Electrical Equipment and Machines: Finite Elements Analysis

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Lecture No 40

Computation of Forces using Virtual Work Method

Welcome to the last lecture of this course - L40. In this lecture, we will study theory related to virtual work method and compare the results of this method with the MST based force calculation and with an analytical formula.

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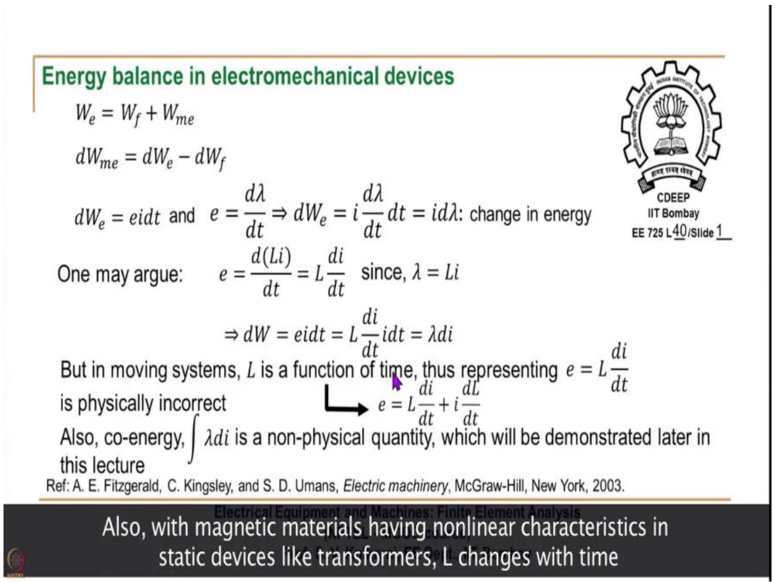

Let us understand the energy balance in electromechanical devices, which is defined by the following equation.

$$W_e = W_f + W_{me}$$

From the above equation, one can say that electrical energy (W_e) fed to any electromechanical device like a rotating machine is equal to field energy (W_f) plus mechanical energy (W_{me}). The model will be more accurate if the losses in the corresponding three components are modelled in the above equation.

So, we can write the change in mechanical energy (dW_{me}) will be equal to the change in electrical energy (dW_e) minus change in field energy$\left(dW_f\right)$. Now, we can write $dW_e = eidt$. Here, e is the voltage, i is the current, ei is the power input, and power multiplied with time $(eidt)$ is energy. We also know that e is the emf. So, induced emf is nothing but the rate of change of flux linkages $\left(\frac{d\lambda}{dt}\right)$ and then we can write

$$dW_e = i\frac{d\lambda}{dt}dt = id\lambda$$

So, $dW_e = id\lambda$ gives us the change in electrical energy.

But one may argue that $e = \frac{d}{dt}(Li)$ and the induced emf can be written as $L\frac{di}{dt}$. If we express $e = L\frac{di}{dt}$ then we are assuming that L is constant and this expression is obtained by substituting $\lambda = Li$.

So, that means we are getting two expressions of change in energy, one is $dW_e = id\lambda$ and we get another expression as $dW_e = \lambda di$. Now, let us understand what is λdi ? However, we should keep in mind that in moving systems L is a function of time and thus representing e as $L\frac{di}{dt}$ is physically incorrect because L changes with time in a typical rotating machine.

Also, λdi is called as co-energy and we will see more about this little later. In basics of electromagnetic we had seen, i corresponds to H and λ corresponds to B. So, $id\lambda$ is nothing but HdB. So, HdB corresponds to the change in energy density and λdi is representing BdH.

So, BdH or λdi is the change in co-energy density. If dH or di is considered, then the quantity is called co-energy. Also, it should be remembered that co-energy is a non-physical quantity which will be demonstrated later in this lecture.

