Electromagnetic Theory
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Lecture - 54
Skin Depth / Effect

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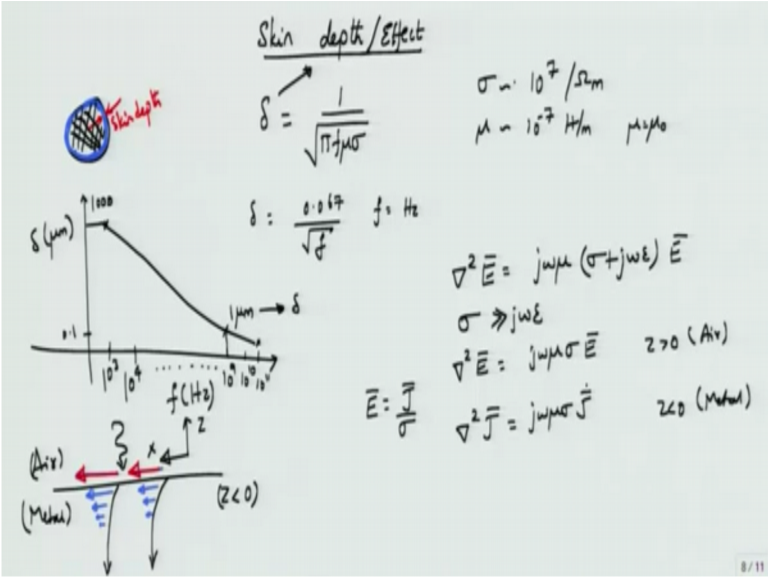

In this module we are going to discuss in depth about one of the most important topic at high frequency wave propagation called as Skin Depth or sometimes called as Skin Effect, okay? Skin effect accounts for all type of major loses in a metallic surface, metallic wave guide or an antenna which is made up of metal but it is driven by some currents at high frequency. There will be loss in the metals because of the skin effect.

So understanding skin effect or skin depth is very important if you want to characterize how much loss you are getting in the metallic materials. So almost all metals exhibit skin effect. In summary or in a kind of an overview kind of thing if I want to tell you, in one or two sentences what skin effect is, it simply tell us that high frequencies when a wave actually falls on a metallic surface then the wave will not propagate complete inside.

It will lie on a small layer called as skin depth layer and almost all of the currents and the waves are actually concentrated in that small layer. Why is that important, consider power cable that you are using to transmit power or high voltage from one point to another point, okay? Although this is not the high frequency effect, the effect is essentially the same because of the lengths that are involved.

What happens is that, the entire electromagnetic energy is concentrated on this very small surface and this surface of this thickness is actually called as the skin effect, okay? So the entire electromagnetic field is actually concentrated on this outer or just slightly inner conductor and only in this particular region, okay? This width is called skin depth.

Almost all the current is concentrated in this skin depth. So if you were to actually take a copper cable and consider a certain radius of the copper cable then what you see is that the only usable area of this copper cable is around this skin depth and all the other areas of the copper that you have used is actually not carrying any current at all. This is especially true at high frequencies. So what is the job of this extra copper?

The extra copper is only giving you mechanical stability. So if you want to actually have a tradeoff between how much mechanical stability you want versus how much copper you use because you know that the electromagnetic fields are all concentrated on the skin depth on the outer to inner layer of the copper. So it is a question as to how much copper can we actually afford to waste.

You can make a mile long or a mile diameter uniform copper rod to carry electromagnetic energy, but that would be totally useless because the electric fields would only be around a small region and will just very quickly estimate what is the order of magnitude of that one here and that order of magnitude you will be surprised to see the value out there. So in that small region is what the entire electromagnetic waves are concentrated on.

So it does not makes sense to use so much of copper to fill in between expect if you are looking for mechanical stability. So with this thing in mind let us try to see what this skin depth is? Now, we have already encountered what skin depth is, right? So we have already written down an expression for skin depth and we denoted that one by a Greek letter delta.

