NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 1

Edge condition for connectivity

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Let us look the very straight forward result that is easy on your intuition, but slightly involved when it comes to bending down the proof, so I say that given any graph G that is connected with N vertices you will always see that the number of edges is greater than N-1,
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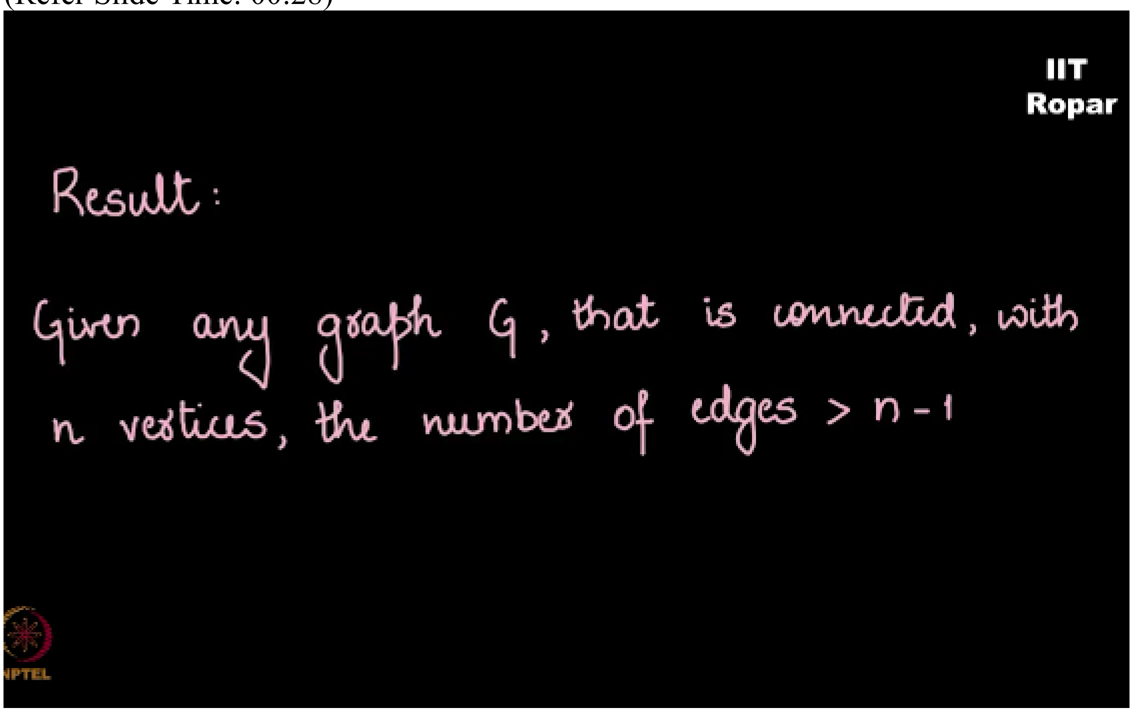

so this is the statement of the theorem, observe the statement and I suggest that you pause the video and try, attempt the proof all by yourself.

I'm sure you're attended, and some of you would have got the proof already, some of you wouldn't, so let us go ahead and see what is my proof for this theorem?

Now look at this, in a graph G if it is a tree, see firstly it's connected and it's a tree, it must have n-1 edges,
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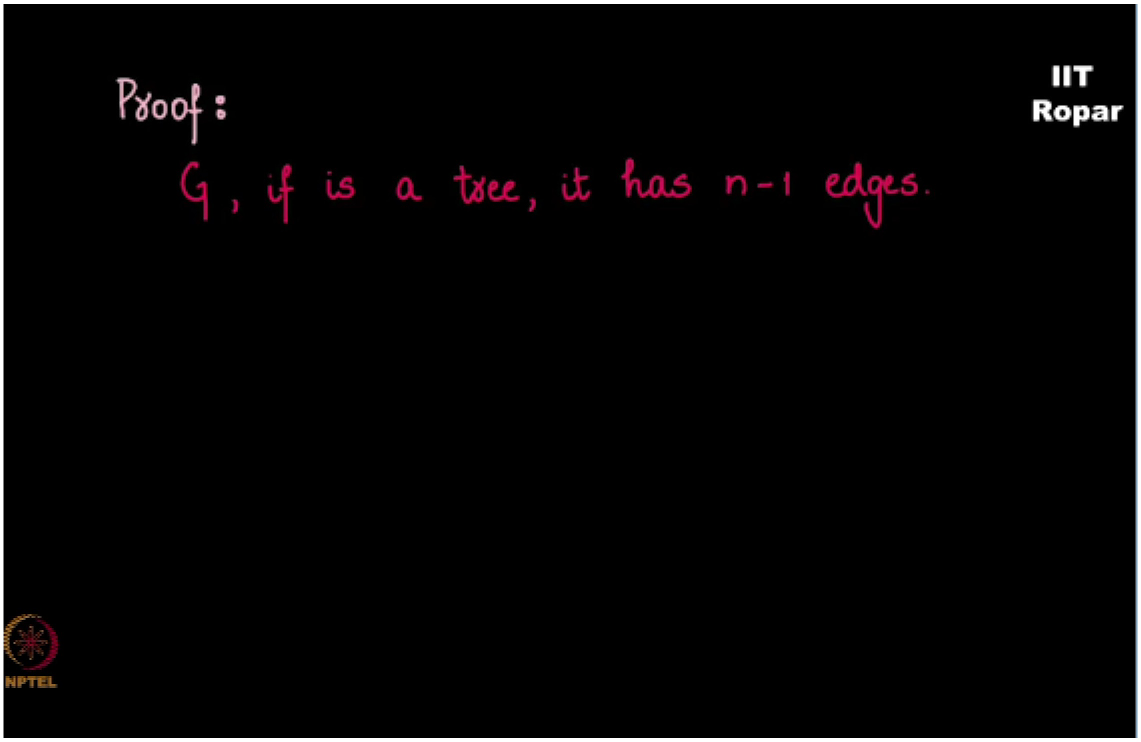

