

Micro Foundations of Macroeconomics
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Lecture – 08
Two Period Model III

Hi everybody, we have started the topics in micro-foundations of macroeconomics and we are covering the two-period model and in two-period models, we have also covered some aspects of the basic foundations of the two-period model, where we define the utility preference of the representative consumer and we also worked out with the budget constraint.

And compared to one period model, the two-period model budget constraint looks even more appealing, because we are not just taking into account the current period but we are also taking into account the future period. I had also given in the last class some examples, For example, we discussed how we can derive the budget constraint from the 2 period expositions.

And all we have tried to understand what happens when we try to optimize the consumption of this representative consumer. are we going to optimize with the simple norm of calculus? Since it involves certain time periods, here we are also taking into account the time period during optimization, and then, we derive the dynamic optimization condition, the other condition.

We also discussed in detail, how we can now further apply this technique, to understand the consumption behavior of the consumer, not only in the current period but also in the future period. In the last session, we also derived the saving function with the help of the two-period budget constraint, and we also try to understand how this saving function can be derived.

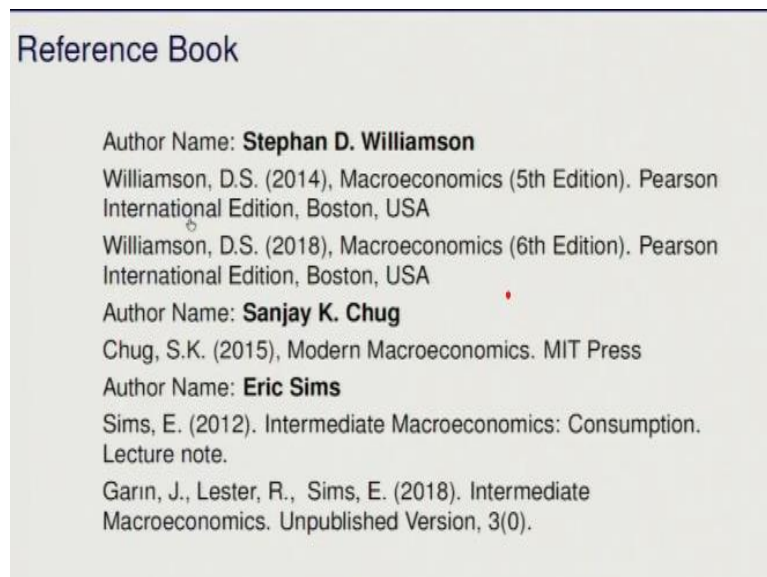
saving is basically income minus consumption. I hope, it is clear to all of you. And now we will be moving ahead with the new dimension. I remember in the last session we also explored to some extent the macro dimensions of the consumption. What do you mean by macro dimensions? I am talking about macro not micro. when I talk about macro dimensions of consumption.

Then the Keynesian concept like marginal propensity to consume and how we measure the change in consumption due to change in income in either a current period or future period.

What happens when we try to adjust the current period consumption, with respect to the interest rate? these dynamics are also important. Because, by these dynamics, you can easily understand the behavior of representative agents.

In the one-period model, it was easier to examine. We will be discussing it further with that intention in mind. Let us start.

