Rock Mechanics and Tunneling
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Lecture 55
Basic Concepts for Lined, Unlined and Pressure Tunnels

Hello everyone. I welcome all of you to the fourth lecture of module-11. In Module-11, we are discussing about the analysis of stresses in tunneling, and today we will discuss about the basic concepts of lined, unlined and pressure tunnel. Basically, this topic we will continue in my fifth lecture also. So, combining these two lectures, I will cover these basic concepts of lined, unlined and pressure tunnels.

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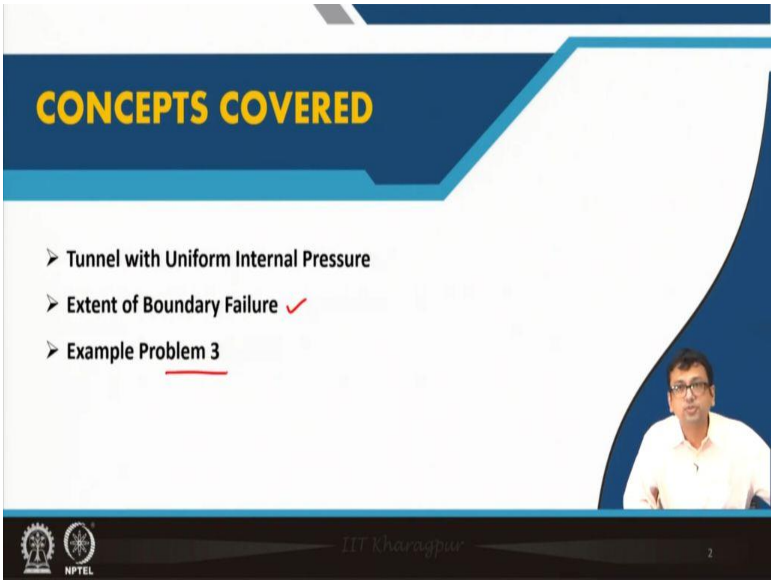

So, we will first discuss about the tunnel with uniform internal pressure, then extent of boundary failure, this is extremely important as far as tunnel design is concerned and related to that we will solve one problem also.

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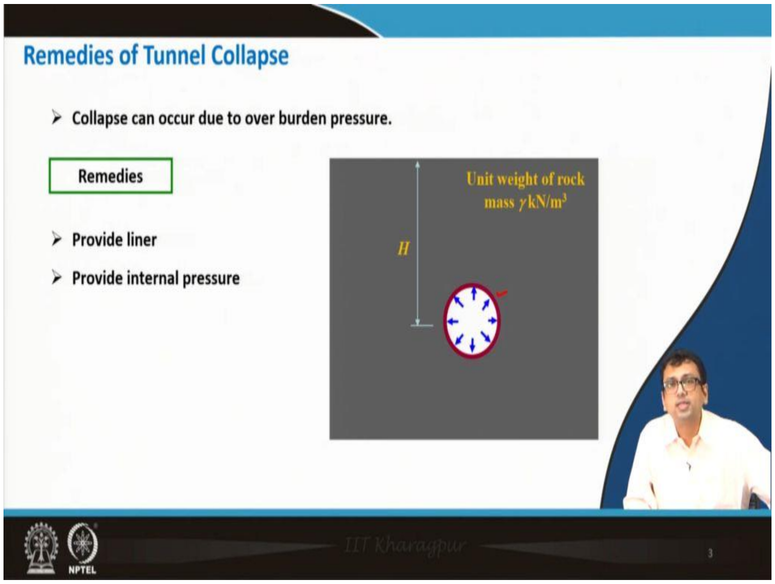

Collapse can occur due to overburden pressure. And you know different stresses are there, because of which it may fail. Remedies are to provide liner or provide internal pressure or both. So, these two are the simple solution what comes to our mind immediately. So, let us learn more about this these remedies.

