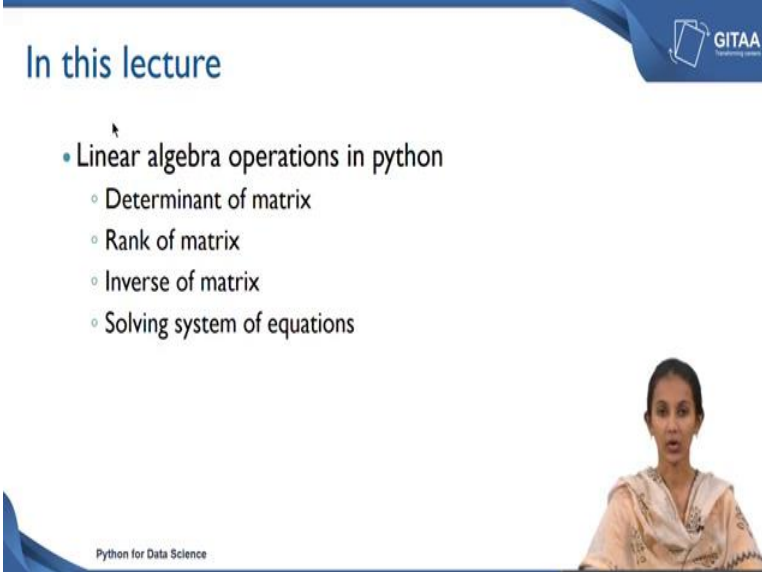


Python for Data Science
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Lecture – 15
Linear algebra Part – 1

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The slide features a blue header with the text "In this lecture" and the GITAA logo. Below the header is a bulleted list of topics. In the bottom right corner, there is a video inset showing a woman speaking. The text "Python for Data Science" is visible in the bottom left corner of the slide.

In this lecture

- Linear algebra operations in python
 - Determinant of matrix
 - Rank of matrix
 - Inverse of matrix
 - Solving system of equations

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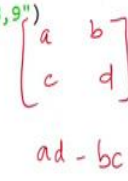
Welcome to the lecture. In this lecture we will see some of the Linear algebra operations in python. So, first one is determinant of matrix, how to calculate rank, how to calculate the inverse of a matrix and also how to solve the system of equations?

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Determinant of matrix

- `numpy.linalg.det()` - returns determinant of the matrix
- Syntax: `numpy.linalg.det(matrix)`

```
x=np.matrix("4,5,16,7;2,-3,2,3;3,4,5,6;4,7,8,9")
In [60]: print(x)
[[ 4  5 16  7]
 [ 2 -3  2  3]
 [ 3  4  5  6]
 [ 4  7  8  9]]
det_matrix=np.linalg.det(x)
```



Python for Data Science 3

So, let us get started first we will look at the determinant of matrix to calculate a determinant of matrix a matrix it should be as square matrix. So, it can be a 2 cross 2 or it can be a 3 cross 3 or it can be a 4 cross 4 matrix. So, the determinant of matrix is very useful in calculating the inverse and also it is used in solving system of equations.

So, in python `numpy.linalg.det` it basically returns the determinant of the matrix. Let us say if I have a 2 cross 2 matrix a b c d, $ad - bc$. So, this is a way to calculate the determinant. So, if you have a 3 cross 3 also you can find the determinant you can also do it for 4 cross 4; as well the Syntax is `numpy.linalg.det` and inside the parenthesis we have to specify the matrix.

So, first we will create a matrix, I am going to create a 4 cross 4 matrix, which means 4 rows and 4 columns you can give values for the first row that is 4, 5, 16, 7 and then after 7 you have to separate it with semicolon. And again you have to specify the values for the second row and similarly for third row and fourth row here I am storing in variable call x matrix.

Now let us print the matrix x so, it has created a 4 cross 4 matrix which is a square matrix. Now let us calculate determinant for this matrix so `numpy`. So, before calling an `numpy` you need to import `numpy`. So, you have to import `numpy` as `np`; `np.linalg.det(x)` and I am storing in variable call `det_matrix` which is it determinant matrix.