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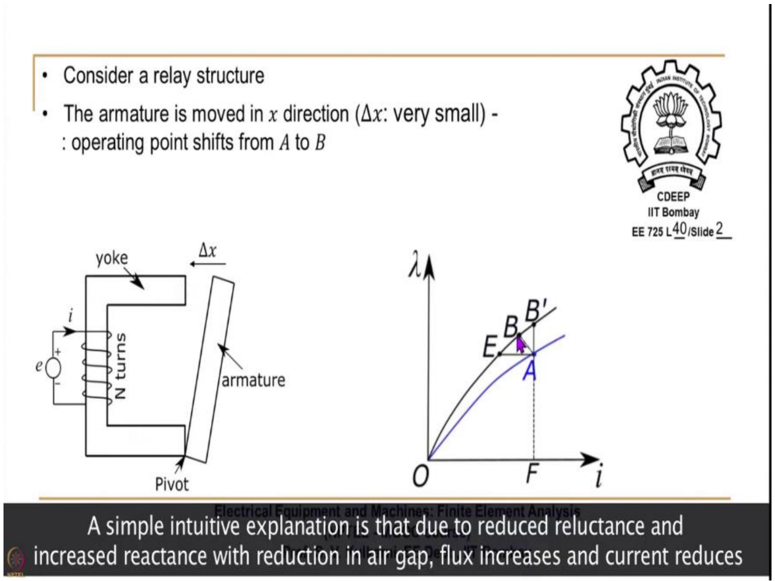
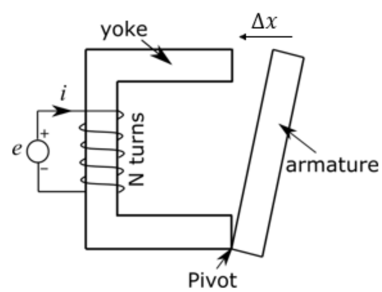
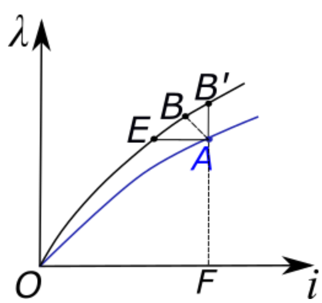


Now, consider a relay structure shown in the following figure with a hinged armature supported by a pivot.



Once the current is passed through the coil the armature will get attracted. Suppose if the armature is moved by a small distance Δx in the x direction (indicated in the above figure) then the operating point in the following characteristics would shift from A to B.



So, initially, if the characteristic curve OA corresponds to a particular gap and after that if the armature is moved by Δx then the operating point shifts on the curve $OEBB'$ through the line AB.

The curve has shifted as shown in the above figure then the value of reluctance will go down because the length of the gap is reducing. So, the flux in the circuit will increase and the operating point shifts from A to B and the value of current to establish this flux will go down. We have two options to simplify the force calculations. In the first option, we can assume flux as constant, that means we approximate AB as AE. So, the shift of characteristics through line AE will be constant flux condition. When the operating point is shifted through AB' then we can approximate the case as constant current condition.

In both these cases, there is an approximation. In the first case we are assuming constant flux condition. So, we are neglecting the area EAB and the corresponding energy. In the second case when we are assuming constant current condition, then there is an additional area ABB' and the force calculated would be higher.

In the first case (constant flux condition), the force calculated would be lower than actual. Now, we will see the advantages and disadvantages of using both these approaches. Later we will eventually understand that constant current condition and considering the shift from as A to B' will be easier to implement and then we end up in using co-energy based force computation method.

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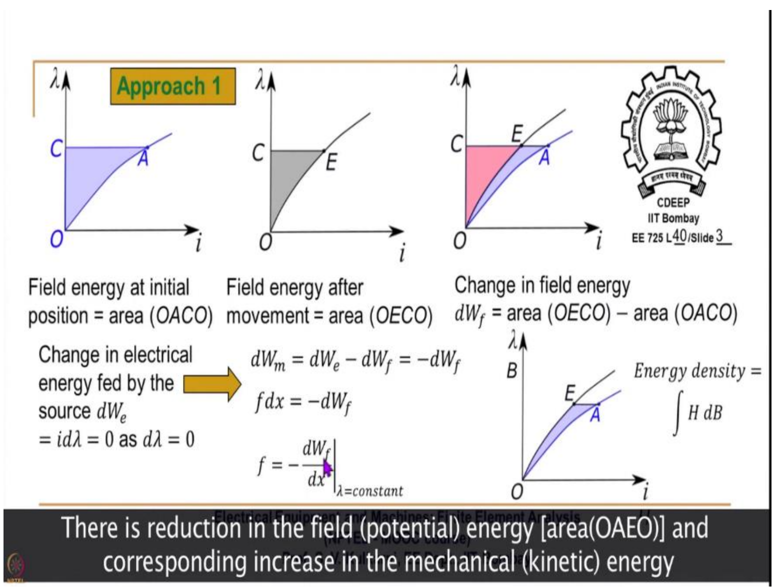
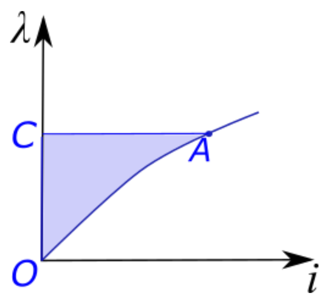
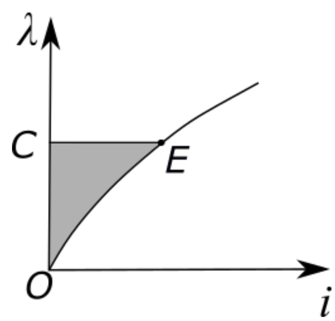


Let us see the two approaches in detail. The first approach is constant flux condition. The characteristics shown in the following figure is for the initial position of the armatures and A is the operating point of the magnetic circuit in this position.