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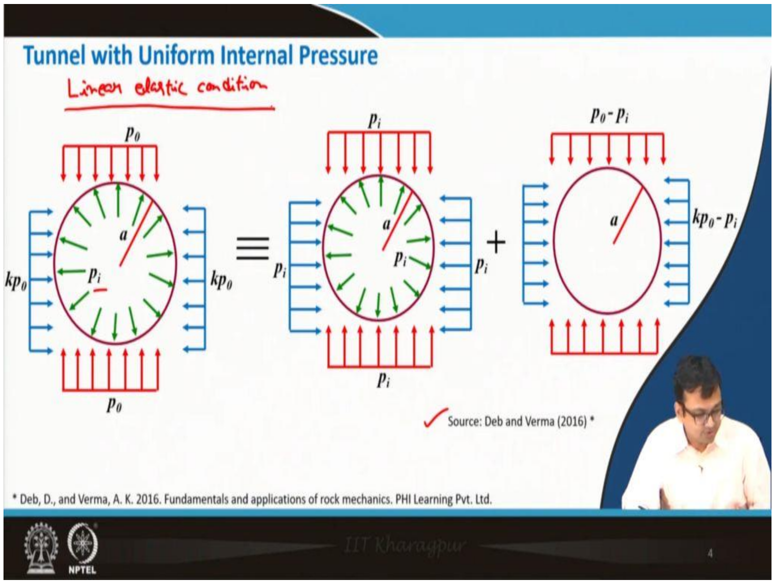

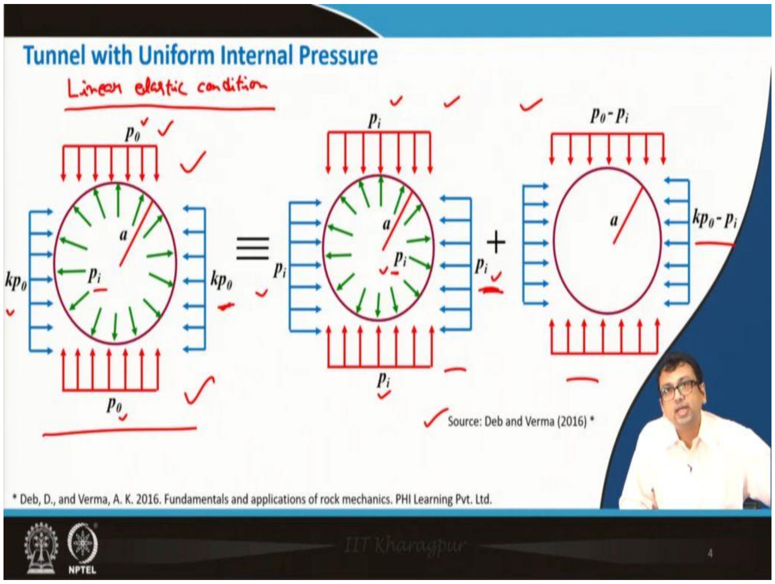

So, first tunnel with uniform pressure. So, assume a tunnel with radius a. Now, let us consider internal pressure p_i is required to stabilize the tunnel, against p_o and kp_o pressure. Earlier when we discussed about Kirsch's equation, p_i term was not present there. Means there was no uniform internal pressure applied there. So, now what to do? We can still solve this problem with our obtained knowledge. We can represent this problem using two figures as shown in the slide and you can definitely refer Deb and Verma's book.

So anyway, let us try to understand why we can do it. You see as it is under linear elastic condition, so linear elastic condition. In Kirsch's equation it was considered continuous, homogeneous, isotropic, and linear elastic material. So, since it is under linear elastic condition, we can apply the method of superposition very easily. This can be to see represented with a figure shown in the slide.

So, we can divide this problem into two parts, two equivalent configurations. If we add the two, we will get the desired case of tunnel with uniform internal pressure. Now, basically we have to ultimately use Kirsch's equations to get an expression for this.