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Determinant of matrix

- `numpy.linalg.det()` - returns determinant of the matrix
- Syntax: `numpy.linalg.det(matrix)`

```
x=np.matrix("4,5,16,7;2,-3,2,3;3,4,5,6;4,7,8,9")
In [60]: print(x)
[[ 4  5 16  7]
 [ 2 -3  2  3]
 [ 3  4  5  6]
 [ 4  7  8  9]]
```

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$ad - bc$$

```
In [62]: print(det_matrix)
128.0
```

Python for Data Science 3


Now, let us print the `det_matrix` the determinant value is 128.0, so this is a value for the 4 cross 4 matrix. Similarly you can create other matrix and you can calculate the determinant as well.

(Refer Slide Time: 02:51)

Rank of matrix

- `numpy.linalg.matrix_rank()` - returns rank of the matrix
- Syntax: `numpy.linalg.matrix_rank(matrix)`
- Consider the matrix **x**

```
x=np.matrix("4,5,16,7;2,-3,2,3;3,4,5,6;4,7,8,9")
In [60]: print(x)
[[ 4  5 16  7]
 [ 2 -3  2  3]
 [ 3  4  5  6]
 [ 4  7  8  9]]
rank_matrix=np.linalg.matrix_rank(x)
```



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Next one is rank of the matrix. So, basically the rank is used to find the number of linearly independent rows or linearly independent columns. So, in python so again in the numpy package, we have the `matrix_rank` which basically performs the rank. The Syntax

is `numpy.linalg.matrix_rank` and inside the parenthesis again you have to specify the matrix name which you are going to create.

Now let us consider the matrix `x` same matrix we will find the rank for this matrix. So, it is a 4 cross 4 matrix right for this 4 cross 4 matrix calculate the rank. So, the command is `np.linalg.matrix_rank`. And inside the parenthesis you have to specify matrix name and I am storing it variable call `rank_matrix`. So, you can also store it in some other variable name as well.

(Refer Slide Time: 03:58)

Rank of matrix

- `numpy.linalg.matrix_rank()` - returns rank of the matrix
- Syntax: `numpy.linalg.matrix_rank(matrix)`
- Consider the matrix `x`
`x=np.matrix("4,5,16,7;2,-3,2,3;3,4,5,6;4,7,8,9")`
In [60]: `print(x)`

```
[[ 4  5 16  7]
 [ 2 -3  2  3]
 [ 3  4  5  6]
 [ 4  7  8  9]]
```

In [64]: `print(rank_matrix)`
4

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So, let us print the rank value so, it shows 4. So, which means it has 4 linearly independent rows.

(Refer Slide Time: 04:04)

Inverse of a matrix

- `numpy.linalg.inv()` - returns the multiplicative inverse of a matrix

$A^{-1} = \frac{adj(A)}{|A|}$

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Next we will look at the inverse of A matrix; inverse formula is A inverse is $A^{-1} = \frac{1}{\det(A)} * adj(A)$ so, this is the formula for the inverse of matrix. So, in python the command is `inverse` it basically returns the multiplicative inverse of a matrix.

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Inverse of a matrix

- `numpy.linalg.inv()` - returns the multiplicative inverse of a matrix
- Syntax: `numpy.linalg.inv(matrix)`
- Consider the matrix **A**

```
A=np.matrix("3,1,2;3,2,5;6,7,8")
```

