost of material of low intact strength then obviously though this is the rock mass, it will it behave something like similar to the soil mass.

So, that is why the circular failure occurs along a circular arc as you can see, this one, so in the rock masses that are either highly fractured or composed of material of low intact strength, so there we can observe circular failure.

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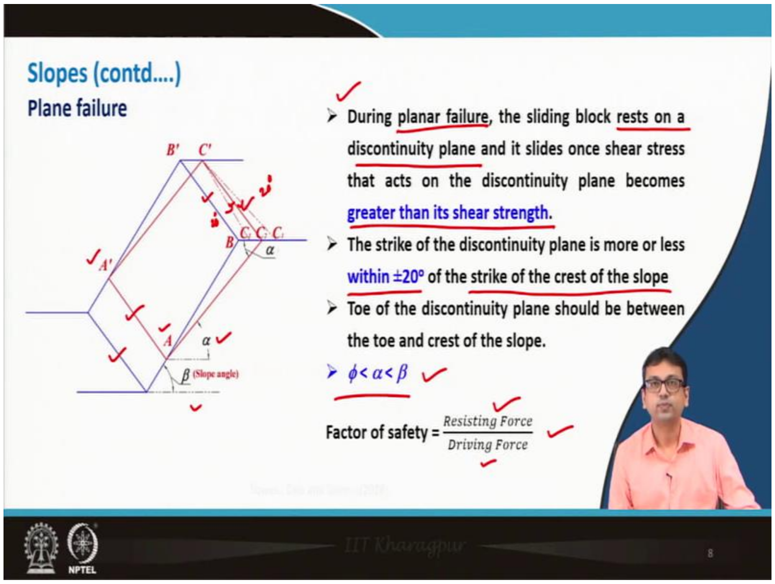

So, now today we will focus on the plane failure. So, there are some of the important points or conditions what we need to understand when slope this kind of plane failure occurs, so few points are like this. So, first one is during planer failure the sliding block rests on a discontinuity plane and it slides once shear stress that act on the discontinuity plane becomes greater than its shear strength.

So, that is important and quite obvious reason. So, during plane failure the sliding block rests on a discontinuity plane as I have shown you earlier also when I have shown you the first picture plane failure, so sliding block rest on a discontinuity plane and it slides on one shear stress that acts on the discontinued plane becomes greater than its shear strength.

Now, next point is what the strike of the discontinuity plane is more or less within plus minus 20 degree of the strike of the crest of the slope. So, crest of the slope is what here? You see this is my crest of the slope means here what we can say is for my diagram, this is the crest of the slope B B’, so the strike of the crest of the slope is obviously, we can find very easily, now what it says the strike of the discontinuity plane maybe suppose it can be C’ C1 or C’C2 or C’ C3. C’C2 is almost parallel to you see B’ B.

But they are and C’C1 and C’C3 they are at some angle, so what it says the strike of the discontinuity plane is more or less within plus minus 20 degree of the strike of the crest of this slope. So, if suppose this is 20 degree and this is 20 degree under that condition this plane failure may occur and then the thing is the toe of the discontinuity plane should be between the toe and crest of the slope.

So, toe of this discontinuity plane is where? AA’, this or A’A whatever this line here the toe, whereas the toe of the slope is where this is the toe of the slope and crest is as we have discussed B’B is the crest. So, now what we can see? This toe of the discontinuity plane is in between this crest and toe of the slope.

So, that is also another condition to occur the plane failure, another condition is ϕ that is the angle of internal friction should be less than α, α is this angle as we can see the and the β is this angle as you can see, so β is nothing but the slope angle and α is the dip of the discontinuity plane. So, this condition also needs to satisfy that is $\phi < \alpha < \beta$. And factor of safety which is very important to find out that is nothing but the same expression we have to find out the resisting force and we have to divide it by the driving force then only we will get the factor of safety.