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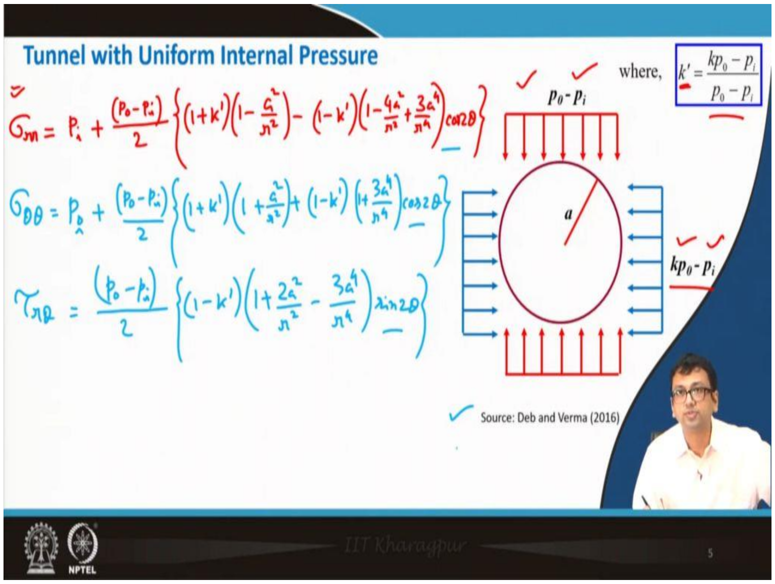

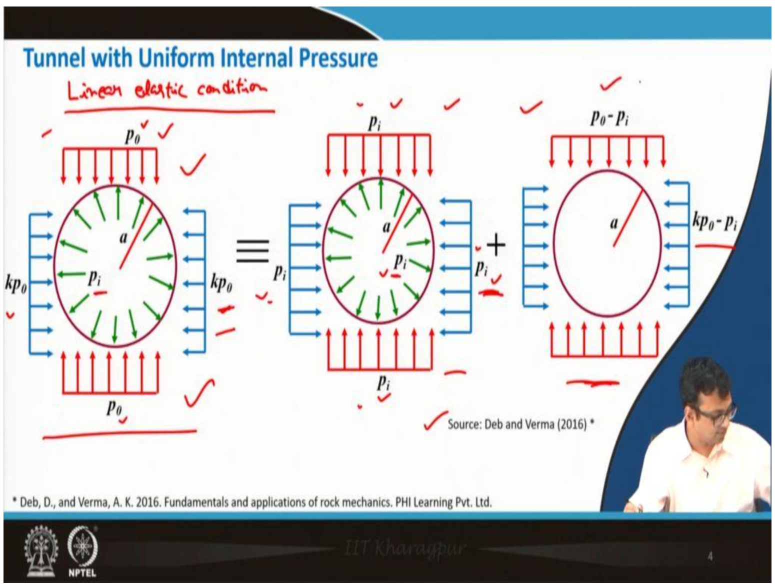

Now, from the second configuration,

$$k' = \frac{kp_o - p_i}{p_o - p_i}$$

$$\sigma_{rr} = p_i + \frac{p_o - p_i}{2}\left\{(1+k')\left(1-\frac{a^2}{r^2}\right)-(1-k')\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\sigma_{\theta\theta} = p_i + \frac{p_o - p_i}{2}\left\{(1+k')\left(1+\frac{a^2}{r^2}\right)+(1-k')\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\tau_{r\theta} = \frac{p_o - p_i}{2}\left\{(1-k')\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right\}$$

For a special case of $k'=1,\ \theta=0$

$$\sigma_{rr} = p_i + (p_o - p_i)\left(1-\frac{a^2}{r^2}\right)$$

$$\sigma_{\theta\theta} = p_i + (p_o - p_i)\left(1+\frac{a^2}{r^2}\right)$$

$$\tau_{r\theta} = 0$$

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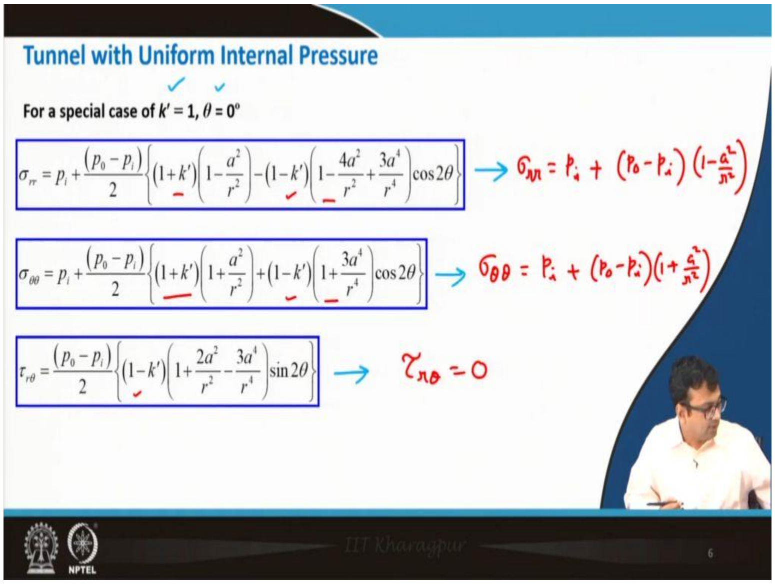

