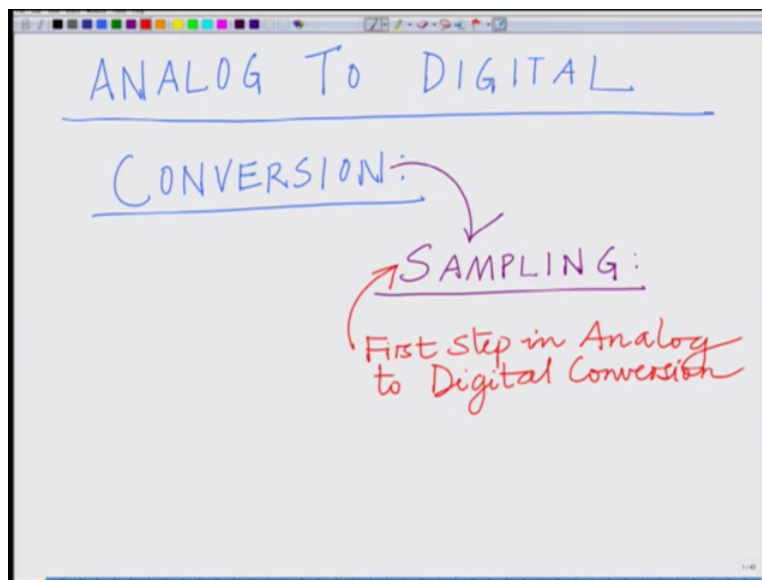


Principles of Communication- Part I
Professor Aditya K. Jagannathan
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module No 6
Lecture 35

Analog to Digital Conversion of Signals and Introduction to Sampling

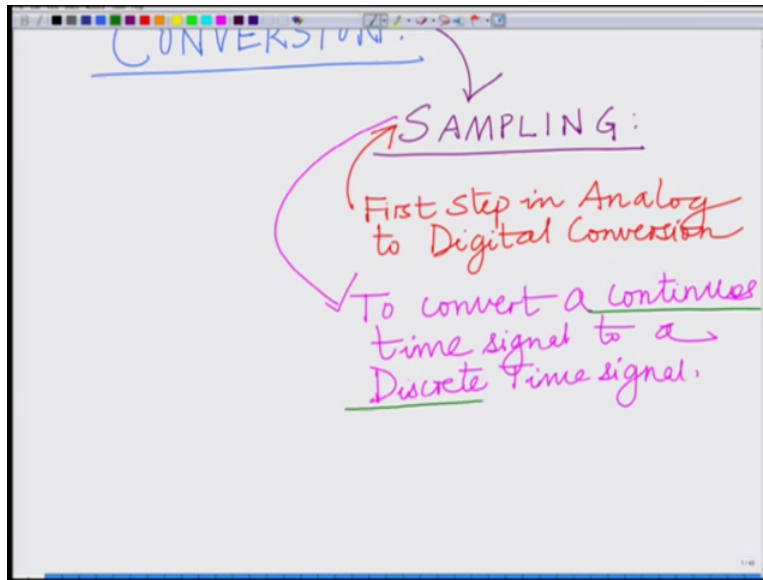
Hello welcome to another module in this massive open online course, so in this module we are going to start looking at new topic that is conversion of analog signals into digital signals, alright. And the first step to convert an analog signal to a digital signal one has to make the signal discrete in time, alright. So we have to sample the signal, alright. So we are going to start looking at sampling to convert analog to a digital signal, okay.

(Refer Slide Time: 0:47)



