Introduction to Soft Computing
Prof. Debasis Samanta
Department of Computer Science & Engineering
Indian Institute of Technology, Kharagpur

Lecture - 29
Pareto-based approaches to solve MOOPs

In the last lecture, we have learnt non-Pareto based approach which is also a posterior approach called the Vega. In this lecture we will learn about other Pareto based approaches, the first we will discuss about the MOGA. The MOGA short form it is called multi objective genetic algorithm, it is a Pareto based approach and also it is a aposteri based approach, because no prior knowledge is required to solve this problem.

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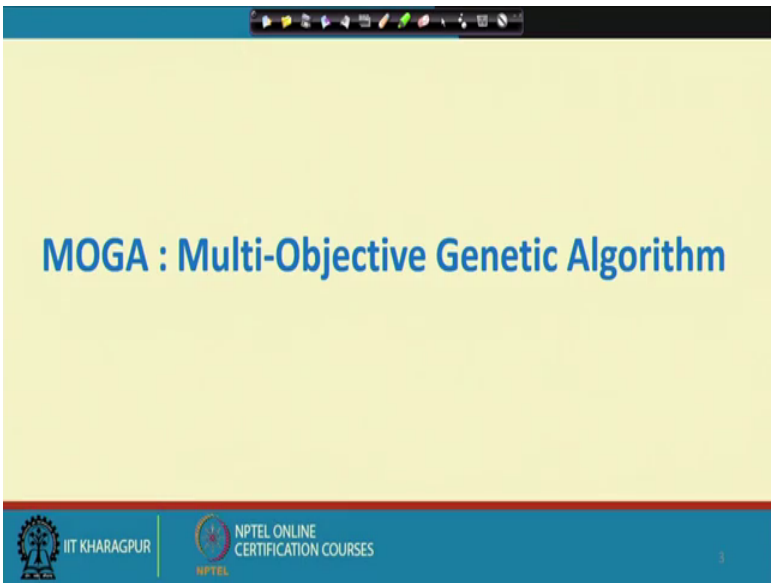

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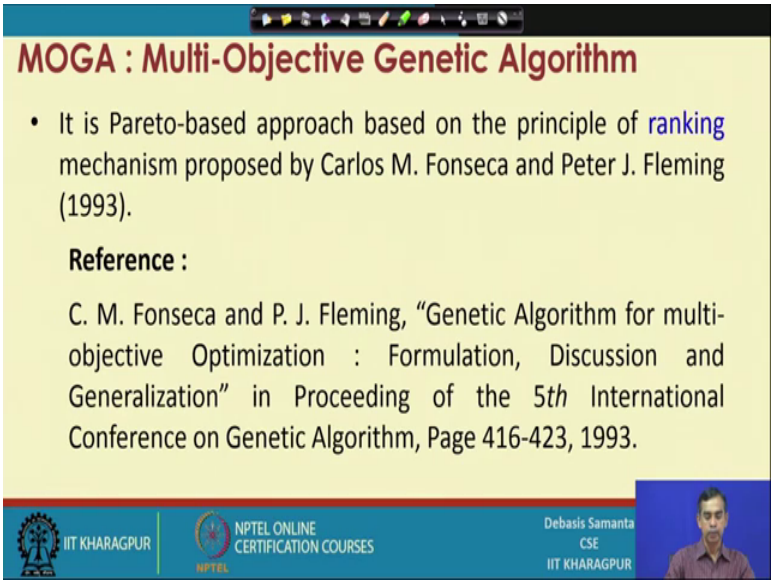

Now, this approach this Pareto based approach first time proposed by Fonseca and Fleming in 1993, they published one work the title of the work was genetic algorithm for multi objective optimization, formulation, discussion and generalization published in the proceedings of 5 international conference on genetic algorithm.

This conference was treated as a base conference in the field of genetic algorithm. Now, Fonseca and Fleming proposed this approach and the basic principles behind this approach is ranking mechanism. So, we will learn exactly what is the ranking mechanism? And what are the different steps, that is there in this approach?

