Introduction to Wireless and Cellular Communication

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Lecture - 35
Fading Channels - Diversity and Capacity
BER in fading, Equal Gain Combining

Good morning. We begin with the summary of lecture 33, but let me give you the highlights of lecture 34. We are going to look at the mathematical and statistical characterization of maximal ratio combining. We will find that the requirements for MRC are quite stringent in order to be able to achieve the benefits. So, as always is there a tradeoff that achieves most of what MRC achieves, but without the complexity of MRC there is a version called the equal gain combining; which we will talk about that.

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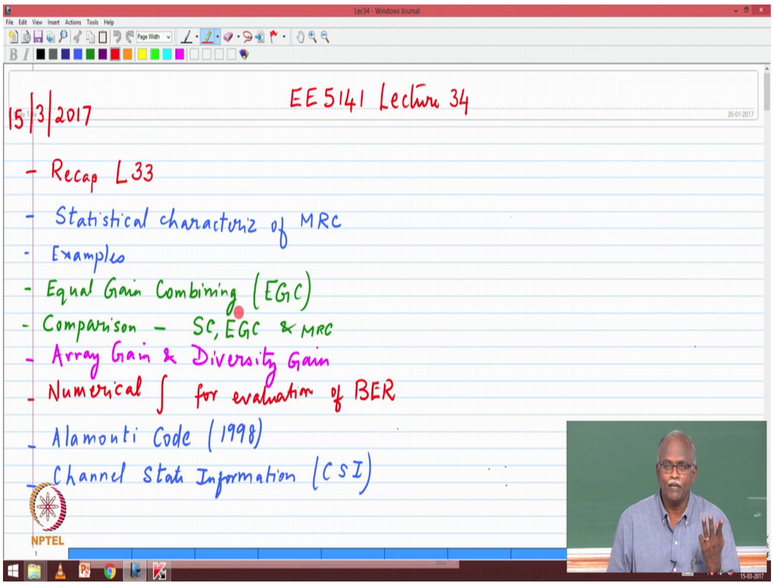

So, that will give us 3 methods of diversity combining one is selection, optimum maximal ratio combining and something in the middle which is equal gain combining. Then we will look at some examples of the BER calculations in the presence of diversity and the different kinds of diversity and what are the ways in which we would get the benefit of that.

I would now then having come to that point of understanding how received diversity is giving us a huge benefit. Then we asked a question what happens if the mobile is does not is not capable of having receiver diversity. Then comes the whole notion of transmit diversity yesterday we talked about one example of transmit diversity, but in 1998 there was a very famous in discovery called the Altamonte code which said that you could get very good performance from transmit diversity and we would like to study that. And of course, methods like Altamonte code needs to need to know what the channel is.

So, which means that there should be a feedback channel and will talk a little bit about that that sets the stage for us to understand wireless channels with feedback because, once the transmitter knows what is the channel conditions then you can optimize the transmission according to the channel condition. Lecture 33 summary.

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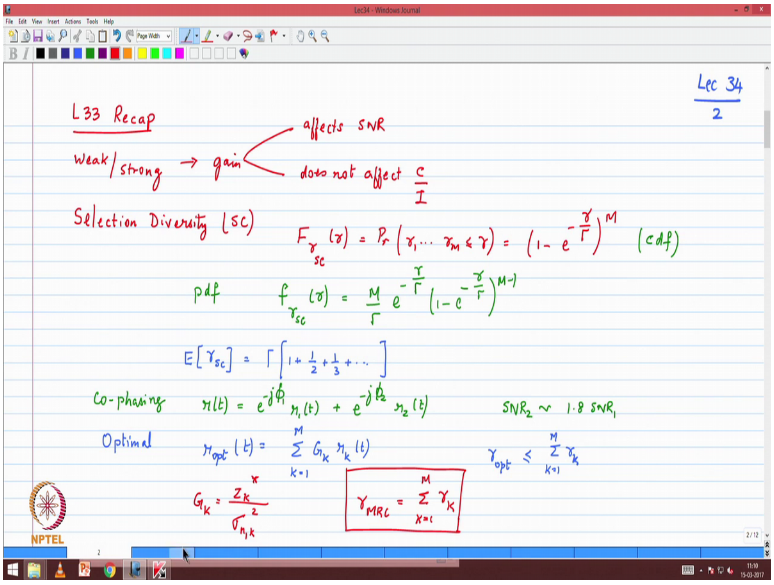

The key high lights from lecture 33. I thought that it would be better for us to write it down from scratch rather than reuse the formula because gives us a chance to go over this information.

So, this notion of antenna being weak or strong, weak or strong has to do with the gain of the antenna is related to the gain of the antenna. And gain affects SNR, SNR, affects SNR affects SNR it does not affect C over I, does not affect C over I because, both the desired signal and the interfering signal pass to the same antenna does not affect C over I. So, that is our, then once have the understanding that we have some number of equally strong antennas we can perform the simplest of the diversity schemes, selection diversity you can call it selection combining usually call it selection combining.

And yesterday the there is these are the results that we have shown or derived in the last class the CDF of the SNR under selection combining gamma sc of gamma this is the same as the probability that all the M antennas gamma 1 through gamma M is less than or equal to gamma, that is the CDF the this is given by 1 minus e power minus gamma by gamma raise to the power M.