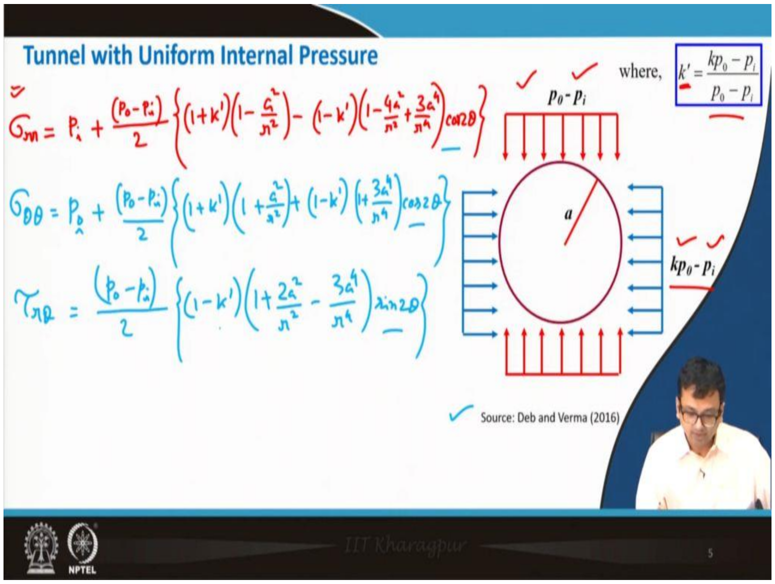

Now, one very important thing we should discuss that is prediction of extent of boundary failure.

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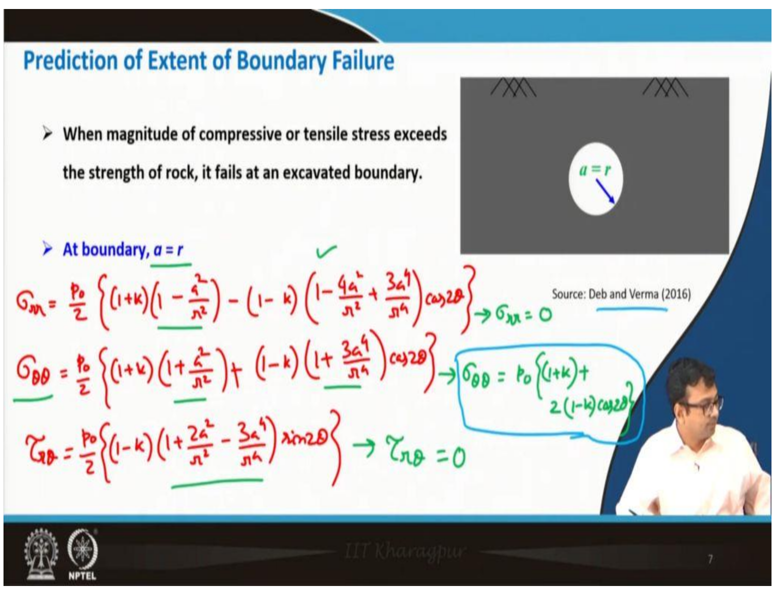

Suppose there is a tunnel, and at boundary a = r, no confusion. Now, when the magnitude of compressive or tensile stress exceeds the strength of rock, it fails in an excavated boundary. Now, we know

$$\sigma_{rr} = \frac{p_o}{2}\left\{(1+k)\left(1-\frac{a^2}{r^2}\right)-(1-k)\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\sigma_{\theta\theta} = \frac{p_o}{2}\left\{(1+k)\left(1+\frac{a^2}{r^2}\right)+(1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\tau_{r\theta} = \frac{p_o}{2}\left\{(1-k)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right\}$$