so we are done, the number of edges is, I have to prove that it should be greater than or equal to n-1, now and if it's a tree it's n-1 I'm done, right, so that's all is the proof I'll go ahead.

Now if the graph is not a tree, then what do you mean by that? Then there is a cycle somewhere in the graph, okay, spot the cycle and remove just one edge from the cycle,

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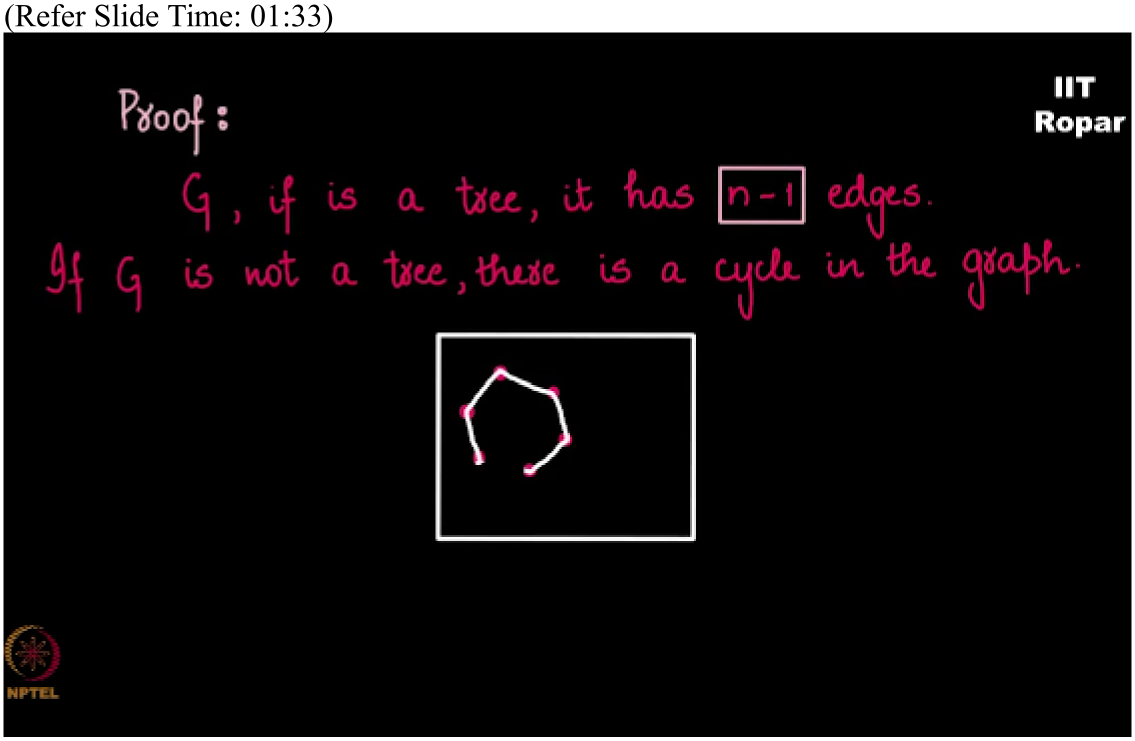

what happens to the cycle now? The cycle now more exists, right, that you know that when you remove just one edge from a cycle in a given connected graph, the graph can never get disconnected, removal of an edge can result in a graph getting disconnected but if that edge is part of a cycle, the graph does not get disconnected, you just now saw the proof while before, right.