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Reference Book

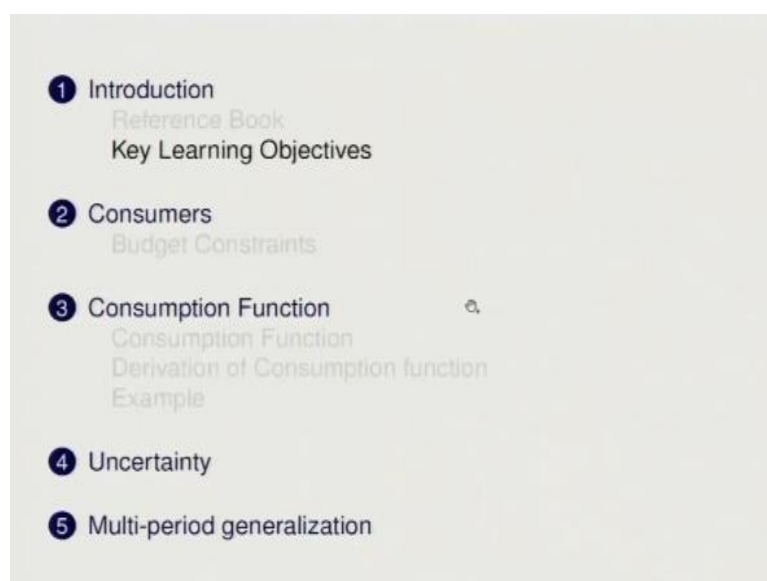
Author Name: **Stephan D. Williamson**
Williamson, D.S. (2014), Macroeconomics (5th Edition). Pearson International Edition, Boston, USA
Williamson, D.S. (2018), Macroeconomics (6th Edition). Pearson International Edition, Boston, USA

Author Name: **Sanjay K. Chug**
Chug, S.K. (2015), Modern Macroeconomics. MIT Press

Author Name: **Eric Sims**
Sims, E. (2012). Intermediate Macroeconomics: Consumption. Lecture note.
Garrin, J., Lester, R., Sims, E. (2018). Intermediate Macroeconomics. Unpublished Version, 3(0).

Here the reference remains the same. I would request all of you to follow the Stephan D. Williamson book and the Eric Sims, because these two authors are important. And for two periods Sanjay K. Chug, he also has given very good technical analysis.

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Key Learning Objectives

- Setting-up the two-period model and how two-period model helps understand the consumption behaviour in the inter-temporal framework.
- The process of consumption smoothing and how consumer adjusts his/her consumption within two period dimension.
- The optimization condition in the two-period model.

Now, coming to the aspects, we had derived the budget constraints in this way.

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Budget Constraints

- The consumer's current-period budget constraint:

$$C_t + s_t = y_t - t_t \quad (2.1)$$

- The consumer's future-period budget constraint:

$$C_{t+1} = y_{t+1} - t_{t+1} + (1 + r_t)s_t \quad (2.2)$$

Simplify

- Solve for s in (2)

$$s_t = \frac{C_{t+1} - y_{t+1} + t_{t+1}}{1 + r_t} \quad (2.3)$$

And then, we had gone by having the tangency condition.

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Consumption Function

- The tangency condition or Euler equation is not a consumption function.
- The Euler equation is a condition between current and future consumption.
- A consumption function expresses current consumption as a function of income, the interest rate and parameters.
- Given our set-up, we are working on three variables: current consumption, future consumption and interest rate.

Here is our budget constraint. Here was the saving function that we had derived. In the last example that we did now, we were talking about the consumption function. Euler condition that we derive that your marginal utility of current consumption is equivalent to the beta multiplied by the marginal utility of future consumption.

And then you have $1 + r$, the reward for future consumption. Both are equal and consumers can decide but that will not be regarded as a macro indicator or the other equation cannot become the consumption function.

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Consumption Function

$$c_t + \frac{c_{t+1}}{1 + r_t(1 - \tau)} = y_t + \frac{y_{t+1}}{1 + r_t(1 - \tau)}$$

$$c_t = C(y_t, y_{t+1}, r_t)$$

where C is a function which maps income and interest rate into consumption.

Certain points:

- $\frac{\partial c_t}{\partial y_t} > 0$ and $\frac{\partial c_t}{\partial y_{t+1}} > 0$, consumption is increasing in both the periods.
- $\frac{\partial c_t}{\partial r_t} > 0$ is theoretically ambiguous (discussed later)

For the consumption function, we have also derived to some extent, this particular part. We had defined the consumption function with respect to y_t , y_{t+1} and r_t . And here y_t means that the consumption depends upon the current income, consumption depends upon the future

income and consumption depends upon the rate of interest. Once we have the function, then, here we have the income and the interest that we try to measure.