In [66]: `print(A)`

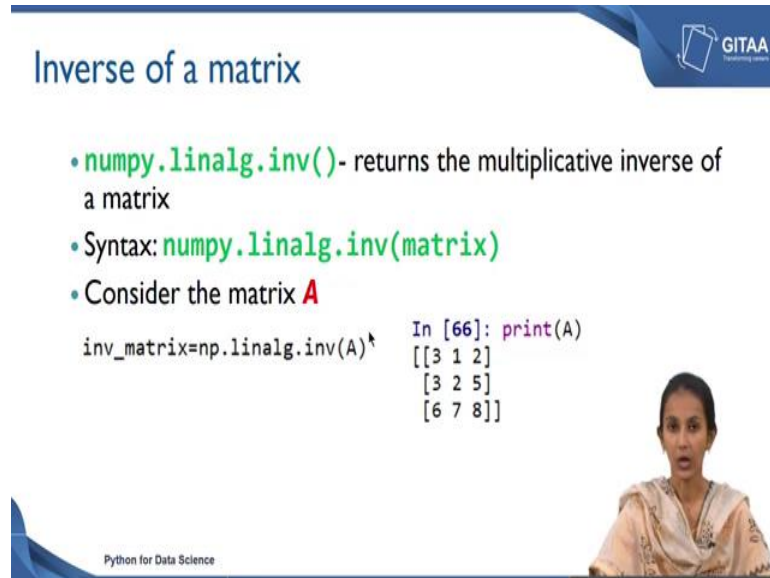
```
[[3 1 2]
 [3 2 5]
 [6 7 8]]
```

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The Syntax is `numpy.linalg.inverse` that is `inv` and inside the parenthesis you have to specify the matrix name. I am creating a matrix 3 by 3 and then storing in a variable call in A. So, before calling `np.matrix` you have to import the numpy package `import numpy as np` and from the numpy package you can call the `matrix.matrix`. So, the values for the

first row are 3, 1, 2; 3, 2, 5 for the second row basically for the third row. So, now, we have printed the values so, this is a 3 cross 3 matrix for this matrix we will calculate the inverse.

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Inverse of a matrix

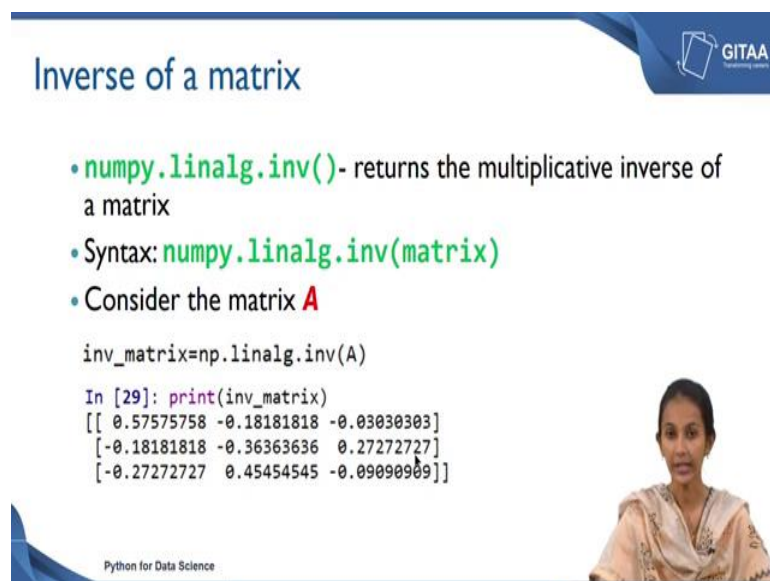
- `numpy.linalg.inv()` - returns the multiplicative inverse of a matrix
- Syntax: `numpy.linalg.inv(matrix)`
- Consider the matrix **A**

```
inv_matrix=np.linalg.inv(A)  In [66]: print(A)
                               [[3 1 2]
                               [3 2 5]
                               [6 7 8]]
```

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So, the command is `np.linalg.in inv` and inside the parenthesis I am specifying the matrix name which is `A`. And you can store it in the variable `inv_matrix`. So, now, let us print the inverse of the matrix.

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Inverse of a matrix

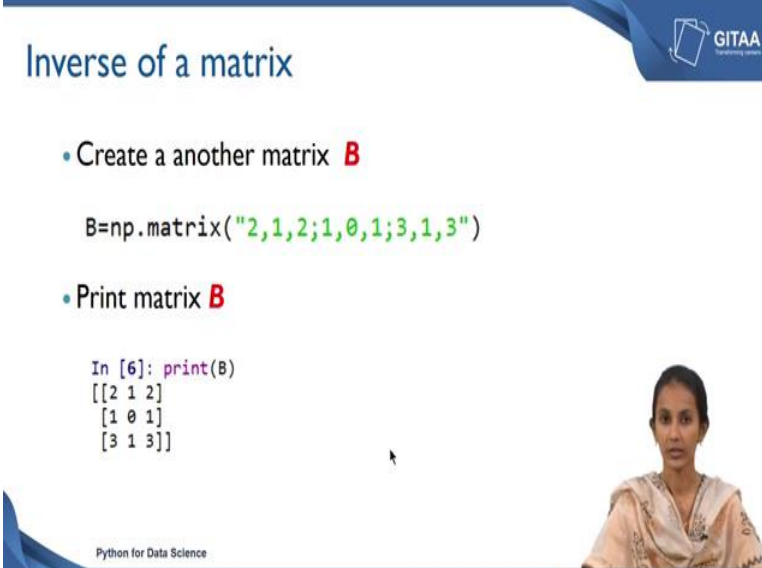
- `numpy.linalg.inv()` - returns the multiplicative inverse of a matrix
- Syntax: `numpy.linalg.inv(matrix)`
- Consider the matrix **A**

```
inv_matrix=np.linalg.inv(A)
In [29]: print(inv_matrix)
[[ 0.57575758 -0.18181818 -0.03030303]
 [-0.18181818 -0.36363636  0.27272727]
 [-0.27272727  0.45454545 -0.09090909]]
```