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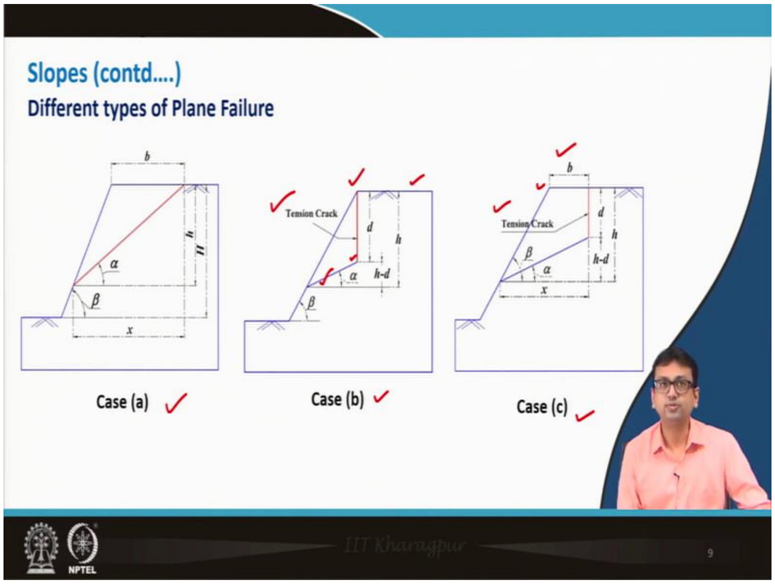

So, now different types of plane failure, there will be actually different types, so here I have considered three very common and important type, so this is the first case, this is the second case, and this is the third case. So, what are the difference we can notice that you see here it is directly this if I see this see the one another thing we have already discussed about the plane strength condition, so remember that have at the time also discussed the slopes or embankments all these things can be as analyzed as a plane strength problem, so and we did detailed discussion there, so I am not going there again.

So, anyway so as we know the slope may be analyzed as a plane strength problem, so if you see over here, so this is a triangle you can see and here this is also triangle but you see here is a tension crack is developing over here, this line is not extending up to the ground surface like these up to these, so here some tension crack is developing which is quite possible in case of rock slope or you see is here the tension crack is where in I mean starting from the crest and bending over here.

But it may happen that this tension crack maybe at a distance b from the crest. So, this is also a tension crack. So, under that condition how to find out the factor of safety also you should know, so as I am telling that there may be few other modes also or different types of plane failure, but these three are quite common, so, I will discuss about these things now. And we will try to derive the expressions for factor of safety.

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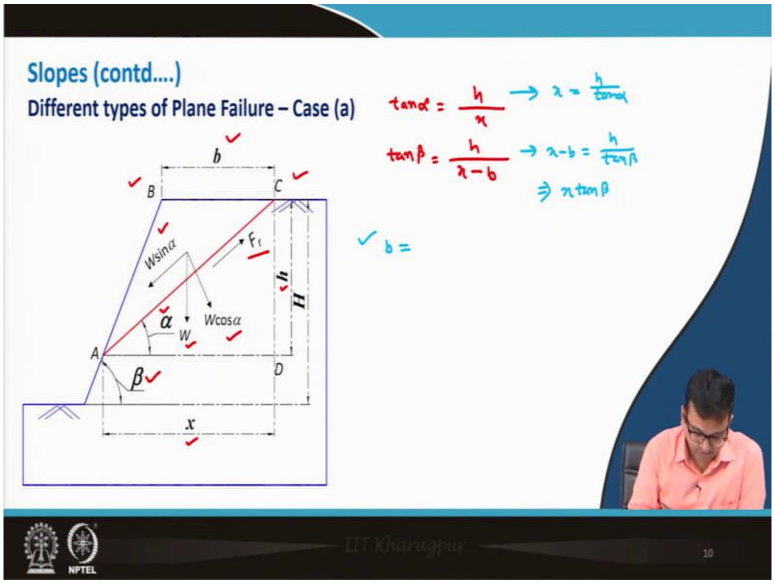