So, the probability that each of those antennas is below the below some threshold gamma. And then differentiating we got the PDF. So, this is the CDF the PDF was by differentiation and that gave us the following result FSC of gamma is M by gamma e power minus gamma by gamma 1 minus e power minus gamma by gamma raise to the power M minus 1. And I hope most of these results are a you would had a chance to look at them and possibly verify.

We also showed a result, we showed it for the special case of M equal to 3, but expected value of gamma selection combining is given by gamma times 1 plus 1 half plus 1 third that is the result that we have. After selection combining we went on to talk about the co phasing, co phasing was to estimate the phase of the channel gain and then compensate for that. So, r of t was e power minus j phi 1 r 1 of t plus e power minus j phi 2 r 2 of t. And we showed that in this case the SNR 2 was approximately 1.8 times SNR 1 that is if you have only a single antenna and you have 2 antennas which you were doing co-phasing you got most of the benefit ok.

The final one which on which we spent the last part of the lecture was on optimal combining. What is the best that you can do if you are not constraint by complexity? The optimal says that I am I can I am allowed to combine all of the available signals of t is summation K equal to 1 through n G K r K of t. And we showed that the upper bound for gamma optimal is less than or equal to summation gamma K, K equal to 1 through M. This was also shown and we showed that a particular choice of g K, G K being equal to Z K conjugate divided by the noise variance on that particular branch n K whole square the first n says it is noise K is the Kth antenna that is and under this assumption we get the maximal ratio combining which is equal to summation K equal to 1 through M gamma k. So, maximal ratio combining is the best that we can do in the context of antenna diversity ok.

So, that is a quick summary of all the points that we had discussed, I again probably more mathematics than what we have seen in the other lectures, but I hope the none of it was difficult you could have you can easily verify the result.

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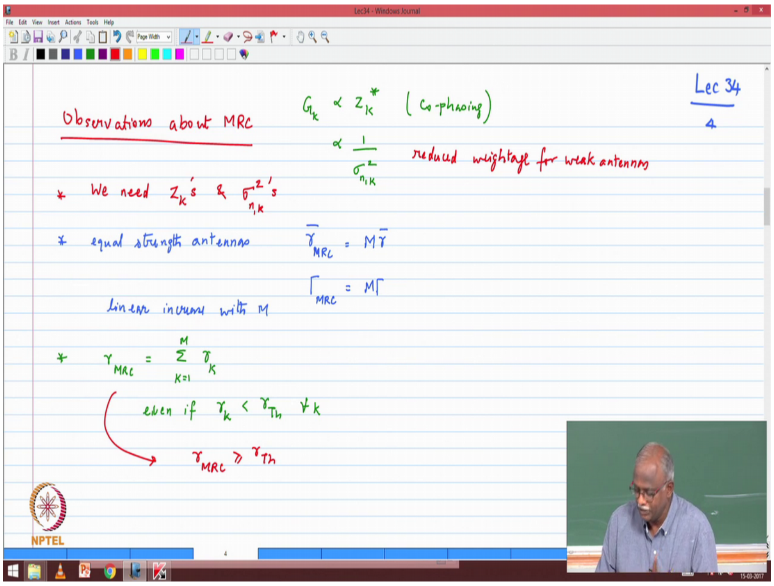

So, let us continue today's lecture by making a few observations about MRC. So, observations about MRC, what are the things that I need to know for MRC? And what is it that makes it a difficult or complex for us to do? So, what is intuitive is the choice of G K g K has to be proportional to Z K conjugate this is from the co phasing aspect of it. For co phasing we need only the argument, but you know if once you take Z K conjugate you will get the negative of the angle as well.

And we also said that when you have a particular antenna which is very noisy, you should not be giving it a lot of weight. And so therefore, this is proportional to 1 over sigma n comma K square. So, that you do not get. So, that you down play the antennas that have got a large noise variance. So, this way the good antennas get the a more of a boost and the weaker antennas get suppressed accordingly.

So, basically this means reduced weightage for weak antennas, reduced weightage for weak antennas. So, maximal ratio combining is our optimal method the form of the weighting functions are intuitive one of them the numerator does the co phasing the denominator does the scaling based on whether the antenna is strong or weak, So for the weak antenna. So, the important thing that we have to keep mind is that we need to have we need estimates of Z K case and sigma n K case. We need this is something that we need. So, given that this is Z K case is channel estimation, but sigma n K square estimation is noise variance estimation.

So, which is a little bit more involved, but something that definitely can be done with the signal processing tools that are available to us; the other aspect that we want to may be observe is that when all of the antennas or of equal strength, if you have all equal strength antennas that when you say equal strength I equal average SNR equal strength antennas then the result gamma MRC bar average value, will be equal to M times gamma, right gamma bar.

So, or may be consistent with our notation gamma MRC is equal to M times gamma. So, your gamma MRC will grow linearly as the number of antennas. So, it does not decrease it keeps increasing as the numbers of antennas grow. So, there is a linear increase which is very good because; that means, by adding more antennas I am going to get better performance. So, linear increase with M that is that is a very useful result, and possibly an observation that we have made once before, but may be worth making again since gamma MRC is equal to summation K equal to 1 2 M gamma K.