This was supposed to represent your skin depth and it is given by pi f mu sigma. For a copper, sigma is in the order of 10 to the power 7 per ohm meter and mu is equal to 10 to the power minus 7 Henry per meter. This is for the case of mu is equal to mu zero which is what we are going to assume and if you substitute these values for mu and sigma, the actual sigma is around 5.9 multiplied by 10 to the power of 7 or something.

So if you substitute all these plans you will get delta to be around 0.067 divided by square root of f, where f is the frequency measured in Hertz. So you can actually plot this, in the form of frequency along the horizontal axis and the skin depth in micro meters along the vertical axis, okay? And you can employ rather than talking in terms of linear scale for f, you can employ a log scale, okay?

I have about 10 the power 9, 10 to the power 10, 10 to the power 11 and what you will see is that at frequencies which are close to kilohertz or less, the skin depth is around 1,000 microns, okay? A 1,000 micrometer is nothing but 1 millimeter. So this is about 1 millimeter thick even at frequencies as low as 1 kilohertz. This value drops down to about 0.1 micron at around 1 gigahertz or rather at around 10 to the power 11.

So at around 11 gigahertz you are very close to 1 micrometer. So this is the skin depth that you are looking for copper at frequency or 1 gigahertz. So if you were to build up your PCB and populate it with gigahertz sources, then this is essentially the region or the thickness of the copper that is being penetrated because of the skin depth. Even though this currents are concentrated in this very small region, the kind of loses they induce cannot be neglected.

So even though they might be propagating only in a short layer around the conductor, you cannot actually ignore the loses because of this one. We are going to come back to this loses in short while, okay? So let us look at certain equations which will describe skin effects for us. To describe skin effect, I actually need to write an equation for current density vector J.

So write an equation for current density vector, I actually start from an equation which I have for electric field which is del square E is equal to j omega mu sigma plus j omega epsilon multiplied by E. And because this is a good conductor, skin effects is observed in those good conductors. Sigma is very large compared to omega epsilon. Therefore, you have del square E is equal to j omega mu sigma multiplied by E.

Let us assume that I have a metallic surface. This is air from which I have some wave coming in and then there is a region that is sufficiently thick metal is kept and then the waves are actually coming up impinging on this metal surface, okay? So once the waves come in and impinge on a metal surface, there will be, assume that this wave is actually a plane wave. So which means that it has x component for the electric field and y component for the magnetic field and the wave is propagating along z direction.

What I am choosing in this particular co-ordinate system is to consider z less than zero, that is negative values of z to be metal and positive values of z to be air and then the z axis is going. It is actually going from metal to air and the wave is going from air to metal. What will happen? As the wave impinges there will be electric field lines and these electric field lines will be along x axis.

Let us assume that the surface is kind of uniform along the xy plane. Therefore, their entire electric field lines will lie on xy plane and they would all be directed along x axis. There would be uniform at a given z equal to zero plane and they would be pointing along x axis. However, as you go deeper into the metal, you will see that these lines of J vector actually start to become small and small.

So these are the electric field lines and the blue colors are the J lines, okay? This electric field at the surface will induce a certain surface current density and this surface current density actually goes exponentially decays. So this current exponentially decays inside that of a metal surface and how do we describe that one, well, for regions z greater than zero, that is for air, the electric field is described by this wave equation.

However, for z less than zero where you have the wave converted in the form of the current density lines, this equation needs to be changed. So for z less than zero, that is in the metal surface I want an equation for J not for electric field. So what is the relationship between the two? I know that E is given by J by sigma. So I can substitute for that E. So when I substitute for E I get an equation for J as del square J is equal to J omega mu multiplied by J.