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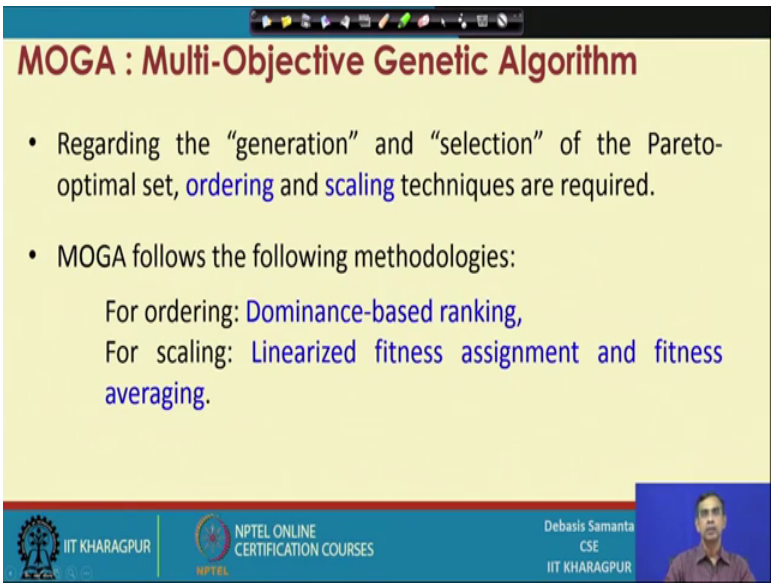

So, here basically the idea about I told you that, all the approaches are same as the genetic algorithm except the selection mechanism, now the selection mechanism towards the next generation population generation or it is basically and it is the generates the next population, so that all the solutions are the non-dominating solution; that means, they are the base solutions.

Now, here regarding the generation and selection of the Pareto optimal set; that means, non-dominating set the approach that is the MOGA approach considers 2 techniques the techniques are called ordering and scaling. So, here for ordering they follow one new concept called the Dominant-based ranking, and for scaling they proposed one idea it is called the fitness assignments and that is also sometimes called the fitness averaging is a linear function of the fitness averaging.

So, learning of MOGA is basically to understand clearly how the ranking is carried out there, and then how they do the fitness scaling or scaling of the objective functions. So, we will learn this 2 steps in the next few slides, and then the MOGA approach can be understood clearly.

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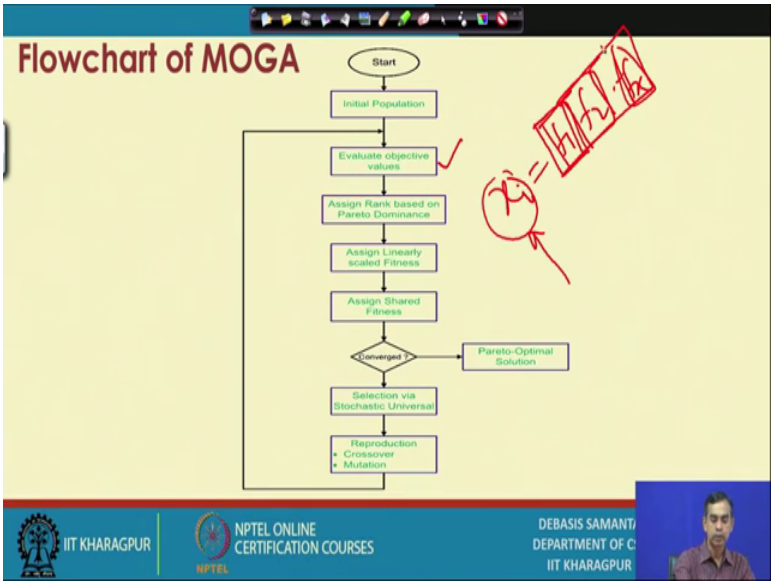

Now, here first see what is the flowchart of the MOGA approach. So, it is basically same as the genetic algorithm flowchart if we see. So, it starts with creating the initial population can be created once the chromosome is decided and it can be created with some random solutions into it.

So, the initial population creation, once the initial population is created it will evaluate the values or object or evaluate each solutions. So, it is basically evaluation step which basically evaluates all the solutions, now again when you evaluate this one it basically evaluate all objective values; that means, if a solution x i is there. So, it basically evaluates f 1, f 2 dot dot f k, if the k vectors are there.

