Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 25
Cylindrical coordinate systems

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to the NPTEL mook on applied electromagnetics for engineers. In this module we will continue the discussion on calculation coordinate system, and then talk about cylindrical coordinates and spherical coordinate system. In the previous module we have already introduced the need for coordinate system, we have also told you how one dimensional, two dimensional, three dimensional Cartesian coordinate systems look like.

In a three dimensional Cartesian coordinate system it is actually the intersection of three planes one plane can be this x z=0, this 0 upon this plane or below this plane you are moving along this z-axis let us say. So then there will be other plane which is you might look at my palm of the hand, this would be x and z. So if you move along this line, which I am showing that would be the x movement, you can move up and down towards the z movement.

Similarly, there will be a yz movement here, so if you move like this, this would be the y-axis and if you move up or down you would be moving along the z-axis. Any point on this three dimensional system can be described with respect to the origin by giving three coordinates, one coordinate will be x which could be the projection down there and then finding out the x component and the y component, and the other will be the z component.

So how much along x, how much along y and how much along z if you move you would be reaching this particular point in the space will be given by the three dimensional coordinate system.
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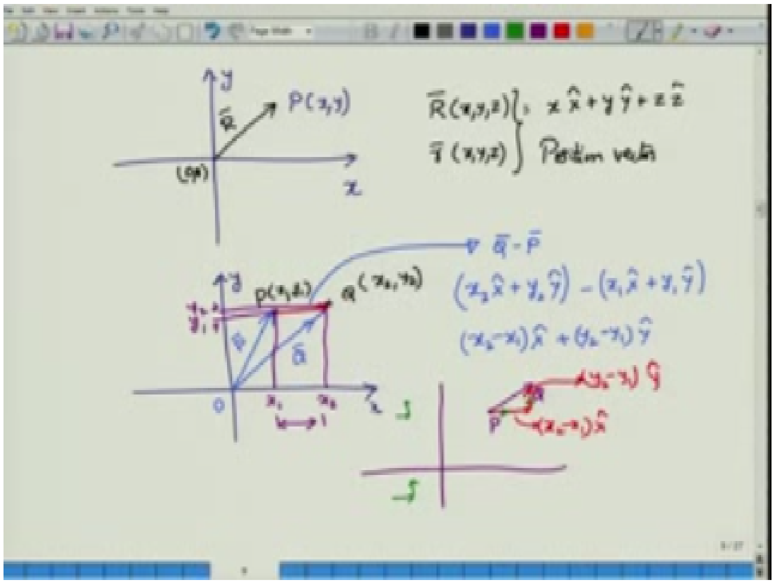

Now it is something that is interesting that even in the two dimensional case such as the one that we have considered x and y every point can be considered to be not just a point, but we can associate a vector to it, this special vector will start at the origin okay of the residence point with 0 and 0 as the coordinate for children and then when you extend the line from the origin to the point p with an arrow at the point p, then this special vector becomes what is called as the position vector.

And if position vectors are either described by r or the r okay with a bar on top of it and this position vector is of course the function of x,y and z. This is the three dimensional which I am drawing, but I am showing you two dimensional one, but I hope we can understand the connection between the two. This position vector is given by x x^+y y^+z z^. So whether if you may use the rotation for r with r this becomes what is called as the position vector okay, which means that at every point this place I can actually imagine the line joining from point j to the center point making a position vector.

And I distinguish another concept with you, suppose I have again the two dimensional case, suppose I have two point, let us call these point as point p and call this point as point q okay. Now I can have a vector whose head is at point p and whose tail is at q that is originating that point p and going to q. Let us say p is described by x1 and y1 coordinates, q is described by x2 and y2 coordinates.

Then what can be use or how can we describe this red color line which is actually a vector from point p to point q okay. You can imagine for that we actually draw the position vectors so this is the position vector from the origin to the point p, this position vector let us call this as p okay. And similarly, we had a position vector which goes from origin to the point q which we will call as q.

Now from the law of addition you can clearly see that this vector which is joining p and q is given by q-p, why, because p+q-p will be equal to q which is exactly the vector q that you are looking for okay. And in partition coordinate system how do I describe the vector q and vector p, the vector q is given by x2xhat+y2yhat in case we were considering a three dimensional scenario, then there would have been a z2 z^ as well.

But this is a two dimensional case that we are considering, therefore the reverse head component. And then what is the p vector, p vector is x1x^+y1yhat normally you would subtract these two, you can see that you can individually now correct the components and you obtain x2-x1x^+y2-y1y^.

Geometrically, what this means is that we shall project this one in a vector the red vector on to the x axis I need at point x1 and x2 and the difference between x1 and x2 if size the length that I have to move along the x axis, similar will be the case for y vector and y2 although in this case we are not looking very different because the vector is almost horizontal to the x axis in general you can have any vector and the difference in the y that you would have to move will be this particular vector okay.

Moreover if vector red vector here itself, then you can imagine that so let us we draw an another vector out there so this is the vector that I am considering whose origin is at p and the ending is that q right, if you look at this vector itself it can actually setup a vector whose length is x2 – x1 into x^ which will be obtained by projecting this vector Q on to the x axis correct, so the length of that projector vector will be x2 – x1 into x^ similarly there will be a vector that you can setup which will be going in the upward direction.

That is along the y directions here and that vector will be y2 – y1 into y^ okay, and notice the important thing the directions of the unit vectors along this x2 – x1 x^ is parallel to the x axis and the direction of the y vector is parallel to the y axis, inside this is very true for a Cartesian coordinate system that no matter at what point you can say may be you consider this point the unit vectors are always directed along, the parallel or directed along the x and y axis which are parallel.

