Introduction to Wireless and Cellular Communication
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Lecture – 28
Wide Sense Stationary Uncorrelated Scattering (WSSUS) Channel Model
WSSUS - Characterization of Time Dispersives Fading Channels

Good morning we begin lecture 27 with a quick summary of lecture number 26; in the last class we have been studying the wide sense stationary uncorrelated scattering model we build on that to understand the tau dimension in today's lecture, but a quick summary of what we have discussed in the last class the characterization of the time variation the temporal variation. So, that we said could be added in terms of two more parameters which we saw in the last lecture that was the level crossing rate temporal characterization temporal refers to the time domain or the time variation.

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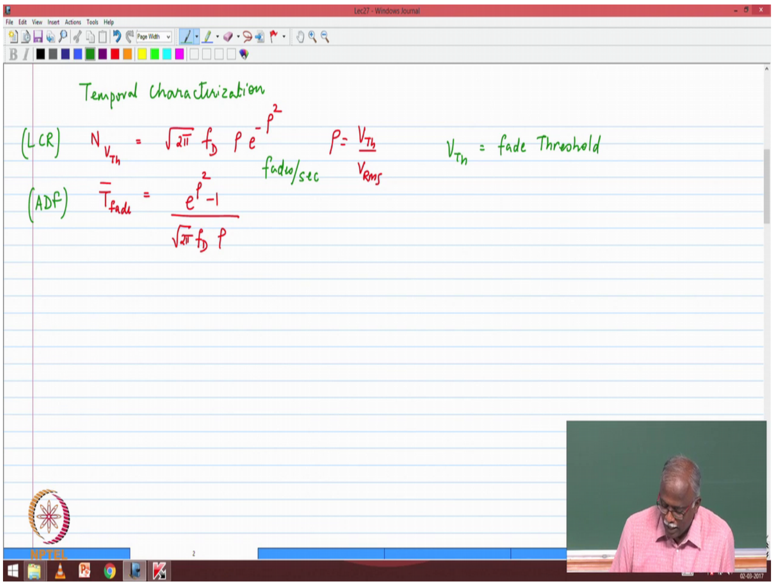

This is the level crossing rate LCR denoted by N v or n V T h basically a threshold where you declared that be the fade threshold. So, V T h is your threshold for fade if the envelop goes below that then you declare the signal to be in a fade. So, that is the and this was the what we derived in the last class was the number of basically the way of counting the fades and the average duration of a fade. So, what would be the units of N V.

Student: Page per second.

Page per second I think almost said it; basically it is a probability multiplies by the time if you take the time to be this one second then you get the fades per second ok.

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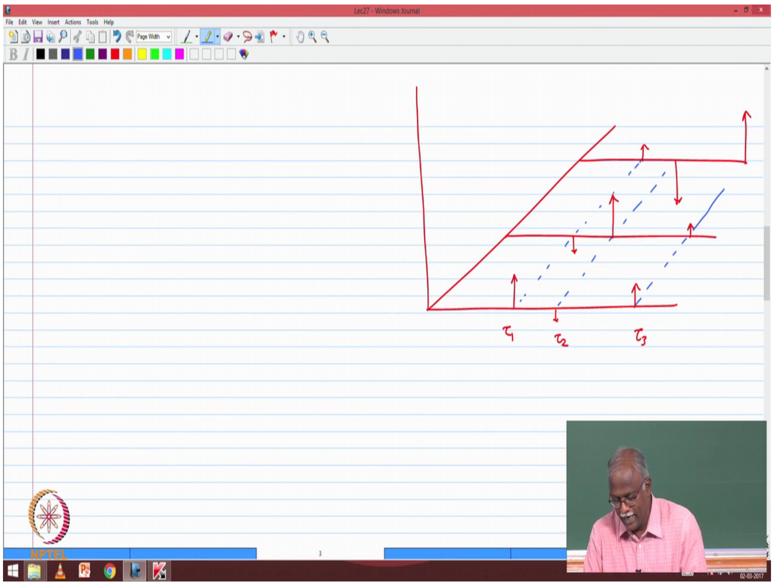

Now this then after this we when moved on to the wide sense stationary uncorrelated scattering model, once again reminding ourselves of the model that we have this is the tau dimension this is the time dimension we are looking at the we are already studied what the time variation looks like on each of the dimensions. Now we moved over to understanding what happens in the delay dimension the tau dimension.

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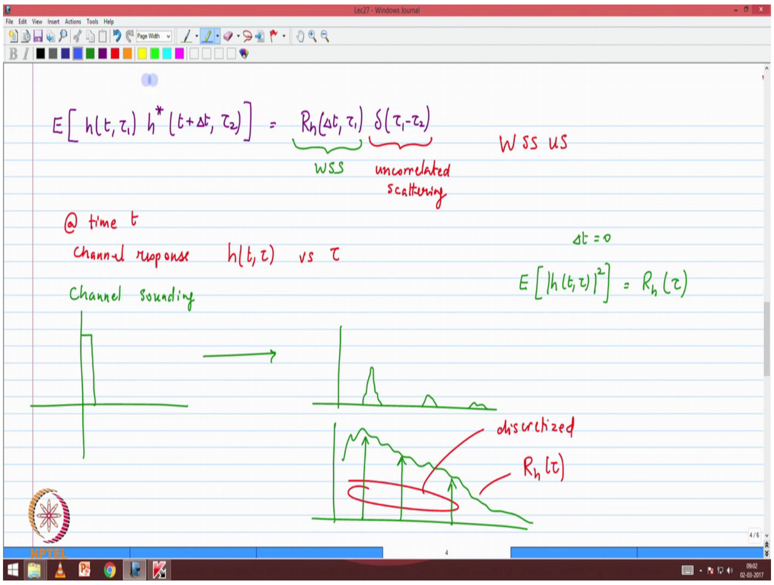

So, the argument that was made was that the interacting objects which create the multi path component at tau 1 are different from that which creates a tau 2. So, therefore, that the there is no correlation between them. So, therefore, the expression for the autocorrelation where I have t comma tau 1 h star, h of t plus delta t comma tau 2 basically boils down to the autocorrelation that we have when we had only a single tap delta times tau 1 minus tau 2.

