Principles of Signals and Systems
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Lecture – 15
Example Problems LTI Systems – Eigenfunctions, System Described by differential Equation, Homogenous and Particular Solution

Hello, welcome to another module in this massive open online course. We are looking at problems on the analysis of LTI systems.

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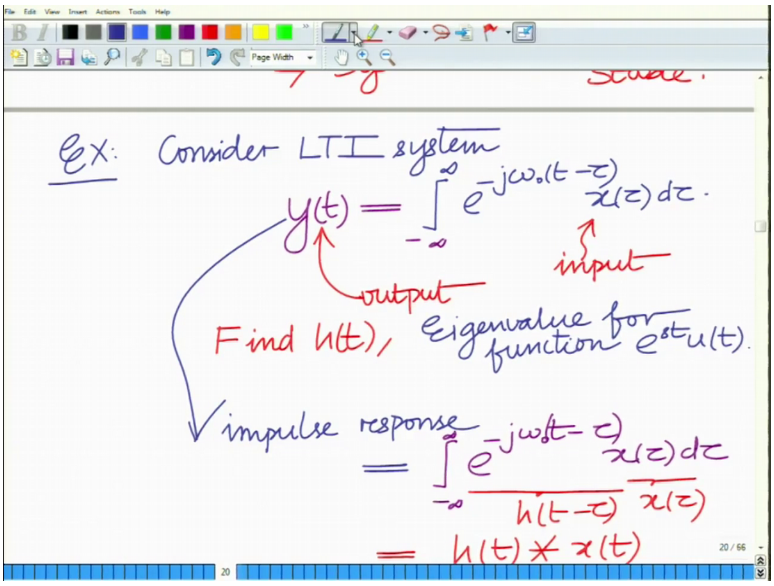

We are looking at some example problems for that to illustrate the analysis and properties of LTI systems. Before I precede further let me just modify the previous problem slightly. What I am going to do is rather than making in type this integral minus infinity to infinity, I am simply going to change this to integral minus infinity to t and now we can write this as this can be written as the following thing.

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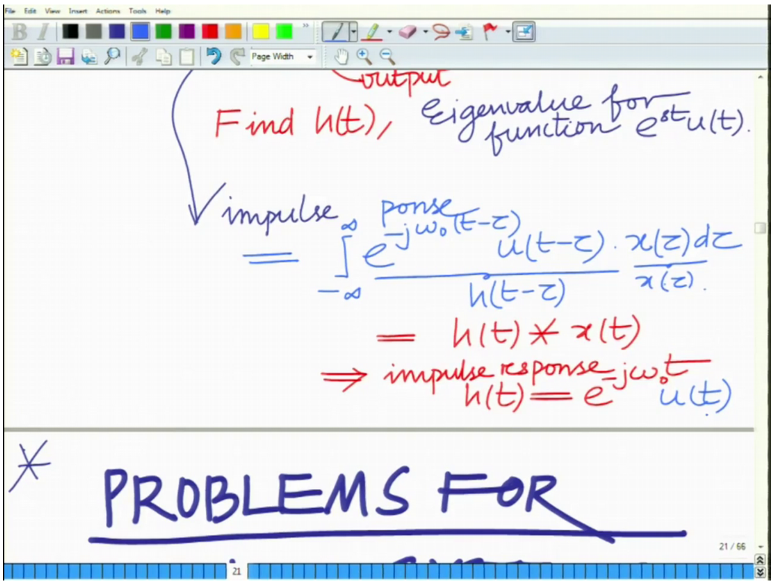

The impulse this can be written as integral minus infinity to t e to the power of minus j omega naught, t minus tau u t minus tau. I can change the limits from minus infinity to infinity tau because I am incorporating u t minus tau times x tau d tau, now you can see this is your h t minus tau this is of course x tau is t minus tau into x tau this is h t convolve with x t.

The impulse response is h t times impulse response h t is e raised to minus j omega naught t times u t, that is the impulse response let us note that down.