Now, when the armature is moved, it goes to the other curve and the new operating point is E as shown in the following figure.

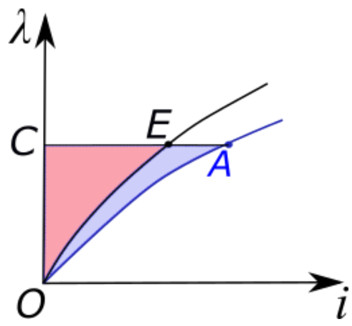


The field energy at the initial position corresponds to the area of OACO highlighted in the previous figure. Field energy after the movement of Δx is represented by OECO area. So, the change in field energy (dW_f) will be the difference in the two areas. Remember that the field energies are calculated by using $\int i d\lambda$. So, $\int i d\lambda$ is the field energy and it is equivalent to $\int HdB$ because i correspond to H and λ corresponds to B.

The $\int HdB$ would represent energy density because its unit of J/m^3 whereas the unit of $\int i d\lambda$ is Joules. The change in electrical energy fed by the source in this case is 0 because λ is not changing in the constant flux case. So, $d\lambda = 0$.

We know from the very first slide that $dW_{me} = dW_e - dW_f$. Since $dW_e = 0$ we have $dW_{me} = -dW_f$. dW_{me} is nothing but fdx. Here, f is the force and dx is the displacement. So, $fdx = -dW_f$ and then eventually we can calculate f as $-\frac{dW_f}{dx}$ with the total flux linkages held as constant.

Effectively we are calculating the difference in two energies (OAC and OEC) which is equal to OAE as shown in the following figure.



In this formulation, there is no problem from the point of basic principles. Here, $\frac{dW_f}{dx}$ is negative because W_f is reducing. So, $-\left(\frac{dW_f}{dx}\right)$ gives a positive value of force and that can be verified.

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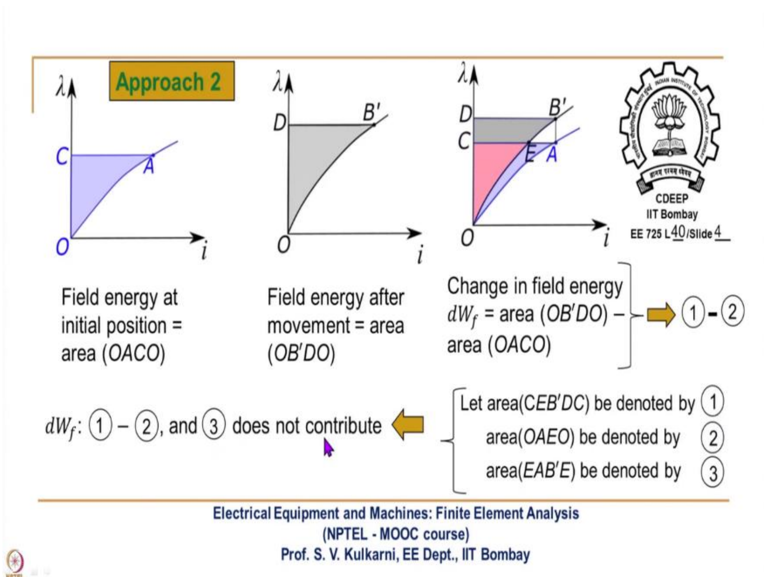
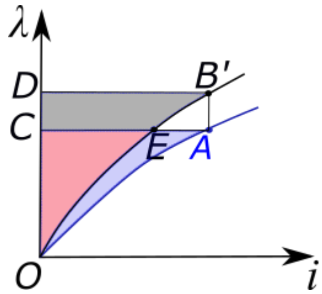


In approach 2, we are keeping current as constant. Here, the characteristics changes from AB′ as shown in the following figure.



So, at the initial position, the operating point of the magnetic circuit is A, then we went to operating point B′ after a movement of Δx. The field energy at the initial position is represented by OACO and field energy after the movement of Δx is equal to the area OB′DO. So, the change in field energy is the difference between the two areas OB′DO and OACO as indicated in the above figure.