Now, for a = r,

$$\sigma_{rr} = 0$$

$$\sigma_{\theta\theta} = p_o\left\{(1+k)+2(1-k)\cos 2\theta\right\}$$

$$\tau_{r\theta} = 0$$

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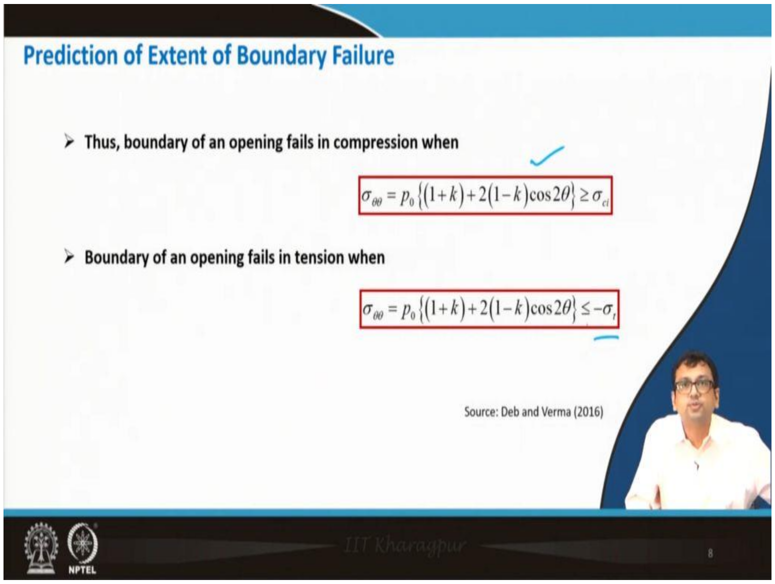

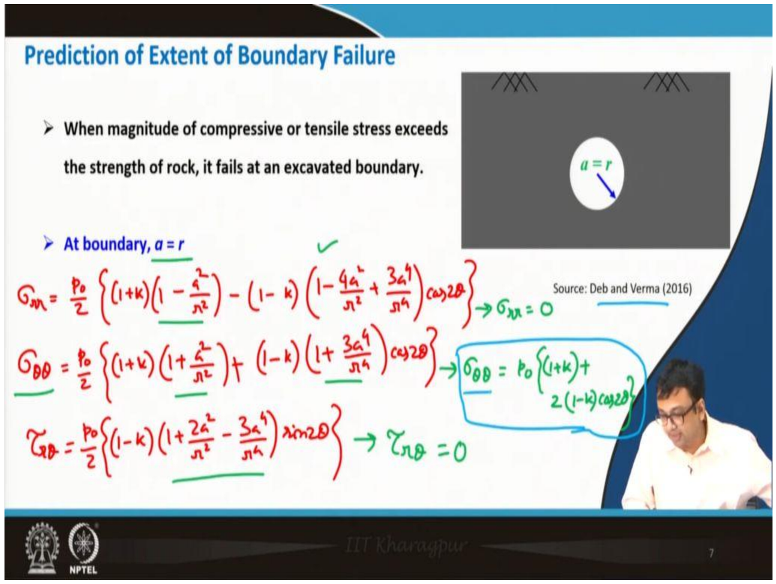

Thus, boundary of an opening fails in compression, when

$$\sigma_{\theta\theta} = p_o \left\{ (1+k) + 2(1-k)\cos 2\theta \right\} \geq \sigma_{ci}$$

i.e. when the generated compressive stress is more than the compressive strength of the rock.

Similarly, boundary of an opening fails in tension, when

$$\sigma_{\theta\theta} = p_o \left\{ (1+k) + 2(1-k)\cos 2\theta \right\} \leq -\sigma_t$$

As compression is positive here. So, tension has minus sign.