Here if you take into account $\frac{\partial c_t}{\partial y_t}$ means rate of change in current consumption, due to change in y_t rate of change in the future and current consumption. Due to a change in future income, you expect that future income is going to increase. So based on that you will try to adjust the current consumption. Similarly, the reward in interest that we are having.

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Derivation of Consumption function

- Suppose we have log utility function $u(c) = \ln c$ with given two-period budget constraint:

$$c_t + \frac{c_{t+1}}{1+r_t} = y_t + \frac{y_{t+1}}{1+r_t} \quad (3.1)$$
- F.O.C. is $\frac{c_{t+1}}{c_t} = \beta(1+r_t)$ and hence

$$c_{t+1} = \beta(1+r_t)c_t \quad (3.2)$$
- Now we plug the c_{t+1} in budget constraint (4.1)

$$c_t + \frac{\beta(1+r_t)c_t}{1+r_t} = y_t + \frac{y_{t+1}}{1+r_t} \quad (3.3)$$
- Now we can simplify and get c_t

$$c_t = \frac{1}{1+\beta}y_t + \frac{y_{t+1}}{(1+\beta)(1+r_t)} \quad (3.4)$$

Once, I have these dimensions then I can get this analysis.

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Derivation of Consumption function

$$c_t = \frac{1}{1+\beta}y_t + \frac{y_{t+1}}{(1+\beta)(1+r_t)}$$

We work now on partial derivatives:

- $\frac{\partial c_t}{\partial y_t} = \frac{1}{1+\beta} > 0$, $\beta \uparrow, \downarrow$ MPC \equiv more patient household
- $\frac{\partial c_t}{\partial y_{t+1}} = \frac{1}{(1+\beta)(1+r_t)} > 0$
- $\frac{\partial c_t}{\partial r_t} = -\frac{1}{(1+\beta)(1+r_t)^2}y_{t+1} < 0$

I think we had done this part and we had derived the consumption function. Now, this particular expression became very important because of the behavioral coefficient that we have so here if we just try to see.

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Derivation of Consumption function

- Suppose we have log utility function $u(c) = \ln c$ with given two-period budget constraint:

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$$c_t + \frac{\beta(1+r_t)c_t}{1+r_t} = y_t + \frac{y_{t+1}}{1+r_t} \quad (3.3)$$
- Now we can simplify and get c_t

$$c_t = \frac{1}{1+\beta}y_t + \frac{y_{t+1}}{(1+\beta)(1+r_t)} \quad (3.4)$$

If we substitute in the lifetime budget constraint that we derived, if you substitute the other condition here then, we ultimately arrive at it on this platform. We get this expression.

$$C_t = \frac{1}{1+\beta}y_t + \frac{y_{t+1}}{(1+\beta)(1+r_t)}$$

Once I have this derivation then it becomes interesting.

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Derivation of Consumption function

$$c_t = \frac{1}{1+\beta}y_t + \frac{y_{t+1}}{(1+\beta)(1+r_t)}$$

We work now on partial derivatives:

- $\frac{\partial c_t}{\partial y_t} = \frac{1}{1+\beta} > 0$, $\beta \uparrow, \downarrow$ MPC \equiv more patient household
- $\frac{\partial c_t}{\partial y_{t+1}} = \frac{1}{(1+\beta)(1+r_t)} > 0$
- $\frac{\partial c_t}{\partial r_t} = -\frac{1}{(1+\beta)(1+r_t)^2}y_{t+1} < 0$

Here you have the current consumption. The rate of change in current consumption due to change in income, it becomes $\frac{1}{1+\beta}$ and once I have $\frac{1}{1+\beta}$ then, since the value is greater than 0 which means that if we have a β , if the value of β increases your MPC comes down.

