Introduction to Wireless and Cellular Communication
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 24
BER Performance in Fading Channels
Ricean and Nakagami Fading, Moment Generating Function (MGF)

Good morning we will begin with a quick summary of lecturer number 22, but before that before I begin let me just sort of highlight the items that we will be covering in today's lecture. We will be looking at another pdf which characterizes the channel characteristics of that we observe in wireless channels it is called the Nakagami-m, unlike the others where we took the theoretical model and then derived the statistical model this was based on looking at experimental data and fitting it to the pdf.

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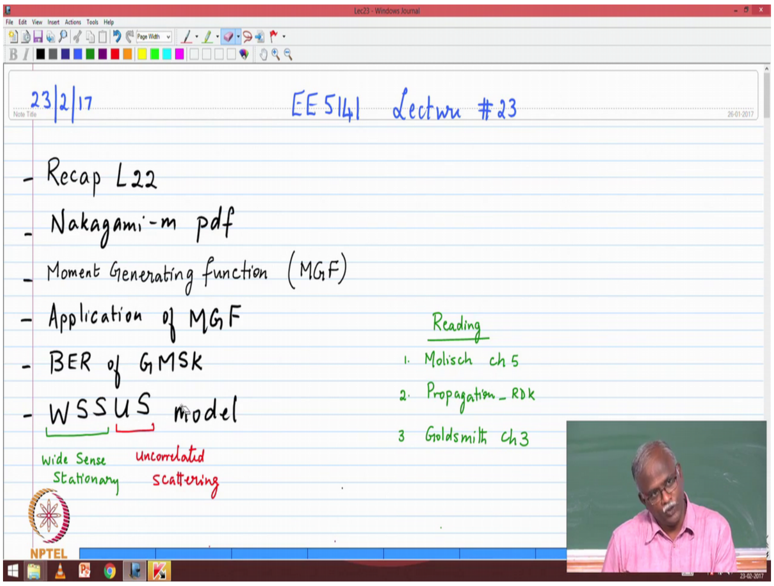

So, basic this is a experimental base experiment based derivation, but it fits very nicely with the Rayleigh and Ricean type of type of statistics. So, we will look at the Nakagami-m distribution. We will look at the use of a tool that you are probably familiar with from your study in probability theory that is the moment generating function. We have a very specific and a very important role for a MGF and we will just highlight that and explain why it is a very useful tool when we are looking at the BER analysis of modulation schemes in the context of a wireless channels, application of MGF will be one of the things that we. Along the way we will just make a comment about the bit error rate of GMSK because that is something that we use quite extensively and we just show that whatever tools that we have developed are sufficient for us to get the BER of GMS case well.

Probably the most important concept that we need to build on and develop in our understanding of wireless channels it is called the WSS US model it stands for 2 components WSS stands for wide sense stationary, the second one stands for uncorrelated scattering. And there are this basically describes the behaviour of a wireless channel particularly the ones that we will encounter. So, WSS US model is probably the most important concept because it links the time and frequency domain characterization of the fading channel, today we will just get an introduction to it and build on it in next lecture. So, that is our goal for today. Let me begin with a quick summary of the points that we have mentioned so far.

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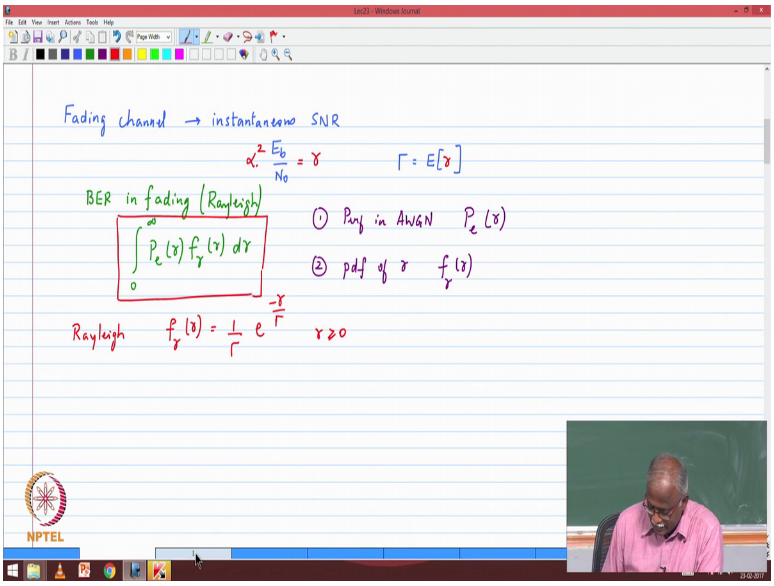

So, I know I am repeating this multiple times, but it is for a very specific purpose. Fading channel we talk about an instantaneous SNR, very important that we keep that all always in mind instantaneous SNR. The signal is fluctuating I can set a nominal level E b by and naught, but the actual the what I am actually going to be seeing it will going to be alpha squared E b by N naught. So, that is a very important element and we denote this as gamma. So, the ones in red are the random variables, blue is it is constant. So, then we say that this is expected value of let me keep consistent with the notation that it is the expected value of a gamma which is a random variable, but that is itself for the constants. So, uppercase gamma.

So, when we wanted to get the BER in fading. So, BER in fading and again notice I have not specified that is BPSK, 16; QPSK or 16 qam. Any modulation scheme characterizes as Rayleigh fading. So, far we have looked at that primarily, but it can be generalized to any fading just in one step we will we will characterize that. So, any modulation scheme in the presence of fading can be written as 0 to infinity, probability of error as a function of gamma this is the performance in AWGN and multiplied by the distribution of the SNR in that particular environment. So, this is a very useful very powerful equation which will be your starting point for any modulation scheme, any type of fading, what are the 2 things you will ask for? The first thing you will ask for is the performance in AWGN, performance in AWGN where you have a fixed SNR.

So, probability of error in AWGN as a function of gamma you will ask for. Second you will say that gamma is not a fixed number anymore because of fading I need to know the statistics whatever it is Rayleigh Ricean Nakagami does not matter, give me the pdf of gamma. So, that is f gamma of gamma once you have those 2 you are you are able to get the representation in any fading system, any fading environment. So, the Rayleigh fading specifically is what we have focused on and we will continue to focus on that is the most important one because we are working in environments where there is non line of sight. So, under this assumption again this is a very important result always keep this in your mind that we are dealing with an exponential pdf gamma greater than or equal to 0. So, this enabled us to get the BER expressions for the different modulation schemes I will not repeat that, just show you the slide from yesterday, basically the modulation schemes that probably are of a lot of interest to us.