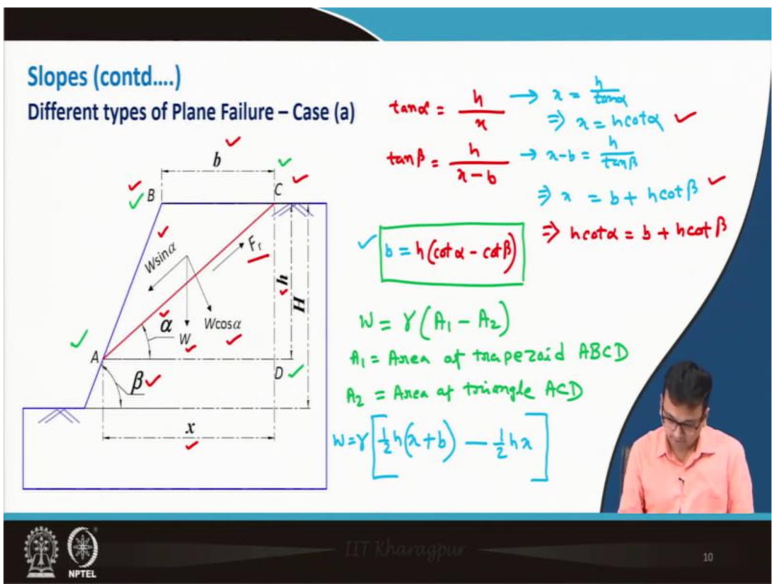

So, this is suppose the first one, what is happening you see the same diagram is again drawn somehow with little more details, so β is my slope angle, α is the dip of the discontinuity plane as we have discussed, b is the this distance from crest to the this crest at B to the C where this discontinuity plane is meeting with the horizontal plane and what we can see if W is the weight of this sliding block then obviously this component is W$\cos\alpha$ and this is W$\sin\alpha$.

So, and this F_r is nothing but the resisting force and obviously here driving forces nothing but W$\sin\alpha$, so now here it is very simple we need to may only find out the expression for weight W, and then we can very easily obtain this factor of safety. So, what we can see from this diagram that we can easily write that $\tan\alpha$ is equal to what? Here it is nothing but h and if it is x, it is h/x.

Similarly, tan beta is equal to what $\tan\beta$ is equal to h and you see and h by total is x, so x-b, so $\tan\alpha$ is h/x and $\tan\beta$ is nothing but you see this is my β angle, so this total angle, slope angle is by β, so it is nothing but h/x-b. Now, what we can little bit do further simplify this one, so maybe let me use some other color.

So, let me then let me write down few other things like b, b is equal to what? b is nothing but okay before b maybe we can simplify it a little bit, so that is what from here you see from here we can write x = $h/\tan\alpha$ and also what we can write this from here is x-b is equal to $h/\tan\beta$, some actually I am interested in obtaining expression for b.

So, what we can do from here, we can see simply play with this equation, so we can write it as x $\tan\beta$ or maybe let me take simply b in the right hand side, so we can write it as simply x is equal to $b+h\cot\beta$ and also from here we have seen x is equal to this, so which is nothing but from here we can write x is equal to $h\cot\alpha$.

So, now using these two this is x this is also x, so what we can write that we can from here we can very easily find that h cot alpha is equal to $b+h\cot\beta$, so that means b is nothing but equal to $h(\cot\alpha - \cot\beta)$. So, this we can keep within a box maybe, so b we could able to find out with the given information like height, this h height and α angle and the β angle with that we can obtain the expression for b.

Now, let us try to see what is w. The w is nothing but the weight of this block. So, this means we have to just simply find out the area of this block, so area W is nothing but w is γ(A1-A2). And what is A1 here? Suppose A1 is the area of area of the trapezoid suppose ABC and D, so this ABCD trapezoid.

So, area of the trapezoid, ABCD and A2 is the area of the triangle this triangle AC and D. So, now let us write down the expression for W let me use some other color maybe, so W is now equal to γ into area of the trapezoid, so ABCD, so it is what half this distance is h into total is x plus this is x and this is b, so x plus b. And the area of the triangle ACD is equal to half height into x, this is what we will get. Now, if we further simplify maybe let us go to the next slide.

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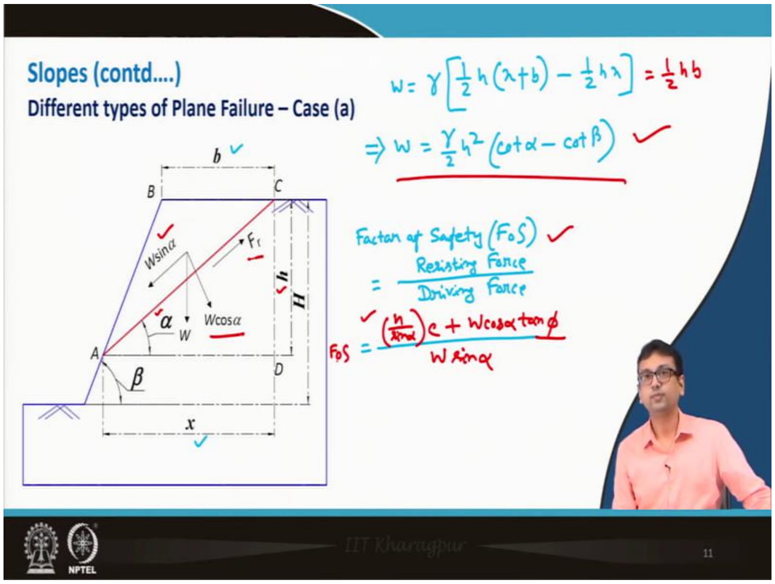