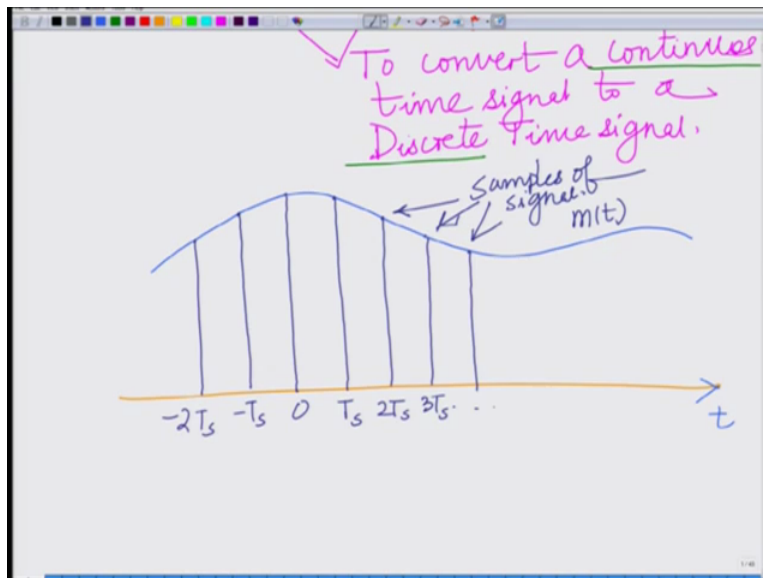
So we are going to start looking at analog to digital conversion, alright. To convert an analog signal to a discrete signal and the first step towards this is sampling, okay. And what does sampling do sampling converts a continuous time signal into a discrete time signal, okay.

(Refer Slide Time: 2:18)



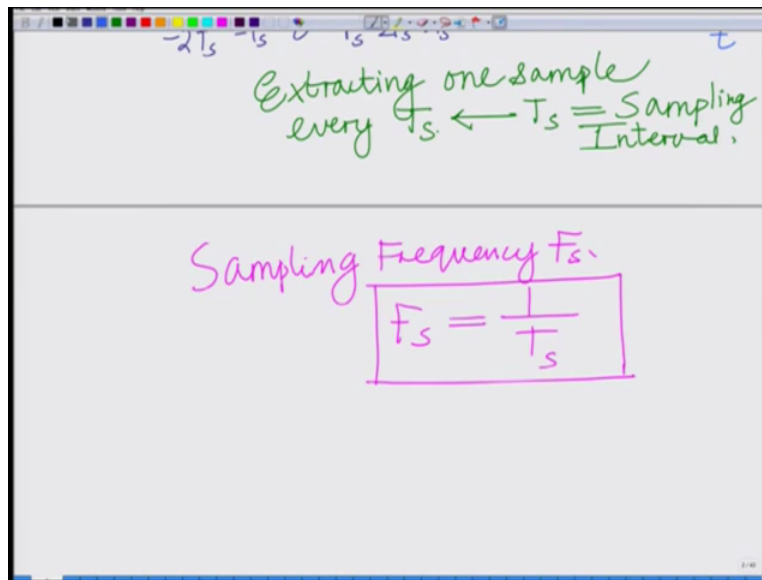
So sampling the basic idea behind sampling is to convert an analog converter a continuous time signal to a convert a continuous time signal to a discrete time signal, okay.

(Refer Slide Time: 2:44)



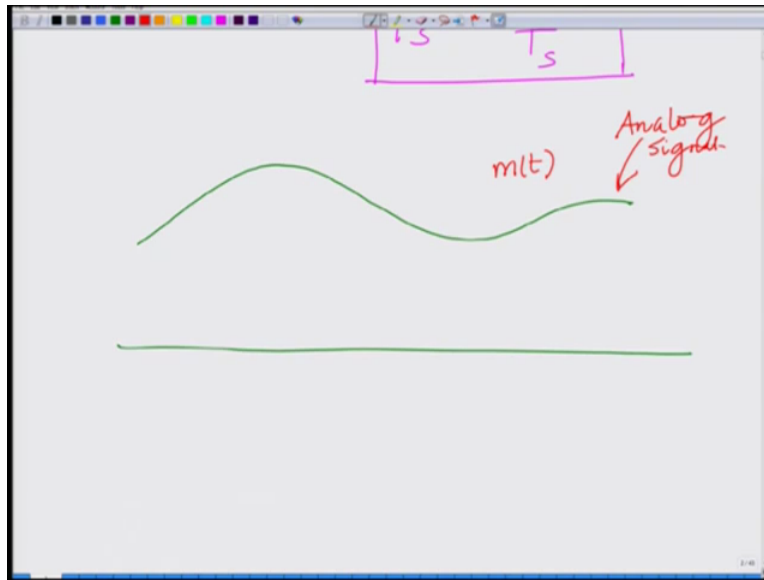
For instance let's say we have our signal let us draw a simple signal here to illustrate this point so we have this signal, alright which is continuous time signal now I want to sample it, alright. So I am going to sample it at certain specific discrete time instance which are also known as the sampling instance. So let us say the sampling frequency or let us say the sampling time is 0 that I