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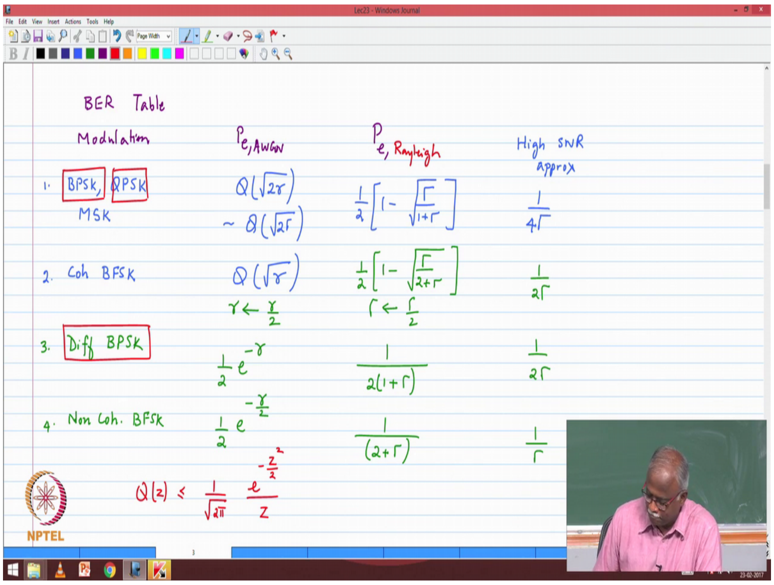

BPSK, QPSK and add differential BPSK, the others not as important, but it gives us a complete characterization that is helpful for us and whenever there is a Q function we also said that there is an approximation that is available to us, there is an upper bound which is given by 1 by root 2 pi e power minus Z squared by 2 divided by Z and we found that that is a good approximation under high SNR or large values of Z. And again it is a good way to get a new quick field for what the numbers are. After this we had looked at the derivation of Ricean fading.

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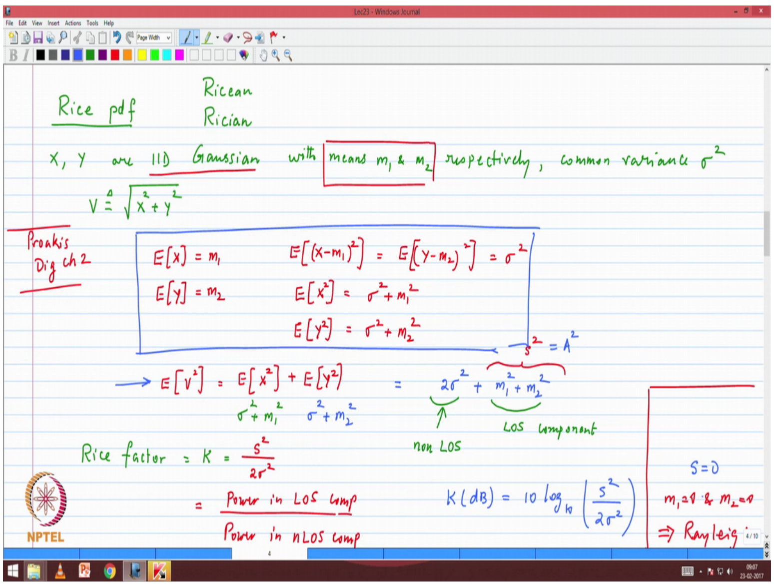

So, we looked at the case where X and Y are IID Gaussian no longer 0 mean, but with means m 1 and m 2 which basically gave us this set of equations which are very very useful for us to always keep in mind, this set of equations which says expected value of X, expected value of Y and the variances and the relationship with the variances. So, this leads us to expected value of v squared the mean of the power of the received signal that is given by 2 sigma squared plus m 1 squared plus m 2 squared.

Now, in case you are following along in Molisch, he calls this as A squared, A squared is what is used more widely. So, I have used A squared in case you are reading do not get confused, he has used A squared instead of S. So, this then we described that there is a factor that describes the pdf call the rice factor ratio of S squared to 2 sigma squared it can be more intuitively visualized as the power in the line of sight components, in the power in the non line of sight component usually expressed in dB special case would be when S is equal to 0 which means K is equal to 0 then we get the Rayleigh distribution.

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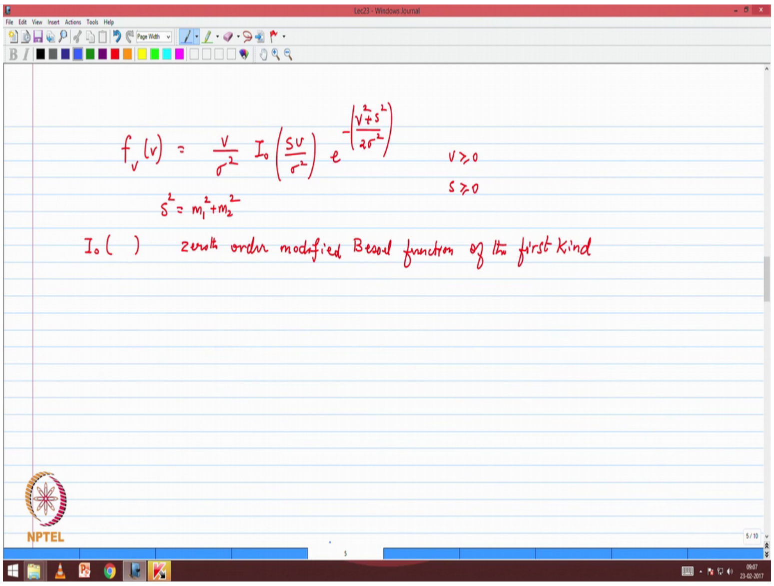

And the expression for the Ricean pdf is given here I naught being a Bessel, a special case of a Bessel function. So, that it is our broad classification of understanding.