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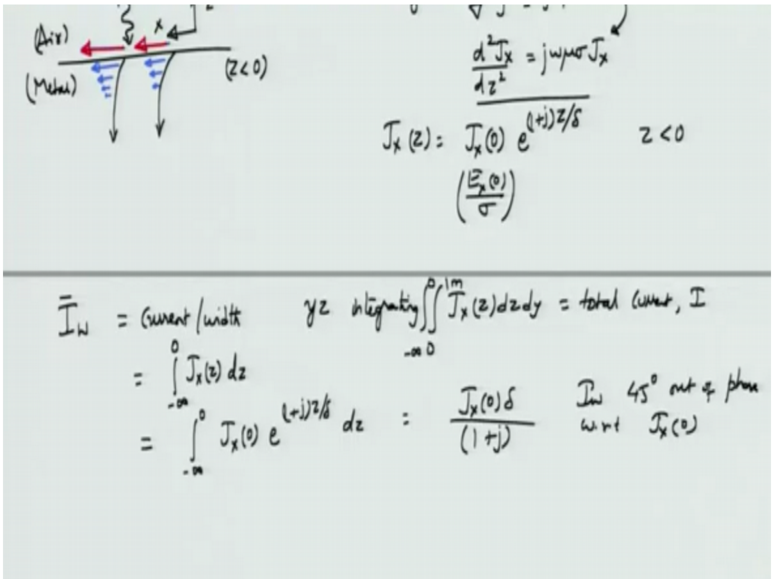

But I already know that electric field was propagating originally along z direction. It was polarized along x direction and since E and J are having the same polarization, I can replace this del square J by d square Jx divided by dz square. This is obvious because the electric field is along x, therefore j must also be along x. This is equal to j omega mu sigma Jx.

So I converted this vector equation into a simplex scalar equation that tells us how the current density vector Jx is actually changing or actually propagating as you go deeper into the metal surface. What is the solution for this one? The solution for this equation is very simple. Jx of z is equal to Jx of zero, this is the value of current density at the surface which will be given by the applied electric field.

Ex of zero divided by sigma and inside the metal this would be going as e to power kz, where k is square root of j omega mu sigma. Therefore, this would be 1 plus j z by delta, okay? Now you might be surprised why do I have a 1 plus j multiplied by z. What happened to the minus sign? And you should remember that this expression is actually valid for the negative values of z. So rest assured your J vector does not grow inside. J vector actually decays.

So you do not have a situation where at the surface you have a certain J vector whereas at a certain later stage you have a J vector which is actually grown, no, what you have wave which is, I mean J vector which is actually decaying exponentially and this decay rate is again given by the skin depth delta and the only thing is that this expression will have a plus sign out there.

Associated with this Jx of z and this Jx of zero, there is also another quantity called current per unit width. What is this current per unit width? Imagine that I have this J lines which are going like this. So this is how the J field lines are all going. This is the x direction which I am taking and then this J lines actually start to decay in amplitude and eventually reach to zero down below. So after a certain amount of depth they would actually be very very close to zero.

This is how the electric field lines are all and then they will reduce, reduce their amplitude and then go to a very small values inside the good conductor. Now what you do is, you consider integration along y axis. So my fingers are pointing along x axis, this thumb is supposed to be pointing along the z axis and therefore I have this direction to the be the y axis. So x, y, well, the other way around. The y axis should be coming out to be like this.

But the point here is that I have x and I have y and if you look at an integration of these J lines in this area which is given by this say some meter, 1 meter, along y axis and then going all the way towards infinity along z. Coming back from infinity and coming up. So you actually have a loop through which the J lines are all piercing out and then this loop has a width of 1 meter along y axis.

So in the y z plane consider integrating this J field to get, so integrate this Jx of z to get what is the total current. So this I am going to get as a total current. So total current I, okay? However, integration along y because Jx is a function only of z, therefore integration along y gives you 1 meter and if you divide this integration along y what you get is current per unit width which is given by integral from minus infinity to zero, Jx of z, dz.