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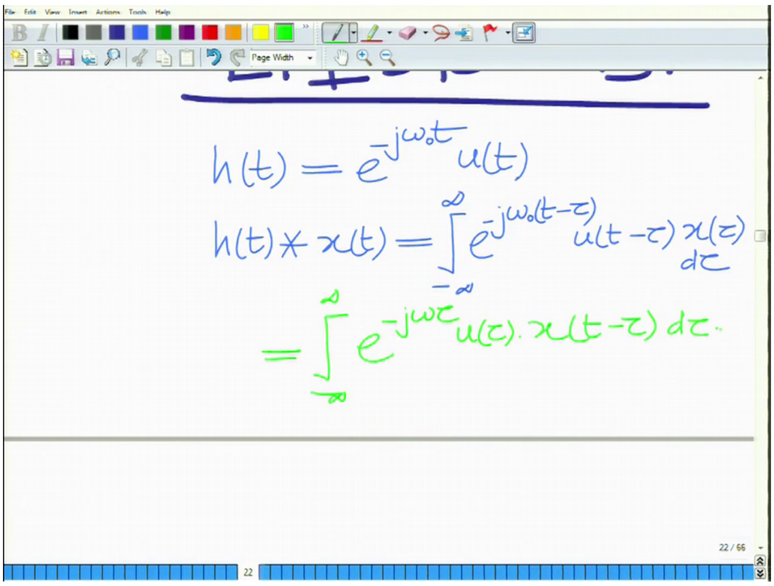

Small correction we have the impulse response h t equals e raised to power minus j omega naught t times u t, now to find the Eigen value corresponding to e raised to s t, we note that this can also be written as the impulse response h t convolved with x t that is e raise to minus j omega naught t minus tau or e is raised to minus j omega naught all x slightly this way it has to minus j omega naught t minus tau d tau x t d tau.

Now, this I can also write this as x t convolved with h t which means this I can write it as e raise to minus j omega naught u tau x t minus tau d tau, now since u tau is nonzero only for t greater than or equal to 0.

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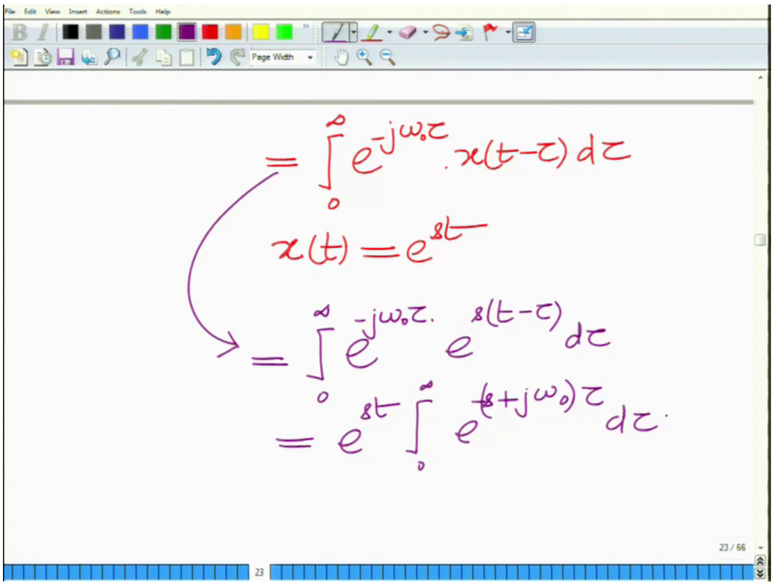

This can be equivalently written as integral 0 to infinity e raised to minus j omega naught tau x t minus tau d tau just equivalently writing this as integral 0 to infinity e raised to minus j omega naught tau x t minus tau d tau and now I am going to substitute our function or signal x t equals e raised to the power of s t and therefore, this becomes equal to 0 to infinity e raise to j omega naught tau e raise to s or e raised to s t minus tau d tau. We bring the e raised s t common outside. This is integral 0 to infinity e raised to minus s this is e raised to minus j omega naught minus s plus j omega naught tau d tau.

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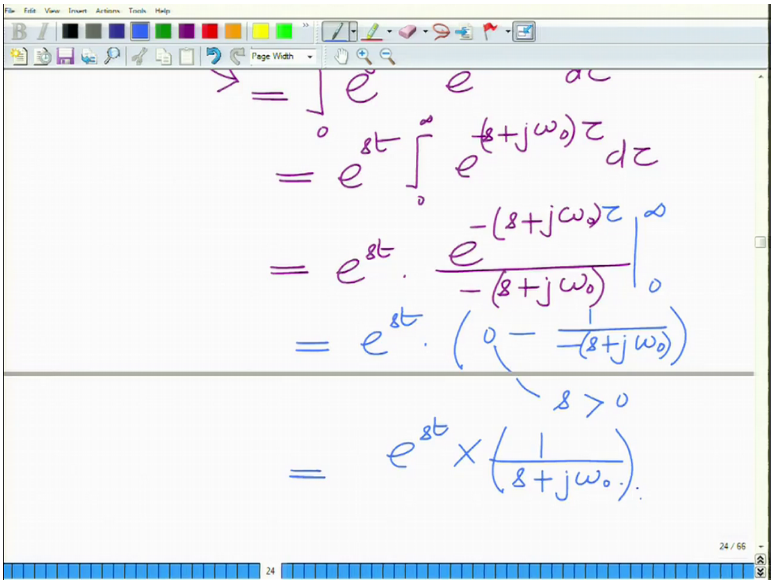