The instantaneous SNR is the sum of the SNRs this even if all gamma K case are below threshold even if gamma K less than gamma threshold for all values of K, all the antennas are you know in a fade it is possible that gamma MRC is greater than gamma threshold or in other words if you have added enough number of weak antennas you can still make a or a reasonable detection of the signal that is that is being transmitted.

So, again that is another useful result which says that we can get the benefit of the antennas in a in a very effective manner. I would like to move on 2 understanding the statistical characterization of MRC. If you have your notes with you please refer to lecture number 22 when we talked about the moment generating functions.

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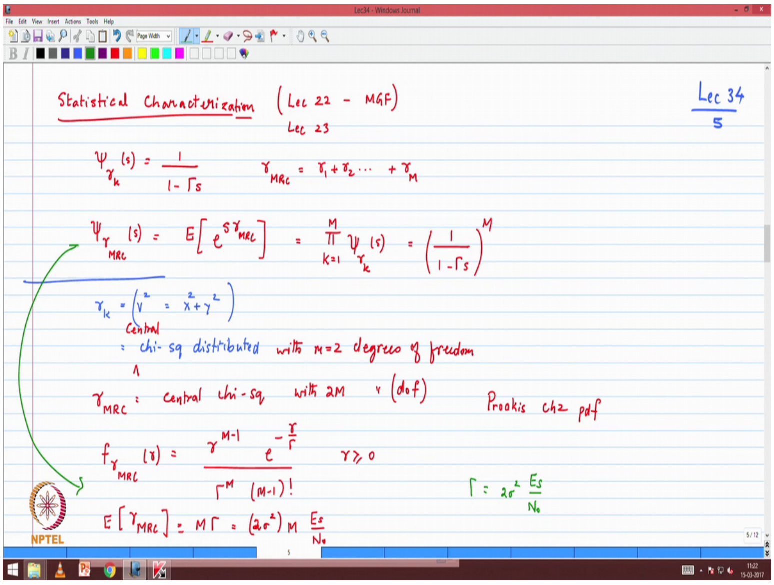

We did actually talk about maximal ratio combining. Statistical characterization, that was the first time we said there is something called optimal combining and gamma MRC will be equal to the sum of the SNRs and we showed how the moment generating function can be used to get the moment generating the expressions for the a PDF of the SNR. But today's thing is actually we excuse me revisiting it, So that we can we can develop it. So, statistical characterization please refer lecture 22 when we talked about the moment generating functions also lecture 23, lecture 23 this material some of it you will see was found there.

So, gamma the moment generating function psi of gamma K of s in Rayleigh fading was given by 1 minus s gamma. And we said that the gamma MRC is equal to gamma 1 plus gamma 2 plus gamma M at that time when we did it we did not have proof, but now we have the proof of this. So, the moment generating function of gamma MRC of s which is nothing, but the expected value of e power s times gamma MRC. Please substitute once and show that this comes out to be because, each of these are independent it comes out to be K is equal to 1 through M psi gamma K of s. And if all of them have got a similar average SNR it comes out to be 1 minus, 1 minus gamma s raise to the power M sorry, it is upper case M ok.

So now, how do I get the PDF of gamma MRC inverse Laplace transform? Now I am not sure if inverse Laplace transform of this function is easy may be you know some close form expressions, but there are; obviously, many ways to solve a particular problem. I would like to take slightly different root. If you go back and look at the expression that we have for gamma K, gamma K we said is a basically is equal to v square this is equal to x square plus y square. This is the background basically, there is a real part and imaginary part I have taken the magnitude square and that becomes the that is that is what is related to the SNR. So, this we said was a chi square distribution chi square distribution because. X and y are 0 mean this is actually central chi square we 2 degrees of freedom M is equal to 2 degrees of freedom alright.

So, now if I want to right down for MRC it is still a central chi square central chi square, but now I have done gamma 1 plus gamma 2 up to gamma M. So, instead of 2 degrees of freedom I have 2 M degrees of freedom. So, it is a central chi square chi square distribution with 2 M degrees of freedom dof and Proakis gives you the expression for the PDF of M. So, refer Proakis digital communications chapter 2 for the PDF, for the PDF of central chi square of this variable.

And please do verify that the following result is something that you obtain you have to basically make sure that you use the correct variables and others f of gamma MRC of gamma is equal to gamma power M minus 1 e power minus gamma by gamma divided by gamma power M, M minus 1 factorial where gamma greater than or equal to 0 expected value of gamma MRC the chi square random variable with 2 M degrees of freedom is given by M times gamma. Where gamma is the of is the average expected value of the random variable with 2 degrees of freedom. This can also be written as 2 times, 2 times sigma square that is the M into 2 times sigma square into E s by n not.

So, if the average SNR gamma average SNR gamma is given by E s by N not and the variance of the chi square random variable is 2 sigma square, we assume 2 sigma square equal to 1 in the general case, but if you wanted to capture the presence of the 2 sigma square that is that is quite alright. So, this is the expression for gamma and this is the. So, this is what you would be able to verify from the Proakis ok.