So, basically there is no correlation across the different delays, but at a given delay you get the Bessel auto correlation function. So, this was the understanding of the why it is called the wide sense stationary uncorrelated scattering. Wide sense stationary in the time dimension uncorrelated in the tau tau dimension. So, basically what we said was if I now set delta t to be equal to 0 that gives us a unique or a very interesting perspective because then we now have what is present in the in the tau dimension, and this was what we this is where we stopped in the last lecture and we were trying to say that we this would correspond to the following; there was a little bit of confusion as to how did you get R h of tau. So, I will explain that in the following example.

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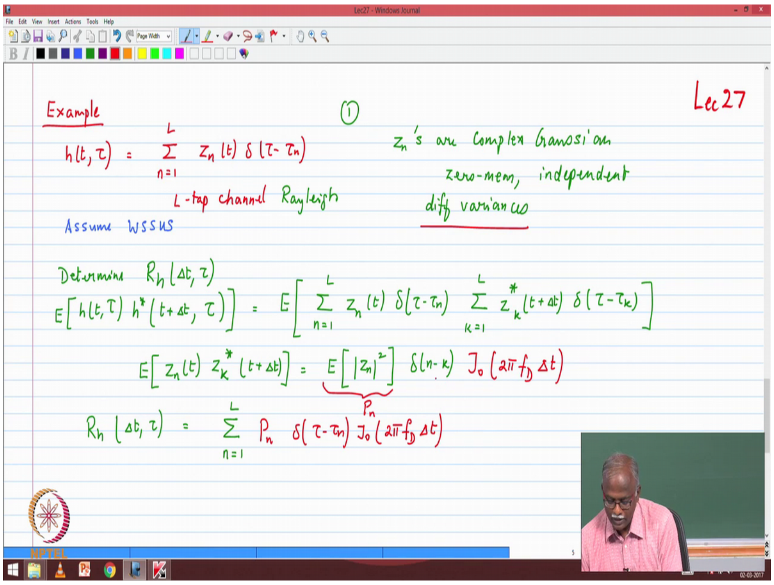

So, let us look at an example may be that will be the best way to answer or clarify any doubts that will that have arisen because of that. So, here is a time varying channel h of t comma tau that is given by summation n is equal to one through l Z n of t delta of tau minus tau n. So, how would do you describe this channel? It is a l tap channel it is time varying. So, L tap time varying with different delays each of the l taps have got different delays. So, it is an L tap channel time varying I would not write that down it is obvious from the and if we are also told that the zns; zns are complex Gaussian complex Gaussian that is complex Gaussian we can also make the assumption that 0 mean and independent not necessary for our discussion.

But they could have different variances there is no assumption about a variances may be we even state that different variances. So, in other words each of the Zns are complex Gaussian random variables 0 mean to so, that they it. So, this can be then described as an L tap Rayleigh channel. So, that is what it is and each of those taps are independent of each other they may have different variances; that means, the power of each of those different L taps may be of different values that is what is a given by the different variances ok.

So, this is a very important model very often assumed and encountered in the study of wireless channels, where you say that there are certain number of taps each of these are independent Rayleigh channel represent independent Rayleigh channels and they all have different they may have different power levels. So, the example then goes on to say that we will assume wide sense stationary uncorrelated scattering model, the task for us is to determine the autocorrelation; determine the autocorrelation R h delta t comma tau.

So, a please write down the expression the autocorrelation is expected value of h of t comma tau h star of t plus delta t comma tau, and substitute from equation one in this expression you will basically get a double summation expected value of the first summation n equal to 1 through L Z n of t delta of tau minus tau n and then I have to change I will change the independent variable to k; k is equal to 1 through l Z k conjugate of t plus delta t delta of tau minus tau k. So, just basically substituted from the earlier expression; notice that the quantities that will in come in to the expectation will be of the form Z n of t Z k star of t plus delta t.

So, this can be written as if we go back to the wide sense stationary uncorrelated scattering model, this will be expected value this this will be nonzero only if n is equal to k. So, this can be written as expected value of mod Z n square delta of n minus k. So, basically the double summation will collapse and what we will get is this expression. The final answer in that case the r the autocorrelation R h of delta t comma tau can be expressed in the following fashion, it will be a summation it will consist of L terms each of these if I denote by P n expected basically is the power of that particular tap. So, it will correspond to P n and you will you will have the Bessel the autocorrelation as a function of delta t wait a minute did I miss something here Z k, Z k conjugate I forget Bessel function J 0 2 pi f D delta t ok.

So, this will be P n delta of tau minus tau n, j naught 2 pi f D times delta t. So, how would we visualizes just a make sure that not missed anything tau minus tau n this is only a nonzero if it is n n and k will collapse. So, basically you will get a single summation with the different delays yes ok.