If we represent the area CEB′DC as 1, OAEO as 2 and EAB′ as 3, then dW_f is given by

dW_f: ① − ②, and ③ does not contribute

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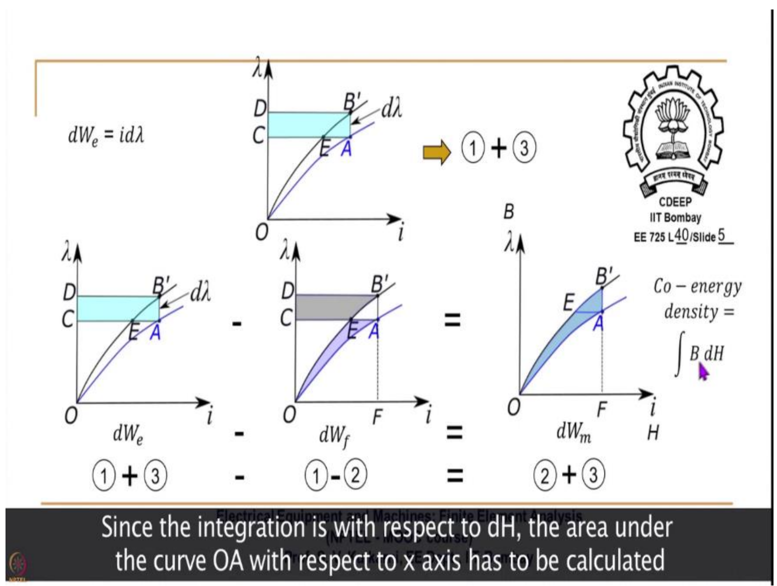
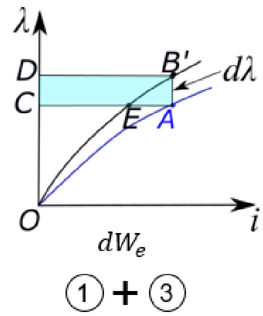
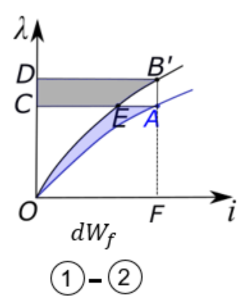


Earlier we have seen that $dW_e = id\lambda$. So, once we move from A to B′ as shown in the following figure then the electrical input has increased.

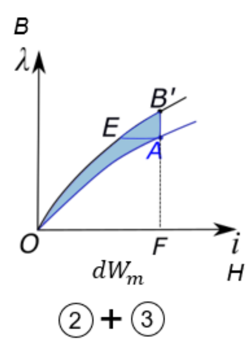


So, the area in sky blue color is representing $id\lambda$ and its value is equal to area 1 + area 3. The change in field energy dW_f is area 1– area 2 as seen in the previous slide.

In the previous case, we saw that dW_f is is equal to the change in mechanical energy. In this case, field energy dW_f is area 1 – area 2 as indicated in the following figure.



We know that change in mechanical energy $dW_m = dW_e - dW_f$ which is equal to (area 1 + area 3) – (area 1 – area 2) = area 2 + area 3 as shown in the following figure.



From the above figure, we can see that the shaded area is representing the change in co-energy. Because if we find the difference in co-energy then we will get the area shown in the above figure.

When we calculate $\int BdH$ or $\int \lambda di$ then the initial operating point was A and the corresponding area or co-energy will be the area under the curve under OA curve with H or i as the reference axis which is the initial co-energy. When the operating point is approximated to be shifted to point B′ then the area co-energy is represented by the area under the curve OEB′.

So, in the above figure, we can see that there is an increase in co-energy. That means the potential energy of the system has effectively increased which should not be the case and so the co-energy based calculation is non-physical. Because, we cannot get both co-energy and kinetic energy increasing.

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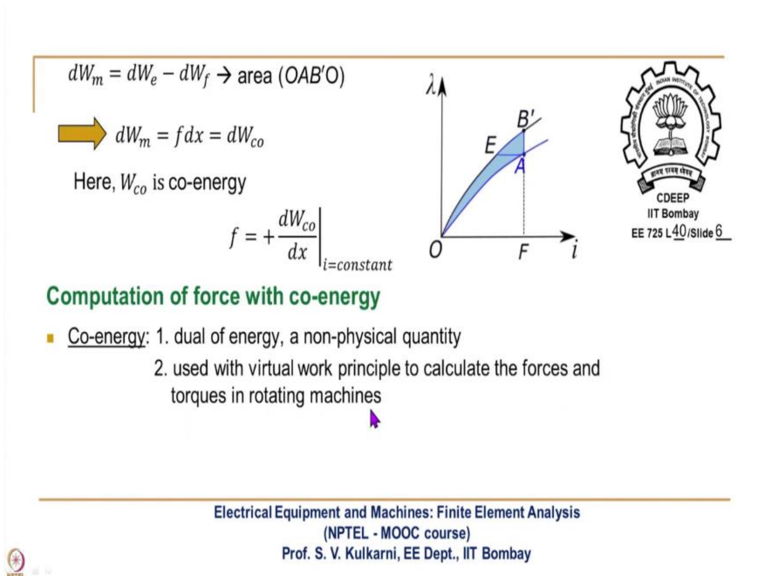

The co-energy based approach has some advantage. Change in mechanical energy is

$$dW_m = dW_e - dW_f$$

This change in mechanical energy is represented as OAB′O as indicated in the figure given in the above slide. This area is nothing but the change in co-energy and the relation between force and change in co-energy is represented as

$$f = +\frac{dW_{co}}{dx}\bigg|_{i=constant}$$

So, finally, *f* is equal to change in co-energy by considering current as constant. Also, co-energy is non-physical and it is dual of energy and it is used with virtual work principle to calculate the forces and torques in rotating machines.