Now, what I would like to do is ask you to verify that this is indeed the. So, basely this 2 must be related correct these 2 must be related. So, if I take the Laplace transform of one I should get to the other.

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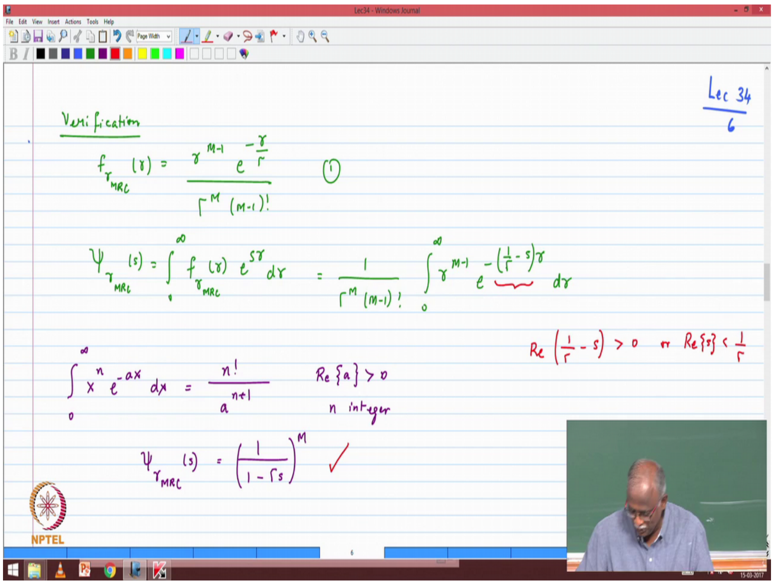

So, verification I will just pose the problem request you to complete the verification. So, the verification is that I would like to take the f gamma MRC of gamma, this is gamma power M minus 1 e power minus gamma by gamma raise to the power M, M minus 1 factorial.

Compute the moment generating function. I know the moment generating function already, but I am I am going to go through the process of computing it. Phi of gamma MRC of s this is integral 0 through infinity, the definition the PDF times e power s gamma d gamma. So, this can be written as integral 0 through infinity let me just a skip a step.

Please do verify that what you will get here is basically pull out all the substitute from equation one you will get gamma power M, M minus 1 factorial the not part of the integral integration, with in the integral is 0 to infinity gamma power M minus 1 e power minus 1 over gamma minus s gamma d gamma. Again I have just regroup the terms. Couple of results that you are already familiar with this is a exponential term where gamma goes to infinity.

So, this will become very large unless what is within the bracket is positive basically it is a negative exponent. So, therefore, it will decrease start decreasing. So, one the real part of 1 over gamma minus s must be greater than 0 or the same as saying real part of s must be less than 1 over gamma. And we have shown that this is this is earlier also we have come in to this scenario when we are computing moment generating functions. So, it is not a new result, but basically this is what we have.

Now, there is a standard result which I am not sure if we have already mentioned it, but if not you can just not it down. Integral 0 to infinity x power n e power minus a x d x is equal to n factorial divided by a power n plus 1. When real part of a is greater than 0 and n is an integer. Which is exactly ours our situation, please substitute this result and verify that phi psi of gamma MRC of s that is the moment generating function comes out to be 1 minus 1 minus gamma s raise to the power M. So verified. So, of course, you could have done the inverse Laplace transform, again it turns out that the forward transform is easier for us to for us to verify all of this is useful only if we can apply it.

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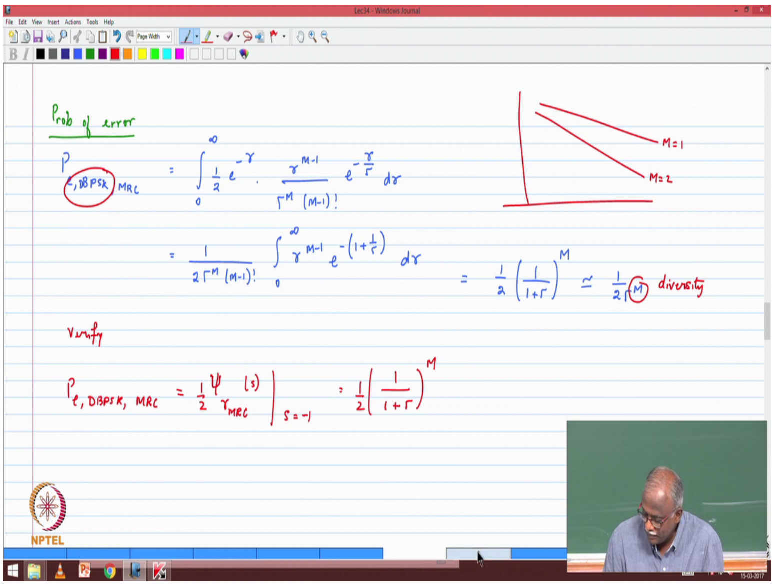

So, let us get to the task of applying it. So, I would now look like to look at the probability of error. So, what is the impact that diversity has had in terms of the probability of error? That is an important result for us to capture and to make sure that we are very, very comfortable with it

So, probability of BER of DBPSK, I choose DBPSK because it is easy for us to do the integration, but I will show you that you can do it equally well for the BPSK as well comma MRC. Let us take the best performance and see what we get. So, this would be integral 0 through infinity the probability of a probability of expression for a DBPSK is e power minus gamma, that is the probability of for a given gamma what is the probability of bit error rate. Then multiply it by the PDF of the SNR gamma raise to the power M minus 1 divided by gamma power M, M minus 1 factorial e power minus gamma by gamma d gamma ok.