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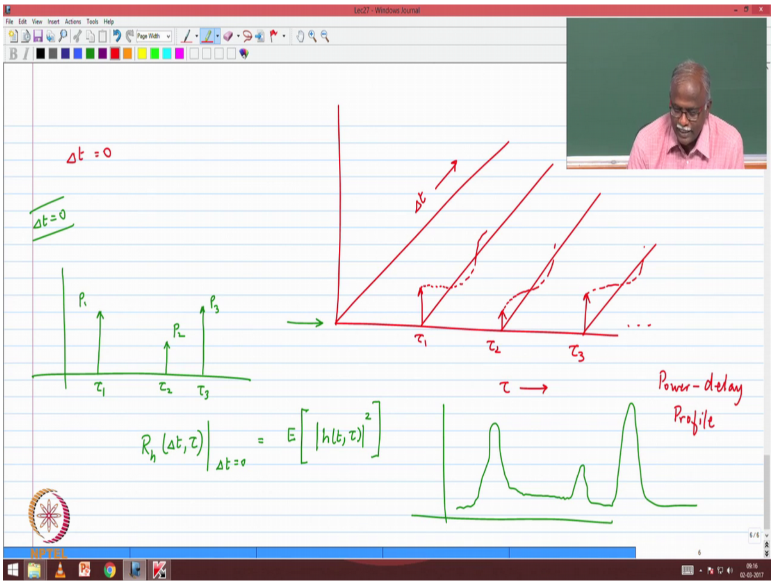

So, how would we visualize this? This is the visualization that hopefully will help us in understanding this corresponds to P 1, something corresponding to P 2, P 3, P 2, P 3 the power levels.

Now, the autocorrelation basically says that there is a Bessel function that that that is in that particular axis and there is another Bessel function here there is another Bessel function here. So, what did the, what did when they have there was only one tap what as the interpretation? Basically you get the power of the tab and if you observe along the time dimension you will see the autocorrelation as a Bessel function. Now what happens if you have a wide sense stationary uncorrelated scattering and you have multiple tabs these tabs do not interact with each other these tabs do not interact with each other that is what the uncorrelated scattering part says, but at any given time you if you observe along the time dimension there will be correlation of that particular tab with itself.

So, this is the interpretation of the autocorrelation when there is time dispersion and you have assumed the wide sense stationary uncorrelated scattering model. So, now, under this once if this example is clear what happens if I set delta t to be equal to 0? So, this is basically not a this is the delta t dimension right it is now that is what we are a representing this is the tau dimension that is delta t dimension in the delta t if I said delta t to be equal to 0 I am basically saying look at it at the at a particular reference in in the (Refer Time: 12:08). So, at this point what I get is I get component with power P 1 and then at some this is tau 1 maybe I should not write P 1, P 2, P 3 here these should be at a delay of tau 1, delay of tau 2, tau three like that there up to tau l there are. So, at tau 1 there is a signal with component with power level P 1, then at tau 2 component with power level P 2, and then at tau three component with P level.

So, this is what is the autocorrelation if I said delta t equal to 0 and this is precisely what is obtained by saying if I take expected value or if I take R h of delta t comma tau if I set delta t equal to 0 what I get is if you if you see with in the expression it is expected value of h of t comma tau magnitudes square. So, it is like the power of the channel response power at each point you at each value of tau. So, in other words if it was the discrete time model you will get something like this, if it was a continuous model what you will see is something where there is some variation.

So, this corresponds to basically there is one may be we will just try to draw this particular itself if we were to if we were to look at in continuous time something very low you see a strong component something very low and a smaller component and then a stronger component. So, basically if you sample this you will get something that looks like the impulses one this is in a continuously as a function of tau if you measure the power levels. So, this is what we refer to in the last class as the power delay profile and the name is descriptive of what it represents? Delay in one dimension power in the other dimension and how would you do that you transmit very narrow pulse and then you measure the pulses or the power that is received at different delays. So, the power delay profile is what we obtain when we set delta t to be equal to 0 in the autocorrelation function.

So, yesterday there was a little bit of confusion saying you know how do you get different values of tau because is not this supposed to be delta of tau because; that means, there is no nothing that only at tau equal to 0 you will get something, but a basically what it says is there is no correlation across these different delays, but within the delays there is the time correlation, but if I am not interested in the time correlation I just want to look at this at any given snapshot in time, what I am saying is a power distribution across the across the tau dimension any questions. So, that was where we stopped in the last lecture lets pick it up from there and. So, our starting point today is to take a look at the power delay profile and then work with a power delay profile.

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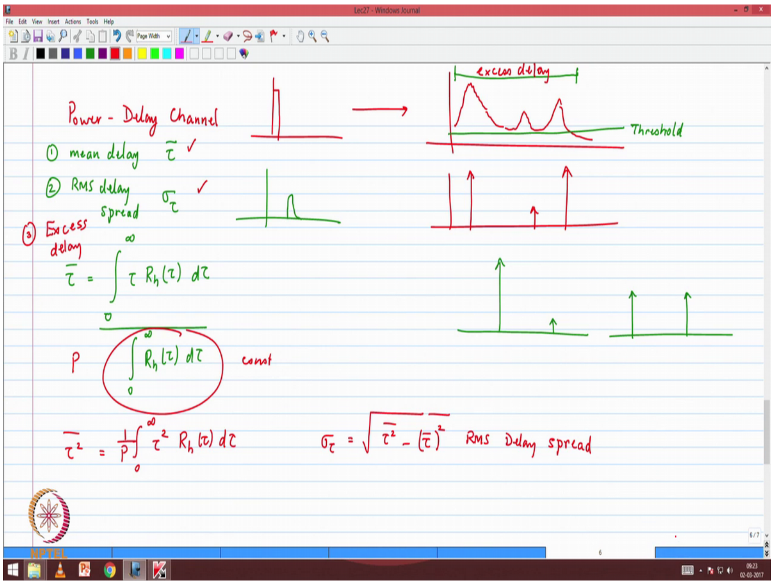

So, power delay profile is a very useful characterization of a channel. In fact, that was probably very widely used in even in static channels if you wanted to look at a how to transmit information over telephone channel or a cable TV channel, the first thing that you would do is to do channel sounding and see what is the response of the channel. So, as we mentioned in the last class typically you transmit a pulse and then observe what comes out at the other end, typically what you will see is that there will be a little bit of a delay because the pulse has to traverse to the receiver and then you will see that there will be some rise of the power that is where the bulk of the energy is coming, sort of drops down then maybe there is one more pulse and then may be another third pulse and then it dies down something like this is what you observe.