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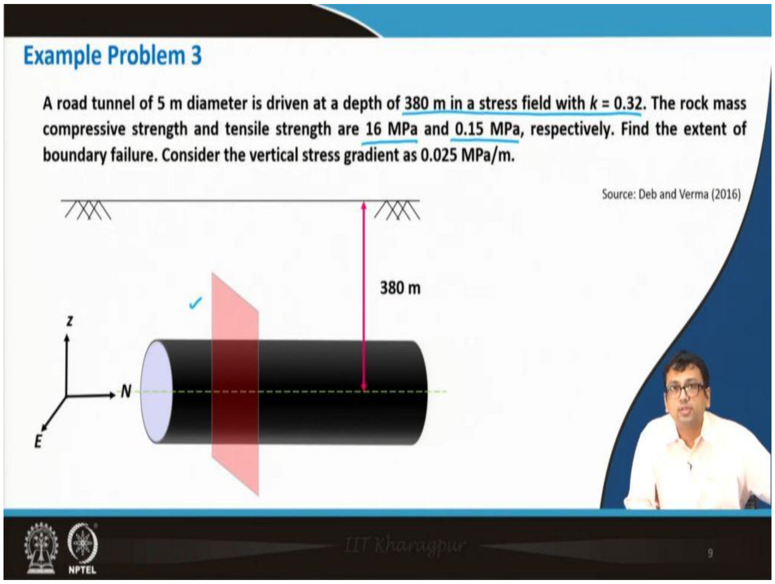

So, now let us solve a problem and let us clear our concept.

The problem says a road tunnel of 5-meter diameter is driven at a depth of 380 meter in a stress field with k = 0.32. The rock mass compressive strength and the tensile strength are, 16 MPa and tensile strength is 0.15 MPa respectively. Find the extent of the boundary failure. Consider the vertical strength gradient of 0.025 MPa/m.

Solution:

(Refer to figure in the slides)

We can consider this problem as a plane strain problem. The vertical stress gradient is provided.

k = 0.32

Vertical stress at 380 m, $\sigma_v = 0.025 \times 380$ MPa = 9.5 MPa

Horizontal stress, $\sigma_H = k \times \sigma_v = 0.32 \times 9.5$ MPa = 3.04 MPa

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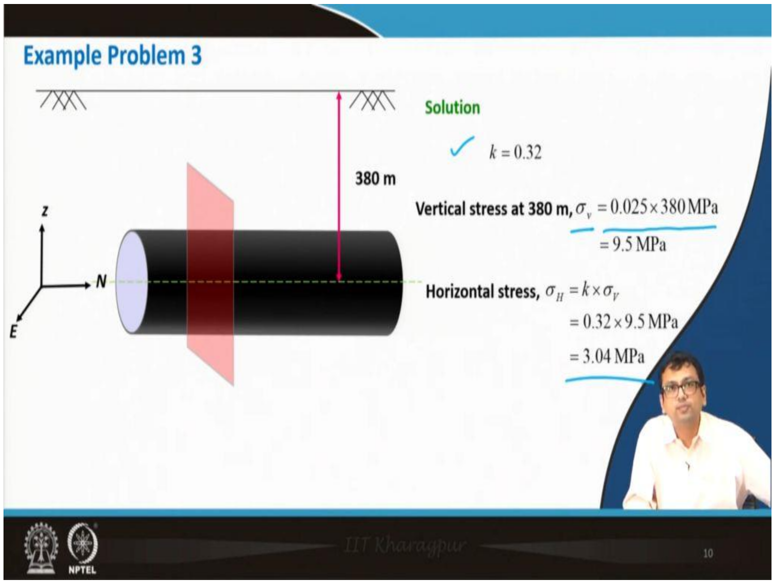

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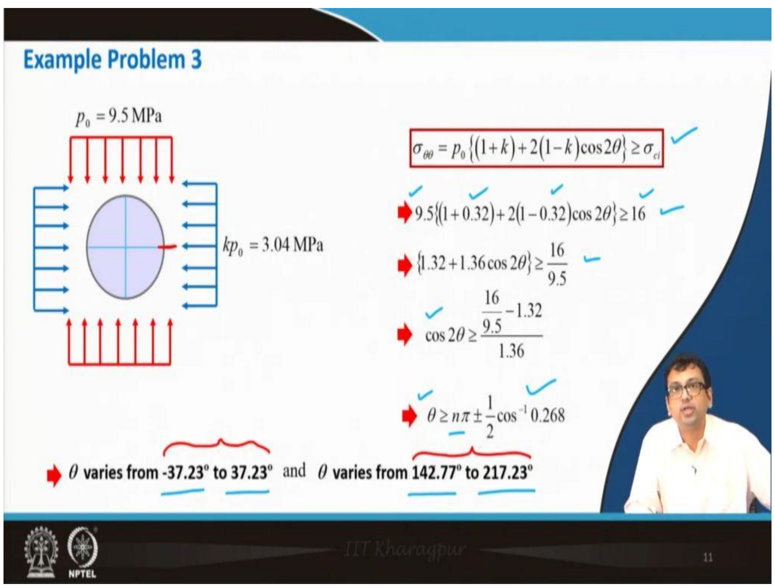