(Refer Slide Time: 5:14)



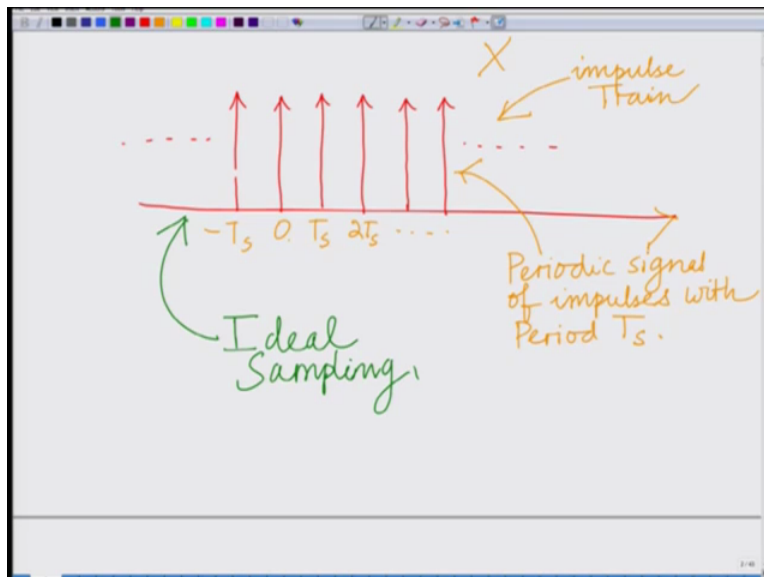
So correspondingly since this is a periodic process sampling frequency which will see is a fundamental quantity of this sampling process f_s equals 1 over T_s , so your sampling frequency is f_s x sampling interval is T_s and therefore sampling frequency f_s is 1 over the sampling duration or the sampling interval a T_s and therefore this is basically you are sampling this at sampling frequency when you are extracting since you are extracting samples once every interval of duration T_s , alright. The sampling interval is T_s the sampling frequency is 1 over T_s which is denoted by f_s .

(Refer Slide Time: 6:09)



And naturally the sampling process can be represented as follows, so I take my signal again, okay. This is my $m(t)$ this is my continuous this is my analog signal I can multiply it by an impulse train, now what I am going to do I am going to take an impulse train, okay.

(Refer Slide Time: 6:29)

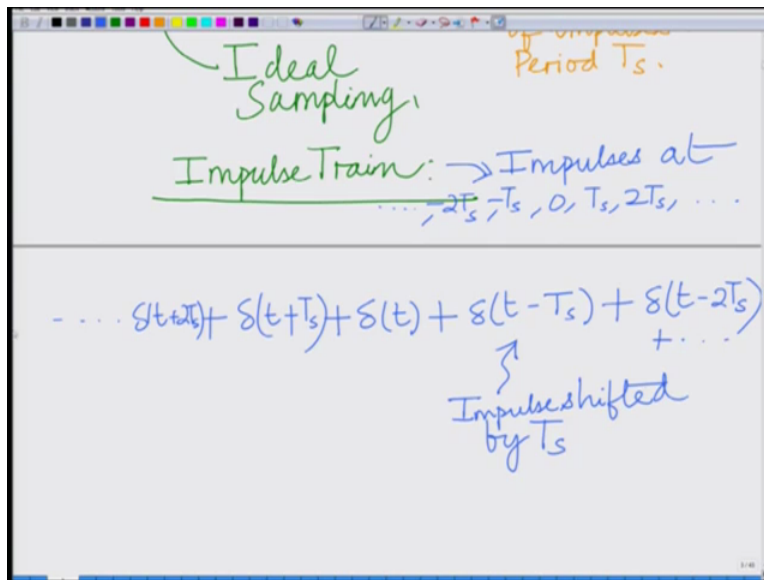


So this is my impulses which are spaced so I take the product of this with this which is my impulse train, now if you look at this, this has an impulse at 0 for instance this is an impulse at 0 , T_s , $2T_s$ and so on minus T_s , so this has this is an impulse train this is a train of impulses which is

one impulse that is it is a periodic signal of impulses with period T_s , so this is a periodic signal of impulses, periodic signal of impulses with period that is you have basically one impulse every T_s . So when you multiply this impulse train with the signal what you can see is each impulse at the point of that impulse for instance impulse at 0 will pick the magnitude will pick the level of the function of the signal at 0.

