Introduction to Wireless and Cellular Communication
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 40
Wireless Channel Capacity - Water filling
Review of L19-36

Good morning. We begin lecture 39. There was a request from the students to do it as a review lecture just like we did last time. So, this is a review of lectures 19 to 36. Just to avoid confusion, the portions are: lectures 1 to 36 are just that we are the review part. Since we have already covered lectures 1 to 18 in the previous review, we are focusing on lectures 19 to 36.

By way of instructions, these are been shared through model just for clarification hand written formula sheet whatever you want to write. You know integral formulas, derivations whatever it is permitted. Please bring your calculator. The question paper will be distributed at 7.50 like last time. As planned we will have a doubt clearing session on assignment 4 today evening 5 to 6 pm, ok.

So, let us quickly run through again. Most of this is a material that is probably things that you will remember, but just sort of putting it all together in single canvasses what we are trying to do. So, I will just write down approximately what there is over lap of lectures, but what the points are in each of the lectures. Lecture 19 was around the time we were finishing up our discussion on Rayleigh Fading.

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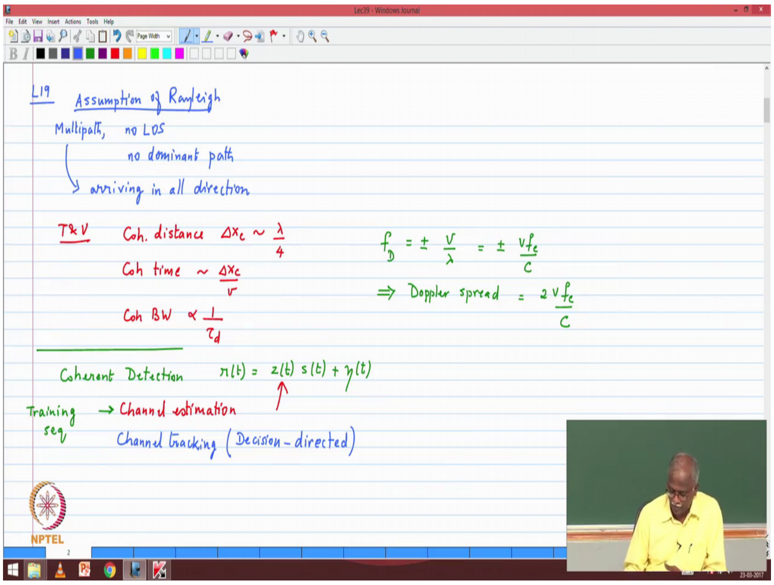

So, the assumptions of Rayleigh fading and they were a few assumptions in the basic model. I thought it is good for us to keep that in mind, multipath large number of components that are propagating from the transmitter to receiver. There is no line of sight, there is no dominant path. That is also part of the model and the multipath are arriving in all directions. I believe these are the essential assumptions that we have made and based on which we have obtained the properties of Rayleigh fading.

This led to a brief discussion of a simple example of a transmitter and a receiver in the presence of a reflecting wall. Again that was a example from PSE and Viswanath and with respect to this example, we observed some results and insights that we obtained was there is a notion of coherence distance. If you traverse beyond a certain distance, then the channel has a substantial amount of change. We call that as delta x c we said that is proportional to lambda by 4 for this example, but in general it is proportional to some fraction of lambda. That is; what is the important thing that we take away.

Once you have a coherent distance, you also have coherence time because coherence time is a delta x c divided by v, the velocity with which you are moving and this also we went back and reinterpreted the result. When there is a multipath, the second copy of the signal through the reflecting wall that there is a notion of coherence bandwidth and this we showed was proportional to 1 over the delay between the first and second paths, but in general it will be inversely proportional to the delay spread. The presence of Doppler is an important element in our understanding of wireless channels. The Doppler frequency can be positive or negative. The maximum possible is proportional to v divided by lambda. This is also written as v writing lambda in terms of the carrier frequency v times f c divided by c which is the speed of light, ok.

So, therefore, any carrier frequency we can assume what is the maximum Doppler and of course, based on the angle between the direction of propagation and the direction of motion, then you get this Doppler times cosine of the angle. So, the maximum Doppler spread or what is the maximum range of frequency is that we can expect in a fading channel turns out to be 2 times v f c divided by c or 2 times the maximum Doppler because it can go all the way from minus f D to plus f D.

Lecture 19 also took us into a discussion of coherent detection basically. Now, that you have a model for the channel, what are the aspects of coherent detection? Coherent detection if you received signal r of t is equal some z of t times s of t plus eta of t, coherent detection requires you to know z of t.

So, we said that there are two aspects to coherent detection. One is channel estimation and you must enable or assist the receiver to do channel estimation and you do that by transmission of training sequences. So, may be this is aided by the presence of training sequences which are inserted at the transmitter. These are overhead. They do not carry information, but just that your coherent detector can do channel estimation.

Now, apart from channel estimation, depending upon the changes that are happening in the channel between your channel estimates, you would also need to do channel tracking and this channel tracking has to be based on the decisions that you have made using your demodulation scheme. So, it is actually decision directed channel tracking. So, you would use the decisions that you have already made and do the channel tracking. So, it is actually called decision directed channel tracking. So, channel estimation and channel tracking, both of these are needed, so that we can build our coherent detection scheme.

Now, if we do not want to do coherent detection, we said that there is an alternative that is the presence of differential modulation and differential detection. So, that was lecture 20 took into the discussion now from coherent on to differential.

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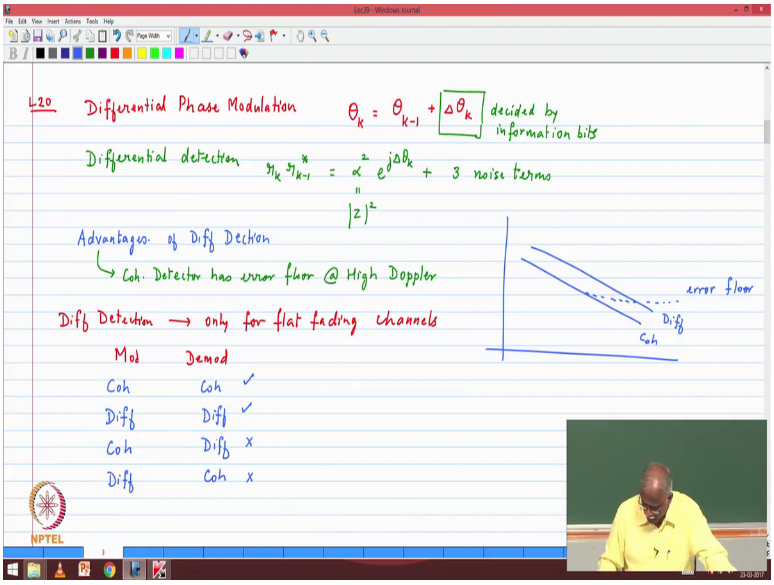

So, in the context of differential, we have to encode the information using a differential technique, differential phase modulation that is a phase modulation is what lends itself very easily to differential modulation and differential phase modulation basically says that the phase that I transmit at time instant k is a phase at time instant k minus 1 plus delta theta k and the information that I want to transmit at the time instant k is in delta theta k. This is what is decided by the information bits. The thetas themselves do not carry the information. The delta theta is what carries decided by the information bits, ok.

Now, how many levels I have delta thetas? I use all of that is part of the choice of the modulation method in information bits and the differential, the counterpart would be that we do not need to do coherent detection, but we can get by with differential detection which is given by r k minus 1 conjugate and we can show that this is equal to alpha square or it is same as mod z square times e power j delta theta k plus there are three noise terms, ok.