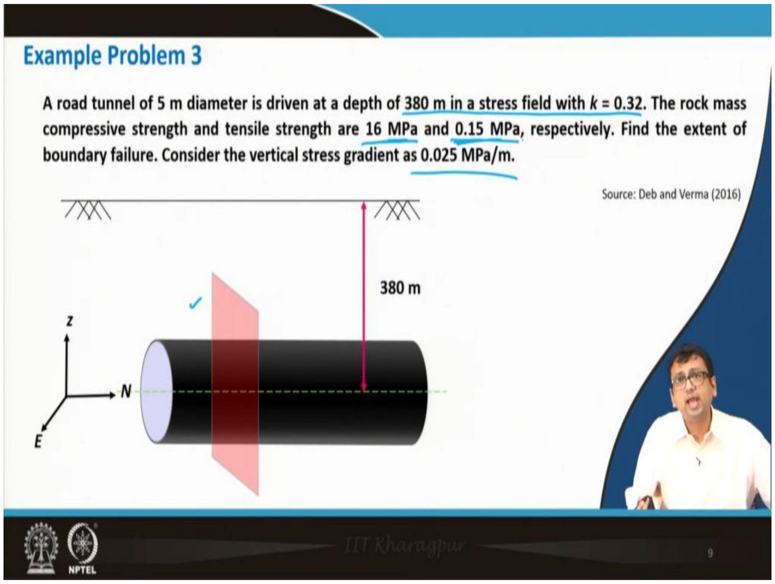

Now, now let us go for the plane strain analysis,

$p_o =$ 9.5 MPa

kp_o = 3.04 MPa

Now, we just have learnt about the condition, i.e.

$$\sigma_{\theta\theta} = p_o\left\{(1+k)+2(1-k)\cos 2\theta\right\} \geq \sigma_{ci}$$

if the above condition satisfies, then there will be some boundary failure due to compression.

So,

$$\sigma_{\theta\theta} = 9.5\left\{(1+0.32)+2(1-0.32)\cos 2\theta\right\} \geq 16$$

From here we can obtain the expression for theta θ

$$\cos 2\theta \geq \frac{\frac{16}{9.5}-1.32}{1.36}$$

$$\theta \geq n\pi \pm \frac{1}{2}\cos^{-1} 0.268$$

n is either 0 or 1,

Therefore, θ varies from -37.23º to 37.23º and θ varies from 142.77º to 217.23º

Refer to the figure in slide 13 for understanding of the compressive regions with varying θ value.

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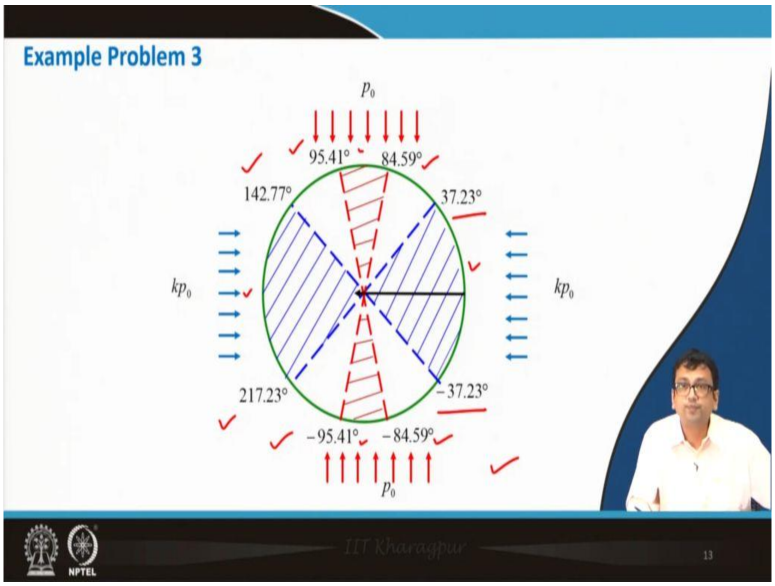