It does not matter more you are located the unit vectors are always located in the directions, so they do not change the direction at all so this is the important point to keep in mind because we will see that in the cylindrical coordinate system, this will not be the case okay.

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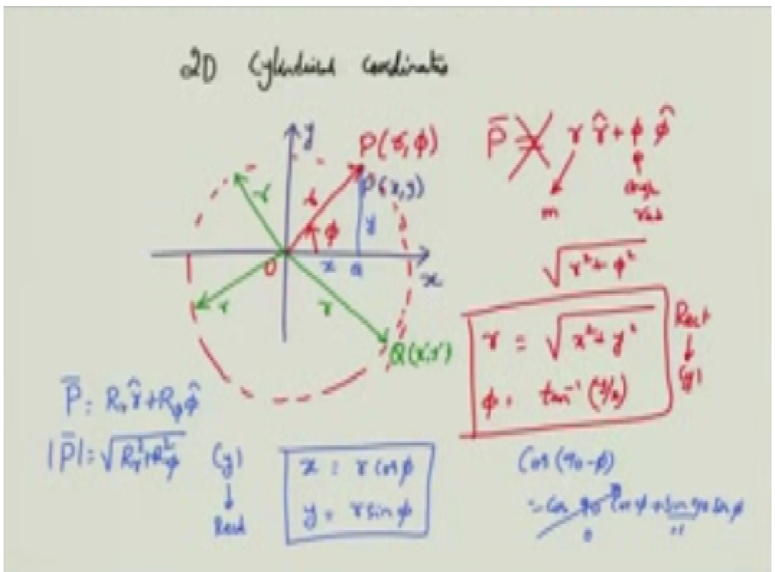

So now let us move on to cylindrical coordinate system we consider the two dimensional version of it first so we are now trying to go to a different coordinate system why would one want to get your different coordinate system, the answer is quite simple there are some cases you know for example like charge distribution that is present along a certain line or a current there is being carried by a particular wire then the corresponding magnetic field will be in the form of circles which would be surrounding this current we will be seeing this one as a consequence of Maxwell's equation.

Later on but any current carrying wire will have magnetic fields which are circling around, and the best way to describe this kind of a scenario and the best way to describe this kind of a scenario is to actually imagine cylinder of may be different radius is so we imagine that one cylinder is kept inside another inside and another of different radiuses okay, so here describing every point in terms of x and y becomes slightly difficult especially a three dimensional case the easier one.

Is to actually describe the distance are and the angle 5 and whether you are up along the positive z axis or negative axis, so in some cases naturally there are situations where a particular coordinate system might be useful, for example then we are dealing with antenna okay, we can see that the fields of an antenna radiate outward so the best way to describe to this kind of scenario is not of course a cylinder but rather is be sphere because the radiation fields start you know for a simple isotropic.

Radiation antenna it will actually come out in the form of sphere so there one has to go to the spherical coordinate system so depending on the symmetry of the problem we will end up of using 1 or the other coordinate system, the one that would be easiest to use will be the 1 that is decided by the problem itself, so we will have a good reason to study this cylindrical and spherical coordinate systems, simply because there will be useful late on okay. So we will look at the two dimensional version of the.

Cylindrical coordinates and then built to a three dimensional version of a cylindrical coordinate what is a two dimensional version of a cylindrical coordinate wait that to the plane in the plane we have set up the x axis and the y axis and described this particular point by giving two numbers x and y these numbers represent, how much I moved along x and how much we will moved along y okay, consider the different way of approaching this one I know that I can actually set up a vector here.

Which is the position vector and the length of this vector we tell meet the particular point where I am looking at okay, so there is a length of the vector which we will call as small r so that would be the length of this lines segment from the origin so from the origin to the point be here of course very different vectors in certain infinite number of those vectors will all be directed at the cylinder point why because you know you can for example consider this green colored vector although it is not looking like that but I have actually drawn a circle over here okay, this will also have an arc this vector will also be R this vector will also be R okay that this the length of this vectors are all equal and they are all R.

If a draw a circle of radius R here, so how do I distinguish a point P which is sitting on this red vector to a point Q which is sitting at different point X' Y' on the coordinate system well one can actually one need to specify second number in order to completely fix this problem and that number will be specified by the angle with which this radial vector or radial directed line segment makes with respect to the x-axis, okay.

So from the x axis which I have chosen as a reference you measure the angle of rotation and this same point P which was expressed in terms of x and y Cartesian coordinate system can equivalently be discovery by specifying the angles R and ϕ, okay. The same thing but P(r, ϕ) does not mean that the vector P the position vector corresponding to P is equal to r, r^ + ϕ, ϕ^ this is simply wrong, why it is wrong?

Well there are many reasons why it is wrong but one important reason happens to be that even though I am measuring the rotation and specifying the point P by the angle ϕ please remember that the units of R is in meters or it is actually a distance measuring variable, ϕ is an angle measuring variable right, so the units of foe is an angle or radians if you want to consider that one, so I cannot just find the magnitude as $\sqrt{r^2 + \phi^2}$ just to had done for the case of a Cartesian coordinate system.

Were I took the length of the vector to be $\sqrt{x^2 + y^2}$ okay, I cannot do this one in fact this value is equal to r which is the radial length, right? And what would be the value of ϕ given x and y the angle through which you will have to rotate in order to reach point P will be $\tan^{-1}(y/x)$ whereas this $\tan^{-1}$ relationship coming from, you just drop a perpendicular clearly this will be x this will be y okay.