So, again we would not worry too much about the noise terms because their presence is known. So, there is a penalty that we are paying when we do differential detection, but the good news is that you do not need to do channel tracking or channel estimation. We also discussed briefly the advantages and disadvantages of coherent versus non-coherent. So, the understanding is that coherent detection will do better than differential detection in a fading channel. So, if this is coherent, this would be differential. However, if you have channel tracking errors, then you will find that you will have a error floor due to channel tracking and since you are not doing channel, this is error floor due to errors in channel tracking and the differential detection does not suffer that.

So, there are scenarios where you may prefer to do differential detection. So, the advantages of differential detection from that figure we can capture in the form of a single statement. Basically the fact that a coherent detector has an error floor at high Doppler’s and that is because of the requirement of channel tracking and the limitations of our ability to track fast fading. This is what makes differential detection attractive. However, we need to keep in mind that differential detection makes the assumption that it is a flat fading channel because if it is a frequency selective, then the differential detection becomes not a feasible option.

So, differential detection is only for flat fading channels. So, if I do coherent, that is a good option. If I do differential, it is a good option. We also looked at coherent and differential. Nothing prevents me from doing that really did not gain anything and of course, if I do differential modulation and try to do coherent detection also, there was no advantage. In fact, there was some disadvantages with the cross combination.

So, basically there is coherent techniques, there is the differential techniques and each of them have a benefit for us in terms of our usage in a fading channel whether I am sort of running through, but if something needs a little bit of clarification or something needs to spend a little bit more time, just indicate. I am assuming this is just a sort of a refresher, so that you get all the pieces together at one stroke once we had our basic hands around coherent modulation and the differential coherent techniques in the differential techniques.

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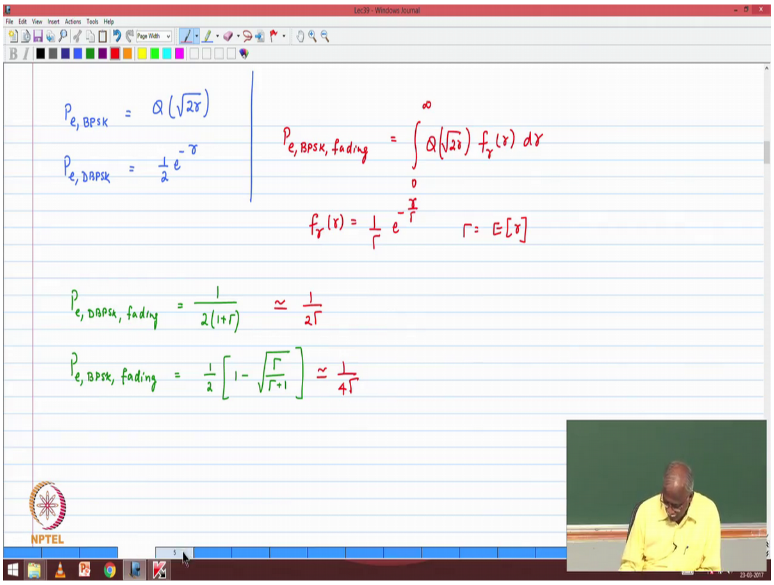

We focused our attention in lecture 20 on Probability of Error. Probability of error of BPSK again this is something that we used from digital communications. We do not derive this probability of error of DBPSK in AWGN channels will be half e power minus gamma and given that you have these types of expressions for the probability of bit error rate, then we said that we can derive the probability of error of these modulation methods; for example, BPSK in fading by using the fading distribution.

So, it will be 0 through infinity the probability of error at a given SNR times, the distribution of the SNR f gamma of gamma d gamma and we also using a change of variables were to show that once you start from Rayleigh distribution, you can then derive SNR in a Rayleigh fading environment which is f gamma of gamma is given by 1 over gamma e power minus gamma by gamma, where gamma is equal to expected value of gamma. That is the pdf I am using that we will plug into this expression and derive.

So, we also went through the exercise of deriving in a lecture 20, the error rates in fading. So, probability of error of DBPSK that was a fairly straight forward integral for fading, basically it was a one step answer. It came out to be 1 over 2 into 1 plus gamma. On the other hand, the probability of error of BPSK because of the queue function did require some additional steps, a change of variables and then, the use of standard results the answer came out to be one-half of 1 minus square root of gamma divided by gamma plus 1 and in both cases, we were interested in the asymptotic approximations. So, the asymptotic approximations in this case were 1 over 2 gamma. In this case, it was 1 over 1 by 4 gamma indicating that BPSK had approximately 3 dB advantage over differential BPSK in the context of a fading channel.

Lecture 21 was the focus or what we highlighted in lecture 21 was augmentation of our design or understanding of wireless channels.

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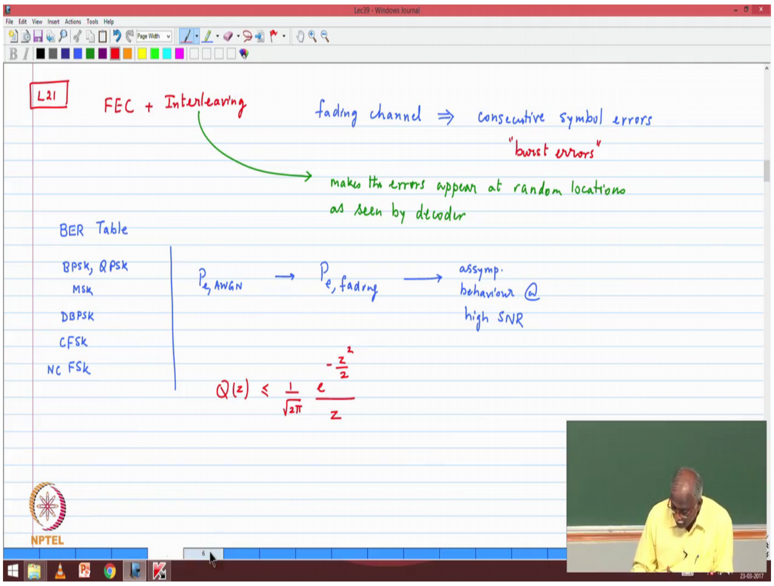

Again we did not spent too much time, but we did indicate that lecture 21, one of the key aspects of wireless channels is the role of forward error correction and we also showed that interleaving is absolutely crucial for the success of forward error correction in a fading channel. We basically showed square interleaver how you would input the data, how you would read the data.

So, the summary of that discussion was that the fading channel because of the co-relatedness of its behavior, once you go into a fade, what you are likely to encounter in coherent detection is consecutive symbol errors which means this is a nothing, but a burst of errors. Most of our codes are designed. So, this would give you a burst error pattern. Most of our error correcting codes like convolution codes works well when the errors are random and the role of the interleaver turns out to be very critical because it makes the errors appear random in terms of position, not in terms of the values. Basically the error has occurred, the position appear at random locations and where does this occur is random. To whom does it look random?

Student: Decoder.

To the decoder; so, basically the receiver is thereafter there is interleaver and then, there the decoder is what is seeing errors which are not burst errors, but they are at random locations as seen by the decoder and this was more to augment our understanding of why we also see interleavers in the context of fading channels having completed that a brief sort of interload. We went back to writing down BER table. We wrote down for I believe BPSK QPSK MSK DPBSK coherent FSK and I think no coherent FSK, ok.

So, basically there was a table and in each of those cases, what we have written down is the probability of error in an AWGN channel. Usually it is a form of an exponent or Q function from that we derive probability of error in fading and from there we wrote down the asymptotic behavior at high SNR. At high SNR again it is useful for us to be able to go from the expression in AWGN to fading and then, an understanding of what the asymptotic performance is going to be.

Somewhere along the line, we also indicated that sometimes it is difficult for us to evaluate the Q function if you do not have a computer. So, therefore, there is an approximation or an upper bound which is a helpful for us 1 over root 2 pi e power minus z square by 2 divided by z and in the assignment number 4, you have seen that you can actually upper bound and lower bound it, so that you can actually get a range of values; now, in order for us to be able to do these computations for the different modulations, different fading environments.