This will be a vector directed along y axis. This is the current per unit width directed along y axis. And this is given by integral from minus infinity to zero. This is Jx of zero, e to the power 1 plus j multiplied by z by delta dz and when you integrate this one, what you see is Jx of zero multiplied by delta divided by 1 plus j. What is the meaning of this?

The meaning is that, the current per unit width is actually out of phase by 45 degrees with respect to the current in the surface, current on the surface Jx of zero is 45 degrees out of phase with the current per unit width. So this is the meaning of this. So Iw is 45 degrees out of phase with respect to Jx of zero which is the current at surface of the conductor.

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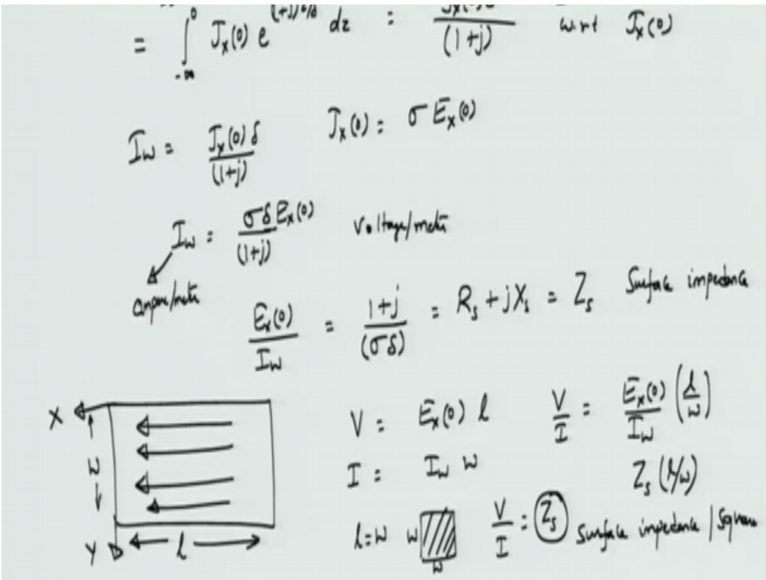

So this current per unit width Iw which we have found out actually can be very useful for us because Iw has units of ampere per meter. Iw has units of ampere per meter. Now if you look at this Jx of zero and relate it to the electric field, okay, so let me write down this one. Iw is given by Jx of zero multiplied by delta plus 1 plus j, okay? I know what is Jx of zero. This is the surface current density at the surface of the conductor which must be equal to sigma times electric field component Ex of zero.

So substituting that what you get is Iw is equal to sigma Ex zero divided by 1 plus j and there is a delta out there. So sigma delta by 1 plus j. Now this quantity Ex of zero has units of voltage or volt per meter or volt per unit length and this fellow has units of ampere per meter, right? Because this is current per unit width. So if you take the ratio of voltage per unit width to current per unit width what you will get is something that would be impedance.

So if you take this ratio of Ex of zero to Iw, what you get is 1 plus j divided by sigma multiplied by delta. And this actually is a complex number Rs plus jXs or I can write this as Zs and call this as surface impedance. Surface impedance is actually telling you that electric field E at the surface to the current per unit width Iw and this ratio is the surface impedance and clearly you can see that this ratio is not real.

That is, it is complex indicating that Ex of zero and Iw are out of phase by a certain factor and because of this out of phase thing the total power carried will not be exactly equal. Only a part of the power will be carried by the wave and the rest of the power would actually be lost to us. So this is what we wanted to write. There is the last matter of actually calculating how much power is getting lost.

To calculate the power loss, first consider a scenario which we want to write down in terms of the voltage and current, right? So this is my x axis. So all the J lines are being uniform on this x axis, of course along z they would actually be decaying. And then I have y axis up here. Now let us assume that this width is w and this length which I am considering is l.

So there is a width of w along y axis and a length l of the conducting surface that I am considering along x axis. What is the voltage that is induced in this length l? Voltage induced will be Ex of zero because this is at the surface I am considering times l. What will be the total current I passing through this width w, that would be Iw multiplied by small w, that is the width here. So current per width multiplied by width will be the total current.

