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NPTEL ONLINE COURSE

Discrete Mathematics

Let Us Count

Problems on Combinations

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So we have seen what is meant by combinations and choosing r items out of n items. We also saw certain results on combinations. Let us now solve some problems. So we'll be going with solving some basic evaluation problems and then moving on toward problems. So let us compute what is nC5. nC5 by applying the formula we get it as n factorial by 5 factorial into n minus 5 which is 4 factorial. So 9 factorial can be written as 9 into 8 into 7 into 6 into 5 factorial by 5 factorial into 4 into 3 into 2. 5 factorial can be canceled on the numerator and the denominator and after simplification we get the answer as 126. What is 11C 3? applying the formula we see that it is 11 factorial by 3 factorial into 11 minus 3 which is 8 factorial. This is same as 11 into 10 into 9 into 8 factorial by 3 into 2 into 8 factorial. 8 factorial gets canceled on the numerator and the denominator and after simplification we see that 11C3 is 165. Please note 11C3 is same as 11C8 and hence 165 is also equal to 11C8.

Now what is 6C2 + 6C1? So let me remind you that we had seen a result earlier n minus 1 Cr plus n minus 1 Cr minus 1 is equal to nCr. So we need not compute these terms individually. So I will directly observe what is n and r and calculate nCr. So by observation we see that n comes up to be 7 and r is

2 and then 7C2 is given by 7 factorial by 2 factorial into 7 minus 2 which is 5 factorial. Now this is 7 into 6 into 5 factorial by 2 into 5 factorial. 5 factorial gets canceled on the numerator and the denominator and hence we get 21. So 6C2 + 6C1 is same as 7C2 which is equal to 21.

Now what is 5C3 + 5C2? Applying the same formula again, let me tell what is n, it is 6 and r comes up to be 3 and hence we'll calculate only 6C3. Now this is equal to 6 factorial by 3 factorial into 6 minus 3 which is 3 factorial. So after computation we see that it comes up to 20.

In a cricket championship there are 21 matches. If each team plays one match with every other team what is the total number of teams? You see there are 21 matches and it is also given that each team has played with every other team. So there are 21 matches in all and hence nC2 comes up to be 21. Now what is n is the question? How did we get nC2 asa 21? you see there are n teams and each team plays with the other team and hence it is nC2 and it is given to be 21. So this nC2 can be written as n into n minus 1 into so on up to 2 into 1 by n minus 2 factorial into 2 factorial. Now simplifying we get n into n minus 1 by 2. This is equal to 21. This implies n into n minus 1 is equal to 42 therefore solving this equation we see that n is 7 and hence there were 7 teams where each team played with the other team and we had 21 matches.

Let us move on to the next question. Find a formula for counting the number of diagonals in a convex n-gon. So we have to count the number of diagonals in a n-gon? So what is an n-gon? n- stands for the number of sides. You see if it is 4 then it is a square, if it's 5 it's a pentagon and if it's 6 it's hexagon and so on. So we have to count the number of diagonals possible.

So you see diagonal is connecting a vertex here with another vertex here. So like this, like this and so on. So we have to exhaust all the possibilities of drawing diagonals. We don't consider these n sides. So what is the number of diagonals? We should connect every two vertex like this and hence it is nC2 and we don't count these sides and hence it is minus n. So the total number of diagonals is nC2 minus n which comes up to be n factorial by 2 into n minus 2 factorial minus n. Now n factorial by 2 factorial into n minus 2 factorial is n into n minus 1 by 2 minus n. Now this is equal to after simplification n into n minus 3 by 2.

Now this gives us the number of diagonals in any n-gon. So you see we have to have a minimum of 3 for n. So if I substitute 3 that is in a triangle we don't have any diagonals as every vertex is connected to every other vertex. Let me tell for n is equal to 4 which is a square so when I substitute n equal to 4 I get 4 into 4 minus 3 by 2 which is 4 by 2 which is 2. So we have two diagonals in a square and so on. So we can compute this for any n.

A question paper consists of 10 questions divided into parts A and B. Each part consists of 5 questions. A candidate has to answer six questions in all of which at least two should be from part A and two should be from part B. In how many ways can a student select the questions if he can answer all equally well?

So you see the question paper has 10 questions which is divided into part A and part B. Each of them have 5 questions and the candidate has to answer 6 in all where he has to answer two from part A and two from part B compulsorily. So in how many ways can he answer six questions? Let us now see what are the possibilities the student has. Let me write down possibilities from part A and part B. So either he can answer two from part A and four from part B. This sums up to be six or he can answer three from part A and three from part B or four from part A and two from part B because of the minimality condition that at least two we cannot have one here and 5 here. So for this first possibility two from part A and four from part B in how many ways can he do this? It is 5C2 into 5C4 by rule of products because there were five questions in part A and five questions from part B and he has to choose 2 from A and 4 from B. So this comes up to be after evaluation as 50 and for the second possibility that is answering 3 from part A and three from part B the number of ways he can do this is 5C3 into 5C3 which comes up to be after evaluation 100. Now for the third one which is answering for questions from part A and two questions from part B again this is similar to the first possibility the answer comes up to be 50. So for possibility 1 he has 50 number of ways. For possibility two, he has hundred ways and for possibility three, he has 50 ways. So the total number of ways the student can select the questions is 50 plus 100 plus 50 by the rule of sum which is 200 because either he can choose this possibility one or possibility two or possibility three. He can follow any of these three and hence by rule of sum we see that the answer is 200.

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