Now, if you were to discretizes it may look like there is a strong eco or the first component that arrives a second component a third component and again depends on how you want to characterize it, and typically the way it is characterized is you say that once the signal level falls below a certain threshold I am not interested in the that I think I am assuming that the all the power has died down. So, there is some threshold below which I am not interested anything above that is of interest. So, basically you want to observe. So, one of the key things that we always what to measure is what is the first when does the energy start arriving, and then till what time does the energy stay above the threshold.

So, that is the amount of time the channel is going to respond to a single pulse. So, that is what is going to give us a characterization of the delay dimension. If it was a non dispersive channel what would you expect to see if I transmitted a pulse I will see another pulse that is it that is that is what you would see, but if it is for the dispersive channel and a very complex dispersive channel like what we you have in wireless, you will see that you know it is totally smeared there are multiple copies they are running one into the other. So, now, in real life if I start transmitting pulses adjacent to each other that is what you will do in transmission, you can see that you will have a lot of inter symbol interfere. So, that is the, it is sort of leading us into understanding that, but for right now we are not looking at the just trying to characterize the channel. So, therefore, that is where there we are.

So, the two characterizations of the power delay profile are the following the first one is a parametrization which says what is the mean delay. No I will explain to you in a moment why mean delay is important, but before that the second one is the standard deviation. So, this is called tau bar tau is the delay tau bar is the mean delay of the channel and then the RMS delay spread and we will see in a moment why these are important characterizations and this is denoted by sigma tau sigma for RMS and a tau for the basically sigma t tau square is variance and this is the r m s delay spread.

Now, one of the reasons why these two are important is may be let me make you think along the lines of digital communications; now we are switching our heads no longer wireless this is digital communications. I have a channel which looks like this it has a certain energied power in the channel the alpha one square plus alpha two square, there is another channel which is of the same power level and that one has got two taps the total powering both are the same.

Now, from a communications perspective what is the difference between these two channels second one has got strong inter symbol interference I will have to have an equalizer to detect it otherwise I will this one I may be able to get away without an equalizer so, but in terms of the total power in the channel both are the same, but this one is a more problematic. Now how do you reflect that and that is what we are trying to characterize that there is a difference between the channel number one and channel two and one of them is more severe then the other and how do we characterize that.

So, the mean delay spread will define it for both the continuous power delay profile and the discretized power delay profile says compute the mean value mean value is exactly like how you would calculate the probabilities 0 through infinity tau, but instead of the pdf we are now going to wait it by the power of that particular delay component. R h of tau d tau we should normalize it. So, that is divided by integral 0 to infinity R h of tau d tau. Integral 0 to infinity R h of d tau this is a constant right this is you can treat this as some constant let us call it as p that is the total power in the channel.

So, basically the mean delay is defined by it looks like a probability calculation, but this is how the mean delay is obtained this is a constant. So, we can sort of you know take it aside, but again it is its part of the expression. So, now, if I were ask you to calculate tau square mean value, mean square value exactly the same it will be 0 to infinity tau square just like the replacing the pdf it is R h of tau, d tau into 1 by p that is a normalization.

So, this is the mean squared value and the RMS delay spread sigma tau is square root of tau square bar minus tau bar whole square, and that is the expression for the RMS delay spread and as I mentioned if you were to calculate the mean delay and the RMS delay spread you will see that these two channels give you very different characterization though the based on the power level both of them will be the same. This is the difference between the first arriving multipath and the last arriving multipath is called the excess delay. So, we characterize a channel by three measures one is r the mean delay the RMS delay spread and the third one is the excess delay that is the total delay that you see excess delay; three parameters which will help us I think. So, examples are before we go to examples let us also look at a couple of the what you do for a discrete time.

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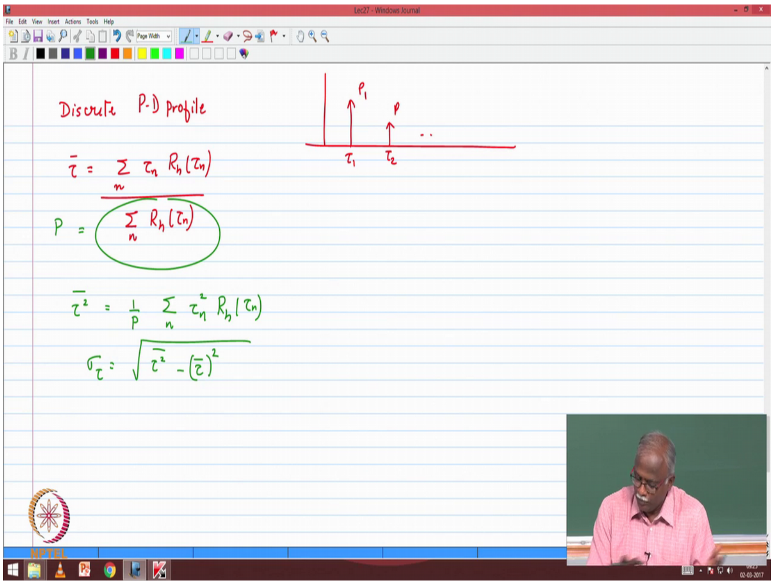

So, it was a discrete discrete power delay profile very similar to the one that we saw in the example. So, there is P 1 here at a delay of tau 1, P 2 at a delay of tau 2 and so on the mean delay would be defined as now a summation over the multipath components tau n R h of tau n. So, basically at that appropriate delay the normalize comes with summation of over n R h of tau n, and again this the denominator is a constant this is equal to P a constant and the mean square value tau square will be 1 over p summation over n tau square R h tau n square R h of tau n, and that would be there and sigma tau would be exactly like before tau square bar minus tau bar the whole square ok.