So, we did spend some amount of time understanding the statistical characterization of the other types of fading. So, Rayleigh fading has been the focus until this point.

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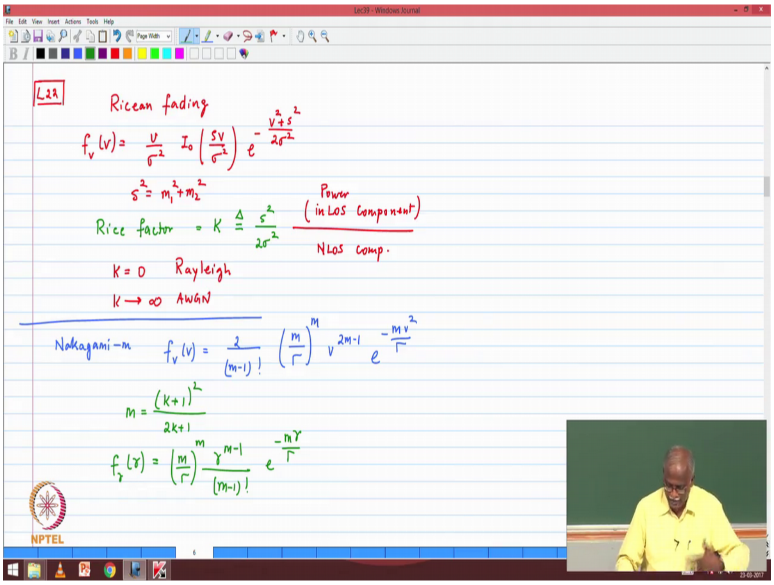

So, lecture 22 we did look at the following the Rice or Rician fading. So, it would be nothing, but a random variable with the Rician pdf f v of v. That is the pdf v divided by sigma square i naught of s v by sigma square e power minus v square plus s square by 2 sigma square. The sigma relates to the non-line of sight component the other what you would call the Rayleigh component and the s relates to the line of sight component.

So, s square is equal to m 1 square plus m 2 square and that is means of the real and imaginary part and using this expression we say that we could characterize Rician fading in terms of a rice factor which is given usually use the notation k which is defined as s square by 2 sigma square. So, in other words, this would be proportional to the power in the line of sight component. So, power or line of sight component divided by the power in the non-line of sight component and we also showed that k equal to 0 means there is no line of sight component that actually does give us the Rayleigh distribution and k tending to a very large number.

So, something of the order of 22 like that and then, we say that the line of sight component starts to dominate. So, the fluctuations tend to become less and less significant. So, it starts to look more and more like AWGN channel. So, Rician fading actually turns out to be a super set of Rayleigh fading. Rayleigh fading is as a special case. We also studied the Nakagami m fading because in terms of the analytical expressions, it is difficult for us to work with the Rayleigh distribution.

So, the Nakagami m is a useful way of describing fading and here again the distribution f v of v, I will just write it down. You can look up in your notes m minus 1 factorial m by gamma raise to the power m v power 2 m minus 1 e power minus m v square by gamma and we have also showed or at least stated in the class that Rician fading and Nakagami m fading actually cover the same range of fading distributions. In fact, the Nakagami m actually has a slightly broader range and we can show that there is a relationship that maps the Nakagami m and the Rician fading very closely. So, the relationship is m is equal to k plus 1 square divided by 2 k plus 1 m is the nakagmai fading paramete. We also were interested to look at the distribution of SNR, not necessarily the fading pdf.

So, in the cases of Nakagami m, we said that f gamma of gamma with this is derived in the class m by gamma raise to the power m gamma power m minus 1 divided by m minus 1 factorial e power minus m gamma by gamma. So, Rayleigh fading, Rician fading, Nakagami m fading, all of them have added different ways of understanding and being able to work with the fading channel. We also indicated that using the moment generating function has got a lot of advantages.

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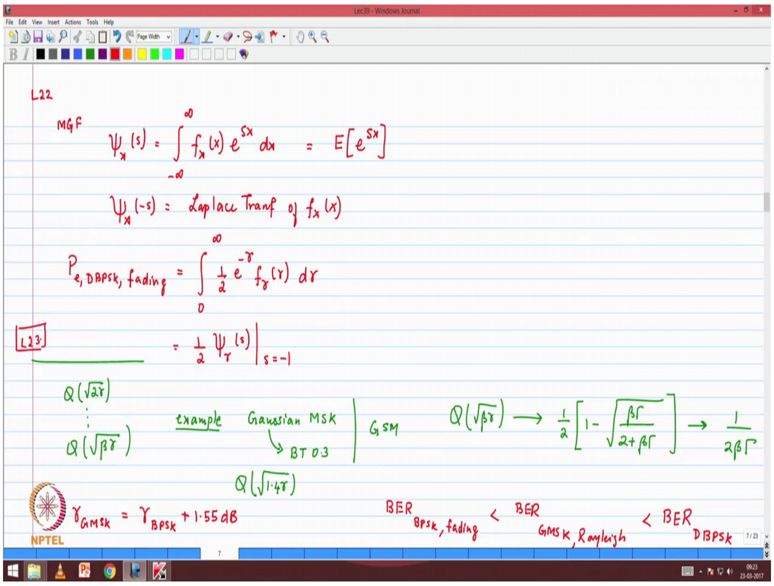

So, again lecture 22 still continuing moment generating function is given by phi x of s equal to minus infinity to infinity f x of x e power s x d x and we say that we could interpret the moment generating function as an expected value of e of s x or that we could also interpret phi x of minus s as the Laplace transform of f x of x, ok.

So, regardless of which interpretation we take the use, usefulness is when we try to develop it and one of the ways, one of the things that is very useful for us is the DBPSK family now becomes very easy for us to evaluate because probability of error of DBPSK in fading becomes just evaluation of the moment generating function. So, let me just write the expression. It is 0 to infinity half e power minus gamma f gamma of gamma d gamma. That would be for any distribution of SNR. So, all we need to do is obtain the moment generating function of that particular fading distribution phi gamma of s and replace it with s is equal to minus 1 and that will give us the expression, ok.

Just as we had coherent detection of BPSK as q of root 2 gamma, there is a whole range of family of modulation schemes for which you will get the expression as some constant times gamma. So, for example, one of the most popular modulation schemes that we encounter in mobile communications is Gaussian MSK. So, its Gaussian filtered MSK and the Gaussian filter has got a bt of 0.3. This is the modulation scheme that is used in GSM and this one has got a error function which is q of square root of 1.4 gamma. So, basically beta is equal to 1.4. Now, since we have already computed it for BPSK, we just by substituting showed that q of root beta gamma if that is BER in AWGN, then the probability of error in fading will be one-half of 1 minus square root of beta gamma by 2 plus beta gamma and the asymptotic behavior will be 1 over 2 beta gamma.

So, actually the results that we have obtained are very useful in terms of a very broad class of functions and using these observations, we did make the following probability or BER of coherent BPSK. BPSK in fading is less than BER of GMSK. That is is very easy to show because what is SNR difference between the performance of GMSK because of them have got a q function in Rayleigh fading and we can show from the asymptotic expressions that this is better than BER of DBPSK, ok.

So, use the asymptotic expressions to sort of see you where these graphs will fall and then, you can show and of course, we did show in class that to achieve a particular BER, the gamma of GMSK based on the expressions that we have is gamma of BPSK plus 1.55 dB. So, it is approximately 1.55 dB poorer than in terms of performance.

So, let us move on to the lecture number 24. What happened to lecture 23? The table and other things came in lecture 23. Lecture number 24: one of the applications of moment generating functions which we saw first in lecture 24, but subsequently also saw it again when we did diversity.