So, from the given solution x i, where the chromosome is known to us, then we will be able to create or evaluate each objective function values, so this is the creation and once the objective values are created then each objective value. So, x i, x j and so, x m, there are m number of solutions are there, then they assign rank to all the solutions. So, the rank means; it can be assigned rank i, it can be assigned rank j, it can be assigned rank k like this on. So, rank will be assigned to each solution, there may be 2 or more solution can be assigned same rank and so on, but no 2 solution will be assigned 2 different ranks. So, this is the concept that is followed here.

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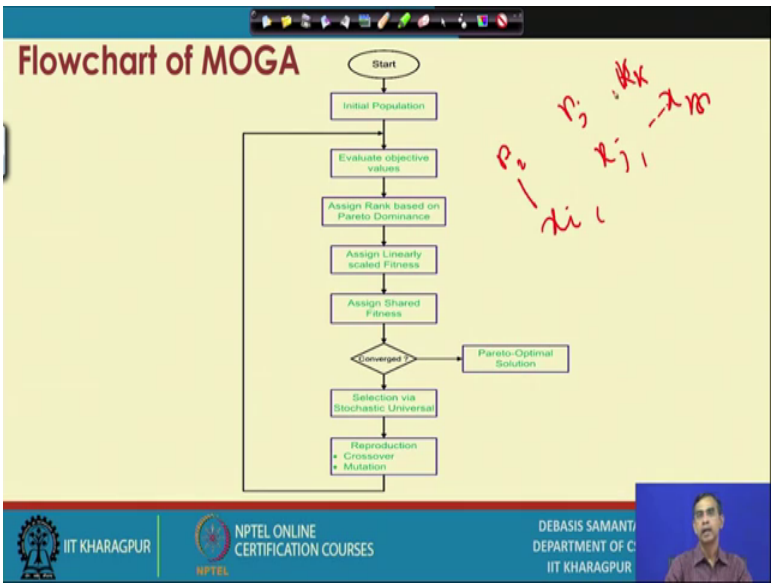

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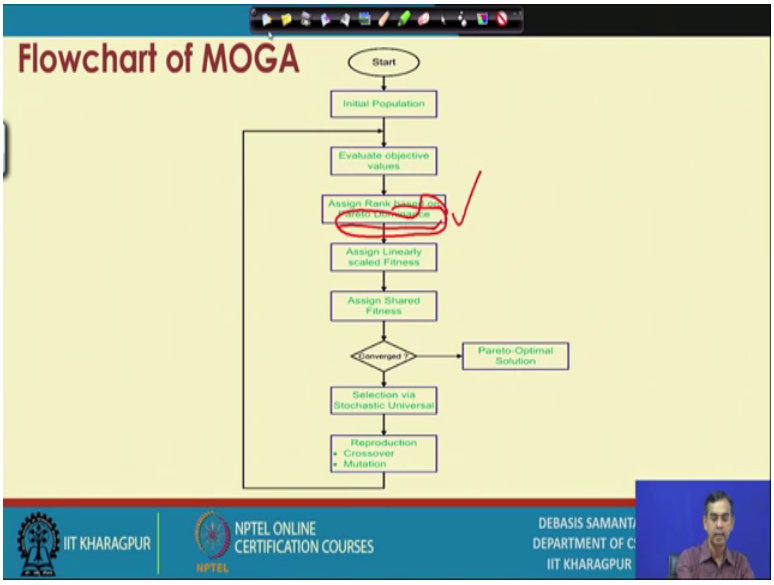

It is a rank basically assigning the rank and this assigning the rank as it is told here, based on Pareto dominance. So, we will learn about what is Pareto dominance? And this concept can be applied to assign the rank.

Once the rank is assigned in the step our next task is to assign the scaling. So, this scaling assignment is basically follows a linearization of all the objective factor; that means, it will follow certain linear function. So, that all solutions belongs to a particular rank can be assigned one unique fitness value. So, this is the concept that is here to, and then it is called the assigning linearized scale scaling fitness, and then assign shared fitness value. So, it is basically after the linearization, we basically give the same fitness values to all the solutions, which belongs to a particular rank. So, that is why it is called the shared fitnessing shared fitness concept.

So, this will give all the solutions, but a modified fitness values like this one. Then the solution that we have to undergo certain convergence test and if they pass the convergence test, then all the solutions that we will be obtained are writtened as a Pareto optimal solution and if it is failed the convergence test is not successful, then we will go for the selection by means of a some probabilistic selection, whatever population based or proportional based selection, whatever the selection that we have learned about it, but it is basically a stochastic selection we will follow, and then this stochastic selection will produce a mating pool, and then from this mating pool we perform the reproduction operation and then next generation will be produced.