So, two based whether you are doing it in the continuous time or discrete continuous variable or a discrete discretized variable, you should get the same interpretation, but the again know that one cases in integral the other one is a summation. Let us look at a couple of examples I think that will be very insightful and also helpful in terms of the understanding of the concepts ok.

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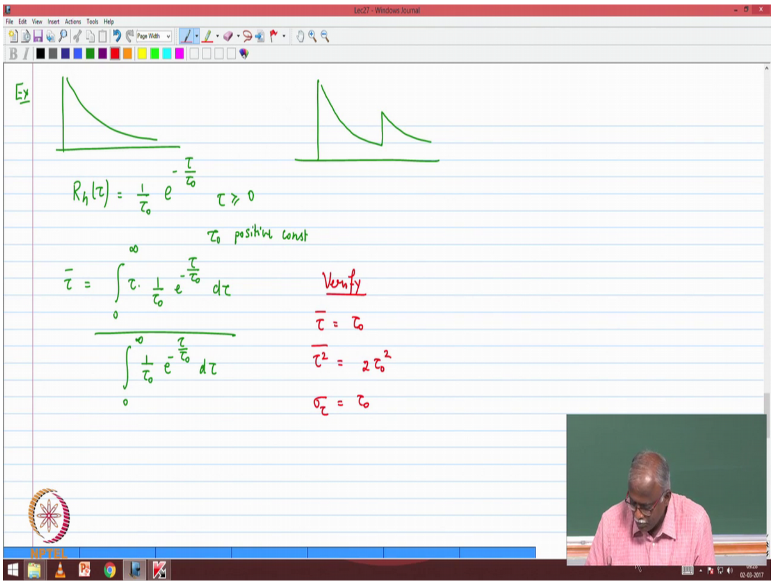

So, one of the common power delay profiles that we will encounter in wireless channels is the exponential power delay profile. So, which means that initially the signal strength is strong and then as time goes on these the delay the components delay die out, this a very common one.

So, that is the early arriving multi path are stronger than the later multi paths and therefore, it dies down. But in wireless channels where supposing you had a building at a distance and that was causing a strong reflection then what you will see is multi path starts to decay and then suddenly it jumps up and then it is starts to decay. So, that is also a very common channel profile that you will see in wireless channel. So, whenever there is a strong component that is it is presence will be felt and again it is not may not books, but it may it will look up of this type.

So, exponential power delay profile is useful for us. So, we can look at an example just a single exponential delay profile R h of tau is given by 1 by tau naught e power minus tau by tau naught. So, basically that and for tau greater than or equal to 0 tau naught is a positive constant. So, this is a very familiar type of graph you know basically capacitor discharging inductor all of those are a exponential. So, we are used to this type of function. So, the integration is a is not difficult. So, tau bar is given by integral 0 to infinity tau times 1 over tau naught R h of tau you substitute, e power minus tau by tau naught d tau and the denominator is integral 0 to infinity, 1 by tau naught e power minus tau by tau naught d tau simple integrals both of them numerator and denominator and you can verify.

So, please verify the following results again through the process of completing the integrals that the answer comes out to be tau bar is equal to tau naught, tau squared bar comes out to be equal to 2 times tau naught squared. So, which then says that sigma tau is equal to tau naught. So, this is a sort of a unique channel where the mean delay and the RMS delay are both the same value tau naught, and you can see where that is a point at which you get one over e as the value of your in the decaying exponential.

Now, let us move on to discrete time example because the that is probably more likely what you will encounter when you are dealing with the 3G 4G types of systems, continuous power delay profile has been sampled and this has been given to us as a example that we have to calculate ok.

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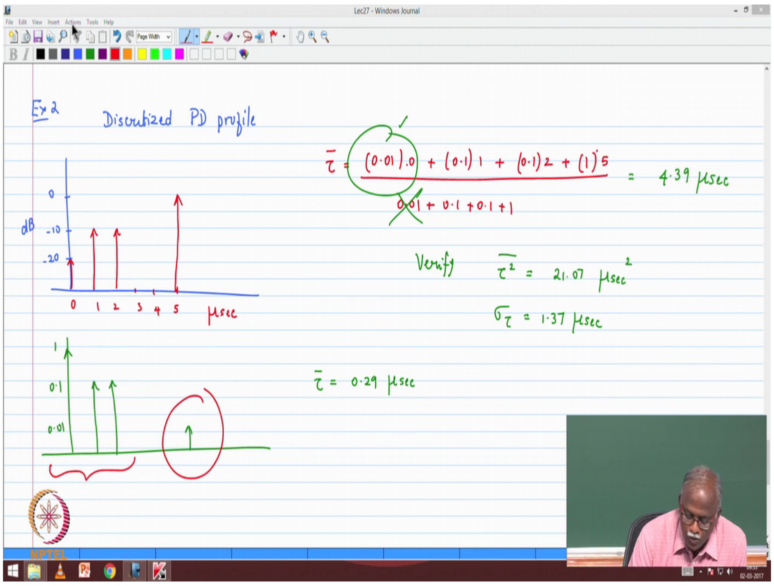

Example 2. So, this is a discretized power delay profile discretized power delay profile and these are scenarios for which we would definitely be interested in calculating the characterization of the channel. So, here these power levels is given in dB. So, there is minus 20 this is in decibels, minus 10 0 the multi the multi path power delay profile is that there is one tap at minus 20 there are two taps at minus 10 d B and then with long delay there is a third a fourth tap which is at 0 dB. So, this is 0 1 2 3 4 5 we will take it as microseconds as our.