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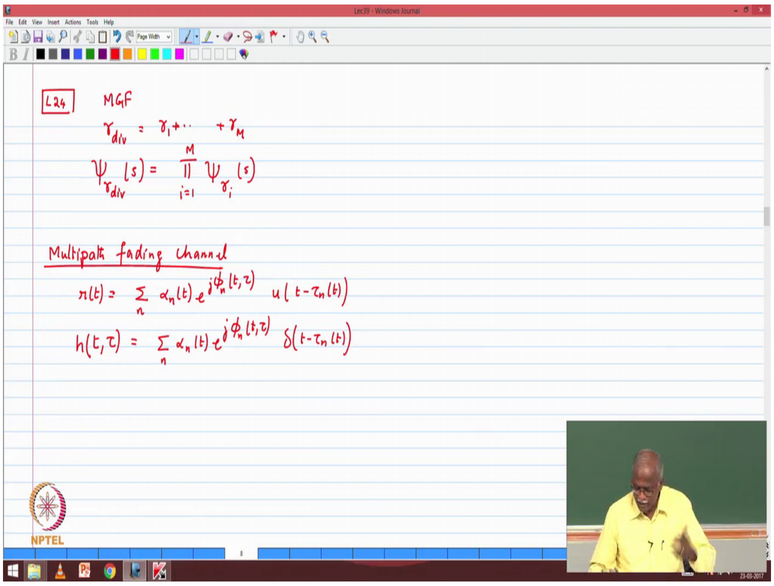

So, lecture 24 and understanding of what the moment generating function is and what is the benefits of that we said that if gamma diversity comes out to be gamma 1 plus all the way to gamma m, if we at that time we did not have a scheme that achieved this, but if this was the scenario, then the moment generating and their all independent variables, then the moment generating function of gamma diversity of s would be a product of the moment generating functions i is equal to 1 through m psi of gamma i of s and once you have the moment generating function, we can use it to compute BER. You can also do a inverse Laplace transform to get PDF. Also both of those methods are used. So, that is one of the reasons why we work with moment generating function.

So, after we finished this discussion, lecture 24 took us back to our understanding of the multipath fading channel. So, the multipath fading channel, we had described earlier want to recapture that in the following manner. The received signal r of t is a super position of many copies of the signal. Each of them has got a complex gain term. The amplitude is alpha n; the phase is e power j phi n. This is the function of t and tau and the received signal if the complex base ban representation was u of t, then this would be u of t minus tau n of t and this also led us to describe the channel in terms of two variables. One is the delay dimension and the other one being the time dimension and this is a description of the channel using a two-dimensional model n times alpha n of t e power j phi n t, tau delta of t minus tau n of t keeping in mind that we cannot call this impulse response or you know treat it like LTI system.

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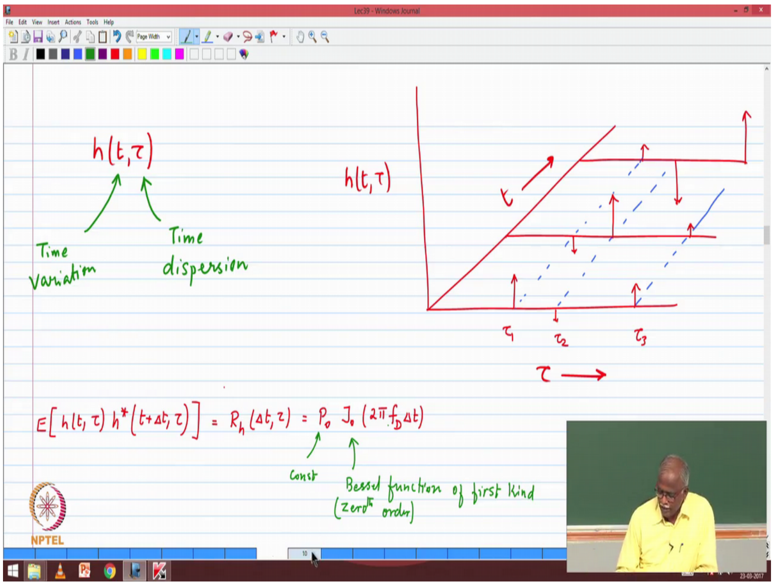

Always keep in mind that our description is that it is a two dimensional description of the channel and what we are saying is at very time instant if you take a snapshot, the channel is going to look like it has got some behavior in terms of the different delays and because it is time varying, these gain coefficience and the delays can change as time progresses.

So, basically what you see is a series of snapshots in time what is capture by h of t, tau and this is our basic understanding of the fading, multiple path fading environment. So, the focus of the next several lectures was to develop a comprehensive characterization of the fading channel. So, the first one that we did was, we took these expression for e of h, tau and we computed the auto correlation e of h of t, tau h conjugate of t plus delta t, tau and wrote down the expressions for that and showed if you denote this as the auto correlation function. It turns out that it did not depend on t and it only depended on delta t and the expression was some constant times the zeroth order Bessel function of the first kind 2 pi f d times delta t, ok.

So, let me just clarify this is constant. This is our zeroth order Bessel function of the first kind. All of these are readily available in tools like MATLAB. We can program and we get the different values.

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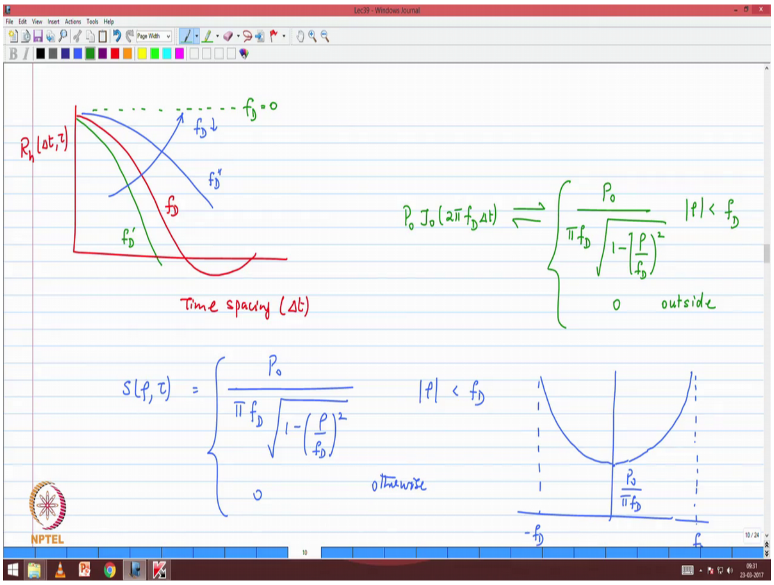

So, the shape of the Bessel function we showed or we sketched in the class looks like a sync function, but you know a focus on the fact that it starts of at a maximum at when the argument is 0 and then, a sort of monotonically drops when you start moving as delta t increases.

So, the focus of interest is primarily not till we reach 0 the crossing, but what is the behavior for small values of delta t. So, that is where the region of focus is and we found that depending upon the Doppler value, the slope of the autocorrelation function will change. In fact, as you keep decreasing the Doppler, the rate of change or the autocorrelation becomes higher and higher and eventually for zero Doppler, the correlation remains constant. Basically it is at a maximum value which means that the channel is not changing at all.

So, the steeper the Doppler or the autocorrelation function, the faster the channel is changing which also means that the Doppler is higher. We also did a Fourier transform of the Bessel function and showed that the behavior since this is the autocorrelation, what you take when you take the Fourier transform. You get the power spectrum, you get a shape which is bounded between plus f D and minus f D. The expressions are given here. So, let us just write down. So, if I have autocorrelation function P0 is a constant j 0 2 pi f D times delta t, the Fourier transform comes out to be P0 divided by pi times f D square root of 1 minus rho by f D whole square for mod rho less than f D and is equal to 0 outside, ok.