So, this will be repeated again and the cycle will be continue to till the convergence test is convergence criteria is met. So, this is the idea about the MOGA approach and we can learn, we can understand that we can see that this MOGA approach has the basic framework same as the genetic algorithm framework, but there are few steps that is unique here, it is here, so basically assigning rank and then linearization. So, these 2 task are different for the selection is concerned or prior to the selection of course. So, it basically make ready that, how the selection can be carried out properly, so that the non-dominating solutions can be selected for the solution.

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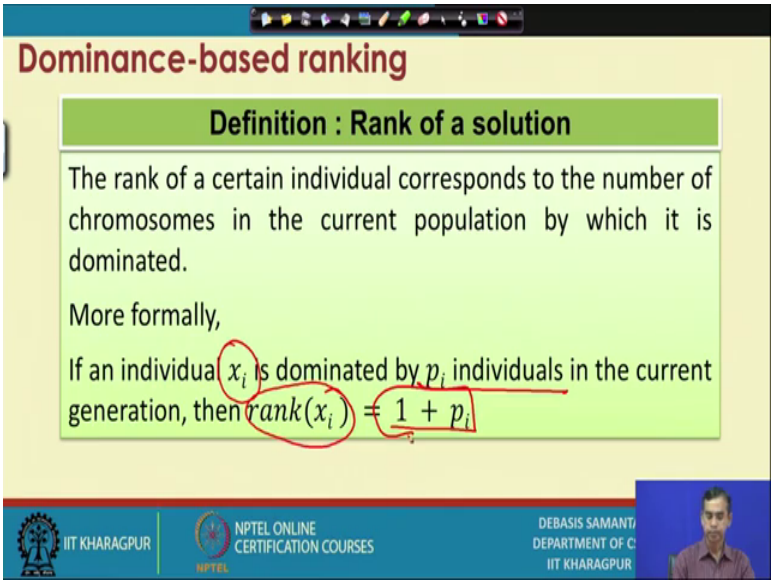

So, this is the flowchart that is followed in MOGA approach. Now, we will discuss about how to assign rank to a solution. So, rank of a solution now here in this MOGA approach, they proposed one criteria or a one concept like the they told that, they assigned a rank like this, they the rank of a certain individual corresponding to the number of chromosomes in the current population by which it is dominated.

So, this is the concept that means, if a solution is dominated by say n number of solutions, then we can assign the rank accordingly; that means, is proportional to n (Refer Time: 9:17) if a solution is not dominated by any other solution, then it is rank will be the lowest one. So, according to this idea about it they defined the ranking of a solution like this, if a solution x i is dominated by p i number of individuals in the current generation, then the rank of x i is 1 plus p i.

So, this way we can easily understand that the rank of the solution which is not dominated by any one is the lowest and the lowest rank is one. So, this is the formal specification by which rank of a solution can be assigned, now let us illustrate the concept with some examples.

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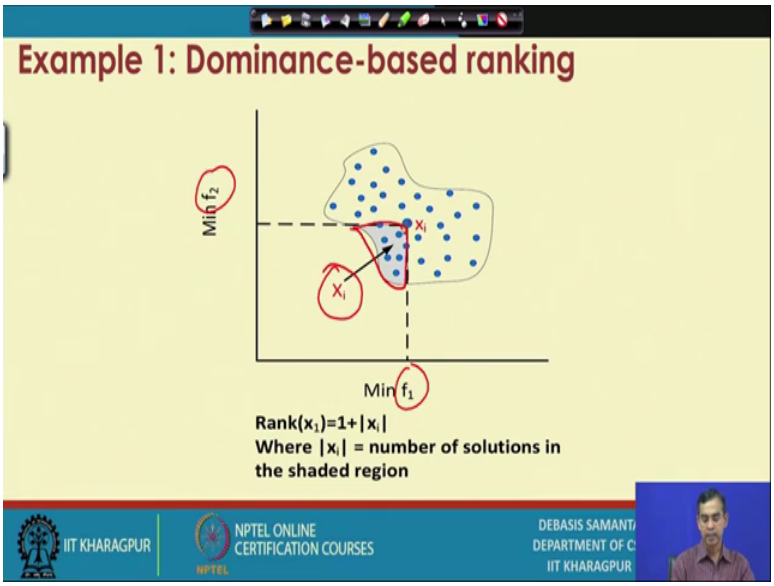