So, tau bar again you may wonder why go through this exercise, but I need you to sort of pay attention to this particular aspect tau bar is 0.0 1 minus 20 dB 0.01 into 0 plus then I get 0.1 times 1, a 0.1 times two microseconds 0.1 within brackets is the strength times the delay by five divided by it is 0.1 plus 0.1 plus 0.1 plus 1. Now why did I make the make this exercise is very often student say you know what this did not contribute anything right. So, therefore, I will leave of that why did not contribute to the numerator why should I contribute to the denominator is that correct no because the otherwise what will happen it will queue your result. So, yes the first one did not contribute the numerator, but it definitely there is a signal component there. So, which means that you have when you normalization you must include that in the denominator. So, that is all I wanted to convey in this one.

So, please verify that what you get is 4.39 microseconds, and that is the value you can also verify the following results verify simple calculations that tau square bar comes out to be 21.07 microsecond units square again it is a we do not use tau square, but if you did use it you will have to indicate it appropriately and RMS delays spread comes out to be 1.37 microseconds RMS delay spread. Now just for illustrative purposes let me do the following I am going to flip those channels around I am going to put the strong channel at 0 delay then these two are at the same point and the weak one goes here just interchange the positions. So, this is at a height of one this is 0.1 0.01 again z0 minus 10 20 dB please do not do it in dB the answer you have to do it in the linear scale.

You will find that the mean delay in this case comes out to be 0.29 microseconds very different from 4.39 microseconds. So, this is why characterization of the channel using these tau bar and RMS delay spread very very important because in terms of the excess delay both of them have got 5 microseconds as the excess delay they have got the same power levels, but the tau bars are very different and the this one this tells us that the channel green channel is much less dispersive than the red channel you may say you know how can you say make such a statement because you know they look the same that is that is how the receiver will see it the to the receiver the green channel is a much less dispersive channel because this particular component can almost be ignored.

So, effectively the dispersive that I have to worry about is this and therefore, I can take that into account. So, again the characterization of the multi path channels in terms of the power delay profiles I hope you are comfortable with this is a useful characterization and it helps us in the second dimension. Now I want to quickly move into another way of characterizing the tau dimension.

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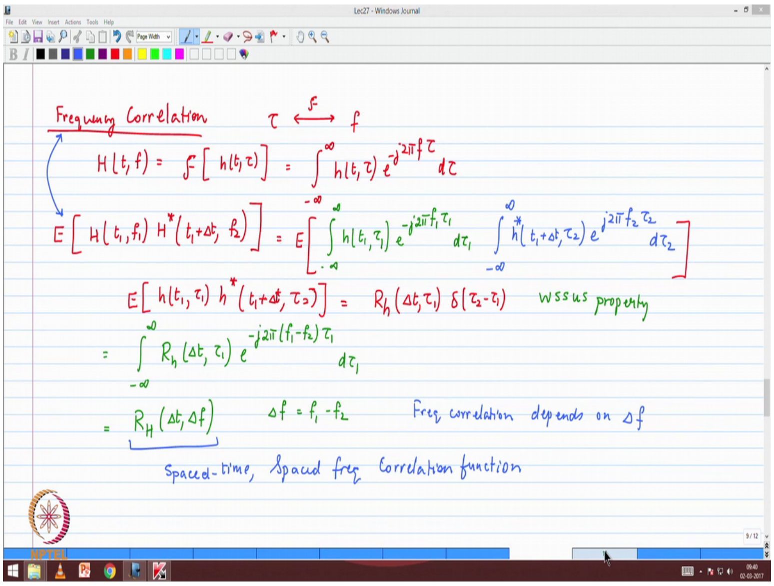

And that we will do using frequency correlation frequency correlation and the name will become clearer once we do the calculation. The first step that we are going to do is in the basically now keep in mind that we are trying to characterize the tau dimension. So, I am going to do a Fourier transform so that there is going to be a change of variables basically I do the Fourier transform for the tau dimension not the delta t, delta t is already completed.

So, tau maps to f in the in the. So, H of t comma f is the Fourier transform of h of t comma tau and let me just right it down. So, that we are no confusion about the notation it is minus infinity to infinity Fourier transform in the tau dimension, h of t comma tau just ignore the fact that there is a t dependence also basically it is a function of tau e power minus j 2 pi f tau d tau assuming that it is a way a function of tau you do the Fourier transform with respect to tau. Now what we would like to do is frequency correlation says what is expected value of H of t 1 comma f 1 some frequency after I have done the frequency come compared with H star of t 1 plus delta t and f 2 notice that I am doing the correlation there are two variables on the time dimension I know that there is a wide sense stationarity. So, I am you calling it as t 1 plus delta t and on the frequency side I am calling it as f 1 and f 2, 2 different frequencies I want to know what is the expression for the correlation ok.