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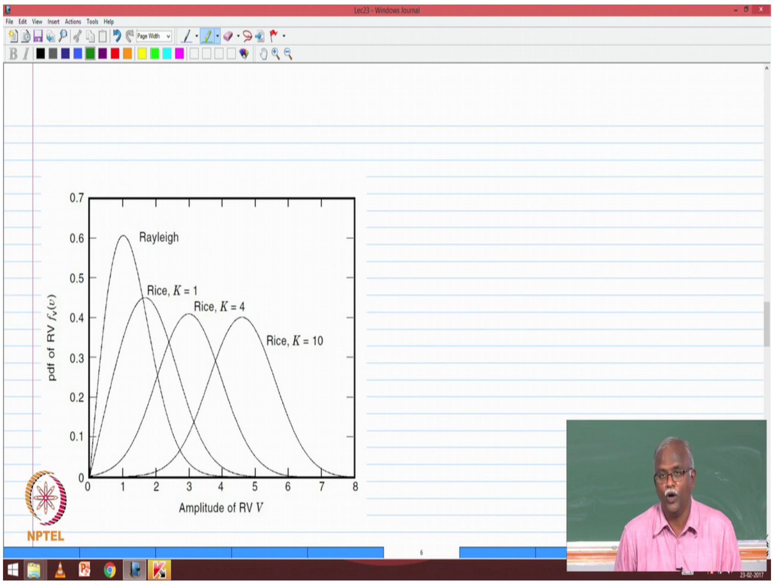

We said that what is the intuitive understanding of the Ricean channel the pdf moves to the right what that tells me is that, what that tells us is that the likelihood of higher amplitudes is higher. So, this is a good thing for us and as the rice factor increases then we will see more and more the channel becoming you know having higher, larger amplitudes. Again this has to be interpreted the best way is if you actually generate it through a MATLAB plot what are the assumptions are we that we are making with respect to the S squared, what is the assumption with respect to sigma squared. So, again we will give this as a simple MATLAB exercise.

So, you can think about you know is the scaling ride why is this shifting to the left, but notice that the centre is now moving towards where your line of sight component is going to contribute the maximum power. So, it is variation about the power that is contributed by the line of sight compounds. So, as the line of sight component increases you will find that the curve shifts to the right and as the line of sight component becomes more dominant what you will see that is the fluctuations start to reduce you know around that. So, again as you play with this generation of these plots you will have there is a lot that you can learn, so I again this is just given you a set of samples of that.

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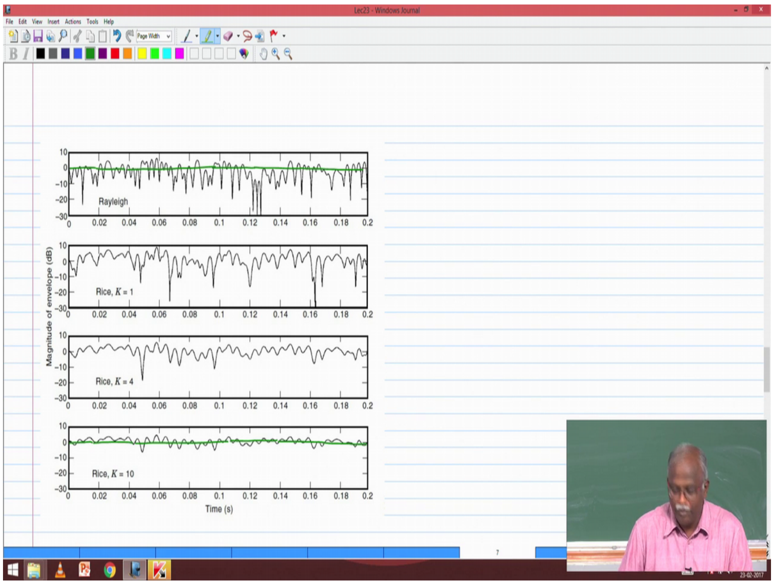

The actual impact is very much seen when you look at it in the time domain, again we will have an exercise to generate this in MATLAB. This is a Rayleigh channel, notice the Y axis very important it is in dB you have fluctuations from 0 dB to 20 minus 25 or minus 30 dB significant level of fluctuation. As you increase the rice factor noticed that the fluctuations are reducing. This can also be seen in the pdf where you will find that the dominant part is going to play an important role and any fluctuations about that you know in the on a large scale looks you know very, does not look like as a its going to be a threat in terms of BER it is also good because it tells you that an amount of margin that you must allow for small scale fading this is small scale fading is now much significantly reduced because the level of fluctuations are not as you would see in a Rayleigh fading channel.

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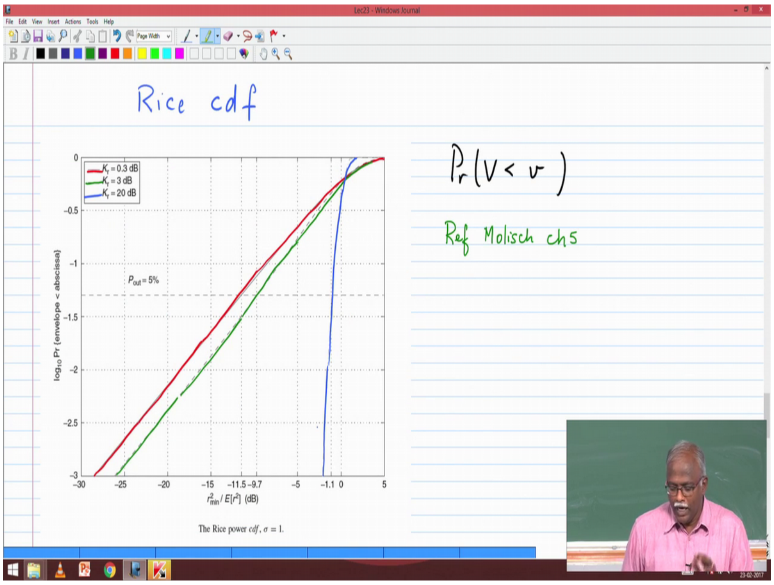

I have asked you a recommended Molisch chapter 5 as a reading assignment, please do take it up it is a very insightful. It is a very insightful chapter.

One of the things that we will not necessarily spend time deriving is the cumulative distribution function for the rice pdf and of course, since the rice pdf depends on the value of K the the cdf will also depend on k. So, basically you will not get a single cdf, but you will get a family of cdfs for the different values of K. Let me just give you an observation. Again the more as you read it and as you if you want to actually generate it you will find that it is a very useful interpretation. So, K going closer to 0; that means, your closer to Rayleigh that is the red graph and if you remember when we did the cdf of the Rayleigh distribution we said that on a log log scale it will look very linear and this is a very, it is very similar to Rayleigh distribution. So, what that tells us is that there is a fairly reasonable probability that you will get very low values of amplitude that is what happens when you have a deep fade. So, this is where the problem happens with the Rayleigh distribution.