Impulse at T_s will pick the level of the signal at T_s therefore they are creating basically a modulated train of impulses the height of each impulse once you multiply the impulse train with the signal the height of each impulse or the or basically the scaling of each impulse will be proportional to the signal level at that particular time of that impulse, alright. So one way to sample is basically by multiplying the original signal to be sampled with an impulse training this is also known as ideal sampling or impulse sampling. This is also multiplying this by a train of impulses this is also known as ideal sampling, okay. Ideally we want to multiply it by a train of impulses, okay.

(Refer Slide Time: 9:13)



Now let us characterize this impulse, train of impulses we have an impulse at we have the impulse train let us look at impulse the impulse train if you look at this impulse train we have impulses at well, they have impulses once every T_s so you have impulses at 0 impulses at T_s impulses at $2T_s$ so on. And you also have impulse at minus T_s minus $2T_s$ and so on and therefore if you look at the corresponding signal that is going to be impulse well, impulse at 0 Δt plus

impulse at T_s Delta t shifted by T_s this is impulse shifted this is your impulse shifted plus Delta t minus $2T_s$ plus also impulses at minus T_s that is Delta t plus T_s and this is continuous so on Delta t plus $2T_s$ and its continuous so on and therefore if you look at this signal train of impulses.

(Refer Slide Time: 10:46)

The diagram shows a sequence of impulses: $\dots \delta(t+2T_s) + \delta(t+T_s) + \delta(t) + \delta(t-T_s) + \delta(t-2T_s) + \dots$. A bracket groups $\delta(t+T_s)$ and $\delta(t-T_s)$ with the label "Impulse shifted by T_s ". A box contains the summation $\sum_{n=-\infty}^{\infty} \delta(t-nT_s)$, with an arrow pointing from the box to the sequence and the label "impulse train".

This can be represented as summation n equal to minus infinity to infinity Delta t minus nT_s this is basically your impulse this is basically your impulse train that is basically you have this sequence of impulses 1 impulse at every T_s or multiple of T_s that is impulse at $0 T_s$, $2T_s$ one impulse at minus T_s minus $2T_s$ so on. So each impulse shifted is at nT_s where n can be any integer is 0, positive, negative each impulse shifted to nT_s is Delta t minus nT_s your (summa) you are considering the summation of all these impulses from n equal to minus infinity to n equal to plus infinity this is your impulse train.

(Refer Slide Time: 11:33)

Handwritten whiteboard content:

$$g_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Annotations: "impulse train" with an arrow pointing to the summation, and "original signal." with an arrow pointing to the δ term.

Sampled signal:

$$= m(t) \times g_s(t)$$
$$= m(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Let us denote this by g delta of t this is your impulse train g delta of t , okay.

(Refer Slide Time: 11:51)

Handwritten whiteboard content:

Sampled signal:

$$= m(t) \times g_s(t)$$
$$= m(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$
$$= \sum_{n=-\infty}^{\infty} \underline{m(t) \delta(t - nT_s)}$$

Annotations: A red arrow points from the underlined term to the following equations:

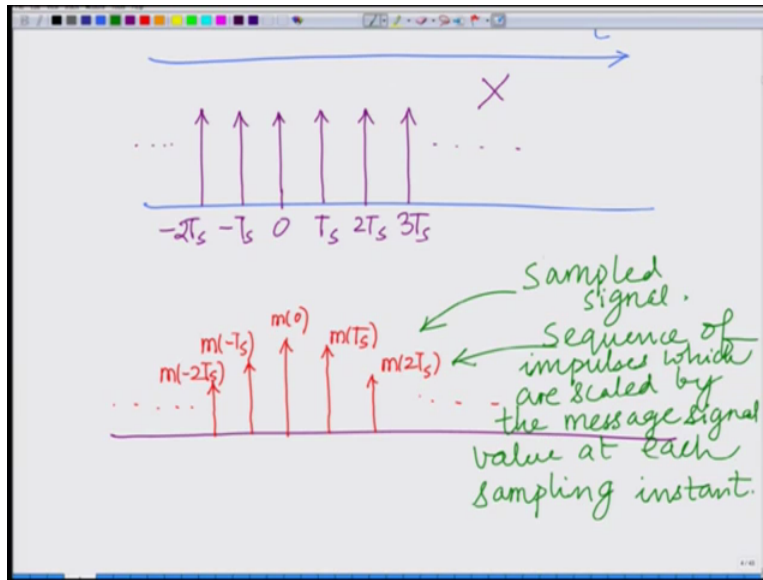
$$= m(nT_s) \delta(t - nT_s)$$
$$= m(t) \delta(t - t_0)$$
$$= m(t_0) \delta(t - t_0)$$

The image shows a whiteboard with handwritten mathematical equations. The main equation is $m_s(t) = \sum_{n=-\infty}^{\infty} m(nT_s) \delta(t - nT_s)$. An arrow points from the text "Sampled Signal" to this equation. To the right, there are three lines of equations: $= m(nT_s) \delta(t - nT_s)$, $m(t) \delta(t - t_0)$, and $= m(t_0) \delta(t - t_0)$. Arrows indicate the substitution of nT_s for t in the first equation, and t_0 for t in the second equation.

And now the sampled signal as we are saying the sampled signal now the sampled signal can be generated this is your $m(t)$ into $\delta(t - nT_s)$ that is product of original signal this is your original signal, signal to be sampled and $\delta(t - nT_s)$ is your impulse train which is $m(t)$ times summation n equal to minus infinity to infinity $\delta(t - nT_s)$ which is equal to summation n equal to minus infinity to infinity $m(t) \delta(t - nT_s)$.

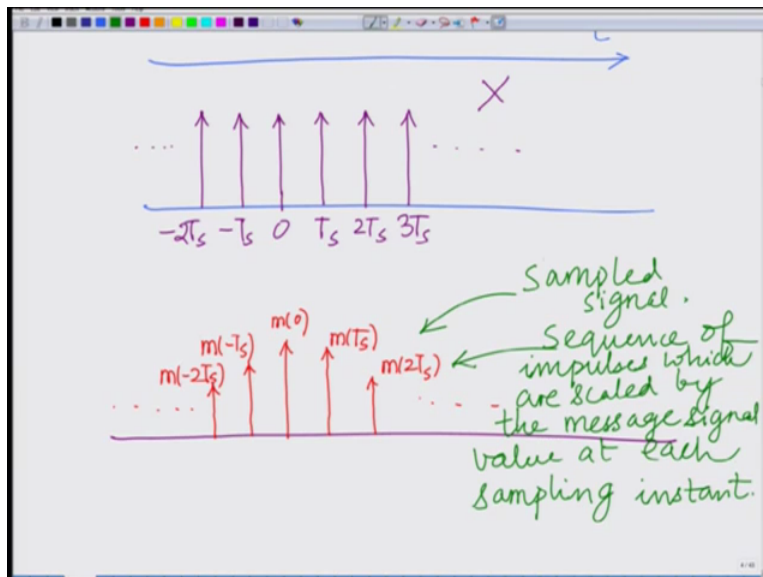
Now you can see $m(t) \delta(t - nT_s)$, now you can use the property that $m(t) \delta(t - nT_s)$ is equal to $m(nT_s) \delta(t - nT_s)$ where we are using the property $m(t) \delta(t - t_0) = m(t_0) \delta(t - t_0)$ basically $m(t) \delta(t - t_0) = m(t_0) \delta(t - t_0)$. It is when you multiply a signal by impulse shifted to t_0 then it simply picks the value of the signal at t_0 , alright. That is how we are basically sampling this segment, alright. So we are able to extract at each nT_s we are able to extract the sample or that level of the signal at nT_s that is your $m(nT_s)$, okay.