The ratio of V by I is given by Ex of zero l by w divided by Iw, right? Now Ex of zero by Iw is something that we have already written down. This is nothing but Zs and then there is a factor of l by w. What happens when you consider l is equal to w, what you are really considering is that of a square whose area is w square, whose sides are w and w and numerically V by I will then be equal to Zs.

Because of this reason this Zs is actually called as surface impedance per square because when l is equal to w, the ratio of V by I, Zs which is in the form of ohm will be numerically equal to the ratio of V by I and this would happen when l is equal to w which is when you would actually have created a square.

Therefore, if you call this Zs as impedance per square or Rs as resistance per square and Xs as reactance per square, then multiplying by the square you are going to get the total impedance, okay? So this is what we wanted to write.

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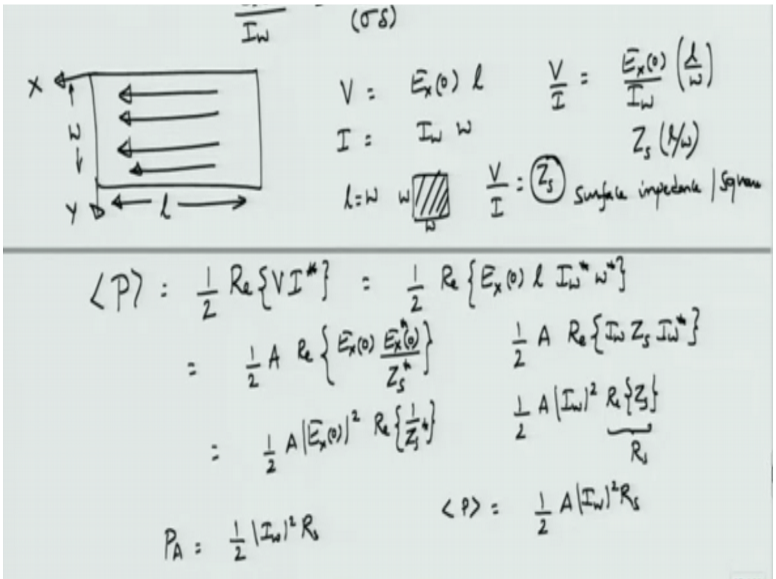

Now for the matter of the power that is lost, we know that the average power for complex V and I that is phasor quantities is given by half of real part of V I complex conjugate. Now I know what is V, I know what is I. So let me write down those expressions here. So this becomes half real part of Ex of zero multiplied by l, I is nothing by Iw complex conjugate, w complex conjugate. I know that l and w are real, therefore they can be put outside.

So I can write this as half area. So area times real part of Ex of zero multiplied by Iw complex conjugate. But I know that Ex and Iw are related to the surface impedance Zs. So I will actually be able to write down two ways. One would be to write this as Ex of zero and then write this Iw square as Ex complex conjugate of zero divided by Zs complex conjugate. Or, there is another form which is I will substitute for Ex.

And write this as real part of Iw multiplied by, Iw because this one, and then multiplied by Zs, there is an Iw complex conjugate, right? So you can see that this form will give you half A, right, Ex0, Ex0 complex conjugate is real. So you can pull this one outside. So you get Ex of mode zero square and real part of 1 by Zs complex conjugate or equivalently you can write this as half A Iw multiplied by, I mean complex conjugate is nothing but Iw magnitude square and real part of Zs.

Now real part of Zs is nothing but Rs. So therefore this is given by half area, Iw magnitude square multiplied by Rs. This would be the average power. But if you are interested in power per unit area, that would be obtained by dividing the average power with area, so you will get half Iw magnitude square multiplied by Rs.