Now again for K is equal to 0.3 it looks almost like Rayleigh and may be really it is good idea to actually plot the Rayleigh distribution, you will find that the Rayleigh is slightly worst that will be you know slightly worse than the red graph. Then you go to something which is mildly a line of sight 3 dB its better than Rayleigh, but it is not substantially different, but when you start to introduce a strong align of sight component notice that the likelihood of low values of received signal envelope or power does not matter both will give you the same interpretation is almost 0 because the you know your cdf starts at a reasonably high value which is centered around the mean received signal power.

So, you can see that the cdf is very steep, cdf being very steep means what is it? Less than that value not possible above that value always guarantee basically your signal will be in that small range. So, that is a good interpretation of what the Rayleigh channel does and its interpretation. Now what was the assumption that we made about the phase of Rayleigh fading channel coefficient?

Student: (Refer Time: 13:25).

Uniformly distributed, now what would is your expectation that would happened to the phase when you start seeing a Ricean distribution, does it still stay uniform what is your expectation?

So, basically think of it like this you have got signals coming from all directions these signals are coming from different directions are you know going up and down based on the fading and there is one line of sight component which is not changing. What is the expectation? It obviously, will get up perturbed from a uniform distribution it is no longer going to be uniform. It is going to be more or less you are going to going to see that constant line of sight component present being present.

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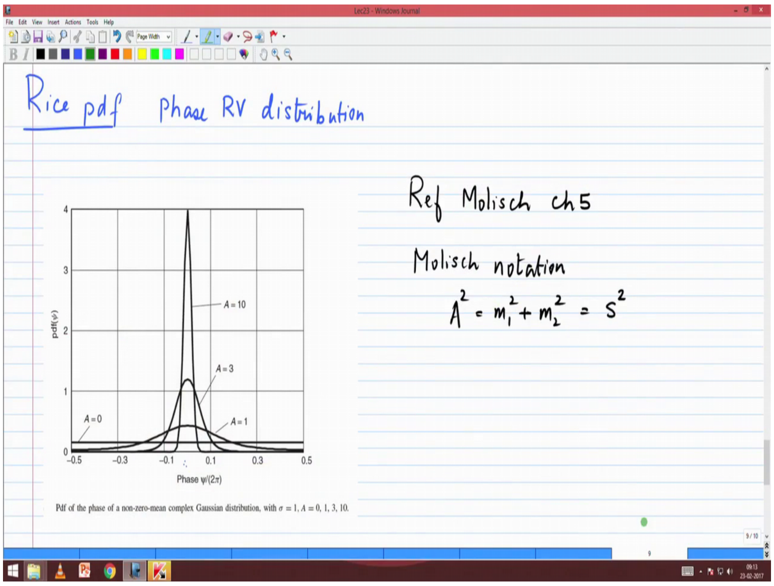

So, here again it is very insightful for us to look at the different distributions. So, A equal to 0 remember A is Molisch’s notation for our X squared, A equal to 0 is Rayleigh notice that graph is straight line it is a flat phase from minus pi to pi. Then as you increase the value of A, the line of sight component he has taken the line of sight component to have 0 phase, basically if you are chosen it to be 30 degrees the peak will have occur around 30 degrees. So, he has chosen it to be 0. Notice that you start to see when A is equal to 1 a slight bump around 0 phase.

Because that means, that that component is starting to become more dominant with respect to the others there is yes presence of signal in the other angles as well. Now as you go to equal a to 3 you will notice that the peak becomes a little bit more sharp and as you go to a equal to 10 you get much much sharper that is more dominant than the other phase angles that are present in your system. Now what is the value will be go back to this expression. What is the value of K that K equal to 0 gives me Rayleigh, what gives me AWGN?

Student: (Refer Time: 15:24).

K is equal to infinity. So, basically A equal to infinity will mean, K equal to infinity means will be more or less like a straight line, the fluctuations of the Rayleigh distribution is no long there, cdf does not make sense because there is no statistical part and this one will become a straight line because basically all your energy is concentrated around the single line of the component which is got a constant phase.

So, again the entire spectrum of the distribution and the statistics that we observe in Ricean Rayleigh, Rayleigh being a special case of the Ricean I hope is something that you are comfortable with that is one of the important thing that we need to take away from this discussion.

So, today’s material we will start with the Nakagami-m distribution.

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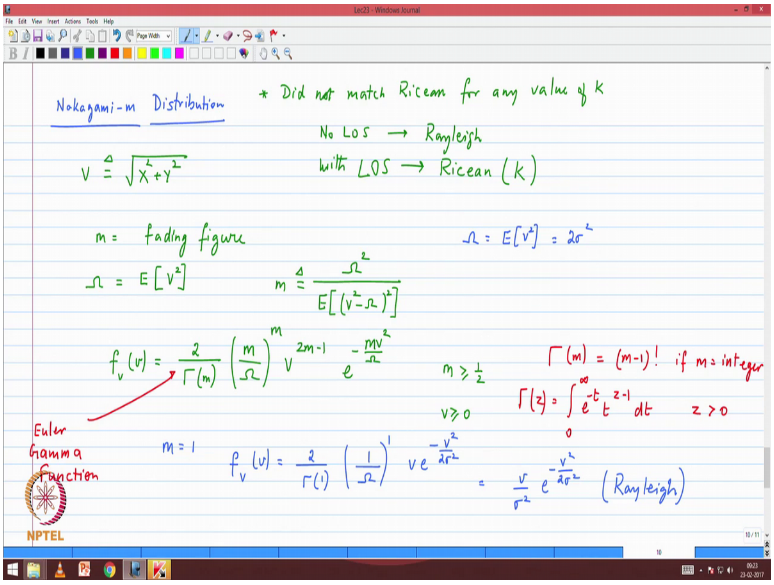

And that is as I mentioned it comes from the world of experimental characterization Nakagami-m, Nakagami-m pdf distribution. Now where did this actually originate. So, people try to do measurements in different kinds of channels and then try to characterize its saying its Rayleigh or Ricean with a certain value of the rice factor. It was found that certain environments did not match Ricean or Rayleigh. So, did not match did not match Ricean statistics for any value of K, when we say that it did not match it did not give a good enough fit to the value of K. Notice that I do not need to say Rayleigh because K equal to, K equal to 1 is a K equal to 0 is a is Rayleigh. So, Rayleigh is the special case of Ricean. So, the observed statistics did not quite fit the Ricean statistics. So, they came up with a distribution which seems to have a framework very similar to the Ricean distribution, but somewhat different and again we want to be able to capture that.