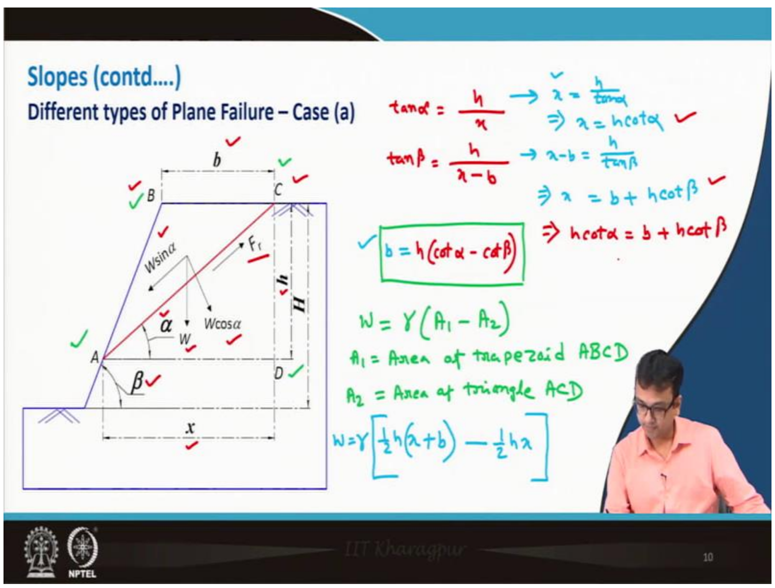

So, let me write down over here once again, w is equal to gamma into half h x minus b sorry x plus b minus, this one minus half h into x. So, now further simplification if we do, so what we will get? So, we have b from here and x we know from here, so if we further simplify this one we should get it as w is equal to $\gamma\, h^2(\cot\alpha - \cot\beta)/2$, so this is what you will get if you further simplify this equation.

Now, but we know the factor of safety is what it is nothing but the safety maybe we will use FoS is equal to the resisting force by driving force, so now here resisting force is what? So, resisting force is if we consider F_r, F_r as it is shown over F_r is equal to what? So, you see one component rating comes due to the presence of cohesion here, so for that the length of this AC and if we call it is a plane strength problem out of plane if we consider one, so basically area of this discontinuity plane will be the length AC into 1, so length AC is equal to what? Length AC is equal to we can write it as simply like it is my h and this is my alpha angle which is nothing but equal to $(h/\sin\alpha)c+W\cos\alpha\tan\phi$.

So, obviously the due to the presence of the angle of internal friction we will get some resistance from this normal force also, so that is nothing but equal to $W\cos\alpha\tan\phi$, suppose ϕ is the angle of internal fiction and driving force, so driving forces what? It is this nothing but this $W\sin\alpha$. So, now you see we know h, we know c, suppose if c is 0 then obviously this component will be 0.

And c is non-zero then we have to consider it, then this is $W\cos\alpha\tan\phi/W\sin\alpha$. So, this way we can simply get the factor of safety. So, now if we and obviously W is already we have obtain which is most important thing we have obtained from this very easily. So, maybe here one step I can probably I can write, half this one obviously one is getting cancelled, so half h into b, half h into b from there it is coming out to be this. Because b already we have seen means that $h(\cot\alpha-\cot\beta)$, so from there this one is coming, this is required for finding out the factor of safety. So, now this is the first case.