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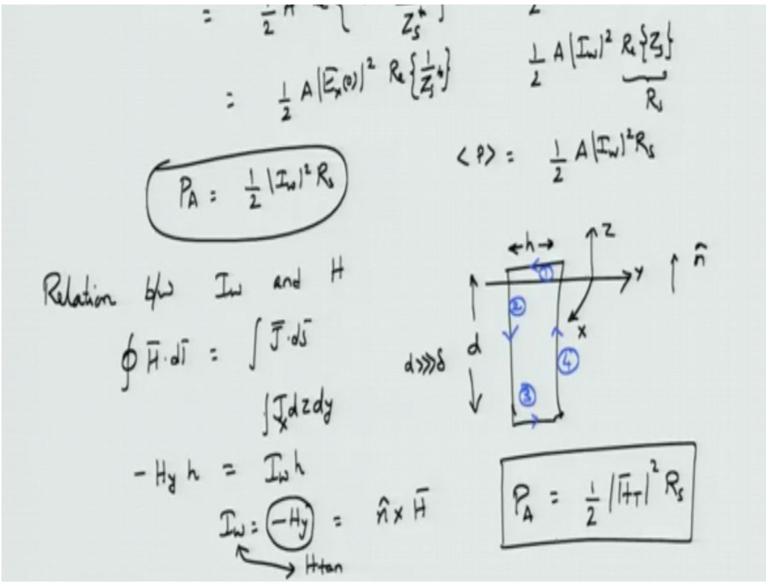

There is one last matter which I want to discuss in which I am going to relate this Iw to the magnetic field. This expression will become very useful for me when I relate the current to the magnetic field because then it will allow me to calculate the losses of waveguide walls, you know how much power is getting inside a waveguide wall. You can also extend that analysis to any other metallic surface.

So what we want now is relationship between Iw which is current per unit width and magnetic field H. In order to get this one, I will invoke Ampere Maxwell law, so I have H dot dl being equal to integral of J dot ds and I know for this case that if I consider this one as say y axis, this as z axis and this as my x axis. So hopefully all the directions which I have written down are correct.

So on the surface if you look and formal path of integration which is having h units along y and having d unit along z where we are going to assume that d is much much much larger than skin depth. So the fields here down in the metallic surface are actually almost zero and we consider the path in this way, okay? Going along the segment 1, segment 2, segment 3 and segment 4. Now this integral of J dot ds we have already written down.

So this is nothing but integral of J dz, Jx dx and dy, right? But integral of Jx dz is something that we have already written down. This is nothing but the current per unit width and times integral over dy will be this integral over h. So this would be Iw multiplied by H, H being the width along the y direction. What happens to the left hand side? Well, for the case of plane wave that we have considered, h will only have y component and therefore segments 2 and 4 will not contribute anything.

Because in these call segments h is along y but the line integral is along z, so therefore contribute. There is no contribution of h in the segment 3 also. Why because H would have decayed so much because of the skin effect thing. The E values and H values depths which are much larger than skin, depth would actually be almost zero. Therefore, h contribution along segment 3 would be nothing.

Therefore, the only contribution of H is that of segment 1, at which point I have minus Hy into small h is equal to Iw h. So this gives me Iw is equal to minus Hy or f you consider a normal along the metal surface, outside the metal surface, right? This can also be written as n hat. So this normal if you consider then it can be written as n cross H. H is along y and n is along the surface z. So you have z cross H, that is what the direction for Iw would be, okay?

So regardless of the fact on the surface, this Hy is nothing but tangential component of magnetic field, right? Therefore, what we have here is the current per unit width or the current on the metallic surface is actually equal to, at least the magnitude of that one is equal to the tangential H component. You can now substitute this expression for Iw into the power lost per unit area.

And obtain the power obtained per unit area is half HT where HT stands for tangential magnetic field. Magnitude square times Rs and Rs is something that is determined by the material properties. It is actually sigma dependent, the conductivity dependent. HT magnitude square is actually dependent on the magnetic field that is induced on the surface. So this expression for power lost per unit area is very important.

And we should keep this expression in mind when we later discuss waveguide losses.