(Refer Slide Time: 13:35)



And therefore this gives us n equal to minus infinity to infinity m of nT_s delta t minus nT_s this is your m delta t this is your m delta t , okay. This is basically your sampled signal, okay. So their sampled signal is summation n equal to minus infinity to infinity m of nT_s into delta t minus nT_s and what we are doing is basically very clear we have this original signal, alright which is varying with time.

(Refer Slide Time: 14:36)

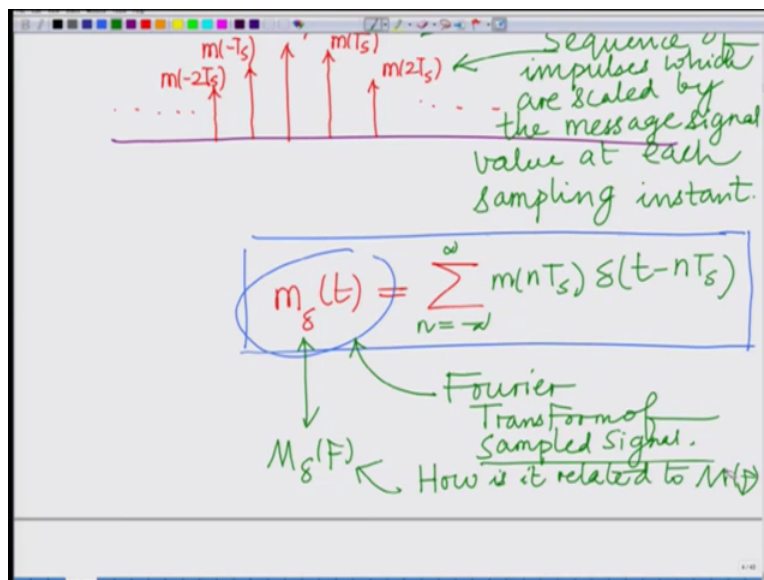


And now you have this impulse train you have this train of impulses at T at $0, T_s, 2T_s, 3T_s$ and so on similarly minus T_s minus $2T_s$ so on, so you have this impulse train which is at $0, T_s, 2T_s, 3T_s$ minus T_s minus $2T_s$ so you take the product basically what you do what you are doing is you are taking the product of the original signal with this impulse train and therefore what you are going to have is basically the signal at 0 will pick m of 0 , so you will have basically you will have signal at 0 picking this value corresponding to this is your m of 0 , so at 0 you will basically pick you will basically pick m of 0 .

At T_s you will pick m of T_s , At $2T_s$ you will pick m of well, whatever is the value here that would be over m of $2T_s$ and of course at minus T_s you will pick this value that is your m of minus T_s at m of $2T_s$ at minus $2T_s$ you will pick m of, okay. So this is basically your sampled signal, okay which is basically again your sequence of impulses this is basically your sequence of impulses this is your sequence of impulses with the amplitude of the impulse, correct?

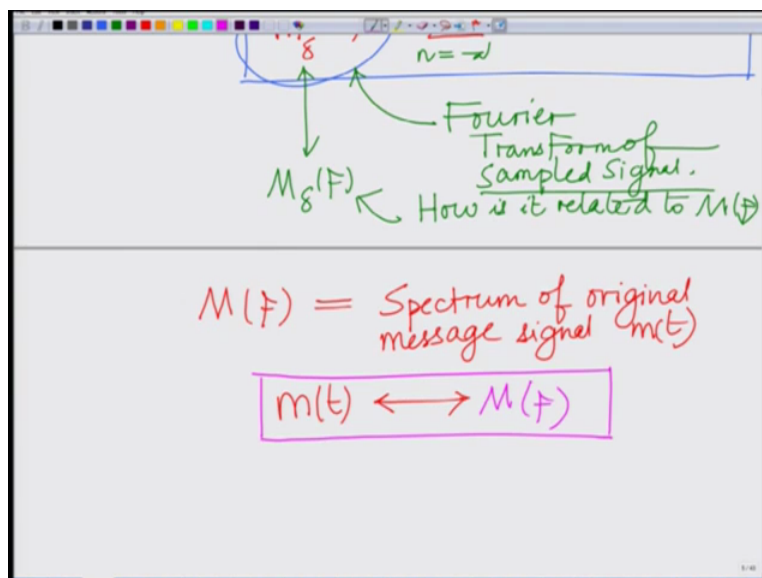
The amplitude of each in sequence of impulses with which are scaled proportional which are scaled by the which are scaled by the message signal value, value of the message signal at the sampling instant at each at each sampling which are scaled by the message signal value at each sampling instant.