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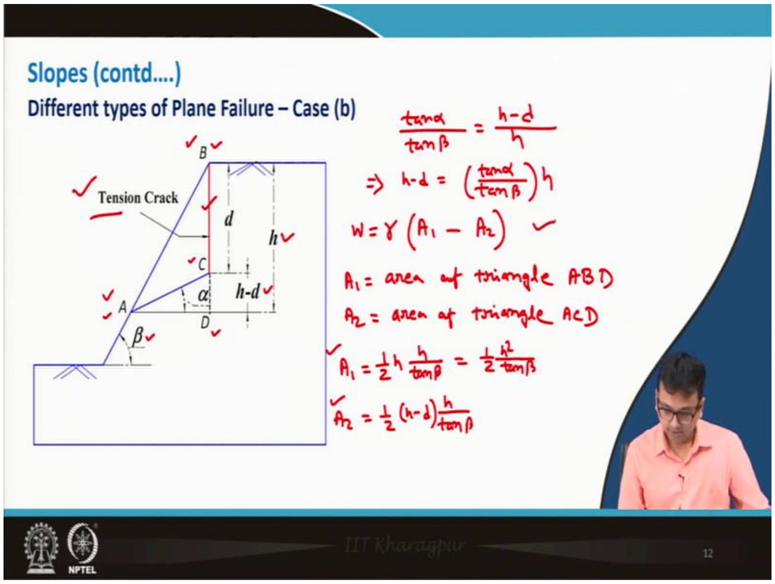

Now, let us come where we have this tension crack is present. So, tension crack is for the first case, it is you see that this is the meeting at the crest, so what is happening because of the presence of tension crack my size of my block is reducing. And now again the main thing is we it is factor of safety nothing but the resisting force by driving force, so now here also driving force will remain like Wsinα, and what is the resisting force? Resisting force will be nothing but the similar to that only you see equation will actually the since this is the tension crack you see their tension crack is there, so cohesion will not play any role.

So, the length this AC we have to find out and that multiplied with c plus w, Wcosαtanϕ/Wsinα that will be my expression, there is no confusion. So, now anyway let us try to see what is the weight of this block. So, from here what we can see that you see directly we can write tanα by suppose tanβ is equal to what, this is nothing but (h-d)/h, that we can very easily, so now from here what we can do h-d can be written as (tan α/tanβ)h, no confusion.

Now, again we have to find out the weight of the this block, so again W is nothing but gamma into the area of the this big triangle, suppose let me write it as A1-A2, so here A1 is nothing but equal to the area of this triangle, this AB and D, so area triangle ABD and A2 is the area of triangle AC and D. So, area of triangle AC and D. So, now what is A1? A1 is very simple, it is nothing but half height is h and this part, so this is nothing but how much? This nothing but you see it is h and this angle is β, so it is nothing but h/tanβ.

So, basically it is half h square by tan beta. Similarly, A2 is equal to the area of triangle ACD, so ACD is nothing but half this distance, the distance to C and D is h minus d, so h minus d into the AD distance, so AD is how much? Again same h by then tan beta, so that is my A2 angle A2 area, A1 area is these, A2 areas these, now we can very easily find out w. So, let us do that.

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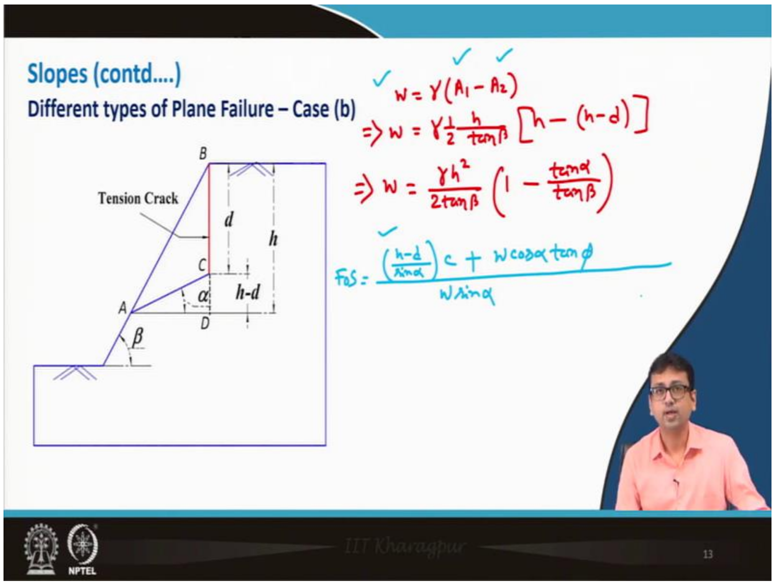