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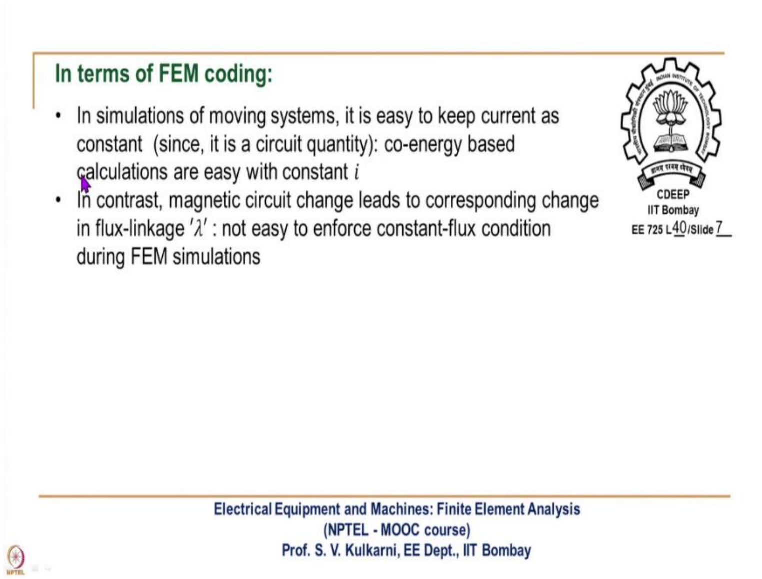

In terms of FEM coding, we will now see the advantage of using co-energy. In the simulation of moving systems, it is easy to keep current as constant because it is a circuit quantity. Therefore, co-energy based calculations are easy with the current being held constant. In contrast, magnetic circuit of a moving system changes with time, this leads to the corresponding change in flux linkage. It is not easy to to enforce constant flux condition in FEM simulations. That is why we used the co-energy based force computation method.

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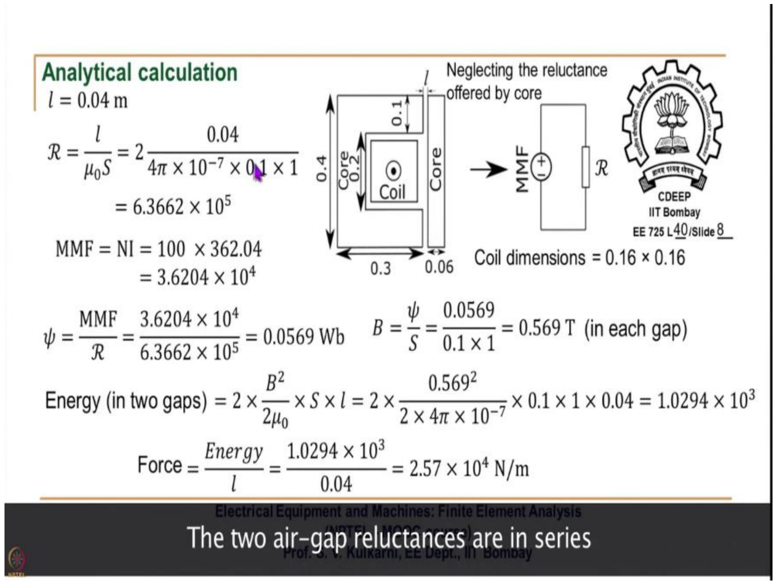
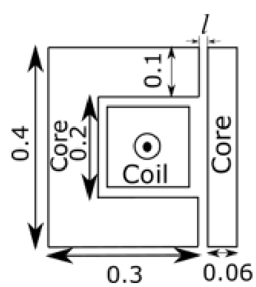


Now, to verify our virtual work based force computation method let us calculate the force analytically. In L-39, we have already calculated force using MST method. So, we will compare the forces calculated using the three methods. We will analytically calculate the expression of force because it is fairly a simple problem. The corresponding geometry given in the following figure was already described in L-39.