Now, say suppose this is the at any instant the solution space and we want to assign rank of any solution, let it be x i, now if this is the solution and this is the 2 objective optimization problem and f 1 and f 2 are both to be minimized, then with respect to x i these are the subset of the solution, which we can obtain solution, which we can denote it as a capital x i and we see that these are the subset of solutions is in fact, dominates x i or we can say x i is dominated by all the solution.

Now, how many solutions by which the x i is dominated, let it be x i is a size then rank of the solution x i it is denoted as rank x i is 1 plus the number of solution which is there in this region. Now we can note that all the solution, whatever the, this region are basically dominates the solution x i. So, this is the concept by which the ranking can be assigned and we can do it writing a very simple program. So, it is not an issue. So, the idea it is like this now. So, this is the concept if it is minimizing f 1 or minimizing f 2.

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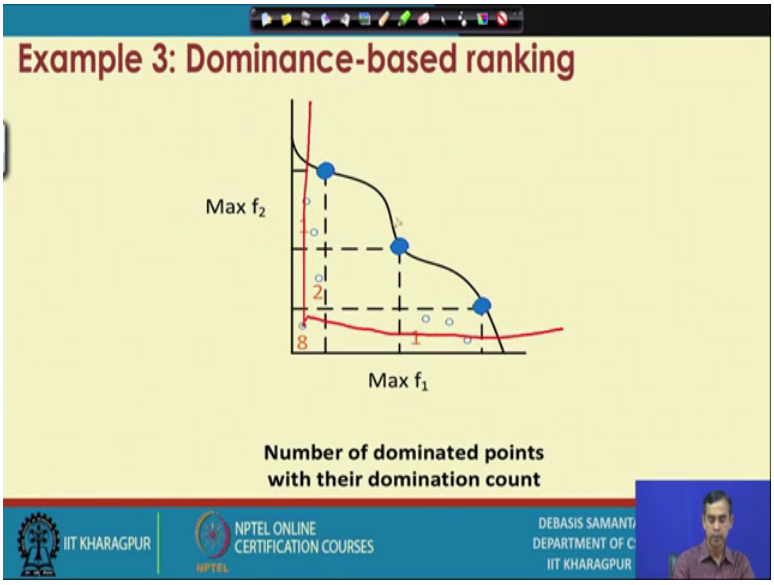

Now, usually the same thing can be explained, if it is maximizing also. So, in that case, if this is the solution x i for which the rank has to be determined, then all the objective all the solution which is in this region are basically dominates this x i or x i is dominated by all the solution which are here. So, the rank will be accordingly the number of all the solutions in this region plus 1 that is the rank of x i. So, this is the concept it is there.

Now, in this particular example, as you can say this solution is dominated by 1 2 3 4 5 6 7 8 9 10 11. So, the rank is, the rank of this solution x i is 12. So, this way it is the idea it is there. So, the rank of the solutions can be obtained and can be assigned. So, this way rank of each and all the solutions can be assigned.

Now, for an example this solution now, this solution is not dominated by any other solution. So, rank of this solution is 1, in other words rank of all the solution which is lying on this font is rank 1. So, this solution we can say this is also nothing, but the font is called the Pareto optimal font. So, we can say that, all the solutions which are having the lowest rank they are the Pareto optimal solution or they lies on the Pareto optimal font. So, this rank assignment can help us to know which solutions are lying on the Pareto optimal font like this one. So, this is the concept of ranking.

Now, another example that I can post it here, we can easily understand that it is both max f 1 and this one and if we say this solution we have discussed about. So, here basically all the solutions this is the one solution, like all the solution and you can easily understand that, how the rank of these solution for example, rank of these all the

solutions are 1, because this solution and this solution only dominated by this one and so on.

So, the rank of all the solutions can be obtained like this. So, rank of this solution it is like this and this one so, the 2 and so on. So, rank of this solution also it can be calculated counting all the numbers here, and then this one this one. So, rank can be calculated by this simple method, and so the rank, this is the idea about ordering all the solution based on ranking.