Which is e raised to s t times e raised to minus s plus j omega naught by minus s plus j omega naught evaluated between the limits 0 to infinity. Now, at infinity this is 0 if s is less than 0 if s if s is greater than 0. This is equal to e power s t and at 0 this; obviously, 1, into s 0 minus 1 by minus s plus j omega naught, but for this to be 0, s has to be greater than 0 that if e to the power of minus s has to be A decaying exponential. So, this is e to the power of minus s plus j omega naught tau. e to the power of minus s tau has to be a decaying exponential which means s has to be greater than 0.

This is basically reduces to e to the power of s t into one plus s plus j omega naught this is the Eigen value.

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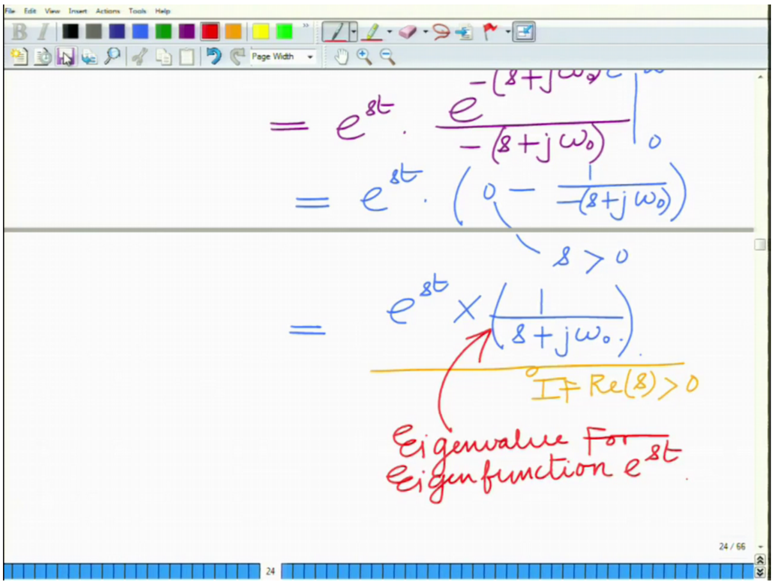

If real part of s is greater than 0, the condition required for this integral real part of f is greater than 0, this is your Eigen value corresponding to for the Eigen function e raised to s t. So, this is the Eigen value corresponding to the Eigen function e raised to s t alright. We have constructed the impulses, we have derived the impulse response of this system the given LTI system and we also derived the Eigen value corresponding to the Eigen function e raised to s t.

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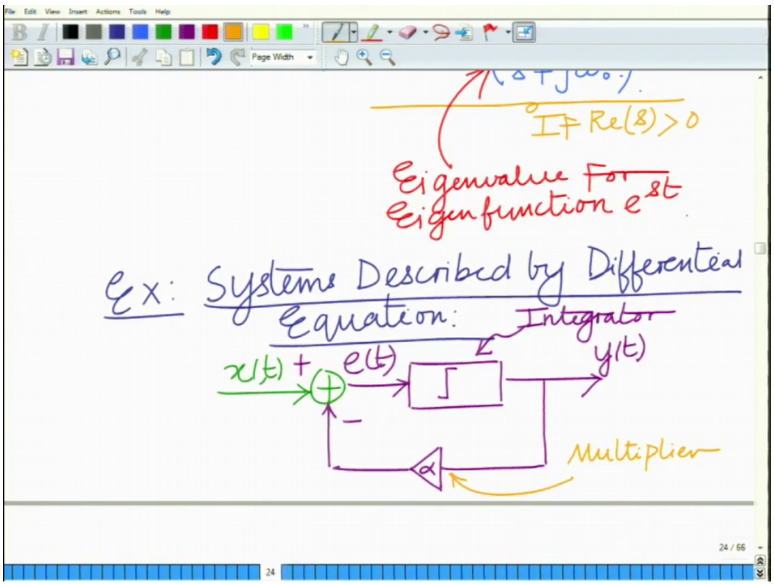

Let us look at another problem which is to determine the differential equation describing a system that is that is systems describe systems described by a differential equation for instance consider this system where I have let me draw a schematic of a system. This system is x t and I have an error signal and this is an integrator and they and the output of the integrator is fed back and subtracted from the input to give the error signal and it is scaled by this is multiplied by a scaling constant this is a multiplier.