This means that your current consumption will come down and you have more derive for the future. Here this is what it leads to. In normal textbooks, if you are reading Mankiw or Dornbusch Fischer or any book, you simply derive the rate of change in consumption due to change in income and then you also have the linear chart and then you mentioned that the slope is this.

But with two-period dimensions, you can add dimensions to this understanding, which means that since we do not talk in detail about the inter-temporal behavior of the representative agent. We just mentioned that this is the change in consumption due to a change in income. But here we are trying to add the dimension of the inter-temporal. This inter-temporal dimension gives you a better understanding of the MPC.

And that is how marginal propensity to consume if you try to put it in the framework of to create model or you can also extend it to an infinite period, how it looks like that becomes the beauty of this course and that you can enhance your understanding about basic theories of macroeconomics with simple mathematical formulations.

Once we have beta increasing your MPC comes down. And once beta is lower then, MPC goes up. This is the understanding which means that if you give less preference to the future, then the current period becomes more important. Similarly, if I go for partial differentiation of C_t w.r.t. y_{t+1} here it becomes $\frac{1}{(1+\beta)(1+r_t)}$

This is what we have here. Then we have $\frac{\partial C_t}{\partial y_t}$. Here we have,

$$\frac{y_{t+1}}{(1 + \beta)(1 + r_t)^2}$$

This is a solution that you get and this becomes an interesting scenario.

And with rate of interest you have inverse relationship of current consumption, with respect to change in current rate of interest which means that if you have interest rate change. If interest

rate is high, we look for less consumptions and because you tend to save more to get more reward. Maybe in the future period you will have better reward. If the rate of interest in the current period is higher you tend to save more.

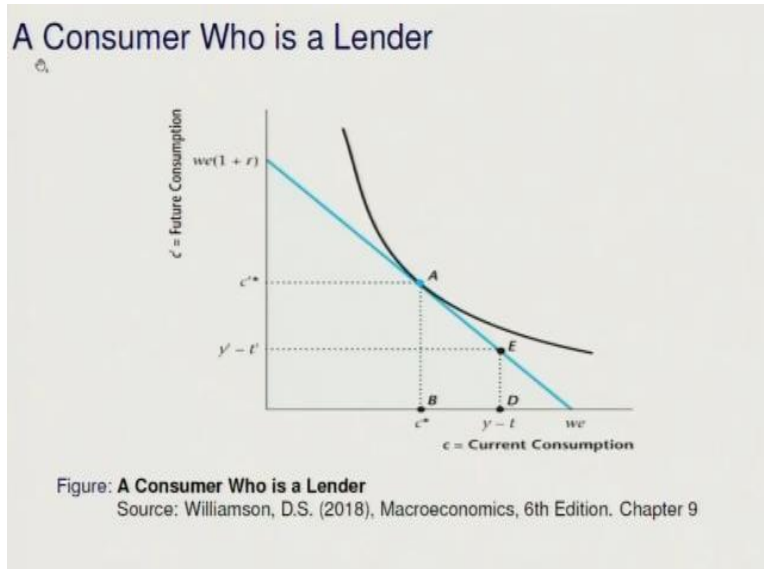
And this higher reward is giving you an incentive or some kind of opportunity cost for you to either go for more consumption in the current period or save more in the current period and then you consume more in the future. Here it is clear that for consumption the current period income and the future period incomes have a reinforcing effect which means that, these two are having direct relationship.

Whereas with inverse relationship this interest rate matters a lot. Once we discuss more about whether you should go for more of consumption or less of savings or you decide about more of saving or less of consumption. This analysis becomes important when you are trying to see the scenarios where you have increased rate of interest. Whether this is also having the dimension that we discussed in the first one period model.

It is where we analyze in detail, the role of substitution and income effects. Once we have role of sufficient income effects playing a very important role, then these dimensions will come again. But, the major attraction of this particular understanding is that, it helps you to understand in a much deeper way, your basic macroeconomics which will further provide new dimensions to understand the certain theories.