And you are looking for the angle ϕ from this right angle triangle OP and may be this is the Q point Q so from this right angle triangle you have $\phi = \tan^{-1} (y/x)$, similarly x in terms of r and ϕ that is if r and ϕ are given to you, you can find what is x and y, x is simply the value of R projected onto the x axis right, so that would be r Cos ϕ and y will be this angle r projected onto the y axis and that would be 90 – ϕ.

So Cos (90 – ϕ) is Cos 90 Cos ϕ + sin 90 sin ϕ because Cos 90 is 0 this is gone to 0 sin 90 = 1 so this is nothing but sin ϕ, the value of y in terms of radial values r and angle ϕ is given by y = r sin ϕ, in fact you can use these two equations to go from 1 point specification in the cylindrical to the Cartesian coordinate system so this is the form of equations that you will use when you have to convert from rectangular to cylindrical.

This would be the form that you are going to use the equation if you want to convert from cylindrical to rectangular, so far we have considered how to represent points on a two dimensional cylindrical coordinate system, what about the vector itself, in general I can write the vector P as R_r, r^ + R_{ϕ} ϕ^ and the magnitude of this vector P will be given by $\sqrt{R_r^2 + R_{\phi}^2}$ okay, where R_r and R_{ϕ} are there are the components of R and ϕ. But here is something that I want to actually distinguish.

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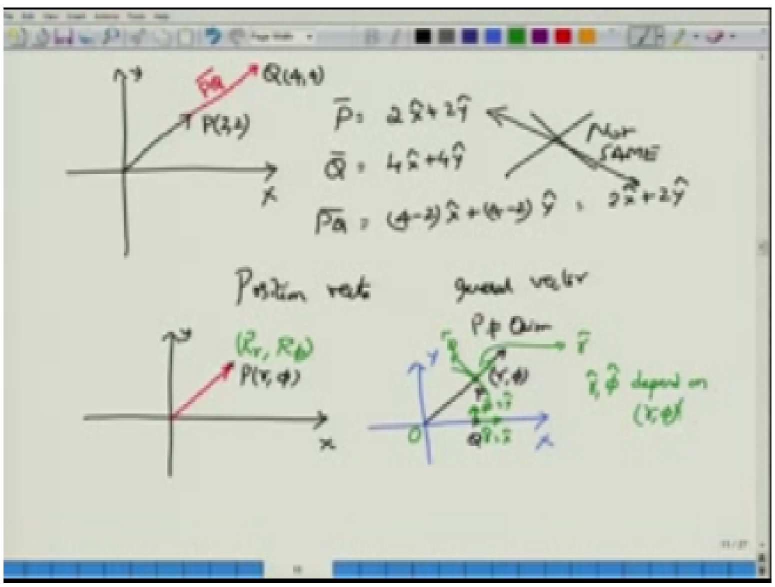

Let us go back to that Cartesian coordinate system for simplicity so I have x and y point here and I have a point P which will be say given by 2 and 2 okay what is the position vector for P the position vector for P is, 2x^ + 2y^ now we can also imagine a point Q which would be given by the point 4 and 4 the corresponding position vector for Q is given by 4x^ 4y^, now imagine that I actually draw a vector from P to Q.

That is I have a vector which originates from P and terminates in Q, what is the vector that describes this one, let us call this vector as some vector PQ, what is the vector PQ given by? The vector PQ is given by (4 -2)x^ + (4 -2) y^ which is equal to 2x^ + 2y^ right, but please note here I have a position vector for P which is 2xф+2yф I have the vector PQ itself which is 2xф+2yф.

But these two are not the same they are not the same at all, why are they not same because 1 is a position vector which actually given from the origin right, so this is given from the origin to the particular point and the other one is a same vector I mean the given is not the same vector because that vector PQ goes from point P and goes to the point Q, okay.

So you have to understand the difference between position vector which always originates from the origin and terminates on to the particular position of a point that you are considering and a general vector that can originate from P which is not the origin in general and it can go anywhere it want, this is a very curial thing that you need to remember one, okay. Now go back to the cylindrical case, on the cylindrical case I still have a plane x and y let us pick a point P which we will label as r and ф.

What is the vector that corresponds to this one in the cylindrical coordinate system, okay if I want to do that one I better first convert how to go from one coordinate system to another coordinate system okay, and from there I will showed in I mean in the corresponding cylindrical coordinate system the answer is not as what you would think, okay. There is in fact no specification of the value of Φ in the coordinator answers, the all the position left as directed only along r which is something that you can see very clearly.

Why, let us see first the corresponding unit vectors rф and фф on the plane that we are considering we will do all the calculations in the two dimensional plane and then go to three dimensional case. So I am going to consider a point here which would be again r and ф and now you actually consider the radial direction here, okay so the point P will have some length but the vector that you can draw from the point here so you have to see this point okay, let us call this point as P from the point P if you draw a unit I am sorry, in the increasing direction of OP that forms the unit vector along the cylindrical coordinates.

How about the ф vector well, you have to draw a circle here or at least a part of the circle and then draw a perpendicular or a tangent to this arc okay, at this point and this becomes the ф vector okay, you can immediately note that both rф and фф depend on r and ф, you are not convinced about it let us look at this point here, I am going to consider a point let us Q on the axis, what is the radial vector here.