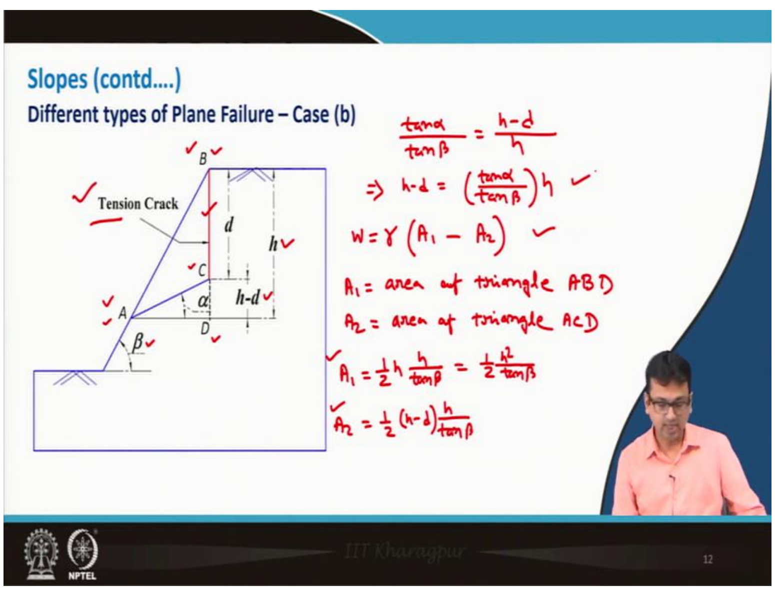

So, next is as we know W is equal to γ(A1-A2), so from here when W is equal to γ simplify we will get it as $\gamma h/2\tan\beta$ and here we will get it as h-d. So, if we simplify we should get it as W is equal to $\gamma h^2/2\tan\beta\ (1-\tan\alpha/\tan\beta)$. So, you see $\tan\alpha$ by $\tan\beta$ already obtain, there is nothing but h minus d by h or h-d is equal to (tanα/β)h. So, that is only what we have used here and that is obtain our weight w.

So, now if I use another color, so finally what we can write factor of safety is equal to, now if AC distance we have to find out, AC is nothing but you see h minus d by h minus d this by my sin alpha into correlation c plus w cos alpha tan phi, so in here it should be w sin alpha. So, simple and no confusion I hope you have understood the derivation also, so this way whatever may be the means though we are discussing three conditions.

But they all I hope you will get even if some other type of failure if you have to analyze then also you should be able to do it because a basic approach is very simple we have to find out the way for that you may have to use the area of trapezoid and then minus area of the triangle or here what we have done we have deducted area ABD from there we have deducted ACD and we have obtain the multiplied that we the gamma we have obtain the w and the length over which the coefficient equation is acting that we are to find out and this way you can very easily obtain the factor of safety.

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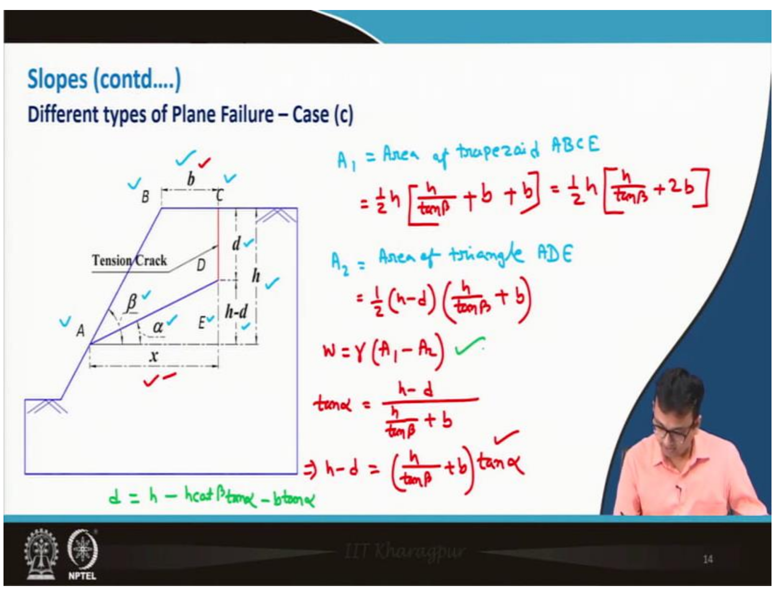