Also, certain aspects of permanent income, relative income and all those theories will be coming up.

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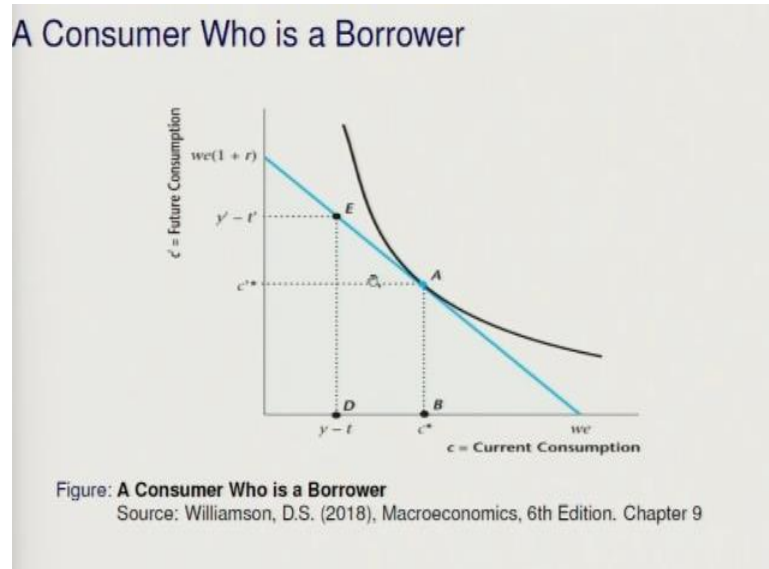


Now, here we have a consumer who is a lender. I am saying a lender which means that he is having income but tries to save more. So here, the condition is that the endowment point is that his income is OD, he is saving BD.

we is the budget line, this is your indifference curve. At point A here, you are consuming only C^* and C^* . Your income is y_t and y'_t which means that this amount of income that you are saving, you can transfer that to future consumption and you will be you can see that the future consumption increasing by a larger amount and this is what it leads to.

For a lender it becomes easier. In this case it becomes easier to understand if you are just focusing on dominant points. This is a current period income and the future period income and this is the consumption that he has in a current period and future period. With this adjustment it plays a very important role.

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Now, here we have a consumer who is a borrower. I think we also had focused on this aspect in the last session where we had mentioned that if he is having the income of E, so here it is $y' - t'$. this is the future consumption and here is the current income. He is consuming this much which means that you are having only an income of 100 rupees, but you are consuming 200 rupees.

The 100 rupees extra that you are consuming it covers here. Here it is $Y - T$ and C^* that you have. At point A what do we infer? We infer that, at this point the consumer is a borrower. He is borrowing in the current period and then he has to pay back in the future period. This is the payback period and this is the borrowing period. This is what the analysis looks like here.

This will be the amount that he is having in the future period, and this is the amount he is going for consumption in the current period.

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An Increase in Current Income for the Consumer

- Current and future consumption increase.
- Saving increases.
- The consumer acts to smooth consumption over time.

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Now, what happens when we have the increase in current income for the consumer? Here we can see that when current and future consumptions increase, saving increases, the consumer acts too. One of the beauties of this particular analysis is that here we have the consumer. We are trying to specify the strategies of the consumer. How this particular consumer will be smoothing out the consumption.

This means that, in the given economic scenario that we are facing whether you have the future period income higher or the current period income lower, it will have the impact. Here the consumer acts to smooth consumption over time.