The radial vector will be directed in this direction so it will in fact be equal to the x vector, what about the ϕϕ the ϕϕ will be directed perpendicular because ϕ=0 which means this is the line which is the arc and if you draw a tangent to that line then you are going to the ϕϕ you can see that the ϕϕ is actually equal to y and the rϕ is actually equal to x, so you see that just between points Q and point P the direction of the rϕ and the ϕϕ themselves are completely different.

Because they actually depend on the values of r and ϕ okay, this fact will be very useful then we want to specify or when we want to find out the corresponding values of Rr and Rϕ and therefore formulate a cylindrical coordinate vector over here, in fact we will show that for this particular red vector that we have drawn the corresponding value of rϕ will be equal to 0, okay. How do we do that?

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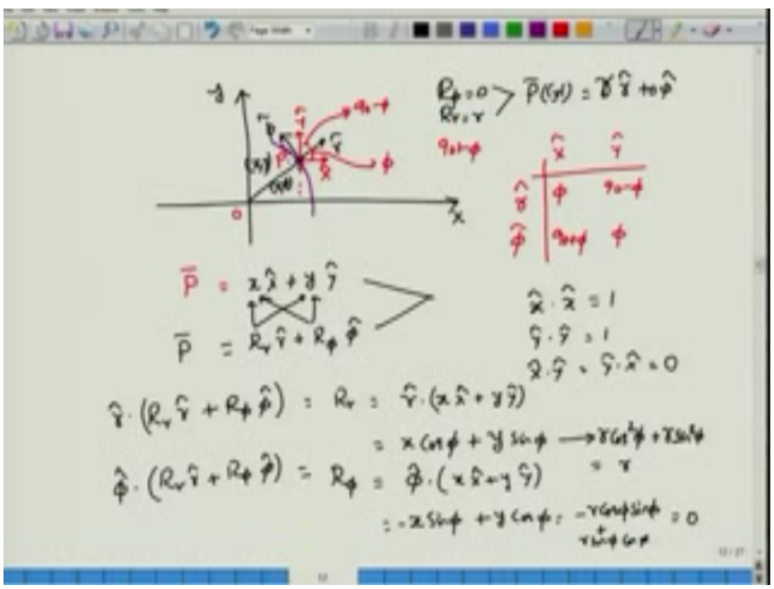

Let us examine the relationship between the ϕϕ, rϕ that is the unit vectors and the x and y unit vector, so this is the r so from this point let us say this is the rϕ and then this is the ϕϕ these are the unit vector that we have drawn but at this point I can also construct the unit xμϕ and the unit yϕ okay, and what is the angle between r and x, the angle between r and x is ϕ, the angle between r and ϕ is 90-ϕ, the angle between y and ϕ is ϕ and the angle between x and ϕ is 90+ϕ okay.

So let us say the angle between x,y,r and ϕ just to put it into a table so that you can easily read of the angle between r and x is ϕ, the angle between ϕ and x is 90+ϕ because there is a 90ϕ to go to y and y+, y to ϕ I mean ϕϕ okay r and y will be 90- ϕ and ϕ and y will be equal to ϕ itself okay now I know how to specify so let us call this point as P right so I know how to specify the position vector P either you specify well I know what is the value for x and y at this point.

So there are those generic x and y points so I can specify this as x x ^+y y ^ but I want to now specify this one in terms of Rr r^ +r ϕ ϕ^ okay were Rr and R ϕ are the components along r direction and the ϕ direction okay so physically these two are suppose to be the same because there are just a same point for this is the same vector it does not matter whether I am considering the vector in the Cartesian coordinate system or in the cylindrical coordinate system.

However I know that I have written down these two by aim is to find R_r in times of x and y $R_Ø$ in terms of x and by how do I do that let us go back to this equation so let us call I mean this is the pollution vector P right so I will write this one to the left hand side okay and then ^r throughout this vector.

So when I take the dot product I already have seen in the Cartesian coordinate that x^ x will be equal to 1 y^ y will be equal to 1 x^ y which is equal to y^ x is equal to right so when I take the product I mean in the similar way is replace x/R and $y_ϕ$ you are essentially get the same violation that is $R_ϕ$ are perpendicular to each other so very well known from this particular thing right.

So now If I take the ^product here what I get is R_r but if I know go to this vector P which is in the red vector and then take the $R_ϕ$ with respect to R what I get $R_ϕ$ xx^+yy^ I know what is $R_ϕ$ x^ I know what is x right x is just x here in this case what is $R_ϕ$x, $R_ϕ$x will be unit magnitude that would be cause site so I will have x cause Ø +R $_ϕ$y, $R_ϕ$y is basically of 19-5 which will already seen to be equal to $\sin_ϕ$ so this would be equal to y $\sin_ϕ$ okay.

Similarly if I take the product which respect to ϕ^ then I will get R_r, R^+Rϕ ϕ^ this would be equal to $R_Ø$ component and now we have to go to ϕ^ x^ so here it could be ϕ^xx^+y y^ so this here you are going to get as x times what is ϕ^x, ϕ^x will be cause of 90+5and that would be –sin ϕ so it would be –x sinϕ and you will have for the component for yϕ^y, ϕ^y will be equal to y cosϕ correct.

So I have got this relationships for R_r and $R_Ø$ from here okay so we see here that R_r is given by x cosϕ=y sin Ø R Ø is given by –x sinϕ+y cosϕ but our job is not complete here because I need to replace x and y by the corresponding values of R and ϕ so the point where I am considering I know that x can be given as r cosϕ and y is given as r sinϕ so what will happen to component R_r it becomes r cosϕ+ y is r sinϕ.