So, so far the options that we had were no line of sight means we said you take Rayleigh, that is your Rayleigh distribution and with line of sight component it was Ricean we had to specify the value of K and now third option is being is being given to us. So, think of it as the same expression that we have for Rayleigh Ricean. So, v is equal to square root of X squared plus Y squared and the Nakagami distribution does not make any specific assumptions on the mean, but it makes assumptions on the received power levels. So, it says that if I introduce something called like the K parameter something called the fading figure which is a characterization of the statistics and we define the following - upper case omega is equal to expected of v squared mean received mean signal power and m is defined involving the fourth power again we are trying to fit a statistics to observe data. So, the exact origins of this formulation not documented, but the results are very very useful. So, expected value of v squared minus v whole square omega squared means it is a square of the square value.

So, basically it is like doing some fourth order statistic. So, m is a fading it is a dimensionless quantity. Now this gives us a measure that can then be used to characterize the distribution of the envelope and given m we can obtain the Nakagami that is why it is called Nakagami-m distribution, it depends on the value of m it says Nakagami-m it says fv of v that is the pdf of the received signal envelope under Nakagami-m distribution is given by 2 times gamma of m, this is not related to SNR and that is one of the reasons why we use a different value this is the gamma function, this is the Euler gamma function.

Again, all of the expressions are given in the books, but it is good for us to just write it down. So, we get a comfort level with that. So, let me complete the pdf, m divided by omega raise to the power m v power 2 m minus 1 e power minus m v squared by omega. So, and it depends on the value of m and m has to be greater than or equal to one-half and basically this is the characterization that we have and of course, v will be greater than or equal to 0. So, this is the Nakagami-m distribution. Let us just write down a few more points that are helpful for us to get the complete picture. The Euler gamma function gamma of m we have come across that in some of the digital communications if m minus 1 factor basically it is a factorial representation if m is an integer, it is also defined for non integer value, so we have to be a little bit careful when they are non integers, but most of the times we will try to find integer values, but if it is non integer its good for us to know the definition gamma of z is given as an integral 0 through infinity e power minus t, t power z minus 1 0 dt for z greater than 0 and that is what we are going to be looking at. Basically looking at some the fading figure by the way this call the fading figure which is greater than or equal to half positive quantity greater than or equal to half and that is what we have and this is the basic definition of that. Now, first thing always few sanity checks.

Let us take a look at the special case if I set m equal to 1, I would like to derive look at the distribution that is a valid fading figure f v of v is given by times gamma of 1, gamma of 1 is 0 factorial which is one the next term m is equal to 1 divided by omega raise to the power 1 and omega is equal to expected value of v squared and so omega is equal to expected value of v squared. So, that will be equal to 2 sigma squared 0 mean Gaussians, 2 sigma squared into v times e power minus v squared by 2 sigma squared. So, basically if you rewrite this, this will come out to be v divided by sigma squared e power minus v squared by 2 sigma squared which is Rayleigh. This is a special case of a really distribution.

Now 2 purposes for making this observation – one is to show that the Nakagami-m distribution also covers the entire span going from Rayleigh all the way to, all the way to AWGN channels that is one aspect and you will find that like the fading fact, like the Ricean factor the feeding figure also has a range of values it covers the whole spectrum and as you increase the value of m you will find that you start to get more and more behaviour like the Ricean. So, the Nakagami-m and the Ricean are like equivalent representations, but there is a very very important difference. What is the worst case that you can get in Ricean fading, worst case Rayleigh?

Student: (Refer Time: 24:44).

Notice that is not yet the worst case for Nakagami-m because m equal to 1 you can actually get certain environments which look worst than Ricean you may asking is no there can I is it possible it was already though Ricean was bad enough, but yes there are certain environments where the propagation is worse than Ricean and even those scenarios are actually captured by the Nakagami-m.

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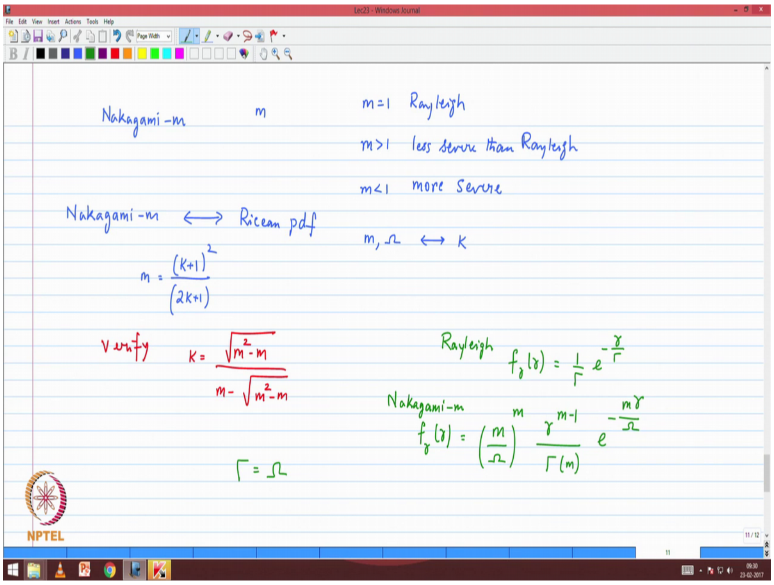

Now you may or may not encounter them, but it is good to know. So, let us build on this little bit more. So, summary Nakagami-m is a very useful form you have to specify the value of m and then the pdf is specified m equal to 1 is Rayleigh, m greater than 1 is milder than Rayleigh that is less severe than Rayleigh and there are scenarios where m is less than one which is more severe.

So, just keep that characterization in your mind before it is good to know that this is a family of functions. Another important thing that we probably need to make note of is look at the pdf of the Nakagami-m distribution has no Bessel functions. So, actually is easier when you want to do integration. So, that is the second reason why we prefer the Nakagami-m, but from understanding point of view Ricean is very intuitive because it is says that the power of the line of sight component, the non line of sight components.