This is a schematic diagram of a system and let us try to determine now what we are asked to do we are asked to determine the differential equation corresponding differential equation representation of the system.

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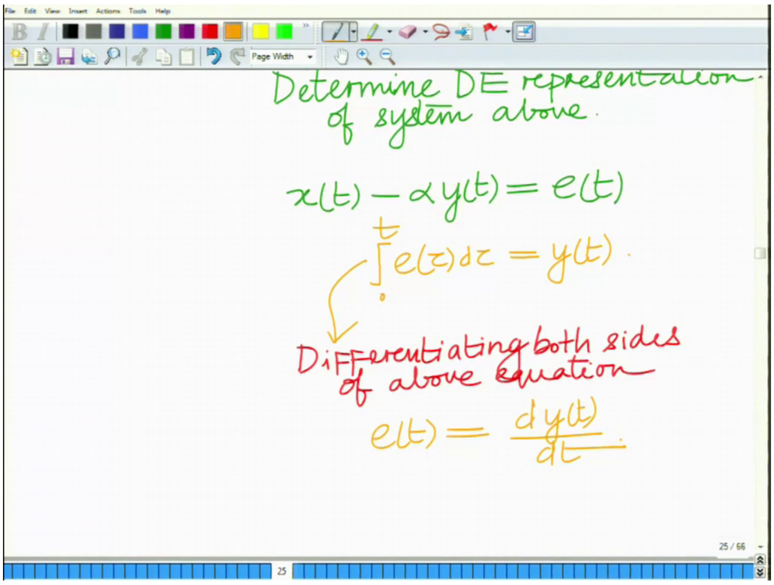

We have to determine the differential equation the differential equation representation of the system above and you can see we have basically x t minus e t x t minus alpha times y t equals e t, we have that. Similarly, e t is passed through an integrator to yield y t which means integral e 0 to t e tau d tau equals y t now what we are going to do we are going to differentiate both sides of above equations. We are going to differentiating, differentiating both sides of above equation we have e of t equals dt of y of t which means if I substitute this over here.

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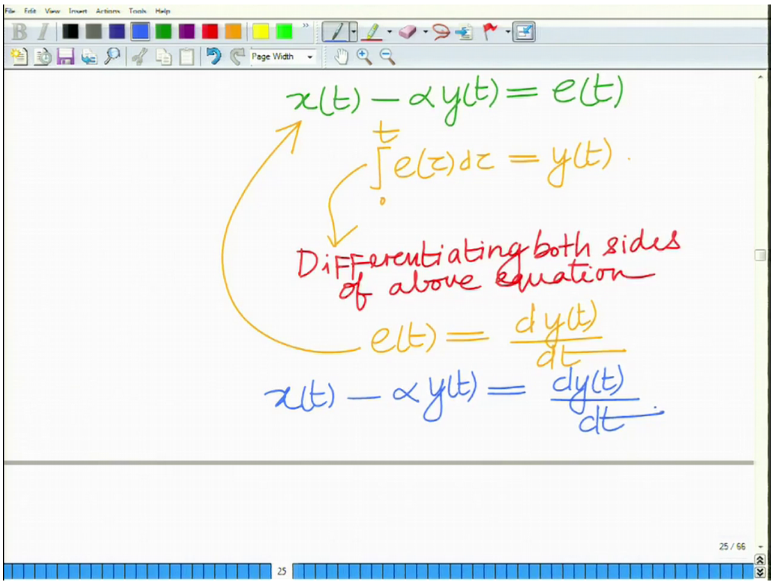

Back over here that gives me x of t minus y of t equals d y t dt.

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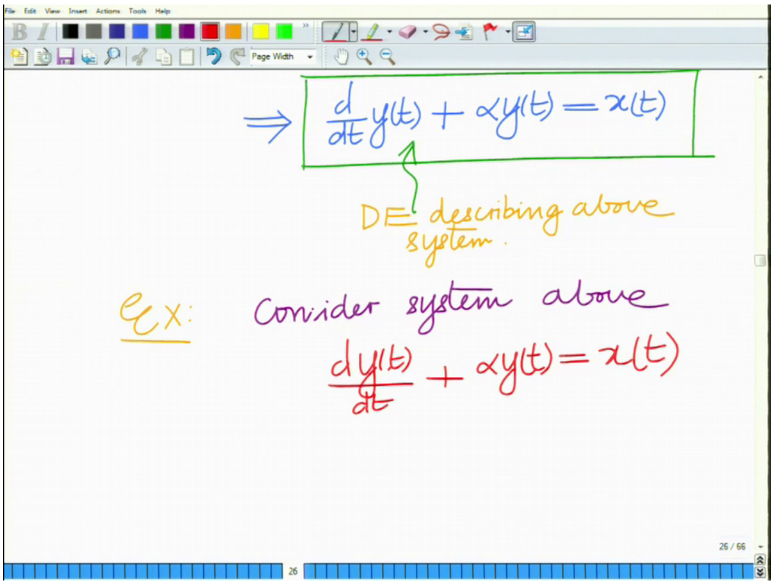

Which basically implies that d t y t plus alpha y t equals x of t, this is the differential equation? What we have is d t y t d or d y t d t plus an alpha time y t equals x t this is the differential equation describing the above system. This is the differential equation this is the differential equation describing above system.