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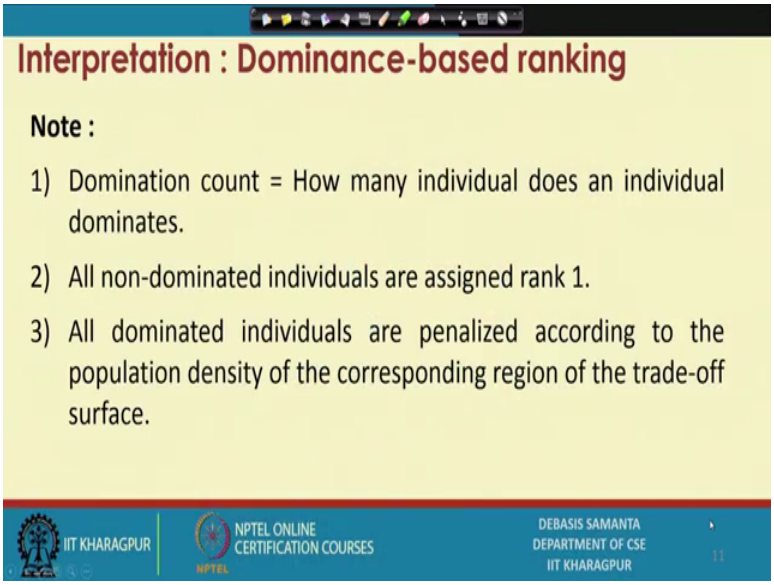

Now, once the ranking is known to us our next task is basically to linearize fitness. Now, again here ranking has certain physical interpretation, I just before going to have the next discussion, let us first discuss what is the interpretation? So, rank is basically domination count, that mean how many individuals does an individual dominates. So, this basically the interpretation, that rank can and as I told you that all non-dominating solutions are assigned rank 1 and the rank is higher; that means, they are inferior solution and that is basically a rank can be considered as a penalty, by which this solution is dominated in the population one solution having higher rank compared to the another solution; that means, it is dominated by more solution then the others.

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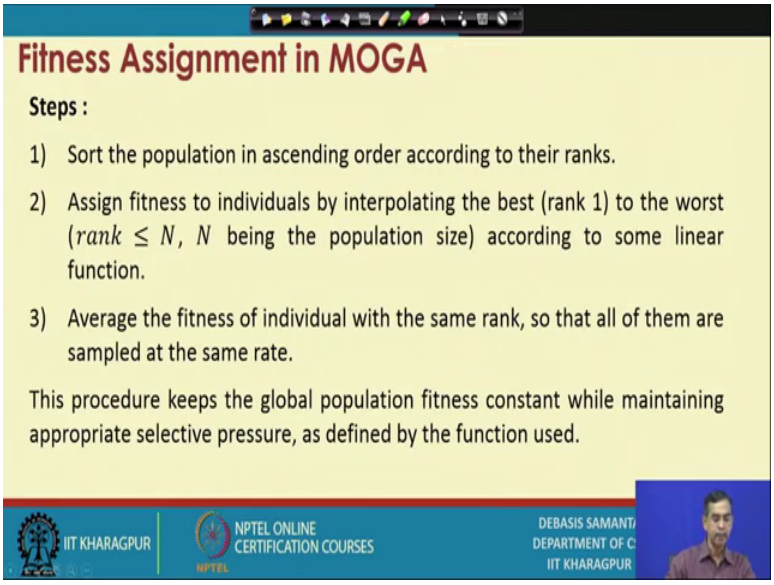

So, this is the interpretation of the concept of ranking here, and then once the ranking is done our next task in the MOGA is basically fitness assignments. Now for the fitness assignment the idea it is like this, it follows few steps 3 steps as we mentioned here, the first step is to sort all the solutions in a current population according to their ascending order of the ranks. So, basically, we sort all solutions based on their rank actually in ascending order.

Then we assign the fitness to individual by interpolating the best rank to the worst rank line. So, worst rank may be as close as N, if N is the number of population size, then we assign all solutions belong to a particular rank in terms of a some linear function, we will discuss what exactly the linear function that it follows to linearize the solutions fitness values, it is basically is a linearization followed by averaging the fitness value. So, this is the main idea about fitness assignment to each solution.