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So, this is a value for the matrix which we have created. So, this as return A inverse. So, it as calculated 1 by determinant A into adjoint A and returns the output.

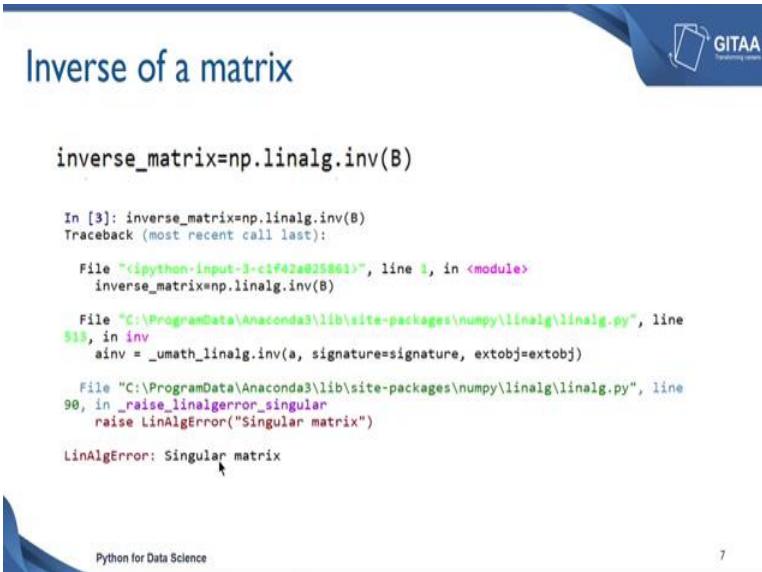
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The slide is titled "Inverse of a matrix" and features the GITAA logo in the top right corner. It contains two bullet points: "Create a another matrix **B**" and "Print matrix **B**". The code for creating matrix B is `B=np.matrix("2,1,2;1,0,1;3,1,3")`. The code for printing matrix B is `In [6]: print(B)`, which outputs `[[2 1 2]`, `[1 0 1]`, and `[3 1 3]]`. A woman is visible in the bottom right corner of the slide. The footer includes "Python for Data Science".

Now, let us take an another example for calculating the inverse. We will create another matrix B. So, this is again a 3 cross 3 matrix now let us print the matrix B. So, the values are 2 1 2; 1 0 1; 3 1 3. So, it is a 3 by 3 matrix.

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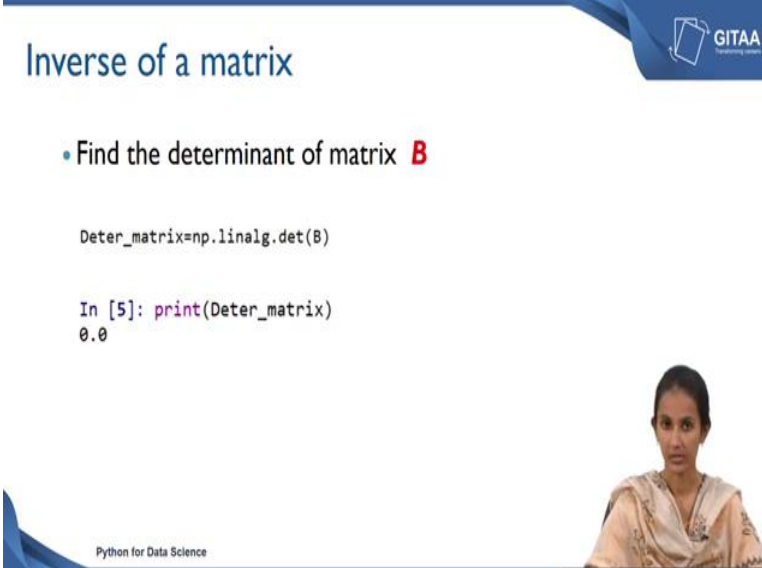