So, that expression this is what it looks like and we have been able to relate the time variation which is caused by Doppler to a spectral characteristic which says there is a variation of the spectral content from zero frequency to plus minus f D. We observed that the channel is a multiplicative channel because when we do fading, it is a multiplicative factor. So, basically in the time domain, it is a multiplicative operation which then says that in the frequency domain, it will result in convolution. That means that whatever is the spectrum you are going to convolve with a shape, that has got spectral content between minus f D and f D which results in spectral broadening.

So, the understanding of a spectral broadening which is caused by multipath fading was introduced in this lecture Spectral Broadening.

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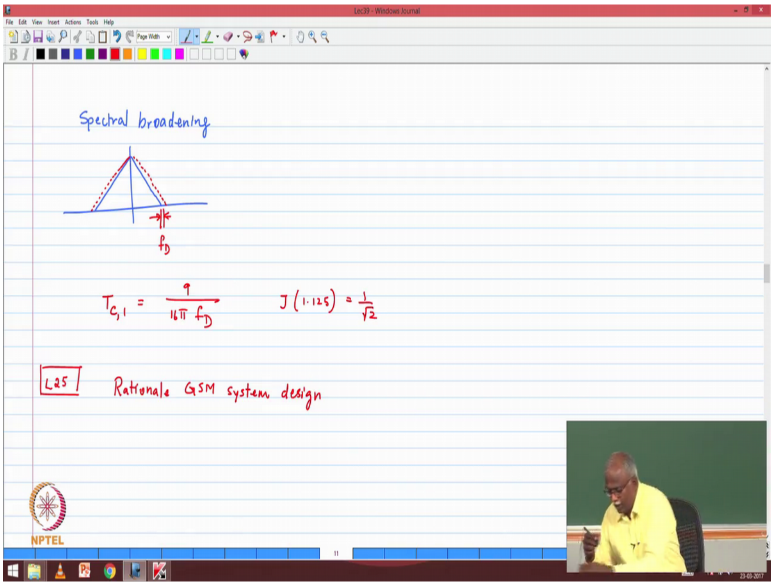

We also showed that for most of the wide band signals, it is very insignificant. It does not matter much because if this was the spectrum, what we are saying is this spectral broadening is going to be that there is some widening of the spectrum on this side and a little bit of a smear on the other side. This range of is basically f D, right. So, the additional shift or the smearing happens with propositional to the Doppler frequency, but then if spectral issue, spectral distortion is not the issue, then what the issue is, it turns out that it is the channel characterization in the time domain.

So, we said that what we need to take away is the coherence time because the channel keeps changing which says how often should I estimate the channel and that is relate to the channel coherence time which is given by 9 by 16 pi f D. This is for the autocorrelation function to reach a value of 1 by root 2. So, that is how we have obtained this expression.

We then computed in lecture 25. We computed the coherence time for different Doppler's. So, basically coherence time is inversely related to the Doppler. So, we also gave the rational for GSM system design. Why did we design the slot of certain duration? Why did we put the straining sequence in the middle? What happens what can you tell about if one burst is affected by a fade? What happens to the next time instant that the same user is going to transmit which will be eight times slots a later? So, rational for GSM system design a lot of it is type II. Our understanding of coherence time and therefore, that is what we are able to do. GSM system design now a long with the temporal variation, we also saw that there were two other elements that are very useful in understanding the channel and those are the characterizations of the time behavior of the channel.

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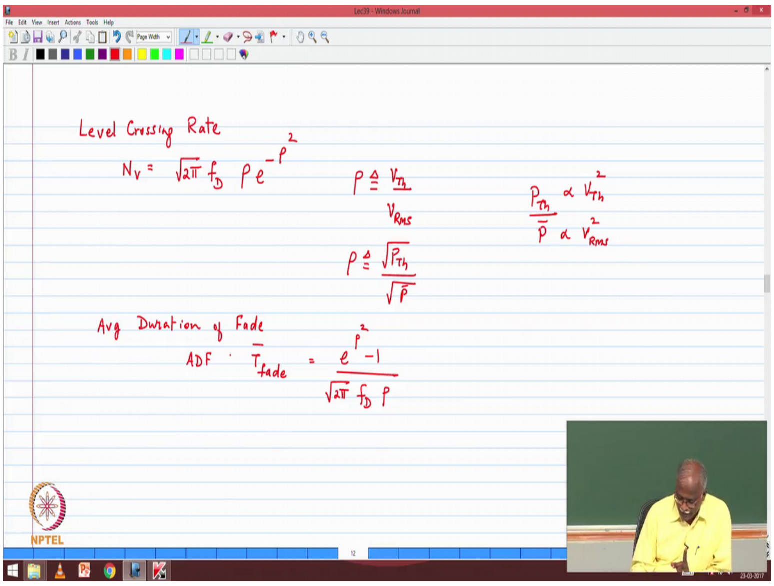

One of them was the Level Crossing Rate. So, given a certain threshold for fading, how often does your fading envelop cross that is given by N v equal to root 2 pi times f D rho e power minus rho square, where rho is given in terms of V threshold divided V RMS, ok.

Now, very often when we talk about fading, we may not write down the thresholds in terms of the amplitude, but we may write it down in terms of a threshold power level with respect to the average power level. So, the threshold power level is proportional to V TH square. This is proportional VRMS square. So, please make sure that you know if you are given in terms of the powers, you can still translate it into correct value of rho. Basically what you would have to do is a rho in that case would become square root of the threshold power level by square root of P and P bar and you should be able to get the value of rho. Once you get the value of rho, you can translate it into the corresponding.

Along with level crossing rate, we also rode down an expression for the average duration of a fade. Now, this is also a useful indication of saying how many symbols are likely to be effected if my signal goes into a fade average duration of a fade denoted by ADF. This is t fade with the bar average. This is given by e power rho square minus 1 dived by root 2 pi f D times rho. Again rho is computed exactly in the same manner as in the previous case. Having fully understood the time variation, we went on to characterize our understanding in terms of describing the fading channel.

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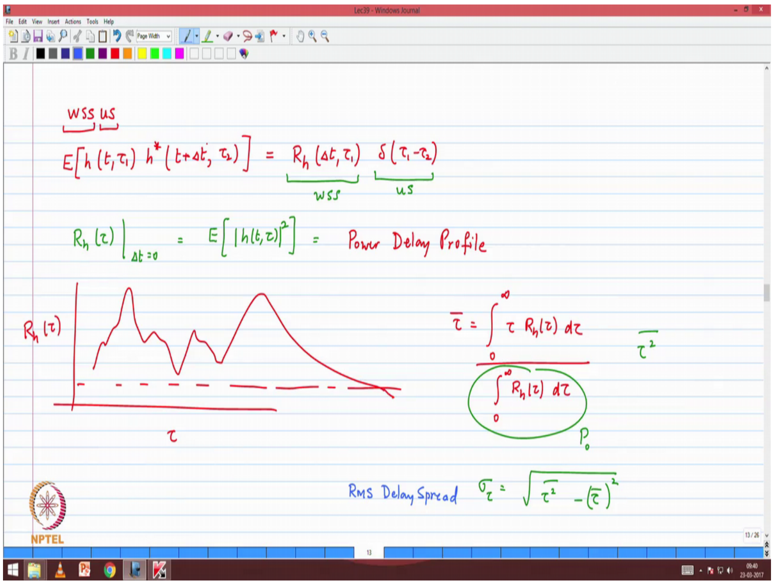

In terms of the wide sense stationary uncorrelated scattering model wide stationary in terms of the time variation uncorrelated scattering in terms of the the time dispersion.

So, this was characterized by this expression expected value of h of t, tau 1 h conjugate of t plus delta t, tau 2. Notice tau 1 tau 2 corresponds to different multi path components coming from different interacting objects. We argued that they are uncorrelated with each other. So, therefore, this autocorrelation becomes R h of delta t, its function depend only on delta t and it is uncorrelated across the different delays delta of tau 1 minus tau 2. So, basically it is only correlated for a particular delay. So, this part was the WSS portion, this is the uncorrelated scattering portion and so, we said having fully understood the time variation which is the wide sense stationary part.