So that becomes r $\sin^2$ ϕ which is actually equal to small r okay. What will happen to this fellow Rϕ you can see that when I substitute for x the first term becomes –r cosϕ sinϕ+ the second term becomes r sinϕ cosϕ but one of them is carrying a negative sign so therefore this becomes equal to or this becomes 0 right so what way I have just found out is that if I go back to this picture but I have x and y is the plane of we have consider and we have considering the position.

That they are of t in Cartesian coordinate the position vector was given by x x^+y y ^ because it point P is described by the points x and y coordinates in the Cartesian coordinate systems but the same point P is described in terms of R and ϕ okay in this case what we have just found out is that corresponding component Rϕ will be equal to 0 while only R_r will be equal to R so the correct position vector for point P in cylindrical coordinates systems is given by r r ^ there is no ϕ^ component over here okay.

It does make sense because r will only give you the increasing directions of r and now if you imagine that you can actually draw circle the direction along increasing r happens to be the direction of P itself right there is no ϕ component require to specify P but you should not hastily conclude that r ϕ will be equal to 0.

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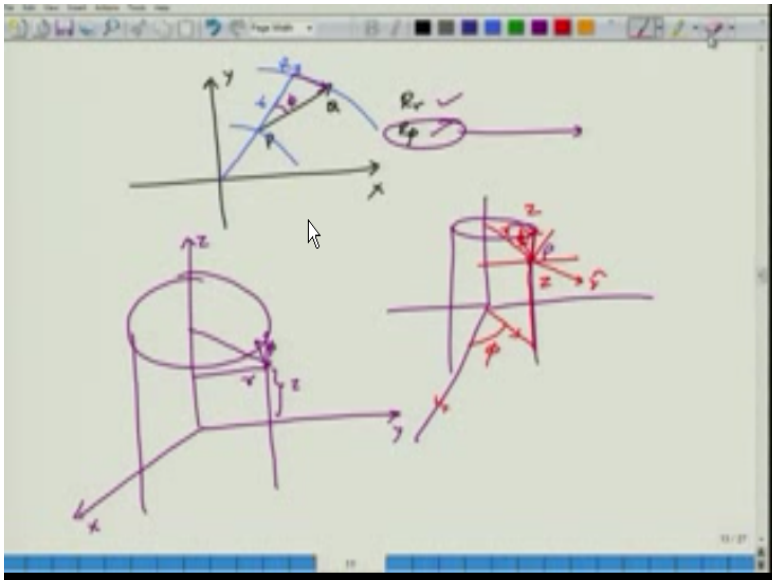

There are many cases when r ф will not be equal to show you that I will not derive it here but you can see that if I consider the general vector which let us say go from, from point P to point Q okay then this general vector cannot be return just component with just rr here r ф should be non zero why because you can again imagine that there is a position vector in the cylindrical coordinates.

There is one more pollution vector in the cylindrical coordinates for P vector and the Q vector this difference vector is still Q-P but you cannot now just go back and say no r-r okay so in days you what you will be generating is the totally different r okay the correct way to solve this problem in a cylindrical coordinate systems which is also the hardware is to first convert this P into the rectangular coordinate systems convert this Q into the rectangular coordinate systems.

Then take the difference oaky that difference that you obtain will be in the rectangular coordinate systems after you taken the difference you can again go to cylindrical coordinate systems there you will see that r ф will be non zero okay geometric way of seeing the same thing will be remove this red color position vectors okay and then just imagine but I have this general vector right.

So which was going from P to Q and if I initially consider going along P. So I will start my journey at P and going in the increasing direction of r the increasing direction of r for point P will actually be along this directions so I made go up to this one so there is a circle here and then there is a circle here so I have actually gone up to this point right.

So let us call this point as capital R I have moved a certain distance let us say this certain distance have moved along R but I do not reach Q I reach a different point r in order to reach point Q I have to move in the direction opposite to ф go over this distance which corresponds to whatever the radius of this Q point circle that I have drawn at Q times the angle through which have moved okay.

So you do see that r Ø and rr both can be non zero but for the position vectors R ф will always equal to 0 okay so that is the difference that you should keep in mind okay we now quickly extend it to three dimensional cases we talk about represent the point on a cylinder okay you can imagine that I actually have a cylinder which is kept along the Z axis okay and from there I can try to find out any point in terms of the cylindrical coordinate systems.

So let us say I find a particular point over here and then the radius distance of this point okay will give me the point on the cylinder oaky so if I have done a cutter some were here I should actually have written a nice cut along this axis but any way it does not matter so in the three dimensional case the their values which are going to tell me what those values are, are given by the radial distance r from the point.

And then the height Z and then the angle ф because on a three dimensional case may be the diagram was not very nice wait let me retry it again I mean you have to first consider point r and then project this point and from the projected point consider that would be the angle sign okay and this would be the Z axis that you are going to see and this entire thing would be the point or the radial distance that you have consider.

So the radial distance is actually coming from this point so this is the radial distance and this is the height over which you have considered or which you have located the point and if you now drop a perpendicular on to this axis and angle which this line this red line make with respect to x axis will give you the angle sign okay.

So you have this three variable to actually tell you the corresponding values at the same time however three different unit vectors for oriented the unit vectors are oriented along r so this would along Z and this would be along ф so this would along ф over here it juts the diagram is not very nice I will show you a proper diagram so this are three directions of R ф and Z may be slightly better way to illustrate the entire thing is to look at this cylinder over here.