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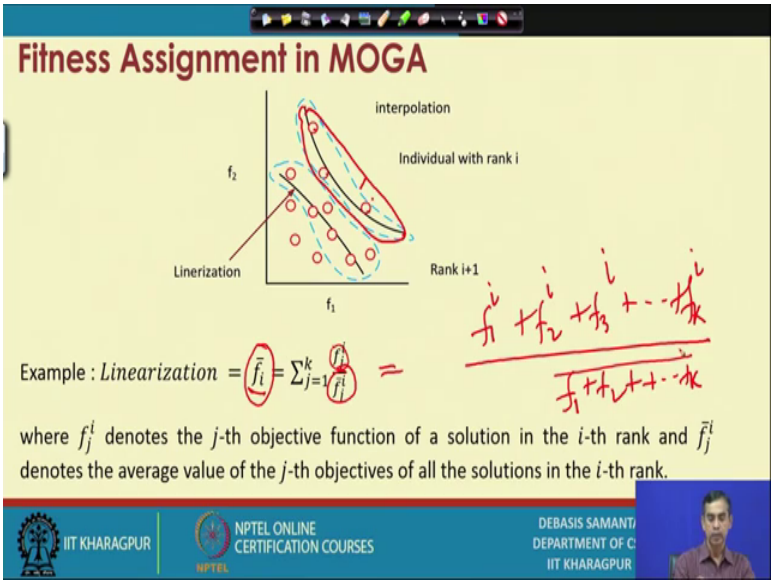

Now, let us understand this concept with an example better we can follow some example and then we follow it. So, idea about fitness assignment I am discussing. So, idea is that the basic concept behind this or rational behind this approach is that, say suppose all these solutions are assigned one rank, then basically the idea is that all the solution which are assigned rank 1 should have only one fitness value (Refer Time: 17:26); that means, all solutions belongs to a particular rank has the same fitness values. So, to do this things it first express all the objective into some linear function here the idea it is like this. So, linearization is like this it is basically, linearizing all the solutions belongs to the i th rank.

We denoted that f i bar, then for all the solutions which is there suppose it is the k number of objective functions are there, then for with respect to each objective function we take the sum of all the objective functions divided by the average objective functions. So, this way we can express a linear functions like this one. So, this can be like this. So, f 1 i plus f 2 i plus f 3 i plus dot dot dot plus f k i and divided by. So, it is basically. So, here and f i j denotes the average value of the j th objectives of all the solution. So, it is basically. So, the j th objective if you say the j, j means f 1 plus f 2 plus dot dot dot dot f k and then this is the average value.

So, this way it basically calculate the linearization of this objective functions. So, once the linearization is done, then the next step is basically to assign the fitness value. So, the assign assigning the fitness value will be like this.

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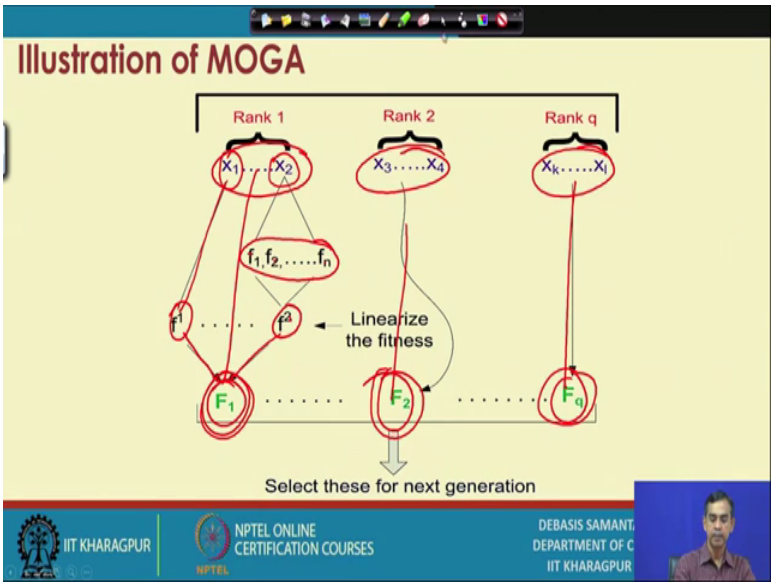

Here this slides can help us to understand how it takes place? Now, first consider these are the solution, which is having rank1 and this a another solution another rank tone. So, there are group of solutions belongs to the different rank like.