The magnetic circuit has a C shaped core and an armature which is hinged. The armature is free to move. The moment we pass the current through the coil the core will attract the armature. In this circuit, we have considered the length of the air gap as 0.04 m. There are two air gaps in the circuit and the total reluctance of the magnetic circuit is given by

$$\mathcal{R} = \frac{l}{\mu_0 S} = 2\frac{0.04}{4\pi \times 10^{-7} \times 0.1 \times 1} == 6.3662 \times 10^5$$

In the above equation, S is the cross-sectional area through which the flux is crossing. So, one of dimensions of the cross sectional area is 0.1 m and other dimension is 1 m depth in Z direction because we are going to calculate the force per unit length. The above figure is in xy plane that is why the value of S is 0.1×1. Then the value of MMF or Ampere turns are calculated as given below.

$$\text{MMF} = \text{NI} = 100 \times 362.04 = 3.6204 \times 10^4$$

The value of flux is calculated as

$$\psi = \frac{\text{MMF}}{\mathcal{R}} = \frac{3.6204 \times 10^4}{6.3662 \times 10^5} = 0.0569 \text{ Wb}$$

The value of flux density (B) in each gap is calculated as

$$B = \frac{\psi}{S} = \frac{0.0569}{0.1 \times 1} = 0.569 \text{ T}$$

The flux density in each of the two gaps is 0.569 T. So, energy in two gaps together is

$$\text{Energy (in two gaps)} = 2 \times \frac{B^2}{2\mu_0} \times S \times l = 2 \times \frac{0.569^2}{2 \times 4\pi \times 10^{-7}} \times 0.1 \times 1 \times 0.04 = 1.0294 \times 10^3$$

Here, $2 \times \frac{B^2}{2\mu_0}$ is the energy density in the two gaps which gives the energy when multiplied by volume. After substituting all the values, we get the energy given in the above equation. Then the force is calculated as

$$\text{Force} = \frac{Energy}{l} = \frac{1.0294 \times 10^3}{0.04} = 2.57 \times 10^4 \text{ N/m}$$

Here, we considered l as 0.04 m because we are assuming that the entire core moves in the x direction by 0.04 m. So, we are not considering any small incremental distance and calculating the force for the complete movement of 0.04 m. We get the force as 2.57×10^4 N/m. As explained earlier, we are calculating force per unit length in z direction because this is a 2D formulation.

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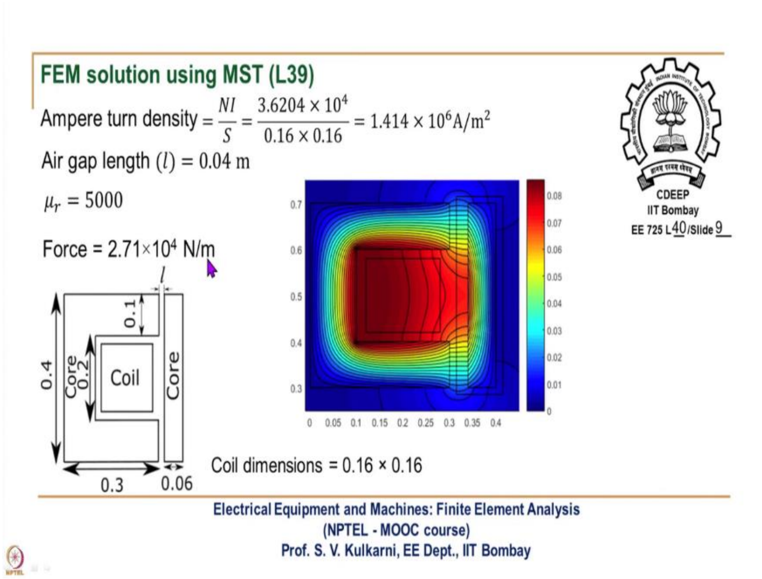

In L39, we have already calculated the force using the MST approach. In the above slide, we have shown the calculations again and the value of force calculated by using this method is 2.71×10^4 N/m.

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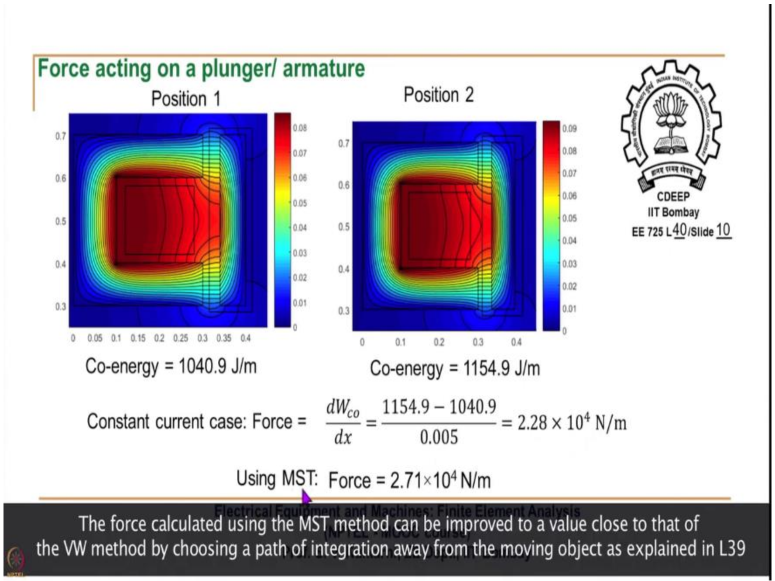