So I have this cylinder over here which is representing a constant radius r okay, and this axis with below z-axis any point on the cylinder, you know you can look from the top here or you can see from this point, let us call arbitrarily this particular circle to be located on the x and y planes which as would as, you know you are taking some kind of a plane and cutting off here.

And measure this height okay, so if you measure this height you are going to measure the value of z and if you start showing this center of the origin and then draw a radius, so it you can imagine that there is a this particular plane is coming down. The length of this one will be the r direction and this would be the direction in which r would be continuing, and this is the z-axis. And then if you project this point down to this x and y plane, and then measure from this, this is the x-axis that I have make sure how much or where is the angle that this projector along the x-axis that will tell you the corresponding value of angle 5 okay.

And this would be the direction in which ϕ increasing, so that is the various method direction. This is the direction in which z is increasing, therefore this is the very quick along z. And this is the direction in which you can imagine from the top and then you are going like this. This is the direction in which the r matrix is located, because we can open at this cylinder and then make a radius of the cylinder larger and larger okay.

So you are actually expanding in this particular direction. At this point I will also like to introduce couple of other method n as which we have not done so in the Cartesian, we will just show you in the cylindrical coordinate system and then go to the Cartesian coordinate system. See suppose I am having this particular cylinder, I start at a point which would be some r, ϕ and z okay.

If I move along this line, along z, so if I move along this line here that we correspond to moving along z-axis right. So if we move and wait of some dx along z-axis a small distance dz along this Z-axis, I can represent that movement as a small vector okay as the movement or the line vector dz, z^ okay. Keeping other long road and small distance z if I start moving along this circle of a constant radius, then I will actually be moving along the ϕ axis right.

But distances along ϕ movement, movement along the circle will move at distance of ϕ will not be measured by the amount of angle itself, but that has to multiplied by ½. So the same angular movement on the shorter cylinder radius will give you a shorter distance than the same movement of an angle. So if you move 30° for the shorter radius, we will move a smaller distance, but the same 30° movement on the larger radius will give you a larger distance.

So the movement along ϕ is actually given by rdϕ, where dϕ is the angle through which you move and r is the constant radius that you have. And you can now actually imagine that you have completed this particular movement move back along the dz direction, move back to the starting point. What I have done is to generate disclaimed surface area which can you observe very closely, you would look like a small plane segment okay.

And this plane segment will be oriented this is at the orientation of this plane segment of area rdϕ dz will be along the z-axis. Similarly, if I move along this side, but then imagine that I am expanding this radius okay, when I move to a certain distance along ϕ which corresponds to rdϕ, and then I move along a certain distance sorry, this is actually the movement along r and z which I have already illustrated.

So first I expand the cylinder, so I move a small distance dr okay, so that is dr along r and then I move a certain distance dz. So again you can imagine that I am moving along dz while simultaneously expanding this one to r. So the correct, the loop that I am going to obtain which would be like this particular loop that would be oriented along the ϕ axis, and you would had a area from here, dr and dz okay.

So what you are generating or what we call as line segments and the area segments or the surface areas. Let us go back onto the board and then show you what exactly what I was talking about.

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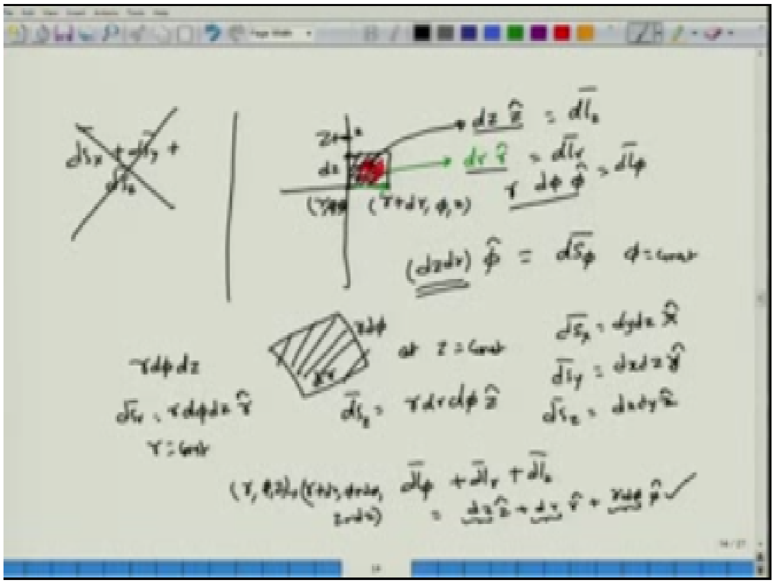

So I have this cylinder right, of some radius r that I had. So if I continue to move along this value r, I would be moving to a new point which will be represented by r+dr all the other points are the same, so all the other points are the same points okay. However, this movement from r point the r+dr point can be expressed in terms of the unit vector along r. So this movement is actually a vector movement given by dr r^ which tells me that I have to move along r direction by an amount of dr, if I move along z so I move a certain distance dz along z axis I can consider this movement so I would be landing from z to z + dz point right, so I would landing up z + dz point and I would have moved a distance of dz, z^.

Now if I consider dz, dr to be the product you know in over which I have actually moved and then what I will actually get is this particular surface area what I am actually forming is what is called as the cross product okay and this product will be oriented along the ϕ axis so the magnitude of that cross product will give me the area or the in decimal area which would be coming out of this plain this would be along the ϕ direction, okay.