So, the question that arises is, is there a way to map the Ricean distribution to the Nakagami-m and it turns out that yes there is a way to do that. So, Nakagami-m can be mapped to the Ricean pdf, Ricean pdf for that we have to choose the following we have to choose the value of m, we have to choose the value of omega basically that is the and this will map to a particular value of K. So, basically on the Nakagami-m side we have to specify something on the Ricean side we have to specify and then we have to make the relationship. So, the relationship I will give you it is derived in the literature, but against more important for us know that it can be mapped. So, m is equal to K plus 1 whole squared divided by 2 K plus 1.

So, if I gave you a value of K and said my channel a behaves like a Ricean channel with this particular value of K not a problem you will just do a simple calculation to get the value of m and then substitute it and do all of your competitions of the BER using the value of m because that is easier for us in terms of the integration. So, if you can relate m to K you should be able to relate K to m please verify that if this is given to us that the inverse relationship is given by 2 square root of m squared minus m divided by m minus square root of m squared minus m. Again its it just to say that you can go from one map one domain to the other, but you cannot get a mapping for those functions which have the fading efficient m less than 1. So, the please make sure that fading figure less than 1 because the those are the ones which are very unique to the Nakagami-m distribution, but again this is something that we may encounter in experiment using when you do experiments.

So, we have the following tools available to us we can characterize the environment very often what we have encounter is Rayleigh distributed. So, the SNR distribution f gamma of gamma is given by an exponential distribution and other wise if it is not there is a line of sight component it will be Rayleigh, Rayleigh means that it will be a Ricean it will have a rice factor once you get the rice factor you can map it to the Nakagami-m parameter and then that says that there is a corresponding SNR in the Nakagami-m. So, what is the pdf of the SNR in a Nakagami-m environment? That is very important to us and this is something that we will derive as part of the assignment, but let me just give you that there is a expression that is easy for us to validate and that is given by m by omega raised to the power m gamma power m minus 1 divided by gamma power m e power minus m gamma by omega and we can go through and validate that gamma what we have used in the Rayleigh environment is the same as omega. So, that is easy for us to verify and you substitute m equal to one we should get back the Rayleigh distribution, but this is the SNR distribution in an Nakagami-m distribution in a environment.

So, if you were to ask, if you were to asked to compute the probability of bit error. So, let us go back to the first equation the probability of error of gamma that is AWGN, now for the second term if it is Nakagami-m you will now substitute the corresponding Nakagami-m pdf and then complete your integration, so that you will get the result that is of interest to us. So, what we have built around, a built around is a different tails that help us to get a complete understanding in terms of insight in terms of the ability to characterize using analytical tools and then and work with the different environment.

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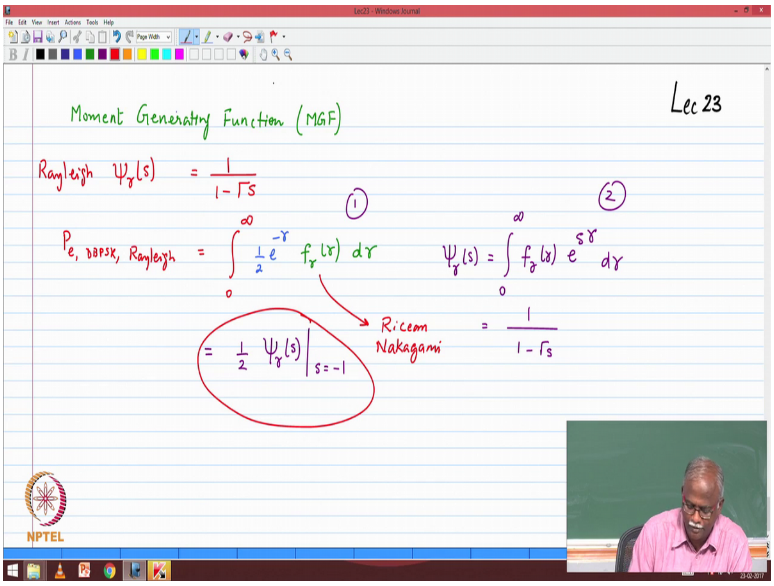

We now we will move into the second part of what I had mentioned today as the moment generating functions usually denoted by the acronym MGF again this is a tool that most students are quite familiar with from the mathematics concept. May be you will when you are studying MGF you will thought you know why is, why even study this is there any use for MGF today we will answer that question its actually very good that you have studied MGF before because we have a lot of used for that. So, let me start with the basic definition and then build on that. I will as always I just give you enough information to understand the concept apply it and then go back to give you the complete a definition.

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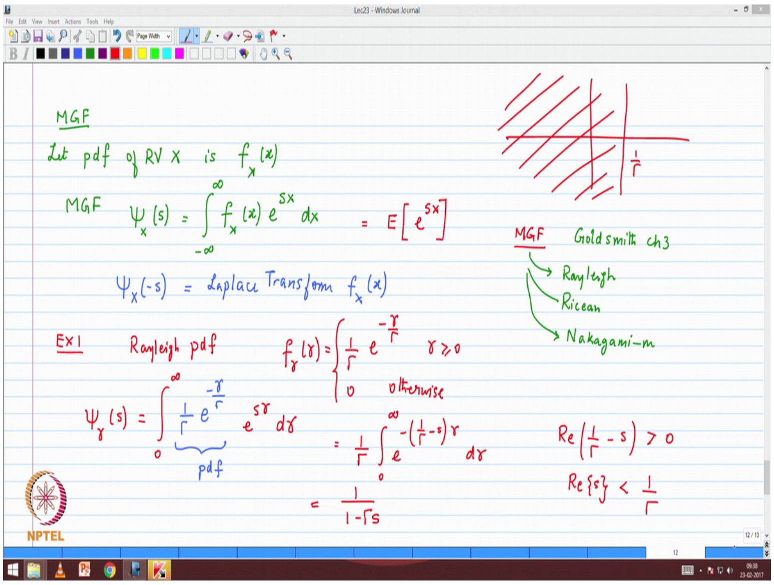

So, let me just this is MGF, the definition of MGF is defined I mean MGF is by this defined for a pdf. So, let us assume that we are given a random variable X and the pdf of. So, the pdf of a random variable X is f X of x then the moment generating function different books used different notations I would like to use psi as my notation for moment generating function. Subscript tells you the random variable and it is usually given in terms of parameter S and you may wonder you know does this mean you are doing Laplace transform the answer is yes it is kind of a Laplace transform yeah I am sure most of you see many of you not doing, but we have to interpret it basically it is just a variable. This is given as minus infinity to infinity and over the range of the random variable X f X of x e of sx dx it is a definition of a moment generating function.