Now, we will calculate the force using the co-energy approach which is nothing but virtual work method. So, in the above slide flux plots of two FEM simulations are shown. The flux plot on the left hand side is for position 1 with higher distance and the second figure is the solution for magnetic circuit with smaller distance which is obtained after virtually displacing the armature or plunger.

Then the difference in energies for both these positions will give dW_{co}. After dividing the change in co-energy by the distance which the armature moved. Consider the distance as 5 mm. In the above slide we can see that the co-energy at position 1 is less than the co-energy in position 2. We can see that the value of co-energy has increased.

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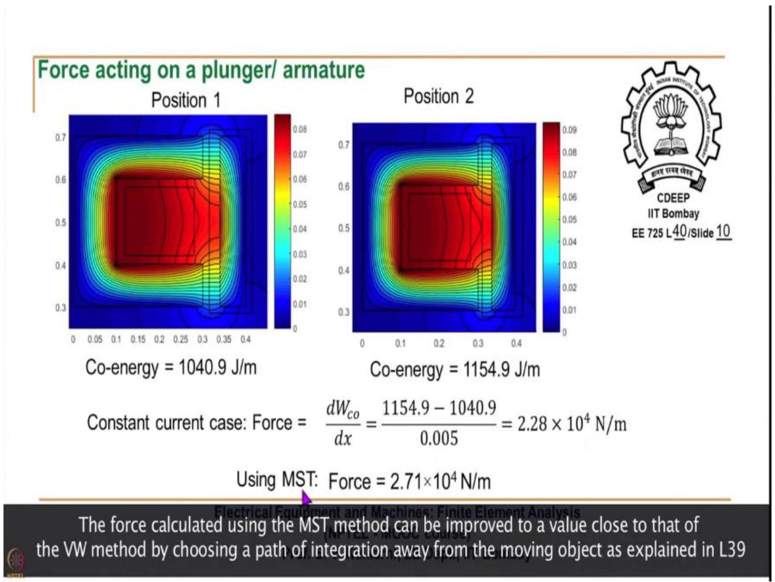
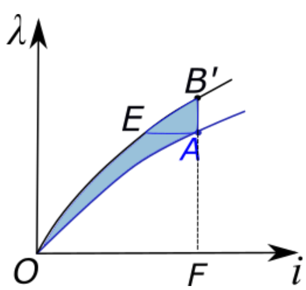


As explained in the previous slide, the operating point is approximated to have shifted from A to B′ as shown in the following figure.



The value of co-energy is represented by the shaded area in the above figure. The same is observed in the two simulations given in the above slide. The value of force is calculated as 2.28×10^4

N/m, whereas the force calculated using the MST based force calculation is 2.71×10^4 N/m. The force calculated using the analytical formula was 2.57×10^4 N/m.

So, the force values calculated using the three methods are in the same range. That gives us confidence in the two calculation approaches. Of course this is a very simple problem to analyze. As mentioned in this course, we can develop a formulation by starting with very simple problems and verify the solution with analytical formula or some other methods to get confidence in the formulation. After that we can adopt the formulation to solve more complex problems.

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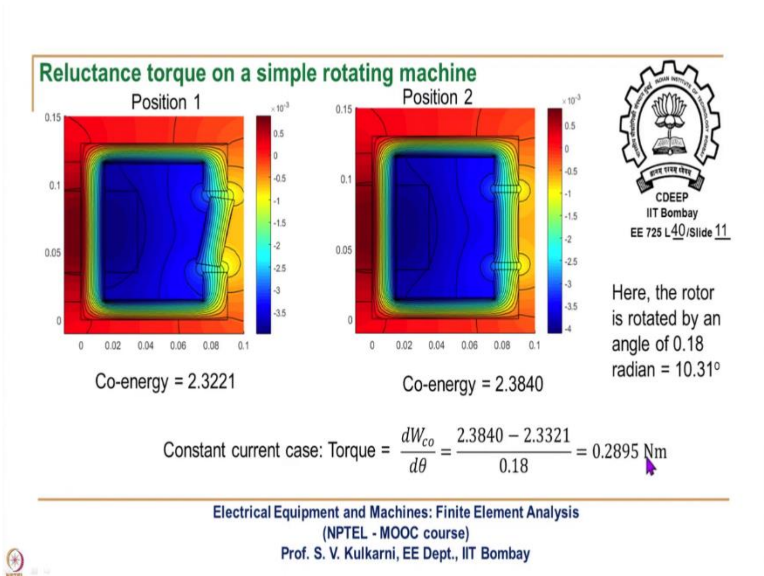