So, like before I would like you to substitute the integrals. So, basically you will get expected value of the first integral in green second integral in blue. So, that we can this minus infinity to infinity h of t 1 comma tau e power I have to tau 1 because basically the independent variable. So, e power minus j 2 pi f 1 tau 1 d tau 1 that is the first integral, second integral I write it in blue minus infinity to infinity h of h star of t 1 plus delta t comma tau 2 that is the independent variable with which I am doing the transformation, e power it should have been minus j because of the conjugation it becomes plus j 2 pi f 2 tau 2 d tau 2 that is the sum within the expectation.

So, again group those terms which will affect the expectation basically you will get expected value of h of t 1 comma tau 1, h star of t 1 plus delta t comma tau 2 this is already a result that we know this is the autocorrelation which is basically the Bessel function R h of delta t because of the wide sense stationary uncorrelated scattering it is tau 1 delta of tau 2 minus tau 1 right this this exactly the expression from the wide sense stationary uncorrelated scattering. So, again I will just write this this is the WSSUS property.

If we now use this result in the substituted in the previously expression what we get is a single integral because the double integral now collapses to a single integral because it is only non-0 when tau 1 equal to tau 2. So, what we get is minus infinity to infinity R h of delta t comma tau 1, the e exponential term e power minus j 2 pi f 1 minus f 2 tau 1 d tau 1. So, the Fourier transform has been computed I am trying to do a correlation operation on the Fourier transform what comes out of this is a very interesting result which says that the result is does not depend on the specific frequencies, it actually depends on the difference of the frequencies very interesting result. So, I will call this as R uppercase h to show that it is a Fourier transform delta t not touched that was not affected by the transformation this is now a function of delta f it does not depend on f 1 and f 2 it depends on f minus f 2, f 1 minus f 2 ok.

So, the frequency correlation like the time correlation has got a very interesting property in the context of a wireless channel. So, this basically says that the frequency correlation frequency correlation is what I mean by this expression, I have I have defined the Fourier transform and I am taking the correlation of the Fourier transforms. So, the frequency correlation depends on delta f and it is a very interesting result we will interpret it in a moment. So, this particular function has got a name it is called the spaced frequency spaced time; that means, it is a it is function of delta t, it is also function of delta f it is called spaced frequency again just to describe what it is correlation function.

So, a correlation function in frequency, but it does not depend on it only depends on delta f and in time already depends only on delta t correlation function again just to describe it, but you know exactly what it means. So, what have we done took the Fourier transform of the channel response with respect to the delay dimension then computed a correlation function and showed that it comes out to be a function that depends only on the difference of the frequencies ok.

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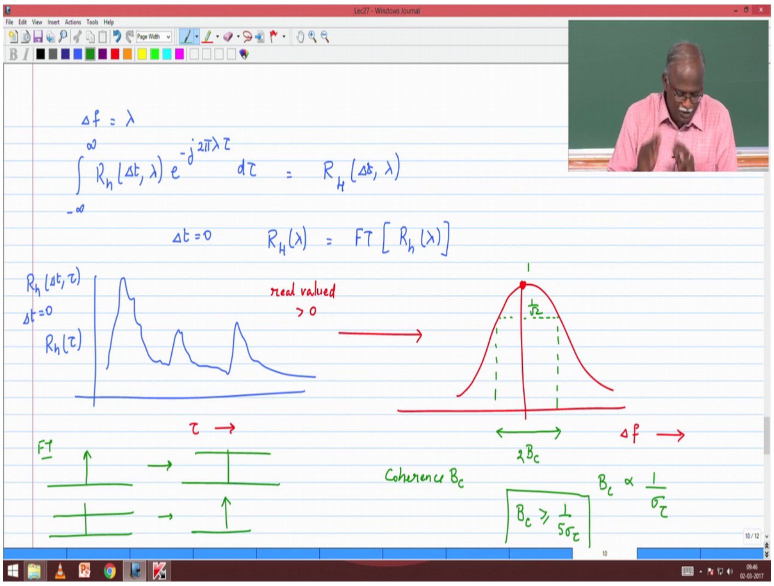

So, here is the how we would like to like to work with this. So, let us indicate or substitute delta f equal to lambda let us just a substitution. So, the result that we now have is says minus infinity to infinity, R h of delta t comma lambda e power minus j 2 pi lambda tau d tau this is e this is what we write as R h of delta t comma lambda that is the space time space frequency and now if I set delta t equal to 0 if I set delta t equal to 0 what is happening what is? If I set delta t to be equal to 0 what did you say at the beginning of the lecture it becomes the power delay profile. So, this space time space frequency which is actually a correlation function turns out to be the Fourier transform of the power delay profile. So, that is the observation that we get.

So, if I set delta t to be equal to 0 I get R h of lambda to be equal to the Fourier transform of the power delay profile R h of lambda Fourier transform. So, let us sketch it the power delay profile something like this what are the so, this is actually R h of delta t comma tau I have set delta t to be equal to 0 that is what and therefore, I am now calling it as R h of tau right the basically the that is what a reason and I am going to do a Fourier transform of this, but I would like to make a couple of observations this R h of tau is a power delay profile. So, I can make the following that it is real valued can I also say that it is greater than 0. So, power delay profile either you get a something positive or not so power delay profile. So, if I have a function that is real valued and positive what can you tell me about it is Fourier transform.

Student: Symmetric.