Now, couple of key interpretations again, the first interpretation is from the mathematics for point of you, the second one is the electrical engineering point of it says I want to look at it as a Laplace transform. The first one is a mathematical interpretation mathematical interpretation says you took some function of X multiplied it with the probability distribution of X and then integrated it. So, what did you do? I computed the expected value that is a mathematical expression e power sx. So, the mathematics interpretation of this equation says that you calculated the expected value of the function e of sx. The electrical engineering definitions says well you almost did the Laplace transform except the sign was wrong.

So, basically if you do replace x with minus s then you have actually computed the Laplace transform, Laplace transform of f x. Now which of these is more useful both are useful, we will use both of them.

The good thing is that the moment generating functions have been already computed for the pdfs that we will encounter. So, I would encourage you that is why I have given you Andrea Goldsmith book as a reference, Goldsmith chapter 3 where you can find the moment generating functions of the Rayleigh distribution can also find it for the Ricean distribution it is a little bit tricky the Ricean distribution and you can also find it for the Nakagami-m. So, it turns out that the distribution that we are interested in are already characterized for us in terms of the Laplace transform of in terms of moment generating function.

But before we even do that why even bother with the moment generating functions let me give you an example and then show you why it is very powerful for us. So, we want to look at the case of the Rayleigh distribution, Rayleigh pdf the moment generating function is what we are asked to evaluate f gamma of gamma is given by this is the pdf of the SNR e power minus gamma by gamma, f gamma is greater than or equal to 0 otherwise. So, when I do the moment generating function I will write phi my subscript will be gamma and it will be s, my limits of integration since it is the pdf is the nonzero only for the in the range 0 to infinity, 0 to infinity, 1 over grammar e power minus gamma by gamma that is the pdf, the pdf part and then I write down a e of gamma of s gamma d gamma. Moment generating function of a Rayleigh distribution is given by this 1 over gamma is a constant integral 0 to infinity e power minus 1 by gamma minus s into gamma d gamma.

Now, s can be in the complex plane. So, I am not necessary assumed to be to be real valued. So, basically I have an integral of this form what is the condition for this integral to converge, what is the condition for the integrand not to explore because the gamma will go all the way to infinity.

Student: (Refer Time: 37:26) gamma.

The 1 by gamma minus S the real the real part must be.

Student: Less than 0.

Less than 0. So, must be.

Student: Greater than 0.

Greater than 0, ok you are paying attention. The real part of 1 by gamma minus S must be greater than 0 which is the same as gamma is already a real quantity. So, real part of S is less than 1 by gamma.

So, just a visualization this is a complex plane if this is 1 by gamma we are saying that everywhere here we are that is sort of the region of convergence that we if you will. So, in this region of convergence I get the expression 1 over 1 minus gamma S and that is the moment generating function of the Rayleigh distribution again keeping in mind that there is a r o c that we have to keep track of its not a problem.

So, let me just remove the clutter and write down the following. So, phi gamma of s for a Rayleigh pdf, for Rayleigh pdf is given by 1 over 1 minus gamma s. So, the question is what is the benefit of this we see in a minute. So, if I were to compute probability of error of DBPSK in Rayleigh fading, Rayleigh fading please ret tell me expression it will be 0 to infinity, the probability of error in AWGN will be half e power minus gamma am I right that is the probability of error of DBPSK in AWGN. Then write down next it to the pdf of the SNR, so the pdf of SNR will be f of gamma of gamma integrate with respect to d gamma.

Now, side by side please write down the expression for the moment generating function and then we are through this is 0 to infinity f gamma of gamma e power s gamma d gamma and this we already know is 1 minus gamma s. So, the question is do I really need to do the integral at all integrals very easy to do not that is because the reason is that once I have this expression you can see that this is nothing, but the constant you pull it out. Now look at what is within the bracket equation number 1 and equation number 2 compare them you can confirm that this is nothing, but phi gamma of s where s is evaluated at minus 1, now is s in the region of s equal to minus in the region of convergence yes because it is the left half plane including the j omega axis no problem. So, this is where the.

So, now the important thing to note is you can change this f gamma of to anything even Ricean no problem Ricean or Nakagami-m, does not matter at all as long as I know phi gamma of s. Any distribution you tell me as long as you give me the moment generating function I will give you the answer in one step I will just substitute the corresponding value and we will get the answer that is of interest. So, this is why MGF is so useful because it is very it helps us in terms of our ability to work with the different types of a environments that we are looking at.

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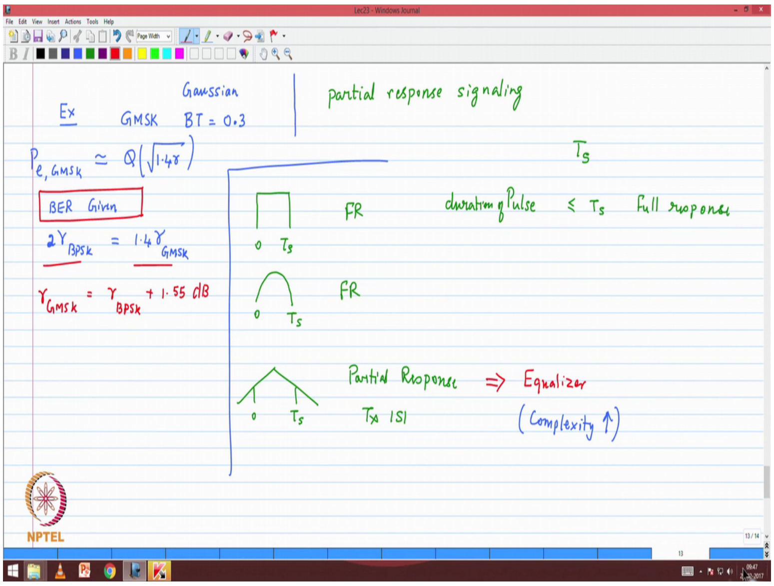

So, let me just move forward quickly to explain one more example this is an example not related to MGF or the pdf it is something that reinforces what we have studied so far. GMSK stands for Gaussian minimum shift keying. So, I will not expand it usually when you specify Gaussian filter you must also specify the shape of the filter or the bandwidth of the filters it is usually given in terms of a parameter call the BT the time bandwidth product and BT 0.3 is the commonly used value, I will make a couple of statements about GMSK BT 0.3. Again this is for you to just keep in mind when we come across it later on we will talk about that BT 0.3 its comes under the family of what is called partial response signaling, partial response signaling. What is partial response signaling? Partial response what is full responsible signaling. If you know only if you full response you will able to what is not full response becomes partial response full response.