So this particular infinity decimal surface area in which ϕ is constant I mean movement along z and r is called as the surface element or the surface area which is oriented along the ϕ axis and this would be the vector surface area which means that you can associate a vector to the area, area being a scalar you actually associated a vector to that particular scalar okay, similarly you can imagine moving along r and then moving along ϕ.

But if you move along ф you would be moving a distance of rd ф, okay moving a dr or along r axis would have given you this one, now if you complete this movement at a constant z so at z = constant what you have just done is to obtain the surface element which we refer to as ds_z and this is given by r dr, d ф oriented along z axis, okay. Finally if I can move r $d_ф$ along the ф axis right so I have actually I need to now find out what is ds_r, right.

So I can move r d ф on the ф axis and then move a distance of dz on the z axis, so when I do that I have what I will get is this element which is oriented along the radial direction, so these are three surface elements which represent ф = constant, z = constant and r = constant surfaces so along these surfaces if you move a certain distances then you would be generating an equivalent amount of oriented vector elements.

This is very important when we consider the surface integrals later on, okay. Similarly I can show or the surface elements oriented in the Cartesian coordinates for the x- axis will be given by moving a distance of dy along y axis, dz along z axis and this would be oriented along x, similarly dx_y will be equal to dx, dz along y and then I have ds_z to be equal to equal to dx, dy along z, okay.

So these are the three surface elements incidentally I have moved a certain distance along r and along z and also I have moved a certain distance which I did not write here but I have moved a certain distance of r d ф along the ф direction, so this is these three movements which are moving along the specified directions along r ф and z are called as line segments, okay. So I have actually moved in an oriented way along r direction, z direction and the ф direction.

So these are the line elements so you can call this as dl_z this as dl_r and this as dl ф okay and in fact if you move a distance of dl fie okay and move a distance of dl r move a distance of dl z you would have actually moved from a point r ф z to r + dr, ф + d ф and z + dz.

so you would have actually moved from this point older point to a new point okay, in this case you can see that the corresponding vectors can also be written, so or the corresponding line element movements are very similar to what we would have seen in the Cartesian coordinate system except you can think of this as dxф, dyф and dzф right. So here you will be thinking this of dzzф+drrф+rdффф so you can see that this is actually a vector in a cylindrical coordinate system with rz given by dz, r given by dr and rф given by rdф okay.

So this is how you can construct a line element and go from any point into any point on the cylindrical coordinate system. But do not make a mistake of combining these and thinking of this as a vector okay, so this is not a vector okay, because these are areas you cannot combine areas which are pointing in three different directions or into three different things and then try to combine them into a vector, okay.

Because here you can move the vector this is not a proper vector, so this is not a correct vector to talk about whereas this is a correct I mean something that would be useful in the line integrals okay, before we break off from cylindrical to Cartesian coordinate system.

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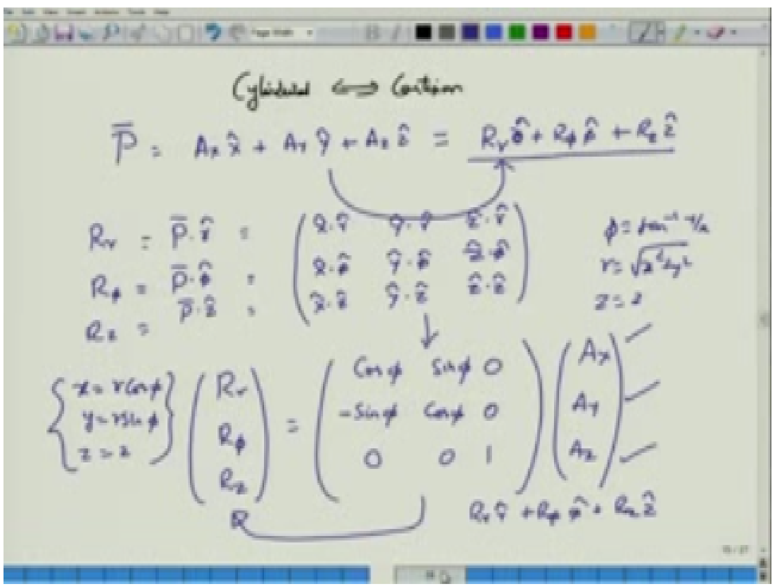

Let us look at the full in a process of converting from cylindrical or Cartesian and back okay, we have already seen a glance of this when we converted the position vector, but let us carry out the analysis in three dimensions and complete this process okay. suppose I have a vector P which is described by giving the elements along x,y and z so it has the components of Ax, Ay and Az okay, let us also assume that the same vector is given by Rrrϕ+Rϕϕϕ+Rzzϕ okay, how do I go from this Cartesian coordinate system to a cylindrical coordinate system.

Well, again I would consider these vectors P.r, P. ϕ and P.z right, when I do that I will in between we will trying to find out what is xϕ.r, yϕ.r, zϕ.r because of the dotting of ϕ with respect to P I will obtaining x. ϕ, y. ϕ and then z. ϕ and then I will have because of P.z I will have x.z, y.z and z.z okay. So once I find this particular matrix kind of thing if you can imagine this will be what is this P.rϕ if you go back to this equation which he has expressing the cylindrical coordinate system this will be equal to Rr this will be equal to Rϕ this will be equal to Rz.

So what you have is a vector whose components are Rr, Rϕ and Rz which you can arrange it in the form of a column vector then there will be this transformation matrix which will consist of the inner products are the dot products of these elements multiplied by whatever the values of Ax you have, Ay you have and az that you have okay.