(Refer Slide Time: 17:54)



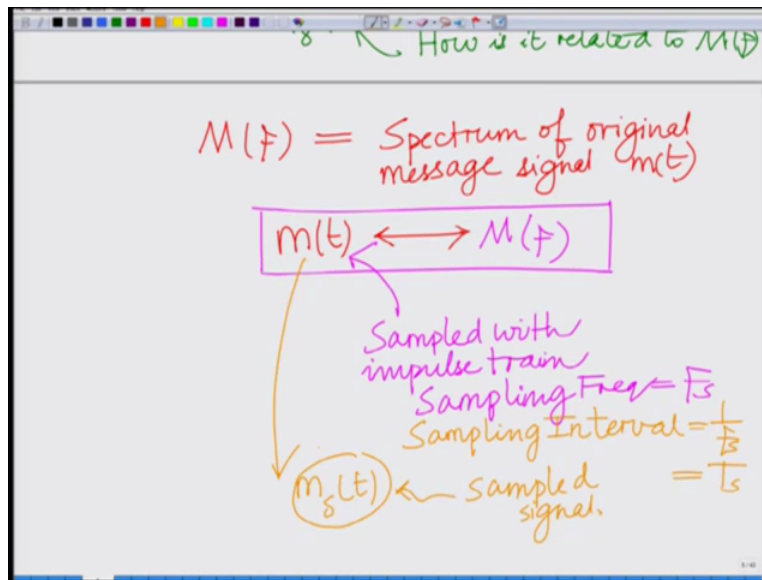
So we have this sampled signal, okay that is your $m \Delta T$ and that is the expression that is given by this expression remember $m \Delta T$ equals summation n equal to minus infinity to infinity $m(nT) \delta T$ this is your this is the this is the sampled signal that we obtained, okay. So this is basically the sampled signal that we obtain and now we want what we want to do? We want to look at the Fourier transform we want to look at this behavior of this in the Fourier domain. So want to look at the Fourier transform we want to look at the Fourier transform of the sampled signal that is what is the Fourier transform $M_s(F)$ of the sampled signal and how is it related to $M(F)$? Where $M(F)$ is the spectrum of the original signal, alright?

(Refer Slide Time: 19:21)



Remember $M(F)$ $M(F)$ is the spectrum of the original message signal $m(t)$ that is $M(F)$ is a Fourier transform of $m(t)$, alright. So we know the spectrum of the original signal, alright. $M(F)$ that is $m(t)$ is the original message signal which spectrum is $M(F)$, we know that we are sampling it with an impulse train at sampling frequency F_s that is sampling interval T_s , alright.

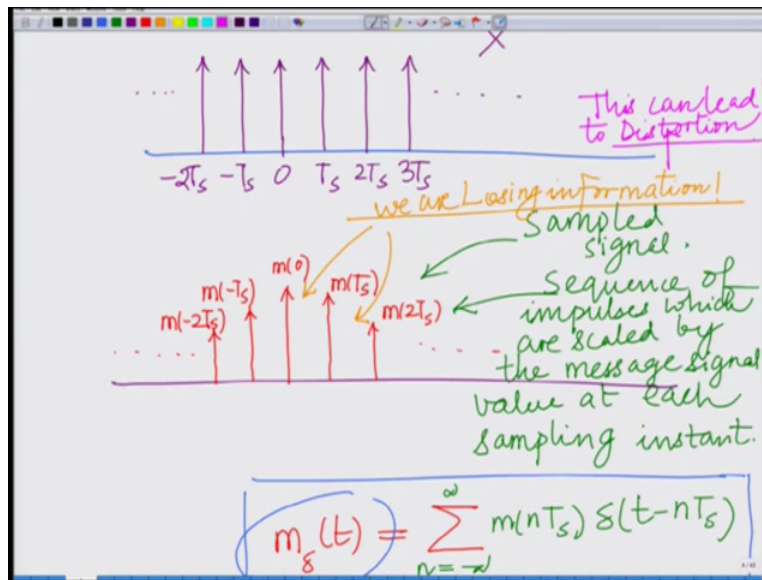
(Refer Slide Time: 20:18)