Symmetric. So, it is going to have a response that looks like this and guaranteed that the peak value will occur at 0, because you are going to add all positive quantities to get the Fourier transform at 0. So, basically this is the peak value that is going to occur it is going to be symmetric I am really not too much worried about the shape for the following reason. So, what is my access this is delta f and this is the tau dimension. So, power delay profile through a Fourier transform operation gives me a shape which is symmetric and the peak occurs at the delta being equal to 0. Of course, if the peak occurs there and it is symmetric it is going to taper off to the other side you may say well what is the guarantee that it is going to taper off what why does not it go do some other some other thing. So, may be at this point another explanation from Fourier transform theory is very helpful to us case one this is just to understand how what how to interpret this result. So, Fourier transforms result if this is time Fourier transform is wide symmetric and wide ok.

Now, if I have a Fourier transform that is this way this is going to be an impulse. So, if something is very wide in the time domain what do I expect to see something which is very narrow if it is very narrow in that in the in the time domain it is going to be very wide. So, it does not matter which way it is it is going to be you know somewhere in between the two. So, there is going to be eventually if this power delay profile went on to infinity then this will become narrower and narrower if it was very short it will become wider, but at the end of the day there is a still a this is a going to be a dropping function.

So, now what I am going to do is just like in the Bessel function case I am going to see where the level crosses one by root two if the I normalize this to 1, 1 by root 2 that will give me a certain spacing between those two points I am going to call this as my coherence bandwidth; and let me just sort of give you a feel for why the name and where it is going to be linked to as I said you know it is a jig saw puzzle where you are taking pieces from different places. Now when we are doing this TSE and Vishwanath example we said that coherence bandwidth is the amount by which you must move in order to see a frequency correlation that is substantially different, this is frequency correlation remember what did we call this calculation frequency correlation.

Now, if I move by coherence bandwidth I am going to see a sufficiently different channel because the coherence or the correlation of the frequency response has decreased by a certain amount. So, this is what we refer to as coherence bandwidth and this is what is going to be characterized in our calculation I believe this has to be two times coherence bandwidth because. So, now; obviously, the wider the power delay profile then narrower coherence bandwidth. So, coherence bandwidth must be inversely proportional to the delays the RMS delay spread correct, it has to be because wider the power delay profile the larger will be the sigma tau narrower will be the coherence bandwidth.

So, the actual relationship again through empirical methods has come out to be is greater than or equal to 1 by 5sigma tau that is a way of characterizing saying. Once you have the power delay profile compute the sigma tau that is easy for us to do and the coherence bandwidth is going to be greater than 1 by 5 sigma tau. So, again it is a characterization as we mentioned this is a way to understand the tau dimension in the channel. So, let me just summarize what we have said so far and hopefully that that will be a good point for us to stop.

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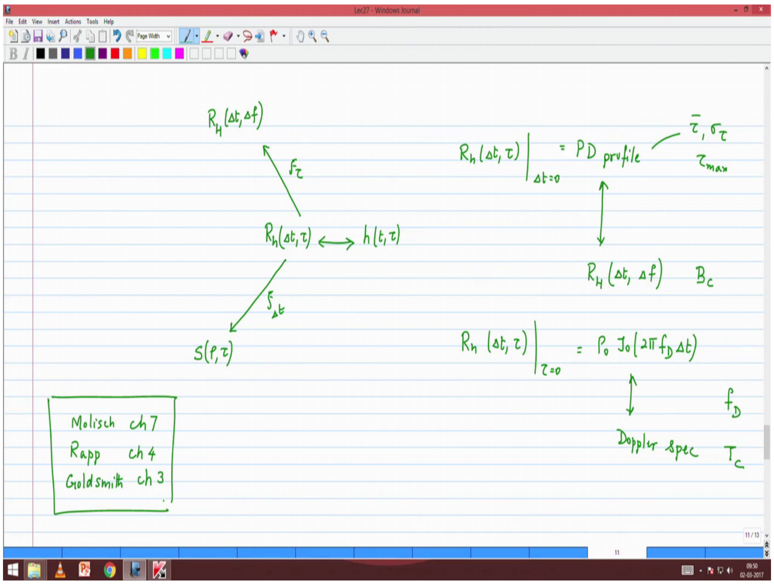

So, we had the channel response h of t comma tau from that we calculated the autocorrelation function R h of delta t comma tau. Now we did a first Fourier transform to get a Fourier transform in the delta t dimension to get the Doppler's spectrum s of rho comma tau. We have done a second Fourier transform and a correlation to get this expression the if I Fourier transform with respect to tau the power delay profile if I set this where delta t to be equal to 0, I will get power delay profile that gave me R h of delta t comma delta f. So, this is the picture that we have so, far I have the channel response I have got the auto correlation function what did we do to the autocorrelation function R h of delta t comma tau we have set delta t to be equal to 0 that gave us the power delay profile. Power delay profile is characterized by three parameters tau bar sigma tau and tau max the excess delay. Now if I take the Fourier transform of the power delay profile I will get this space time spaced frequency function delta t delta f and this tells me how to get the coherents bandwidth.

Now, the same thing I take the autocorrelation function delta t comma tau and I set tau equal to 0; that means, I am not worried about the delay dimension only the time variation this came out to be P 0, J 0 2 pi f D delta t this is the Doppler spectrum when I do the Fourier transform, and this this is characterized by f D and this also is related to the coherence time. Lots of pieces around we need to put all of them together to get the complete picture.

So, mixed class with this picture will become crystal clear in terms of all the pieces and how they fit together. So, as of now we have got all the information. So, the key points that I would like you to do is review what we have covered in the class, but also make sure that you have a chance to look at the following reading assignment Molisch chapter 7 all of this is covered there also covered in Rappaport chapter 4 and goldsmith chapter 3 whichever is your favorite book if preferably all three you should if you can at least take a look at you will find that this information what we have covered in today's class we will know integrate it into a single composite picture which gives us a complete characterization of the wireless channel.

Thank you we will see you tomorrow.