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The Effects of an Increase in Current Income for a Lender

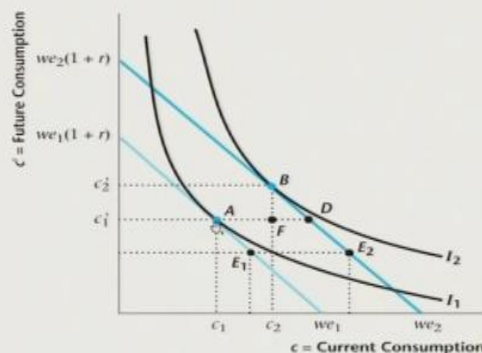


Figure: **The Effects of an Increase in Current Income for a Lender**
Source: Williamson, D.S. (2018), *Macroeconomics*, 6th Edition. Chapter 9

Now, the effects of increasing current income for a lender. So here, this is what we have for the lender. If the current income of the lender is increasing, we can see that this is the original

budget line of the representative consumer. This is the first endowment that he has. How much is the income? They see the income this is consumption which means that he is saving some amount and transferring it to the future.

If this amount is going to increase then he of course would like to move up. Now, his endowments since the current period income increasing his endowment moves from E_1 to E_2 . Now at point E_2 you can see that this representative consumer is now having the consumption at point B and his consumption is increasing. He was earlier at E_1 consuming C_1 but now he is at E_2 consuming C_2 .

This means that this much amount of money he is saving and can be transferred to future consumption. Once I have these two dimensions into account then it plays a very important role. Because there you have the role of current and future. But here we can see that these two lines we are drawing. Here this line is coming and then again, we have the parallel move which means that the slope of the budget constraint remains same.

$1 + r$ remains same in the current and the future period. This is what it means about. If you have the current period income increasing the consumption increases in both the current period and the future period. This is partly because of the income effect that this representative consumer is seeing and we are not seeing any change in the interest rate. If we can infer any change in interest rate then it will play a very important role.

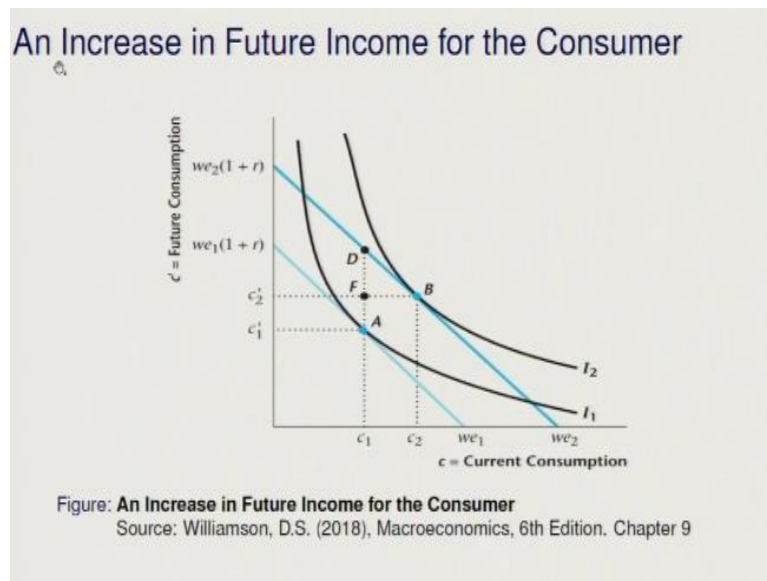
Here I hope it is clear. Here you can think about E_1 . E_1 is consuming only this much which means that this amount is being saved. He is lender so he can go for, if E is seen increased so this will further be repeated in the future period. Here it is getting repeated. Here is at point E_2 and then here we have the consumption by this much amount. The amount of the consumption that you have here.

This will have the effect on the future. Over-all the conclusion is that because of this current period income increase, since the slope of the budget line remains same, no change in the budget line slope. Here you have the consumption moving from A to B. This analysis makes the overall understanding better about the current and future income.

Any increase in future income what do you do when you go for increasing the future income? You have the ability that suppose, you are studying in your college and you are being placed. You are in your last semester and you have got the opportunity you know that, once you start your job you are going to see an increase in income.

Because whatever income that you have at present it is going to add value. This is going to have the impact here.