So if there is a cycle I remove an edge, thus spoiling the cycle, the cycle is no more a cycle, now if there is another cycle I go to that cycle and then remove an edge in the cycle, so removal of an edge from a cycle results in the cycle not being a cycle anymore,

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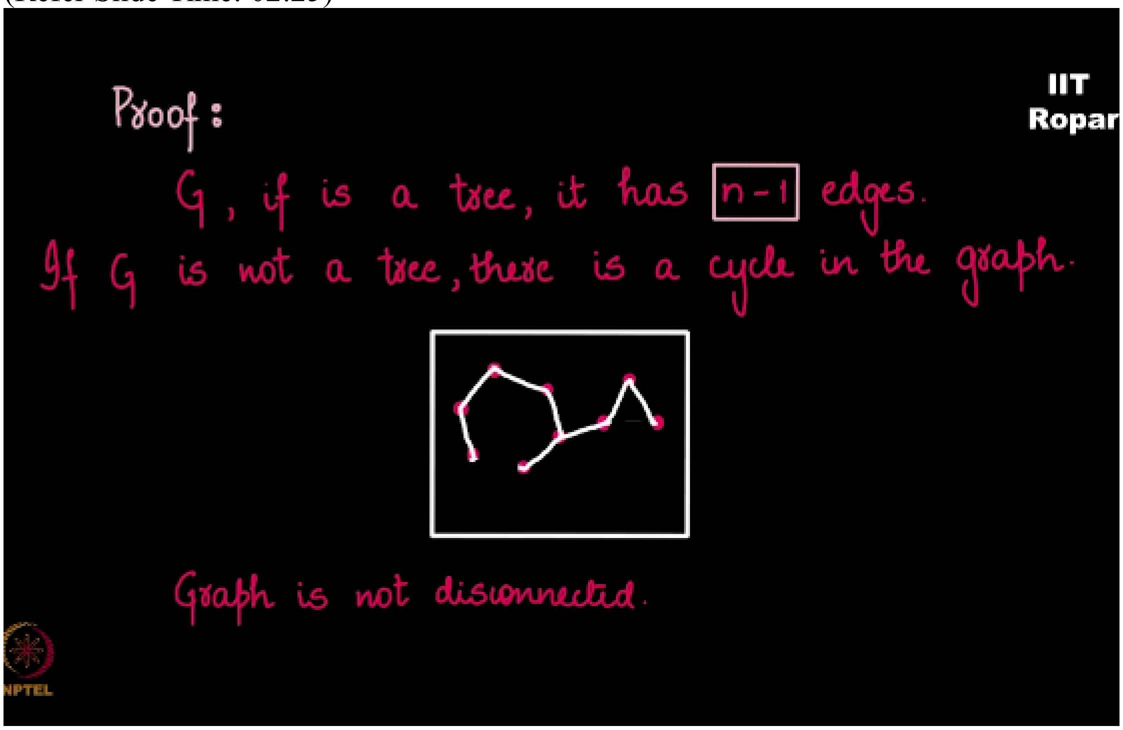

but at the same time the graph stays connected, I keep doing this for every single possible cycle in the graph, and what will happen, at the end I will observe that there are no more cycles, but the graph is still connected, why? Whenever I remove an edge, it’s an edge from the cycle, so it can never result in a disconnected graph, so this way I’m killing all the cycles and ensuring that the graph is connected,