Again pull out the terms that are not part of the integration 2 times gamma power M, M minus 1 factorial, what is within the bracket integral 0 through infinity gamma power M minus 1 e power minus 1 plus 1 by gamma d gamma sorry, d gamma is not there, it is the same formulate that we use last time. This is straight forward for us to apply it in this case please verify that this gives us, 1 by 2 into 1 plus 1 plus gamma raise to the power M. Which is at high SNRs can be approximated as 1 by 2 gamma raise to the power M, previous it was 1 over 2 gamma.

Now we have it with as 1 over 2 gamma raise to the power M and that is diversity the benefit. That is why the graph is going down with the difference slope. So, if this was the graph for M equal to 1 it is going to go with the different slope this is M equal to 1, M equal to 2 and M equal to 3 the slopes are going to change. So, that is the diversity benefit ok.

Now, we also said for DBPSK, we actually do not need to do the integration. How do we do BER calculation without doing the integration? Take the moment generating function and substitute. So, just verify that verify that we can estimate the BER probability of error of DBPSK for MRC can also be obtained from psi gamma MRC of s you replace s with minus 1. So, this actually go back and get the expression for the moment generating function it comes out to be 1 half there is a 1 half outside. So, 1 half 1 by 1 plus gamma raise to the power M.

So, do it the integration to just for verification just. So, that we are comfortable with the benefit the benefit of it, but of course, the short cut method would be to use the moment generating function. Everyone is comfortable with what has been mentioned so far? So, what have we said so far, there is selection combining we optimal is maximal ratio combining we have developed the PDF of maximal ratio combining we has also shown what is the moment generating function for maximal ratio combining and shown that you can use that for computing the probability of bit errors, ok.

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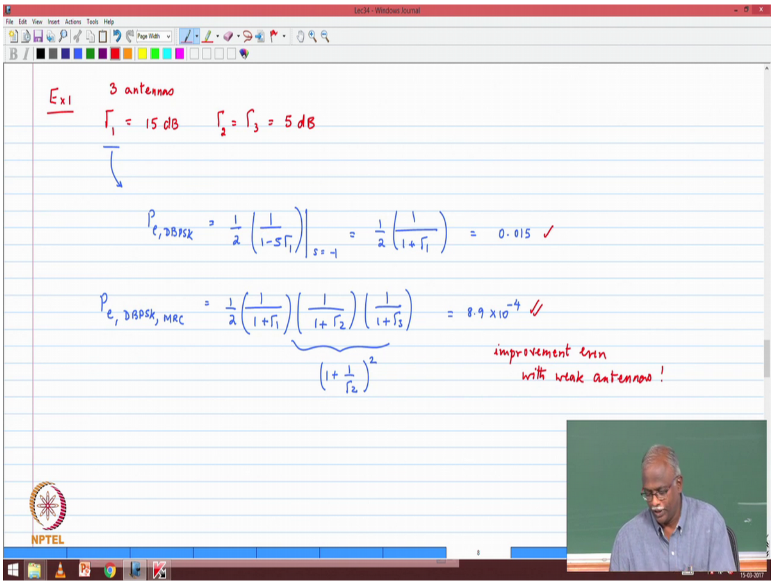

Here is a example that I believe you will find very useful in terms of reinforcing the concepts that we have talked about. So, let us assume that there are situation where there are 3 antennas, 3 antennas gamma 1 means the average SNR of the first antenna is 15 db, strong antenna. Gamma 2 equal to gamma 3 is equal to phi dB reasonably strong, but compare to gamma 1 antenna one these are definitely the weaker antennas. So, if I had only one antenna.

So, basically if only antenna one was present probability of error of DBPSK this would have been 1 half of 1 minus s gamma 1 I should substitute with s equal to minus 1 that would have been 1 half of 1 plus gamma 1. This if you verify basically convert 15 dB to linear scale and then do the conversation this comes out to be 0.015 that is the bit error rate that you would expect in fading with an average antenna of average SNR of 15 db. Not a very good performance even though this is the.

Now, if you were to combine it with the other 2 antennas probability of error of DBPSK with MRC. Now how do we do that? Because these are antennas with different SNRs, but no problem gamma MRC is gamma 1 plus gamma 2 plus gamma 3. So, moment generating function will be the product of the moment generating functions. So, this is going to be 1 half of 1 by 1 plus gamma 1, 1 over 1 plus gamma 2, 1 over 1 plus gamma 3 or I can write this as 1 plus 1 over gamma 2 whole square because those 2 are of the same value. And go head and substitute and you can verify that this is actually 8.9 into 10 power minus 4.