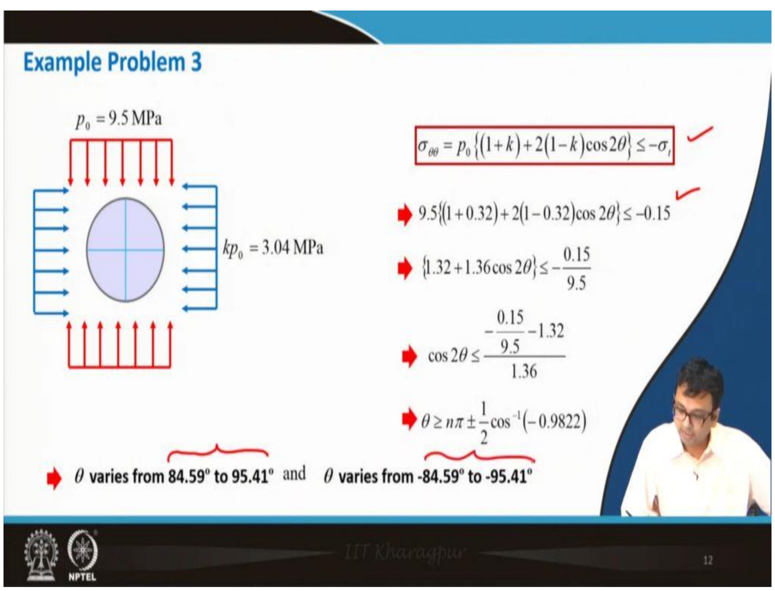

Now, similarly we can see it for the case of tension also

$$\sigma_{\theta\theta} = p_o\left\{(1+k)+2(1-k)\cos 2\theta\right\} \le -\sigma_t$$

$$9.5\left\{(1+0.32)+2(1-0.32)\cos 2\theta\right\} \le -0.15$$

$$\cos 2\theta \le \frac{-\frac{0.15}{9.5}-1.32}{1.36}$$

$$\theta \ge n\pi \pm \frac{1}{2}\cos^{-1}(-0.9822)$$

Therefore, θ varies from 84.59º to 95.41º and θ varies from -84.59º to -95.41º.

It is very important, while you design the liner.

So, depending on the compressive strength and tensile strength of the rock mass if you perform this simple small exercise, you will be able to find out in which region there is a chance of potential failure.

After identifying the regions of potential failure, you can provide your liner. Type of liner to be used can be decided very easily if you can get plot diagram as shown in slide 13.

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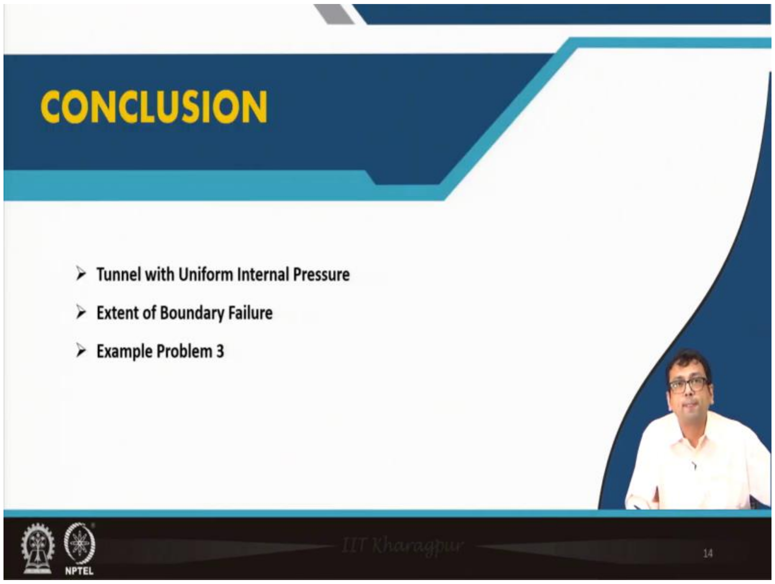

So, with this I am concluding my today’s lecture. Thank you.