Now, what about understanding these scattering parts or the delay dimension? So, for that we computed R h of tau. Basically we have set delta t equal to 0 in the previous expression. This comes out to be expected value of mod h of t, tau whole square and this is what we referred to as Power Delay Profile. this is usually obtained through channel sounding, but again assume that R h of tau as a function of tau just says where are the multi path components, where are the strong ones and then, eventually the multipath components died on below certain threshold and we now are able to characterize this as the multipath behavior of the channel.

Now, the characterization of this in terms of the average delay for a continuous power delay profile, it would be 0 through infinity tau times r h of tau d tau normalized by the total power delay area under the power delay profile graph. R h of tau d tau, this is a constant. Let us call that as some P naught and basically you can compute tau square average and then, sigma tau as square root of tau square average minus tau average whole square and this is RMS delay spread and this is a very important parameter that characterizes the time depressiveness of the channel RMS delay spread.

This is what also tells us whether we need an equalizer or not and therefore, this characterization of the power delay profile is very important for us. Most often we do not have a continuous power delay profile, but we have a discrete power delay profile.

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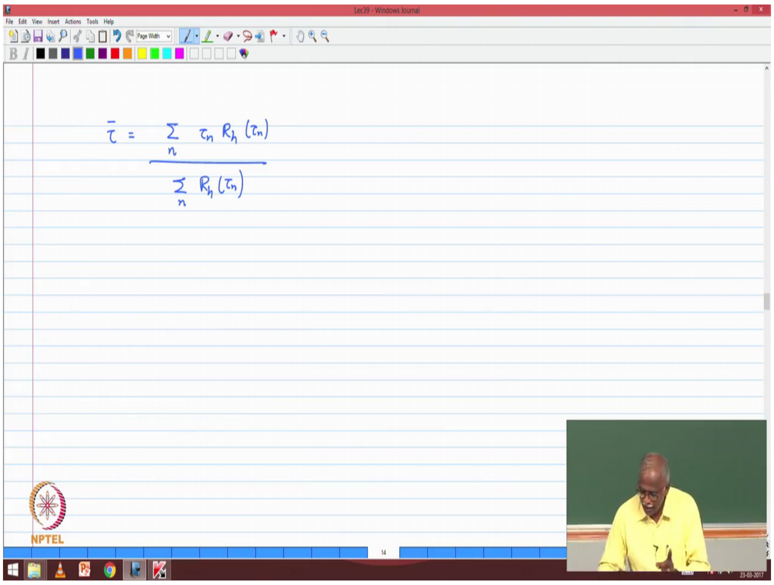

So, in that case, the delay computation will become a summation rather than a integral basically tau n R h of tau n divided by summation over n R h of tau n. Similarly you would compute tau, the average of mean square value and then, you would also compute RMS delay spread, ok.

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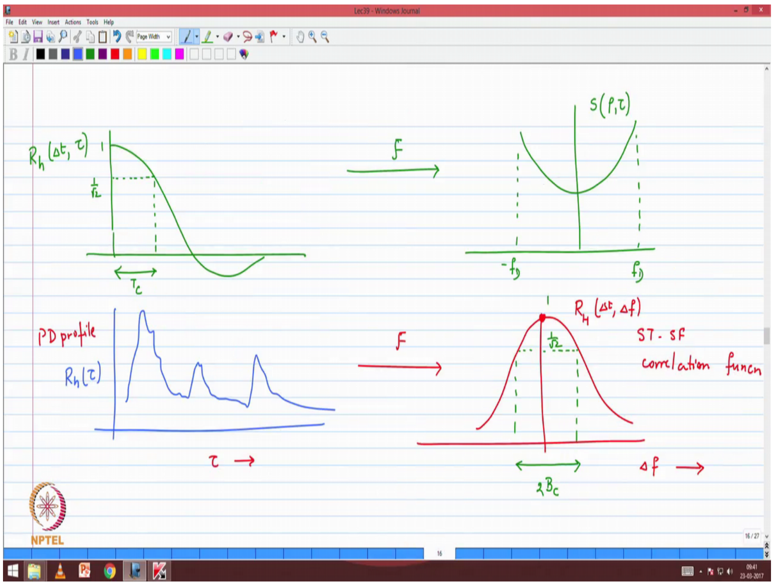

So, basically we have characterized the time variation, the depressiveness of the channel in two forms which then leads us to the complete characterization of the channel, the time variation using Bessel expressions relating to the power spectrum, the power delay profile. If I take the Fourier transform gives me a correlation function which tells me how the correlation of the frequency response changes. So, that then tells me how to compute something called Coherence Bandwidth.

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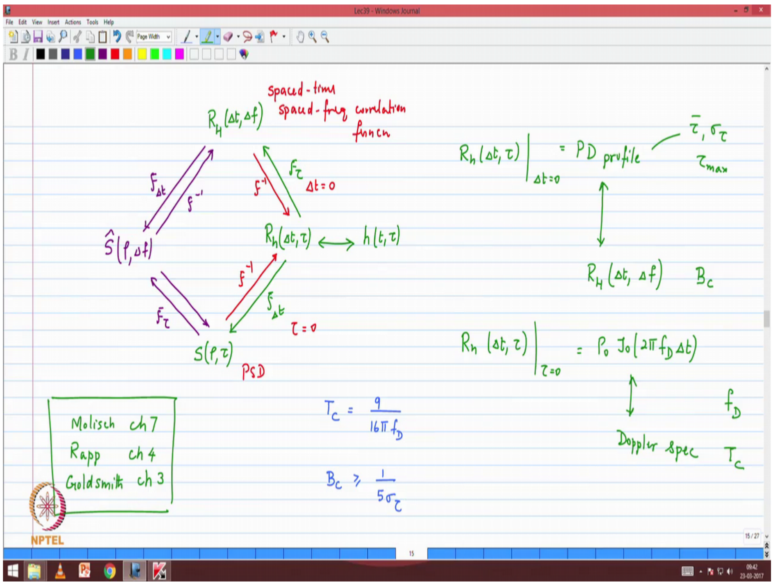

So, the important two parameters that we need in our discussion are coherence time 9 by 16 pi f D, the coherence bandwidth 1 by 5 times sigma tau and these are the key parameters which will characterize both the time variation also, the variation across the different frequencies.

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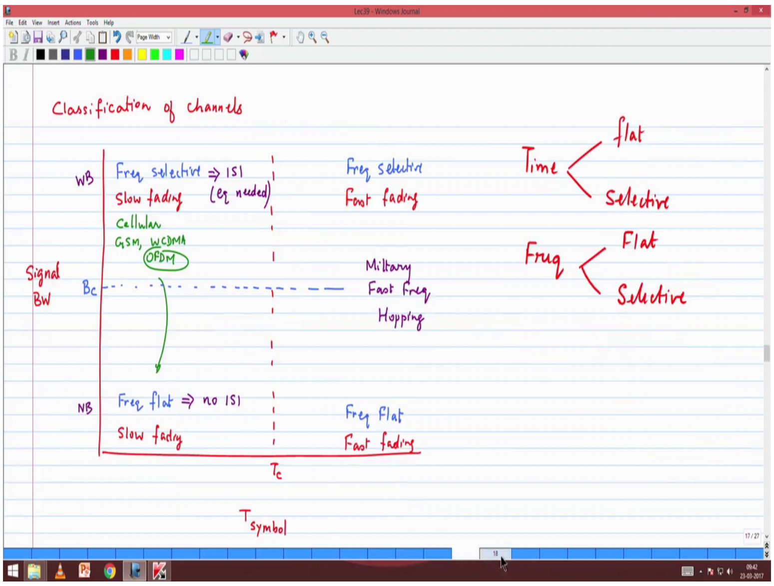