Finally, we will see how to calculate reluctance torque of a simple rotating machine. The rotating structure in the above slide moves in the gap. The magnetic circuit has a core, an excitation coil, and a rotor structure. When the coil is excited, the flux is established and the rotating structure tends to rotate towards the minimum reluctance position. So, when the rotor is at the position shown in the figure on left hand side, then the torque would act on the rotor to move and it becomes vertical.

Again, we can develop our own code or use a commercial FEM software to model this problem. Using a developed code based on the theory that was discussed we calculate the co-energy for positions 1 and 2 as indicated in the above slide. Then the torque acting on the rotor structure is

$$\text{Torque} = \frac{dW_{co}}{d\theta} = \frac{2.3840 - 2.3321}{0.18} = 0.2895 \text{ Nm}$$

So, the difference in co-energy divided by the angular displacement which is equal to 0.18 radian or 10.031°, we get the value of torque as 0.2895 Nm. This is how we can calculate torque using the co-energy based approach.

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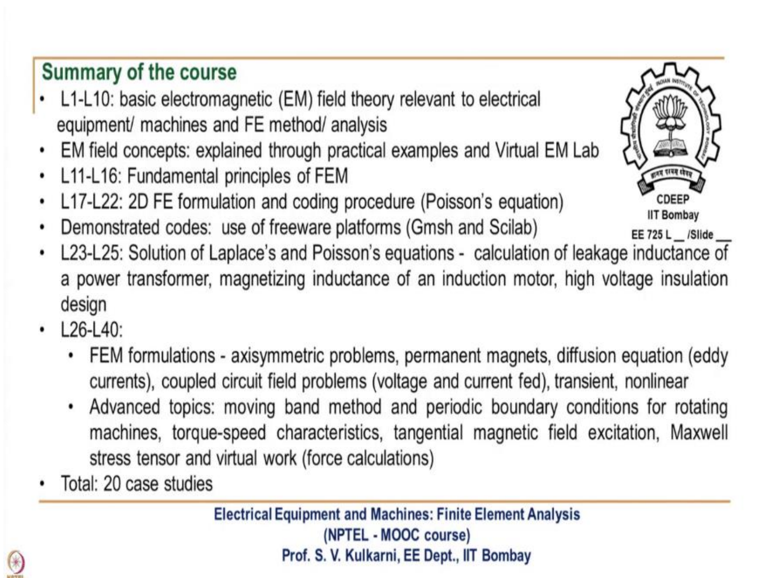

With the case study that we saw in the previous slide our course is completed. In this slide, all the concepts that we have covered in this course are summarised. In the first 10 lectures, we covered the basics of electromagnetics relevant to electrical machines/ equipment and finite element analysis. In these 10 lectures some of the concepts were explained using practical examples and virtual electromagnetic laboratory developed in EE department of IIT Bombay.

In lectures 11 to 16, we understood fundamental principles of finite element method. In the next 5 lectures, we studied 2 dimensional finite element formulation and the corresponding coding procedures, and we studied the formulation with reference to Poisson's equation. In these lectures, we demonstrated few codes using freeware platforms like Gmsh and Scilab. Then we went on to solve Laplace's and Poisson's equations in next 3 lectures and we calculated leakage inductance of a power transformer. Then we calculated the magnetizing inductance of an induction motor and we understood how we could use finite element method effectively for high voltage insulation design. In the further lectures from 26 to 40, we analyzed many case studies and some extensions of standard simple FEM formulations for Laplace's and Poisson's equations to more complex cases involving axisymmetric problems, permanent magnets, eddy currents, etc. Then we also studied how do we couple circuits and fields to solve practical problems that involve transients and nonlinearities in magnetic circuits.

Thereafter, we also got exposure to some advanced topics in which we studied how do we model rotation of rotating parts that is the rotor structure of a typical machine. We also studied how to impose periodic conditions, which reduces computational burden. Then we saw how to compute torque-speed characteristics of a rotating machine. Also, we saw how to impose tangential magnetic field excitation for some specific problems. For example, we studied how to calculate effective complex permeability to analyse core joint of transformers. Finally, we studied how do we calculate forces using Maxwell stress tensor method and virtual work based approach. In the entire course, we have seen about 20 case studies. This course will be useful to not only undergraduate and postgraduate students but also for practicing professionals for solving real life problems either by developing their own codes or by effectively using the available commercial FEM software. Thank you very much.

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