The slide is titled "Inverse of a matrix" and features the GITAA logo in the top right corner. It shows the code `inverse_matrix=np.linalg.inv(B)`. Below this, a Jupyter Notebook cell shows the execution of `In [3]: inverse_matrix=np.linalg.inv(B)`, which results in a `LinAlgError: Singular matrix`. The traceback includes the following information: `File "C:\ProgramData\Anaconda3\lib\site-packages\numpy.linalg\linalg.py", line 90, in _raise_linalgerror_singular raise LinAlgError("Singular matrix")`. A woman is visible in the bottom right corner of the slide. The footer includes "Python for Data Science" and the number "7".

Now, let us calculate the inverse. So, the command is `np.linalg.inv` and inside the parenthesis here I am specifying the matrix B. Now, let us print the value of the inverse.

So, it has thrown a some error saying linalg error and it is singular matrix; its showing some error. So, we need to find what is that error right. So, the singular matrix in sense the determinant will be 0. So, if the determinant is 0 inverse is does not exist. So, we will not be able to find the inverse.

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The slide is titled "Inverse of a matrix" and features the GITAA logo in the top right corner. It contains a bullet point: "• Find the determinant of matrix **B**". Below this, a code snippet is shown: `Deter_matrix=np.linalg.det(B)` followed by `In [5]: print(Deter_matrix)` and the output `0.0`. At the bottom left, it says "Python for Data Science". A woman is visible in the bottom right corner of the slide.

So, let us look at the determinant for this matrix. So, `np.linalg.det` which is a command for the determinant and inside the parenthesis I am specifying the matrix **B**. So, now, let us print the determinant value, the determinant value is 0 for in this case. So, that determinant is 0; so, the matrix will be a singular matrix condition to find the inverse is. So, determinant should not be equal to 0.

(Refer Slide Time: 07:14)

The slide is titled "System of linear equations" and features the GITAA logo in the top right corner. It contains the following content:

- Consider a system of linear equations
$$\begin{aligned}3x + y + 2z &= 2 \\3x + 2y + 5z &= -1 \\6x + 7y + 8z &= 3\end{aligned}$$
- Now we can write the equations in the form of $Ax=b$
$$\begin{aligned}3x + y + 2z &= 2 \\3x + 2y + 5z &= -1 \\6x + 7y + 8z &= 3\end{aligned} \quad \rightarrow \quad \begin{pmatrix} 3 & 1 & 2 \\ 4 & 2 & 5 \\ 6 & 7 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

A **x** **b**

$x = A^{-1} b$

Python for Data Science 9

Now, let us look at the system of linear equation. So, if you have 2 or more equation linear equation then it is called as a system of linear equations. So, if you solve 2 or more equations, you can get the unique solution or there might be a no solutions or are there might be a infinitely many solutions. So, let us see how to do it in python. So, we will consider a system of linear equations. So, in our case taken 3 equations which is $3x + y + 2z = 2$ and the second equation is $3x + 2y + 5z = -1$. Third equation is $6x + 7y + 8z = 3$.

So, these are the 3 equations, so we need to solve these 3 equations to find the value of the x, y, z . So, which are the basically the unknowns; so, we will write it in the form of $Ax = B$. So, these are the 3 equations now will write it in a x coefficients for x is 3 and respectively the coefficient for y is one and respectively the coefficients for z is 2 so, that has been written in the first row which is the values are 3 1 2. Similarly, you have to write the coefficients for the x, y, z for the second row and similarly for the third row.


So, A are the coefficient values and x, y, z are the unknowns and b are the constant values which is 2 - one and three. So, now, we have written the equations in the format of $ax = b$ now we will solve. So, we will keep the x on this side and will bring the A 2 on the other side so that becomes A inverse of b . So, we need to solve A inverse b ; so, we can find the A inverse and then you can multiply with B or else there is a direct command which does the a inverse into B .