Let us say this is sampled with the sampling frequency is F_s sampling interval equals 1 over F_s that is equal to T_s , now we get $m \Delta t$ which is your sampled signal which is your sampled signal. How is the Fourier transform of $m \Delta t$? That is if you call the Fourier transform of this as your $m \Delta F$, how is it related to M of F ? Alright, what is the relation in the, what is the relation of the spectrum of $m \Delta F$ to M of F ? And remember also more importantly when we are sampling we are losing the information because we are only extracting the information of the symbols, alright or the signal original signal at the sampling instance that is multiples of T_s $0T_s$, $2T_s$, minus T_s , minus $2T_s$ so on. So we are losing the information contained in the in the intermediate duration, alright.

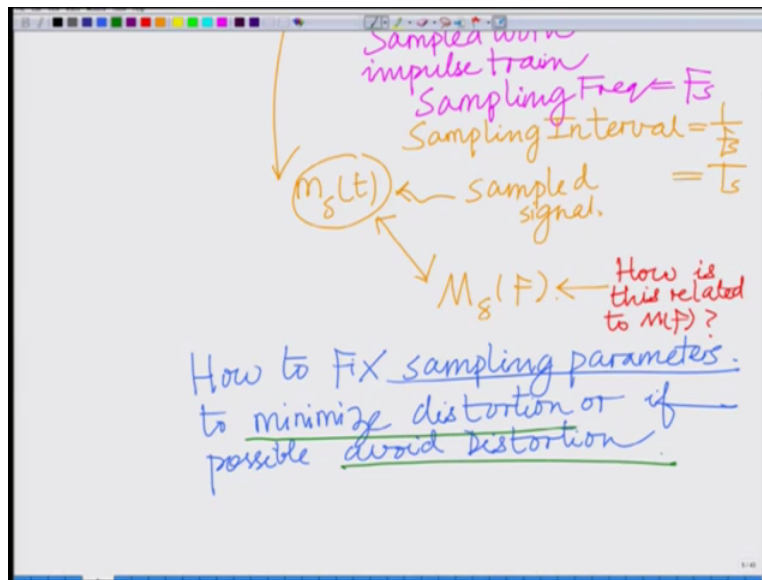
In the intervening (inter) in the intervening duration that is we are losing the information corresponding to the interval 0 to T_s , alright. The information in that particular interval, alright and that leads to distortion, alright. That can lead to distortion, alright. So we are going to see when does that lead to distortion? And what kind of distortion?

(Refer Slide Time: 22:27)



So basically if you look at, go back and look here you can see that basically we are losing information, in sampling we are losing information because we are extracting only the information at the sampling instant so we are losing information this can cost of distortion which means this can lead to this can lead to distortion, alright. It can lead to distortion if you are not able to recover all the information from the signal that is information has been lost then this can lead to distortion, alright. So we want to see under what conditions what is the type of distortion? And under what conditions, that is what is the, what can how can we fix the nature of sampling? Such that the distortion is minimize and if possible there is no distortion at all, right?

(Refer Slide Time: 23:40)



How to fix the parameters of sampling to avoid to minimize distortion or possibly no distortion? So the question we want to answer is how to fix the parameters of sampling? How to fix sampling parameters to minimize or if possible avoid distortion? Either we would like to well ideally we would like to avoid distortion, correct? Ideally we would like to avoid distortion but if avoiding distortion that is making distortion completely 0 is not possible then at least we would like to minimize the distortion.

That is caused by loss of information which is arising due to this sampling process, alright. So these are some of the questions that we want to answer while sampling while considering the sampling of this analog signal which we will consider in the subsequent modules thank you very much.