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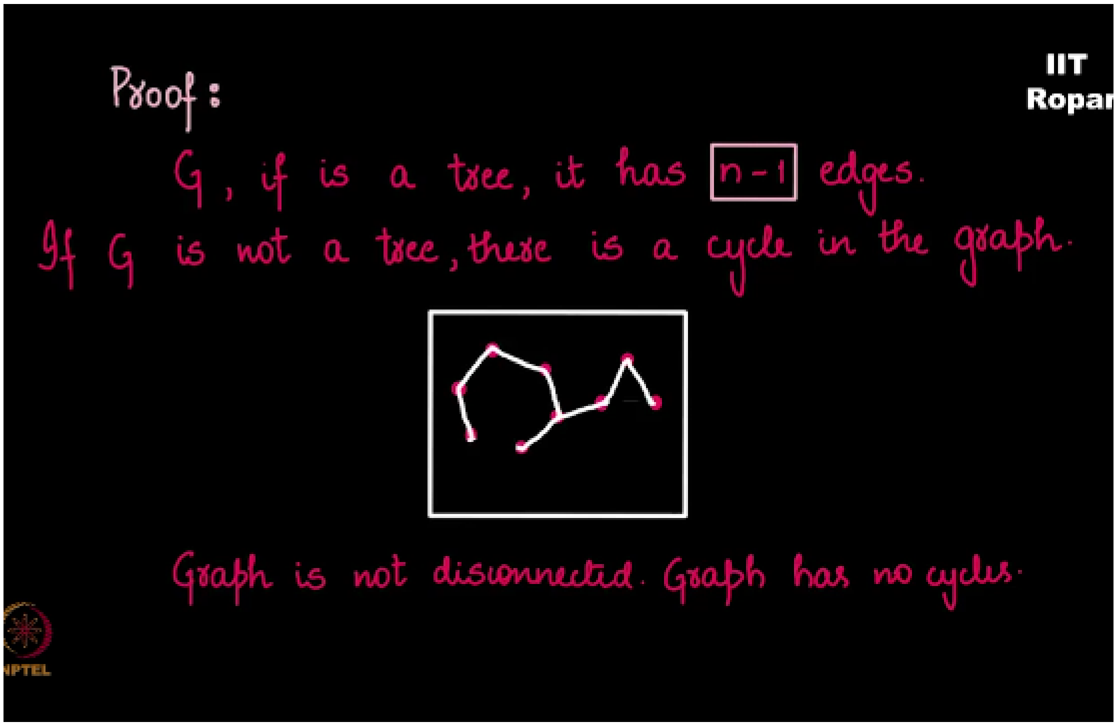

so at the end the moment I observed that there are no more cycles I stopped this process, no more cycles the graph is still connected, which means the graph is a tree, a graph is a tree then the graph should have n-1 edges.

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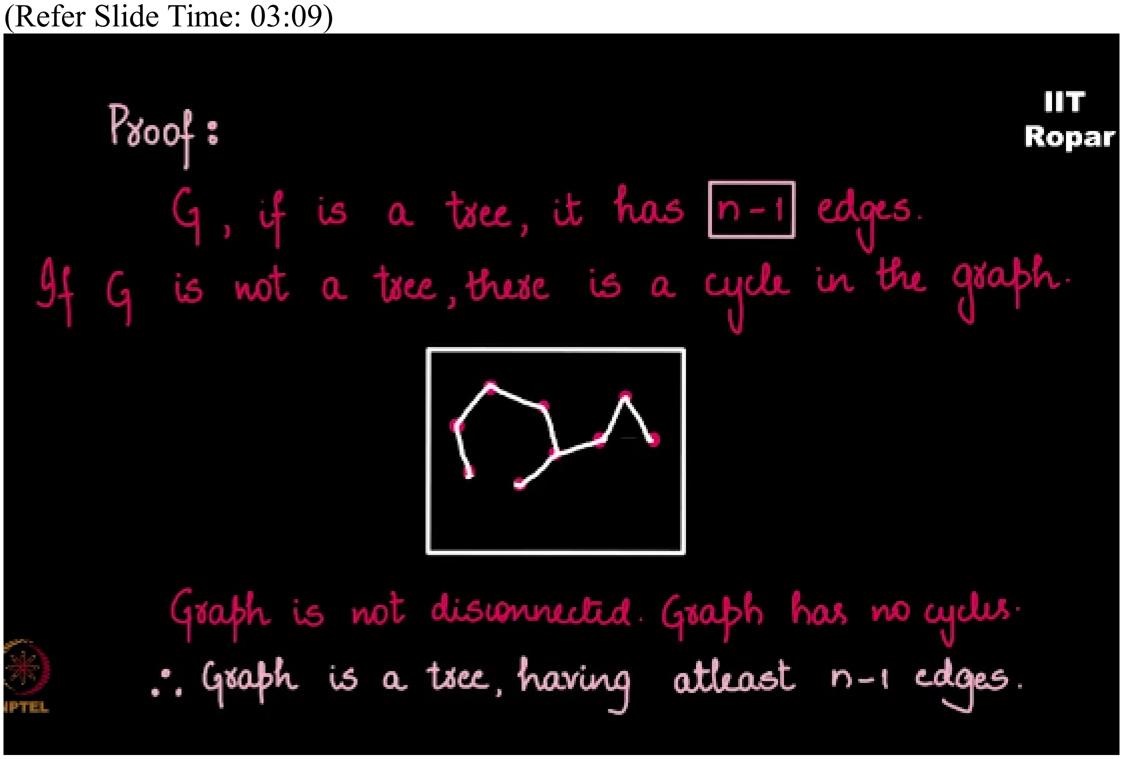

Now I've just now shown that there is a tree which is a subgraph of the given graph, which means the graph should have at least n-1 edges, I was little fast here maybe you should go through this proof once again by playing this video.