Pulse shaping everyone is familiar with why did you pulse shaping to keep the spectrum very compact, now once you have done the pulse shaping you have a symbol duration. So, T s is your duration of the symbol that is you have the symbol rate 1 over the symbol rate is the duration of a symbol. Now if your pulse shape, duration of the pulse duration of pulse, so basically if I were to use a rectangular pulse 0 to T s that is full response, I can do sinusoidal thing between 0 to 2 s that is also full response, this is full response this is full response, if the duration of pulse is less than or equal to T s it comes under full response.

Now, if I have a for some reason I have a pulse shape which exceeds T s let us say 0 to T s it goes little bit outside of T s this becomes partial response, partial response what is the penalty that I have I am paying for partial response, why do I do partial response first of all to make the spectrum more compact, because more I have wider in time narrower in frequency. So, it is more compact in terms of spectrum. So, the reason we even talk about partial response is because of the benefits in terms of spectrum, but what does it do in the time domain. It already introduces ISI at the transmitter whether the channel causes ISI or not your transmitter is caused. So, there is a transmit ISI.

Now, what will happen if you have ISI in the channel? That ISI in the transmitter will get compounded by the ISI in the channel and you will get a composite ISI which you will have to equalize even if you do not have seen the channel you still need an equalizer. So, this one is requires an equalizer to get rid of the inter symbol interference that you have introduced to the partial response. So, again we need not worry so much except make the observation that complexity is going to go up, complexity is higher than if you did a full response system. So, that is all that we need to understand in terms of the partial response.

So, response signaling basically says its ISI is there in the transmitter for the purposes of spectrum, complexity will be higher we will live with that, but now given that GMSK BT 0.3 is a partial response signaling, the question is what is the probability of error of the GMSK. People have characterize this and I have said it can be very closely approximated by 1.4 gamma Q of 1.4 gamma. Notice that there were 2 Q functions will take the BPSK case it was square root of 2 gamma. Now is GMSK is better than BPSK or worst than BPSK.

The way to look at it is if I were to specify a given BER level, given BER level that corresponds to some SNR. So, basically from the Q function you can find out that 2 times gamma BPSK which is what that is what will give you that BER Q function that will map to 1.4 times gamma GMSK right because the Q functions argument of the Q functions map to the same BER value to for a given BER and if I want to satisfy it with both BPSK and GMSK I must satisfy this condition. So, this clearly tells me that gamma GMSK is higher than gamma BPSK. So, that is gamma BPSK plus 1.55 dB. So, it is a slightly worst than BPSK, but it is something that is better than the noncoherent schemes. So, therefore, this is a useful candidate especially because the spectrum is very compact.

Now let us complete the discussion just one more observation that I will give you.

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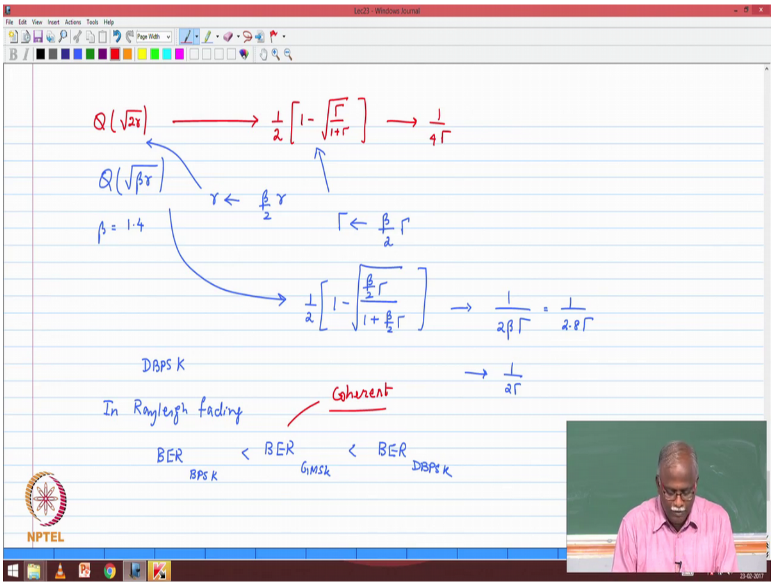

Now if I had the relationship for BPSK Q of 2 gamma that is in AWGN in fading we say that this maps to one half of 1 minus square root of gamma divided by 1 plus gamma and under high SNR this maps to 1 over 4 gamma. Now without re doing all of the calculations can we get the expression for GMSK where GMSK is equal to square root of beta gamma where beta is not true, but is equal to 1.4. So, we have already done a case like this. So, basically what we would like to do is in this expression replace gamma with beta by 2 gamma right. So, basically get it into the form, if I do this kind of scaling where beta is the constant 2 is the constant, so there is no issue with that. Then in this graph I would have to do the scaling gamma as beta by 2 gamma and then redo the calculations we do not need to do the integration this will become one half of 1 minus beta by 2 gamma under the root sin divided by 1 plus beta by 2 gamma.

Again you can simplify that you can also verify the highest SNR approximation comes out to be 1 by 2 beta gamma which is the same as 1 by 2.8 gamma. It is not as good as asymptotically it is not, it will not touch the BPSK it will always be shifted slightly shifted, slightly worse, but it is a very good modulation for us to work with.

On the other hand the DBPSK the asymptotic value is 1 over 2 gamma, so which means that the DBPSK is slightly worse than GMSK and so we can make the following statement which I believe is correct, under assumptions. So, in Rayleigh fading Rayleigh fading when there is no channel tracking errors the BER of BPSK is less than BER of GMSK is less than BER of DBPSK and since I need equalization there is no question of non coherent detection of GMSK it has to be coherent. So, this has to be coherent detection, this has to be coherent detection. So, again it is was a little bit of a detour, but to tell you that the tools that we have developed are very broad and very useful. Now combine the tools that we have developed with the moment generating function you have all the tools that you have to need for understanding and working with the BER expression.

So, what I would like you to do is a refresh your memories about the memory about the MGF if you have recently studied it no problem, if you have not studied it recently look it up in Proakis or any of the standard books we will use MGF in a some in a very clever ways just to help us in understand and derive the bit error rate in fading environments. With that we will then move to WSS US model in the next lecture.

Thank you.