So, all the solutions belongs to one rank, all the solution belongs to another rank, all the solution belongs to another rank. And here the solution any solution belongs to this rank has the fitness value f1 f 2 dot dot f n the fitness functions and we can express all the solutions that belongs to this 1 by means a linearized one. So, f 2 similarly for x 1 this is the f 1.

So, it is called the linearization using the previous step, that we have discussed once the linearization is done, then we can take the average values of all those. So, it is basically averaging. So, these way all solutions which belongs to a particular rank has a fitness value f 1, similarly all solutions which belongs to this rank has the fitness value f 2 and all solution belong to this rank has the fitness value f k. So, what you can understand is that, all the solutions belongs to a particular rank has the one fitness value and another fitness value and this is the another fitness value.

So, this way we assign the fitness values to the same fitness values to all solutions belongs to a particular rank; that means, if you have if the solution has the same rank, then they have the same fitness values. So, it is basically ranking followed by the assigning the fitness value is the step.

Now, once this fitness values are there, then based on this fitness values we will go for selection; that means, that selection can be any selection may be say proportionate based selection, like say roulette wheel selection or rank selection or in this case one particular selection is called the stochastic selection. Stochastic selection just like a proportionate based selection, but it is basically random selection, it basically generate a random number and then in general in the range of the fitness values of their and then, it basically selects a particular solution based on the random number that is generated. So, is a stochastic this one, but other then the stochastic also, we can follow any standard selection that is used in case of simple genetic algorithm also there.

So, this selection will create a mating pool by from where the conventional reproduction method can be applied, and then next generation can be obtained. So, this is the idea about this MOGA approach here.

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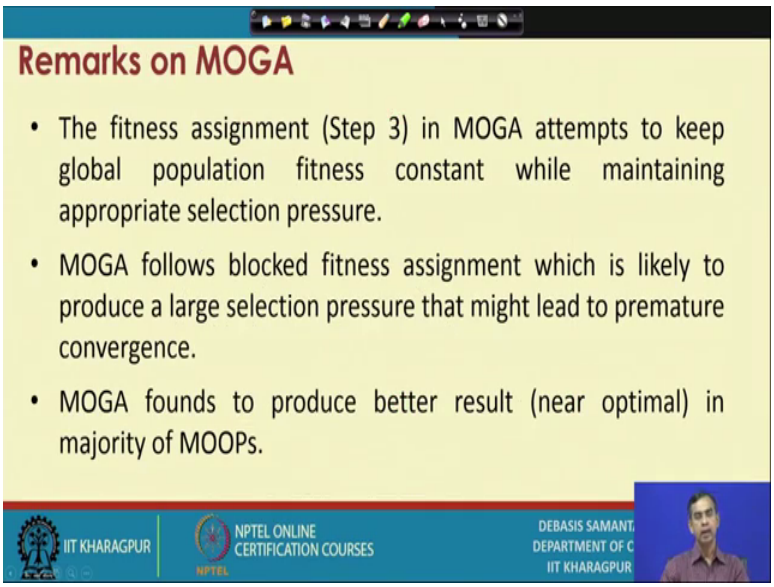

So, this way it can help to I mean have the dominant solution; that means, it always gives more preference to the solutions, which are basically non-dominating actually or dominated by lesser number of solutions. So, this way it searches to that direction, which basically towards the Pareto optimal solutions; now, here the main idea about fitness assignment, that we have this learned about that it basically the objective of the fitness assignment is to keep the global population fitness constant, while maintaining

appropriate selection pressure; that means, we will select all the solutions which has the lowest rank.

So, for the dominant a dominant solution is concerned and it also follows the blocked fitness assignment, because it is called the block because a solution belongs to a particular fitness has the same what is called the fitness values. So, this basically produces a large selection pressure and that may be sometimes to lead to premature convergence; that means, it can terminate giving non-optimal solution or a local solution.

However, it is observed that this MOGA approach founds to produce better result, near optimum or the global optimum solution in many of the multi objective optimization problem. So, this is the one approach the MOGA approach, MOGA is basically out of the different Pareto based approach, except the Vega approach is one of the simplest yet more effective approach known so far now there are many other approaches also known which are basically more elegant, more efficient and gives better result and all this approaches we will discuss in next class.

Thank you.