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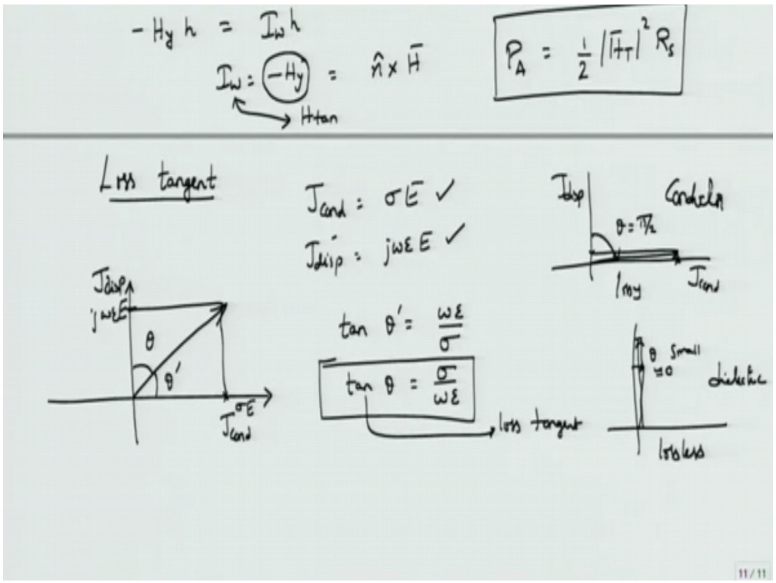

There is one other concept associated with wave propagation inside that of a imperfect conductor or a dielectric. The question is how do you consider which one is a conductor, which one is a dielectric and this can be considered by considering the ratio of how much conduction current is there to the displacement current and this ratio of conduction to displacement current is actually captured by what is called as loss tangent.

What is the total current inside that of material when there is conduction as well as displacement current? You have conduction current given by sigma multiplied by E and the displacement current is given by j omega epsilon E assuming that electric fields and the other field quantities are going as e power j omega t. So this is the conduction current, this is the displacement current.

You can actually plot them on their x and y axis. So I can actually plot this one as the displacement current. This will be given by, in terms of j, this would be omega epsilon and the magnitude of the conduction current I can plot here. This would be the conduction current which is given by sigma multiplied by E. And then the total current is actually the vector sum of these two.

So if you have seen that these are two are the two vectors, then the total current will be of the vector, okay and it would be making an angle of theta here and an angle of theta prime with respect to displacement axis, right? So what is this angle? This angle theta can be obtained by looking at what is tan theta. Tan theta is nothing but omega epsilon divided by, so do not worry about this j, so you just look at only tan theta over here.

For one second let me may actually call this as theta and call this as theta prime. So tan theta prime is omega epsilon by sigma because electric fields on both sides will actually cancel. So both sides will actually cancel or if you measure theta with respect to the displacement axis which is what more commonly is done, then tan theta is given by sigma by omega epsilon. And this tan theta is what we call as loss tangent. Why is it called as loss tangent?

If you look at two cases, in one case let me assume that conduction current is very large and the displacement current is very small, then the total current actually points very close to conduction current and theta is around pi by 2. So this would be the case when tan theta is getting very large, right, because theta is becoming pi by 2. So therefore tan theta is very large which means that there is complete loss because sigma is much high.

Then the propagation constant as you have seen, attenuation co-efficient as you have seen will depend mainly on the sigma and it would be a very lossy situation. So this is your conduction current and this is your displacement current, okay? On the other hand, if the material is a very good dielectric and having very little amount of conducting loses, then that particular material will have no attenuation.

So this would be the electric field which is very close to this one and the angle theta would be very small implying that theta is small. So theta is approximately zero. So in this particular case, the wave will actually be propagating without any attenuation and this is a case where you do not have any loss or the loss is very small. So depending on which one is larger you can actually consider this as dielectric or as a conductor.