So what is the matrix here, x.r we have already seen this is nothing but cosϕ, this value is sinϕ y.r is sinϕ, what about z.r luckily for us z and r are perpendicular to each other therefore this would be a 0, x. ϕ we have already seen to be equal to –sinϕ this y. ϕ we have already seen to be equal to cosϕ and then this is 0 x.z is 0, y,z is 0, z.z is 1, so given any vector with Ax, Ay and Az you can use this matrix to get to the values of Rr, Rϕ and Rz and further write x as R Cos ϕ y as R Sin ϕ z of course is the same z.

So if you use these relationships, and the values ax either and az then you can consider the vector P in the cylindrical coordinate systems which will have RR as the R component Rϕ as the ϕ component and Rz as the z component you can actually try this example with the position that vector itself that is if I consider the position vector.

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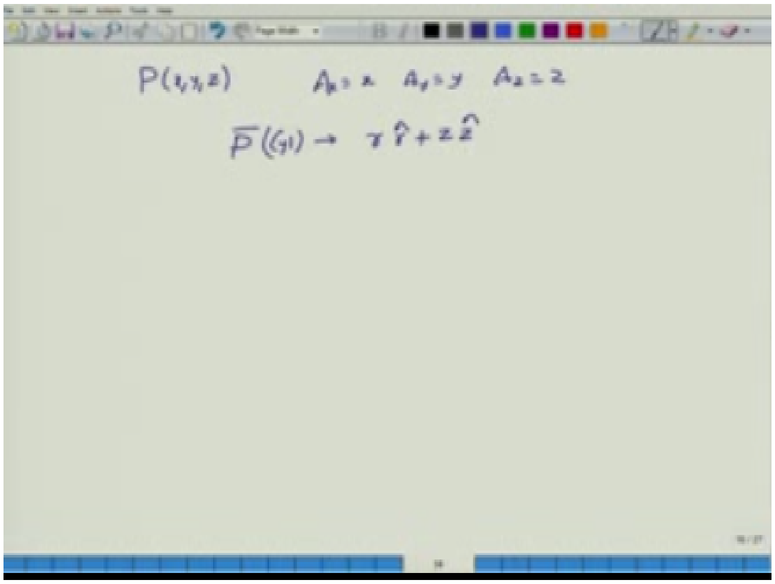

Xyz meaning ax = x ay = y and az = z we will see that the same vector P in the cylindrical coordinate system will be described by RR^ + z z^ again there would not be any find into that expression okay, so you can do this one on machine transform the cylindrical or triangle cylindrical if I actually go back and do the same thing. So you can go to case of cylindrical to Cartesian coordinate you just have the invert this particular matrix and replace ϕ / and inverse y / x R / √ x2 + y2 and az of course will still be = same value of z, okay so to summarize in the Cartesian coordinate system.

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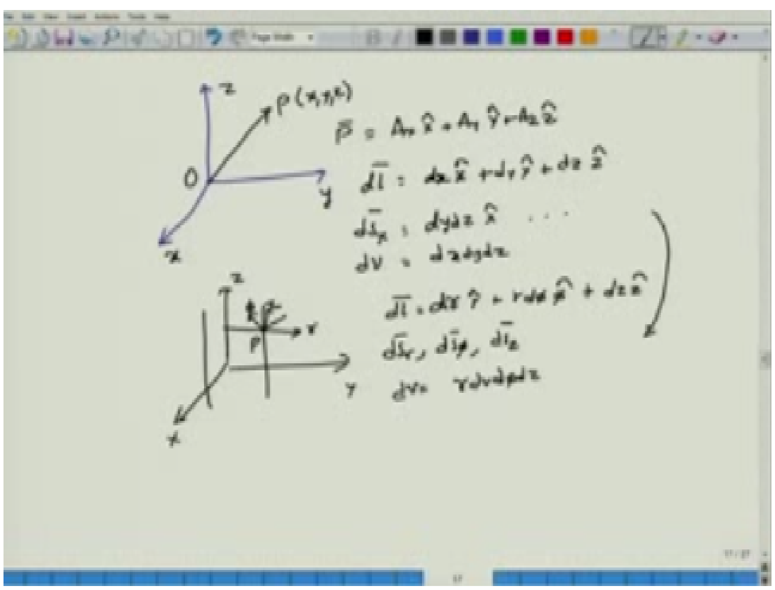

Which is formed by 3 mutually perpendicular axis x y xy and z the point where they all mate is called as the origin n point p can be described by the three points x y and z and a corresponding position vector okay which will be described by ax x^ + a/ y^ + az z^ and if you move a certain distance along x y or z the line element in the three dimensional coordinate system will be given by dx x^ okay + d / y^ + dz z^ similarly the surface element will be given by depending on this surface you are considering that x element will be given by.

d/ dz x^ and so on okay, so the cylindrical coordinate system before the cylinder coordinate system there is one more element which is the volume element which is given by dx d/ and dz and likely this is just a scalar for us okay. So if I go to the cylindrical coordinate system and describing this one by 3 coordinates R ф and z okay so Rф and z, so any point p can be described by 3.Rф and z again we have seen what is the line element this would be R or rather d_r r^ + r dф ф^ + dz z^ we can also now write down what is ds_r dsф.

And dsz as the 3 oriented area elements and the volume is of course given by rd r dф and dz okay, so this is the rectangular which allows you to go from one coordinate system to another coordinate system we would not talk about the spherical coordinate systems simply because the corresponding elements are slightly complicated. So I will refer the text book for you to understand the spherical coordinate system the corresponding lines surface and volume elements than you very much.

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