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An increase in future income for the consumer. Here it is. Here once I am mentioning about you can see that this representative consumer will also have this similar movement. Because, the budget line does not change its slope. Once the budget line is not changing its slope then it is obvious that this representative consumer will be having the increase in both period consumption. This is how it looks like.

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The Effects of an Increase in Current Income for a Lender

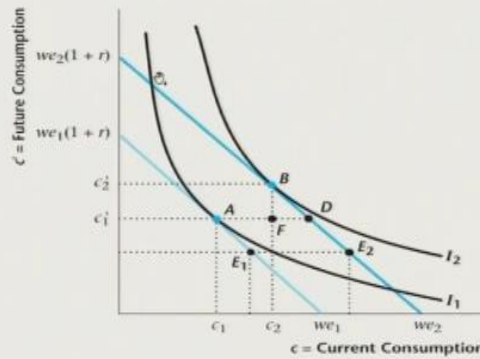


Figure: The Effects of an Increase in Current Income for a Lender
Source: Williamson, D.S. (2018), Macroeconomics, 6th Edition. Chapter 9

In both cases what happens is that whether you talk about the increase in current income or you talk about the increase of future income in both cases you have both current and future period income increasing? Current and future period income increasing as a result you have both current consumption and future consumption rising.

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Temporary and Permanent Increases in Income

- As a permanent increase in income will have a larger effect on lifetime wealth than a temporary increase, there will be a larger effect on current consumption.
- A consumer will tend to save most of a purely temporary income increase.

Now, in most of the macro textbooks you read about consumption theory. There you have the permanent income hypothesis by Modigliani and this permanent income hypothesis mentioned about how the representative consumer will decide about whether his current or the future income or consumption is increasing or decreasing. If you are talking about the temporary and permanent income, what do we mean by temporary and permanent income?

Temporary income could be in the form of some kind of short-term a very short-term gain. Maybe it could be that you won a lottery. Maybe it could be that the government has given you a tax relief in a budget year. So maybe this year budget if it is going to be announced and you get the tax relief then it is not guaranteed that this will continue even next year.

A next year budget may continue the same kind of relief measures.

That we were experiencing or going to experience in the coming budget. If you are thinking in that direction then the basic understanding is that given these two situations that we have that Representative agent is going to get income in the current or for the very short-term period. And the permanent income means that this particular representative agent is employed and so maybe 10, 15, 20 years whatever is the retirement age.

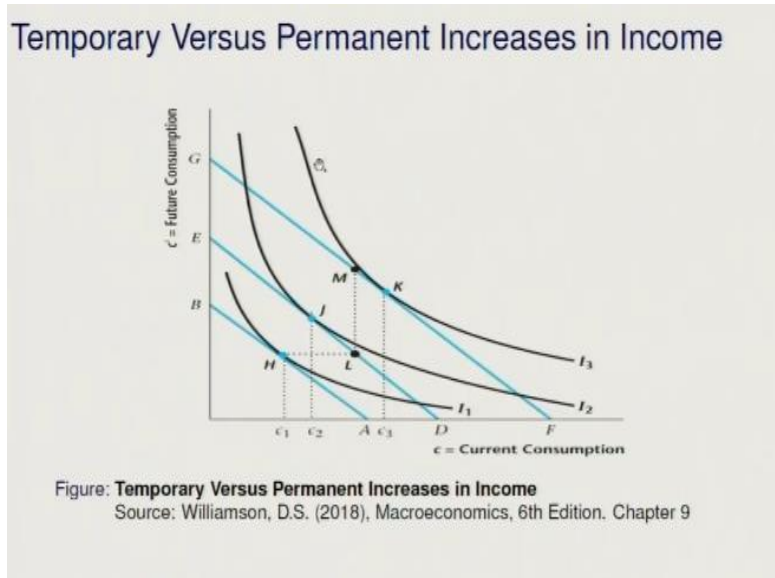
He starts his journey at the age of 25. And then his retirement age is 60. Whatever income he is going to get during this period is called the permanent income. The temporary income as you mentioned I hope it is clear that, what do you mean by temporary and permanent? Temporary permanent incomes also have implications on the current and future consumption.