And the difference there were 2 weak antennas right, one strong antenna, but that itself did not give you very outstanding performance, but MRC even though it was being combined with weak antennas actually improved it by order of magnitude and therefore, we start to see the improvement even with weak antennas; so improvement even with weak antennas because of the power of MRC, even with weak antennas weak antennas.

That is important point for you to keep in mind and always keep sure that you are. One of the things that we said as the benefit of the moment generating function was the following we said that once you have PDF and you know it is moment generating function then it is easy for us to develop the bit error rate expressions.

So, I am going to look at a case where I have a combination of an antenna that is seeing Nakagami M fading and another antenna that is seeing Rician in fading. 2 different antennas and I want you to get a feel for which is bad which is better? You given a choice between Nakagami M and Rician which would you take first of all you will ask for the M value. If it is M equal to 1 no difference both are the same or you know. So, that is the thing, but first with Nakagami M fading.

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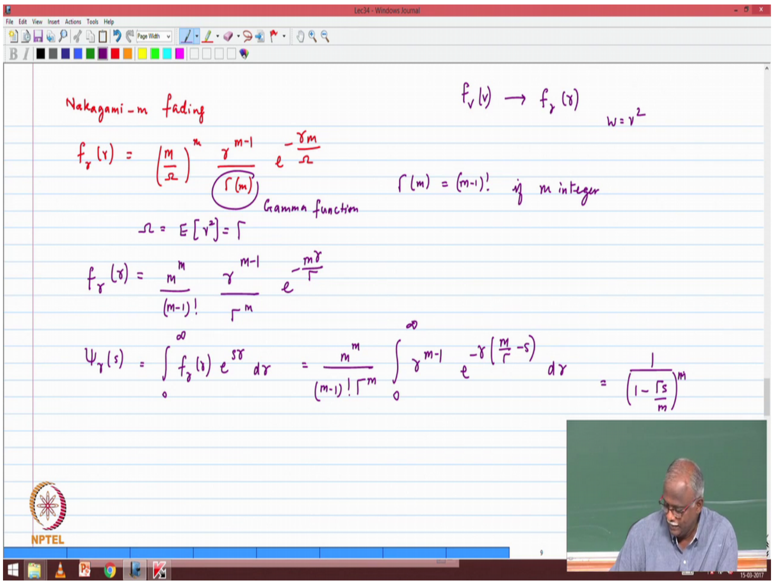

I think we gave a result which I believe which we, I am I am not sure if we actually derived it, but I thought it would be good for us to at least as an exercise for you to drive that.

So, under Nakagami M fading the SNR is expression is given this is given by M by omega raise to the power M gamma power M minus 1 divided by gamma of M e power minus gamma M divided by omega. Now there is a little bit of confusing notation this is a gamma function not SNR; so gamma function. So, if M is an integer than we can we can get rid of the gamma function because for integers gamma of M is equal to M minus 1 factorial if M is integer. So, we will look at the case where M is an integer. So, therefore, we can we can simplify that. And we to avoid confusion we used this notation omega is equal to expected value of v square if this gamma function is removed we can replace it with our gamma which is the standard form.

So, once you have this you can please verify that f gamma of gamma is given by M raise to the power M divided by M minus 1 factorial gamma power M minus 1 divided by gamma power M e power minus M gamma by gamma. I think we gave the Nakagamim fading in 2 forms one with the envelop given the expression in the form of the envelop. So, if it is in the form of f v of v, how do we go to f gamma of gamma? Basically take random variable v is w is equal to v square do the transformation and then make sure that you are able to get the result. So, please go through that transformation of variables to make sure that we are getting this.

So, now to get the expression for the Nakagami M: the moment generating function f gamma of s this is integral 0 to infinity f gamma of gamma e power s gamma d gamma. Now substitute and verify that you get something of this form for the third time in today's lecture it is a form that you will recognize immediately M minus 1 factorial gamma raise to the power M integral 0 to infinity gamma power M minus 1 e power minus gamma M by gamma minus s d gamma. Again it is in the form of a standard integral which you can verify and this comes out to be 1 by 1 minus gamma s divided by M raise to the power M.

So, this is the moment generating function of an antenna with Nakagami M fading. And it is useful for us because the example that we are going to be looking at has one antenna which is got Nakagami M fading. Everybody is with this result? Let me just move on to the example, example there is 2 antennas I want to do optimal combining.

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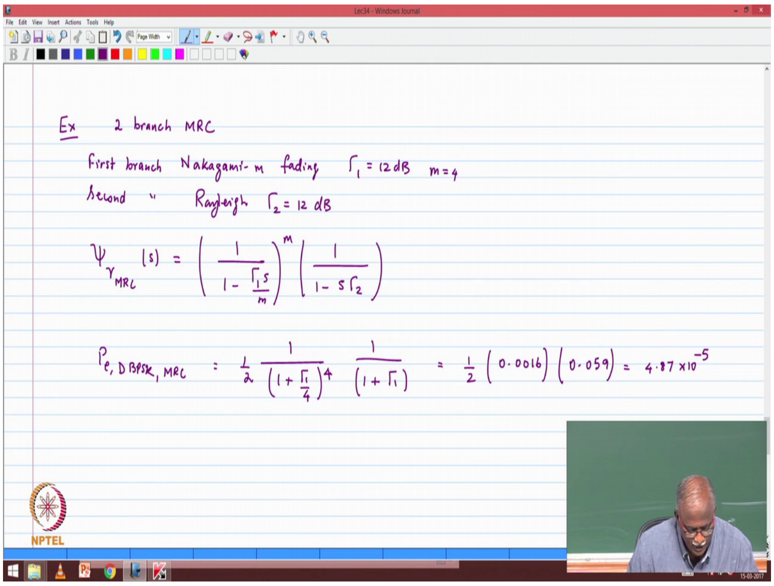