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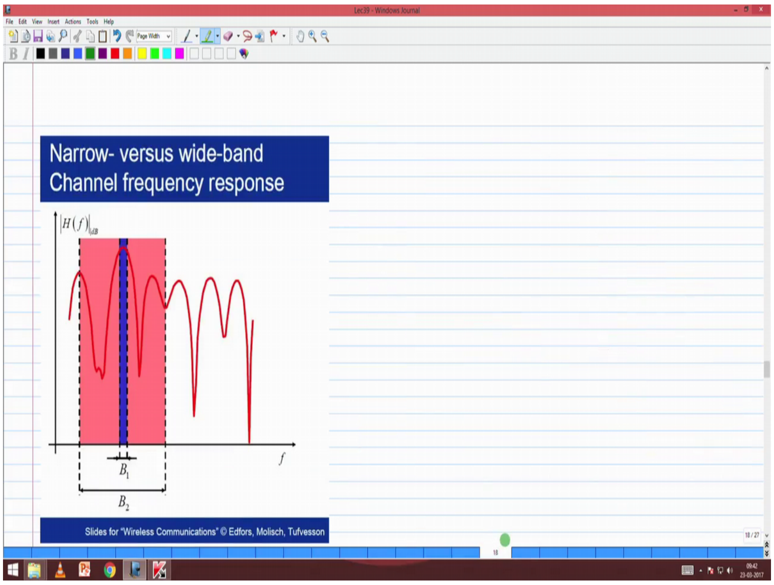

This is useful for us to look at this figure because the channel response is given by the red line, but depending upon the bandwidth of your signal, the channel response may look flat or it may look that it is changing. So, frequency selective basically that then leads us to a classification of channels based on the symbol duration compared to coherence time. If your symbol duration is much less than coherence time, we call it as a slow fading channel. If it does not satisfy that, then it becomes fast fading.

Now, if the signal bandwidth is much less than the coherence bandwidth, the frequency range over which the frequency response is correlated, then it becomes a frequency flat fading channel. Frequency flat means the frequency response is flat. The impulse response is single is a delta function. So, therefore, there is no inter symbol interference. So, that is a desirable element to have, but on the other hand if you have frequency selective fading, then you would have some form of equalization which led us to the following terminology that channels may be flat or selective in time. That means, not changing or changing in terms of the frequency response over the bandwidth of the channel, it could be flat. That means no si or if it has frequency variation, then you would need an equalizer for those particular types of channels, ok.

So, that more or less gave us a characterization of the wide sense stationary uncorrelated scattering model and helped us to understand.

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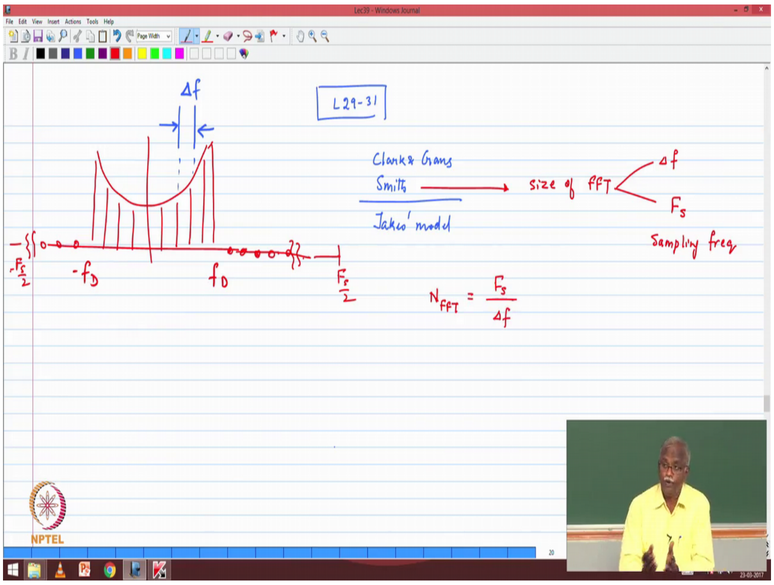

Next few lectures basically lectures 29 to 31 had to do with our ability to generate Rayleigh fading in on a computer. So, that was lectures 29 to 31. So, we had the following three models; the Clark and Gans, Smith model. Smith model, this figure you will remember. Basically you did completely in the frequency domain and of course, the time domain approach which was Jakes model.

So, the Smith model basically had to deal with the size of FFT that was an important element that would come out and the size of FFT depended on two parameters. One was delta f, the frequency resolution that you wanted for the Doppler spectrum and the other one was controlled, was not in our control. It was basically the sampling frequency of the signal that you were trying to generate. So, this is the sampling frequency, ok.

So, the size of FFT if we were to think of this all the way up to F s by 2 and on this side to minus F s by 2, the size of FFT n FFT is equal to F s divided by delta f and this number could be very large depending upon how much resolution you want to achieve in terms of the representation of the Doppler’s spectrum.

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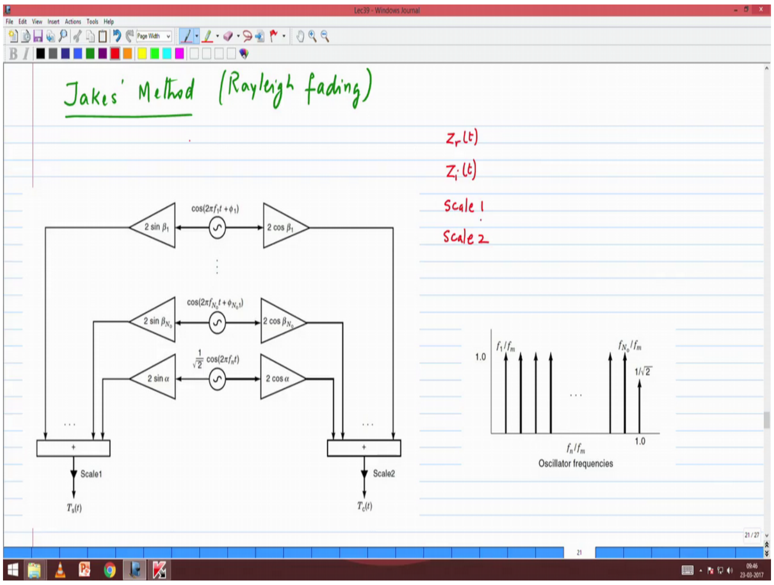

On the other hand, Jakes model gave us a very compact yet very accurate way of working with the signals. So, we wrote down expression for Z r of t Z i of t obtained what should be the scale factors to get unit variance. So, it is variance of one-half actually scale 1 and scale 2 and then, verified all of the properties.

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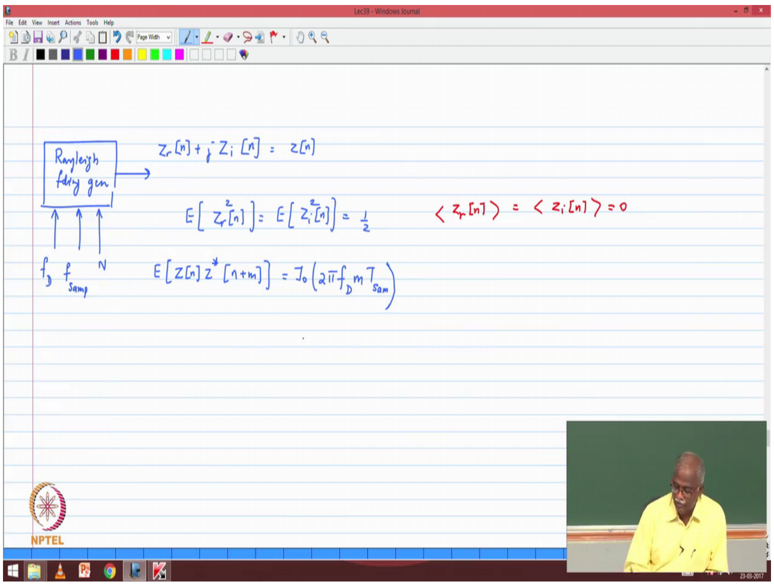

So, Jakes model if you have given the Doppler frequency, what is the sampling or the spacing between the fading way form that you want, what is the number of samples that you want and then, it will generate with the guarantee that the real and imaginary part are uncorrelated. Each of them has got a variance of one-half. The expected value of Z r of n is equal to Z i of n is equal to 0. They are zero mean and they have the desired correlation in terms of the Bessel function and this we verified through the trapezoidal approximation. So, basically this is what helps us generate this wave form on a computer, ok.