And the concept that we are going to see is as we mentioned the smoothing of consumption. once I am mentioning about smoothing of consumption and this leads to how the representative agent is going to decide about the consumption in current and future period given these situations about the income. Let us talk about that. An increase in income will have a larger effect on the lifetime wealth than a temporary increase.

And there will be a larger effect on current consumption. this is what we experienced. A consumer will tend to save most of the purely temporary increase while we are arguing on these 2 lines because as I mentioned about the budget, everyone has some kind of learning experience. So based on the past and this is also linked to some extent with the taxation strategy that we will be discussing after this which is called the Ricardian equivalence.

We always try to smooth out the individual strike to smooth out the burden of tax by adjusting the current period and future period consumption. And in most cases it has been examined and it has been proved by most of the researchers that temporary increase in income is being saved in most of the cases either you take one or two.

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This is the case that we deal now. Let us see whether we can understand in a much better context. Here it is. If we are going to talk about the current period let's suppose the individual is here, he is also going for the increase in the period here. If I am going to see the increase in the current period consumption. For example here we see this is what he consumes right?

If he is at this point consuming this much because of this, the temporary increase in his consumption level mostly from E to D. But here you can see that the change in this income that is the current period consumption that we are seeing. It is less than the future period consumption, which means that the individual to some extent tries to save some amounts. Here you can see this particular gap is much larger than this.

But if you think about the permanent increase then the permanent increase, we are seeing is consistent. Here we are seeing that the current period, the consumption what we are inferring here the gap is much smaller than here then this shows that some amount of the income that he has received as a temporary increase in income has been saved in the current period and it is being used in the future.

So that is what we are trying to understand with the temporary and permanent income. Temporary income increase will have the consumption increase but it will not be as much as we are seeing at point K. At point K we are seeing that it is not increasing that much. The consumption has increased substantially. The future income is also increased. So here in both cases we are seeing the increase.

And we can see that from C_2 to C_3 the jump is much larger compared to what we see here. We can easily say that from C_1 to C_3 the impact is more or less smooth. The same amount is going but when we are seeing the temporary increase then, this temporary increase is having a lot of, role to play. This temporary increase means that, some amount of this income that this representative agent is going to have.

It is being transferred to the future period. So overall, what it means is that the representative agent is also rational. This representative agent whether he or she knows about the movement how to smooth out consumption. In most cases that is why governments do not go for promoting a lottery win or any kind of lottery system because they know that, this is not going to give a permanent boost in income.

Otherwise, instead of creating employment governments would have gone for a simple lottery system that if won the lottery, it is more than enough for you to survive. They go for the creation of employment. Because the creation of employment creates permanent income kind of scenarios. And this is having a more positive impact on the economy compared to a temporary one.

Similarly, in the policy setup also when government goes for decreasing the tax then, you should be ready that in the future you are going to pay a higher tax. Because the government will simply transfer the wealth from current period to future period or from the future to the further next period, if we are going to see the more than two periods scenario. There it helps.

The government will also go for adjustment in the tax structure, adjustment in the income generating sources, in the same way that we have for all the other schemes. I hope it is clear to all of you. In the next session we will be also discussing about, how certain dimensions of the representative consumer is going to work more in favor to the current and future period consumptions.

How we can decide further about the changes in such framework. What happens when we are going to see the change in the interest rate? How the interest rate change is going to play an important role? Can we also add the dimension of borrower or lender? Then, we will see that how we can add the dimension of uncertainty. We can also see that, how we can generalize the budget constraint.

In a more general context, how we can extend the two-period consumption into finance. So those things are important to note and I am stopping it here. Thank you. Thank you so much.