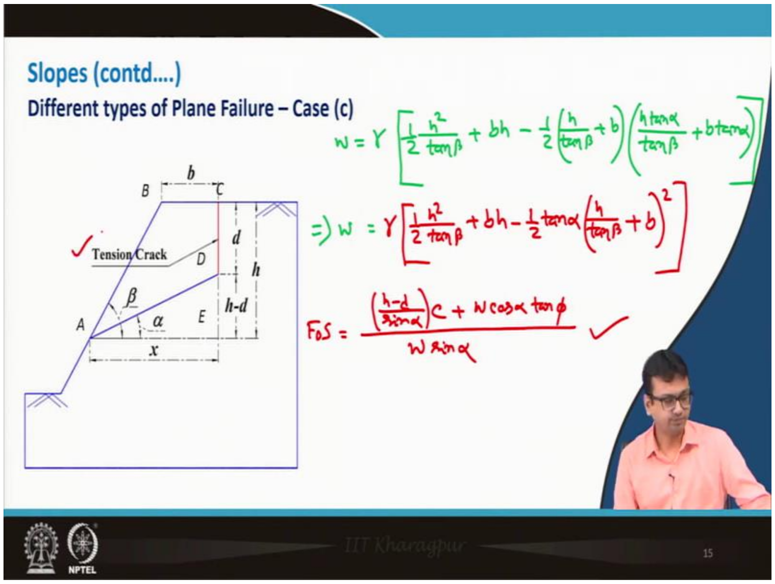
Now, just to understand another case also what we can see here the tension crack is at a distance b from the crest and remaining things or notation are same, it is h, it is h-d, obviously it is d, β is the slope angle and this is your α angle, so with this let us see how can we derive the expression for the factor of safety.

So, here we write down suppose the area of this A1, suppose A1 is the area of trapezoid suppose ABCE, so what it will be? So, this is nothing but equal to (h/2)[(h/tan β)+2b].

Now, similarly if I am interested I means I need another area also A2, A2 is the area of ADE, so area of triangle ADE is nothing but equal to half distance now this height is h-d and this x is again same, so x is again (h/tan β)+b, this is my area of triangle ADE. And W is equal to A1-A2.

Now, another thing what we can notice from here is suppose the tanα, tanα is nothing but equal to (h-d)/x, so x is again we know it is nothing but (h/tan β)+b. So, this is my tanα, so from here I can little bit play with this one and we can obtain h-d=[(h/tan β)+b]tanα, that we can write form here.

So, also you see from here we can also get some expression for d also, if you see if you take this entire thing on the left hand side you can get the and d in the right hand side you will get the expression for d also. So, that is one important thing also, maybe we can write over here, so maybe let me use some other color to write the d, so d=h-[(hcotβ)+b]tanα. Anyway we are interested in h-d, so let us see what we can do further because we have to ultimately obtain this one, and we have obtained A1 A2. (Refer Slide Time: 39:55)



So, if we see now,

$$W = \gamma\left[\frac{1}{2}\frac{h^2}{\tan\beta} + bh - \frac{1}{2}\left(\frac{h}{\tan\beta} + b\right)\left(\frac{h\tan\alpha}{\tan\beta} + b\tan\alpha\right)\right]$$

$$W = \gamma\left[\frac{1}{2}\frac{h^2}{\tan\beta} + bh - \frac{1}{2}\tan\alpha\left(\frac{h}{\tan\beta} + b\right)^2\right]$$

so now we can write factor of safety is equal to again the in the resisting force we will have the contribution from the equation part acting along AD, so length AD or the area of that plane is that AD into 1 is nothing but equal to you see it is,

$$F = \frac{[(h-d)/\sin\alpha]c + W\cos\alpha\tan\phi}{W\sin\alpha}$$

So, that is what result, so W we know, α if it is known and ϕ and c are known then we can easily find out the factor of safety of this failing of this slope, where this type of tension crack is developing. So, okay, thank you, let us conclude here today. In our next class we will start with the problem on plane failure thank you.