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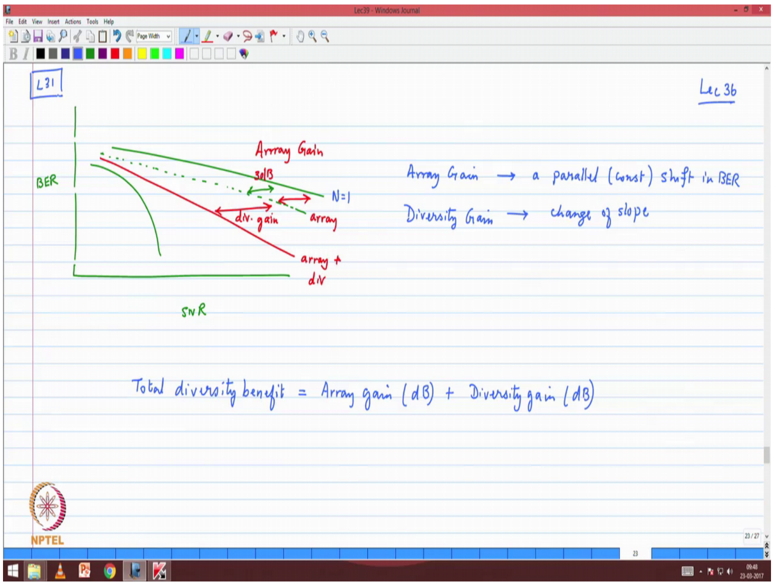

Last few lectures we entered the domain of diversity by looking at the benefits of diversity. We showed that may be will just summarize before we get to this graph. The different forms selection diversity was the simplest form that we had discussed. This was basically choosing the best of the antennas that are available to us of gamma M.

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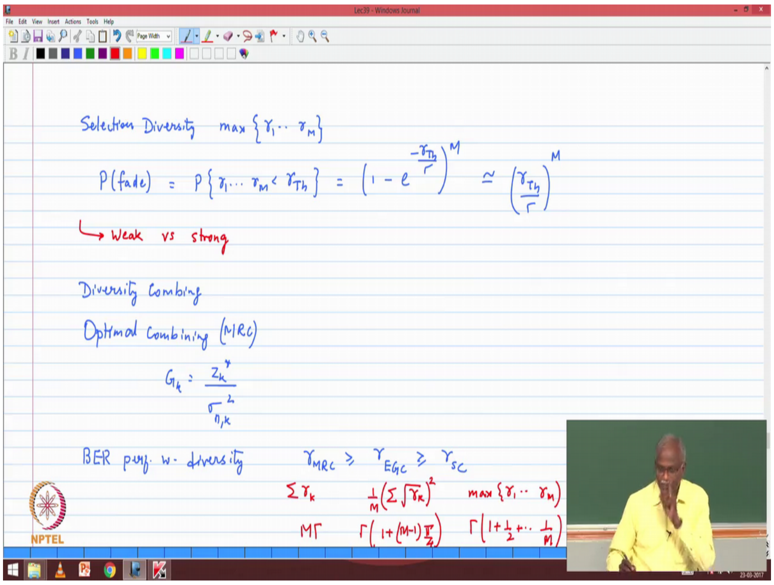

We showed that even selection diversity gave us a change of slope and therefore, our understanding is that diversity is going to change the statistics of the fading and help us achieve closer to the performance of AWGN channel and the statistics of a of fading channel.

So, what is the probability that you are in a fade when you are doing, when you have selection diversity? This is the same as the probability that all your antennas are below the threshold for fade. This we showed is 1 minus gamma e power minus gamma TH e power minus gamma TH by gamma raise to the power M and this is approximately gamma threshold divided with gamma raise to the power M. In the context of selection diversity, we also made reference to weak antennas as opposed to strong antennas and when do weak antennas are useful, when are they of not much use and what are the benefits and the tradeoffs we had discussed that.

Better than selection diversity was diversity combining, where we saw that we could get the better of, we can get the benefit of both channels and this then led us to the optimal combining which was maximal ratio combining MRC and Maximal Ratio Combining required us to use the gain coefficience to be Z k star divided by sigma n, k whole square. So, basically it was co-phasing plus waiting based on the noise variance of that particular channel.

For both selection diversity and for maximal ratio combining, we obtain the statistics and then, basically were able to show the performance of BER performance with diversity and let me just write down one last result. Basically we also showed the following that gamma MRC is better than or equal to gamma equal gain combining which is better than gamma selection combining and this one achieves some, instantaneously achieves some of the different SNRS.

This one achieves something slightly less 1 over M summation root gamma k root gamma k whole square and this one basically selects the best of all the antennas gamma 1 to gamma M, and we also showed that when you take the average value, this achieves M times gamma. This achieves M gamma times 1 plus M minus 1 pi by 4 and this one achieves a diminishing written 1 plus 1 by 2 all the way to 1 over 1 over M and in the interest of time, let me just summarize the last statement that we showed that when you will take the MRC expression, you can split it into array gain and diversity gain and recognizing that received diversity techniques gives you combination of both benefits.

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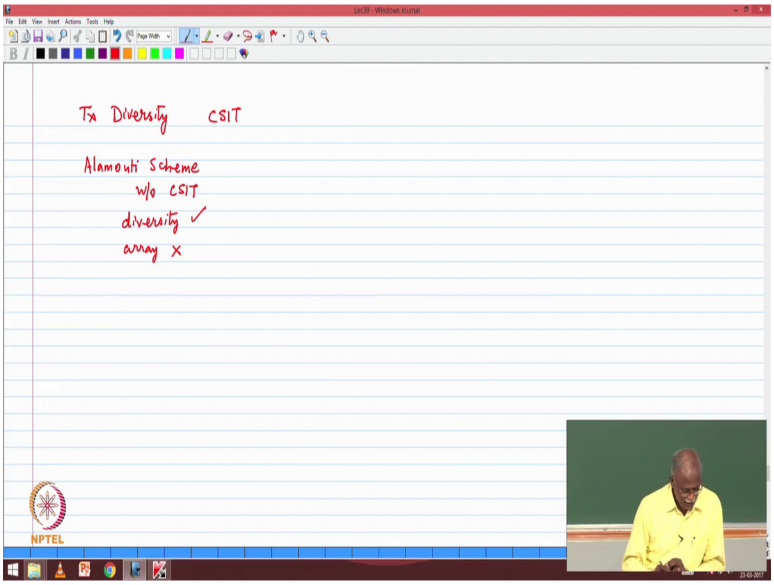

On the other hand, if you are limited to T x diversity, then you will give up the array gain, but you will still get diversity gain, but you will need the knowledge of the signal channel at the transmitter. The only exception is the Alamouti Scheme. Alamouti scheme is a transmit diversity scheme without CSIT which achieves the diversity gain of a two branch system. Diversity gain of course is being a transmit diversity scheme. It does not achieve the array gain, ok.

So, that pretty much was the span of what we had covered all the way up to lecture 36 in the context of diversity; hopefully that sort of helps in sort of putting the pieces together. We will look at assignment 4 and any doubts to be cleared on assignment 4, please come to ESB 242 between 5 and 6 pm.

Thank you.