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System of linear equations

- `numpy.linalg.solve()` - return the solution to the system $Ax=b$
- Syntax: `numpy.linalg.solve(matrix_A, matrix_b)`
- Create matrix **A** and **b**

```
A=np.matrix("3,1,2;3,2,5;6,7,8")  
b=np.matrix("2,-1,3").transpose()
```



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So, in python `numpy.linalg.solve` it basically return the solutions for the system of equations on the format of $Ax = b$. The Syntax is `numpy.linalg.solve` it basically solves the 2 matrices which is matrix, we have to specify the matrix A and the matrix b. Now, let us create a matrix A and b. So, A is 3 cross 3 matrix, so we will supply the values. So, after the end of the each row you have to specify the semicolon. Similarly you have to create a matrix b; in our case so, we had values 2, - 1 and 3 which is along the columns, which means we had 3 rows and 1 column so, that is why we have used the transpose to create a matrix b.


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System of linear equations

- Print matrix **A** and **b**

```
In [66]: print(A)  
[[3 1 2]  
 [3 2 5]  
 [6 7 8]]  
  
sol_linear=np.linalg.solve(A,b)
```

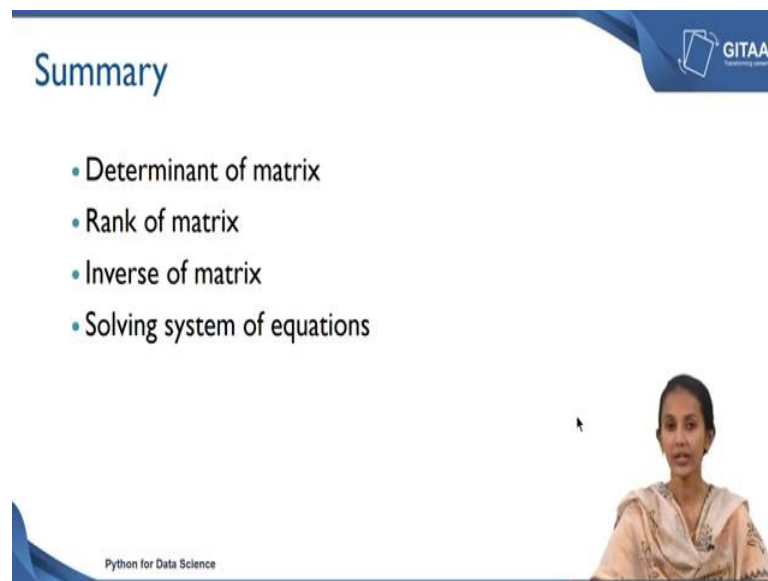
```
In [68]: print(b)  
[[ 2]  
 [-1]  
 [ 3]]  
  
In [27]: print(sol_linear)  
[[ 1.24242424]  
 [ 0.81818182]  
 [-1.27272727]]
```



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So, now let us we will print the matrix A and b which we were created. So, the A matrix will be a 3 cross 3 matrix and the b matrix will be a 3 cross 1 matrix. So, the command is `np.linalg.solve` and inside the parenthesis we have to specify the matrix which we have created earlier. So, we have created a A matrix and the b matrix; and we can store it in a variable. Now, let us print the values. So, when you print `sol_linear`. So, it has printed these 3 values. So, these are the values for the x y and z.

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The slide features a blue header with the word "Summary" in white. To the right of the header is a logo for "GITAA" with the tagline "Empowering women". Below the header is a bulleted list of four items: "Determinant of matrix", "Rank of matrix", "Inverse of matrix", and "Solving system of equations". In the bottom right corner, there is a video inset showing a woman with dark hair, wearing a light-colored patterned top, looking towards the camera. The text "Python for Data Science" is visible in the bottom left corner of the slide.

Let us summarize. So, first we saw how to calculate the determinant of matrix and we also saw how to calculate ranks. So, rank it basically gives the number of linearly independent rows or columns. And we also saw how to calculate the inverse. So, $A^{-1} = \frac{1}{\det(A)} * adj(A)$ and then we also saw of how to solve the system of equations.

So, system of equation sense $Ax = B$. So, you have to keep the x on one side. And you have to take the A on the other side. So, it becomes $x = A$ inverse b and then you can get the values for x y and z.

Thank you.