So, it is 2 branch maximal ratio combining first branch has Nakagami M fading, has Nakagami M fading, fading with M equal to 4. So, it is milder than Rayleigh gamma 1 equal to 12 dB SNR M equal to 4. And the second branch second branch has got Rayleigh fading Rayleigh fading with gamma 2 equal to 12 db. Now how much worse is the Rayleigh fading compare to the Nakagami M? Let us look at it.

So, the for the maximal ratio combining psi of gamma MRC for the 2 branch MRC is given by the product of the moment generating functions for the Nakagami M antenna it is 1 minus gamma 1 s divided by M raise to the power M, for the r Rayleigh function Rayleigh fading 1 minus s gamma 2. And probability of error of DBPSK with the 2 branch MRC. Basically you will substitute s equal to minus 1, this comes out to be 1 half divided by 1 plus gamma 1 by 4 raise to the power 4 and this one is 1 over 1 plus gamma 1, because both are 12 dB ok.

If you just do the numerical calculation it helps us to sort of get a feel for it; the first antenna error the contribution to the error 0.0016. The Rayleigh fading is 0.059. So, clearly the Rayleigh fading is the one that is has poorer performance, but because you have been able to combine it with another antenna which is got better performance, the net result you can verify is 4.87 into 10 power minus 5. So, I think it is, it is it is very powerful to see the benefit is of diversity and just to see how much you can gain just by combining antennas and seeing the benefit is that we can get. Any questions? Ok.

So, now I would like to move on to answer another important question. Is there something that I can do which is not as complex as MRC? So, here is the problem statement.

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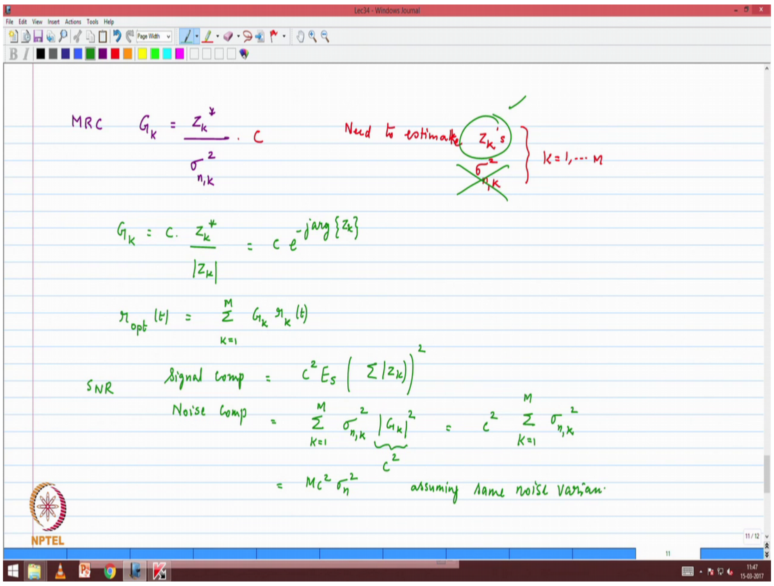

The problem statement says that we have MRC which is the optimal it requires us to compute the channel coefficient Z K take Z K conjugate, compute the SNR noise variance for each antenna for each of the K M antennas. And again one observation you can multiply this by some constant C it does not affect it because the co phasing and the waiting all of it. So, the C is it does not matter, but. So, to within a constant the scale the MRC coefficients are unique. So, what we need is, we need to estimate the z case, estimating the z case also you need to estimate sigma square n comma K for K equal to 1 through M. That is where the complexity arises ok.

Now, if we do not want to estimate the noise. Of course, the z case we will we will have to estimate, but let us say I do not want to estimate this, I do not want to estimate this. So, what do you do? Assume that the noise variances are the same. So, well what else can you do? You do not want to compute noise variances you want to. So, you assume. So, we will let us see; what is it that we can give.

So, G K is equal to sum constant times Z K conjugate. And let us assume that we are going to do co phasing because, we did see that co phasing actually did fairly good performance. So, we will do Z K conjugate divided by mod Z K, because that that will basically tell you that this is equal to C times e power minus j argument of Z K. So, which means basically some constant C times the negative the conjugate of the angle ok.

So, let us say that this is our scheme we are trying to do co phasing r optimum of t is equal to summation K equal to 1 through M G K r K of t. And the these terms do not affect the noise variance because it is just a complex rotation this is these are unit vectors. So therefore, the signal component I want to compute the SNR, SNR the signal component. Signal component says that you must take the expression for the g case and then and then square it.

So, what you will get is C square times e is the Z K star will multiply with Z K you get Z K magnitude square, but then there is a mod Z K at the denominator. So, what you will get inside the bracket is summation mod Z K. And I have to and this will be the scaling factor for the signal component. And this I will have to square, that is a signal component this is different from the earlier case where we were getting mod Z K square inside the summation. But now it is only mod Z K because of the way that we have written it ok.

So, the noise component, noise component can also be obtained as follows. It will summation K equal to 1 through M sigma M comma K square whatever was the original noise component multiplied by mod Z K mod, mod one second, I will just make sure it should be mod G K square; sorry, let me just get this correct. Mod G K square, but that will be equal to C square that is all. So, what this comes out to be is C square times summation K equal to 1 through M sigma n comma K whole square.

Now assuming that all of the antennas are equally or have the same noise variance, this will be equal to M times C square times sigma n square. Assuming all have now all, assuming same noise variance last step and some interesting results that that will come out ok.

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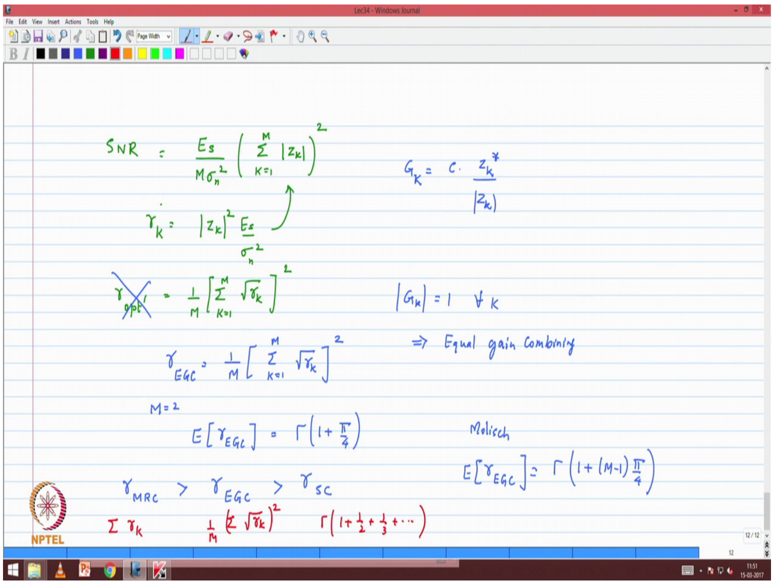

So, now SNR will be the ratio of these 2 terms the C square term goes away what you will get is E s by M sigma n square summation K equal 1 through n mod Z K whole square. Now please note what is gamma K? Gamma K, gamma K is gamma K is mod Z K whole square times E s by N not or E s by sigma n square I will just write it. So, that we do not confuse by sigma n square. Now, I want incorporate the gamma case into this expression.

So, this gamma combining that I have done, let me call it as opt bar it is not optimal it is some sub optimal scheme, is equal to 1 over M summation K equal to 1 through M square root of gamma K whole square. So, basically you take this E s by sigma n square in to the in to this, in into the squares term and then what you will find is that you can write it in the in the following form ok.

So, basically what we find is that this is the expression. So, let us not call it optimum anymore what did we do we all the G Ks where some constant times Z K star divided by mod Z K. So, basically all of the antennas got a weighting which was equal to 1, magnitude equal to 1. So, magnitude of G K equal to 1 for one for all K equal to c. So, basically it is a, so his is why this particular method of combining instead of calling it co phasing, we call it equal gain combining. All of them have the same magnitude, but different phases just to do the co phasing. So, equal gain combining.

So, gamma EGC is equal to 1 over M summation K equal 1 through M square root of gamma K whole square. And if you remember for the case that M equal to 2 we actually did compute the expected value of gamma EGC for the co phasing, for the 2 branch co phasing we showed that this is equal to gamma into 1 plus pi by 4. In molisch we have the general result if I have M antennas expected value of E gamma EGC is given by gamma into 1 plus M minus 1 pi by 4. And maybe it is a good exercise for you to verify the following result gamma MRC is greater than gamma EGC greater than gamma selection combining; the corresponding expressions summation gamma K. This is 1 over M summation square root of gamma K whole square and the last one will be gamma into 1 plus 1 half plus 1 third dot, dot, dot ok.

So, basically this is the final result that we have for diversity combining. Selection combining the simplest of the methods MRC the best equal gain combining which will give you most of the benefit and is equal to the optimal combining if the SNR if the noise variances are all the same. So, in the absence of it you make the assumption that all the noise variances are the same and you will take a performance hit if they are not and that that is when.

So usually, if you have all the noise variances to be the same MRC and EGC will be the same. So, therefore, let us make it greater than or equal to because under some scenarios these 2 will be very close and can also be actually equal to each other. So, hopefully you get a good view of the diversity schemes the books by Goldsmith and Molisch are very good for this please read the corresponding chapters. Chapter 7 in Goldsmith and Molisch, I do not, I will put it up on in module.

So, we will end here and we will pick it from here and continue in the next class.

Thank you.