Now, let us try to solve this to get the output response. Let us consider the as another example the same system that we have derived above consider system above which remember is described as d t y t plus alpha y t equals x t and with auxiliary condition remember to describe the system.

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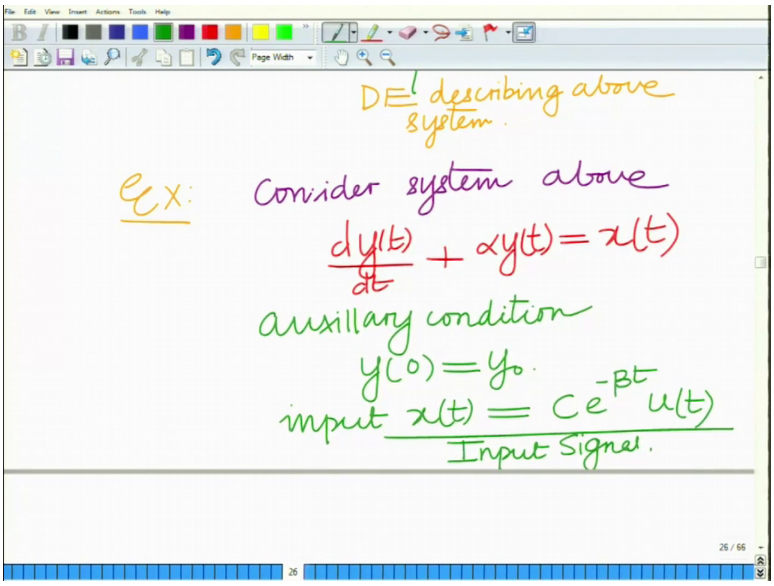

The auxiliary condition auxiliary condition is y naught equals y naught and corresponding to the input x t equals C e power minus b t x t equals.

This is basically your input signal this is basically your input signal where the auxiliary condition y 0 equals y naught and we have the system that is described by the differential equation d y t d t plus alpha y t equals x t and the way to approach this as we have already seen before is that the solution to any differential and their solution of the output y t to any system described by this differential equation can be described as the sum of 2 components by p t and y h t where y p t is a particular solution and y h t is the homogenous solution or y h t is the homogenous component.

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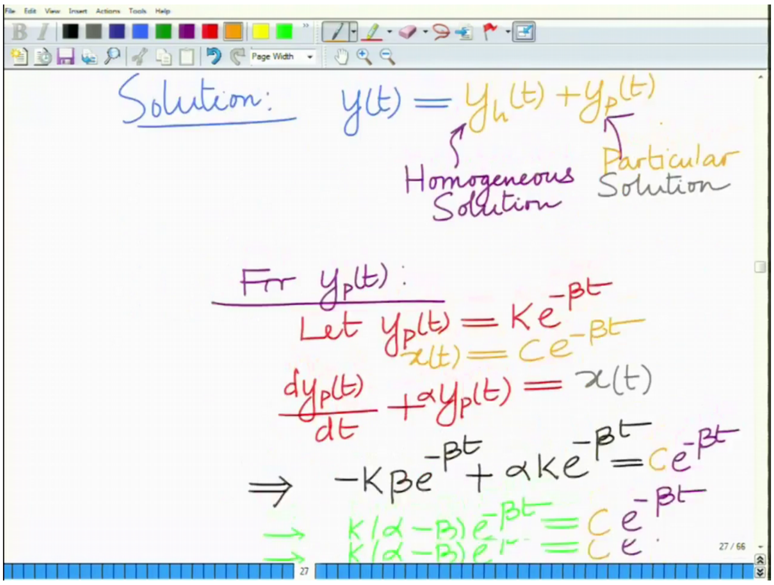

We have seen that y t can be expressed as y h t y p t where this is the where this is the homogeneous solution and this is the particular solution and therefore, now for the particular solution now what we are going to do is to find the particular solution let we are going to start with the assumption that y p t has the structure same structure as the input signal that is K e raised to minus beta t.

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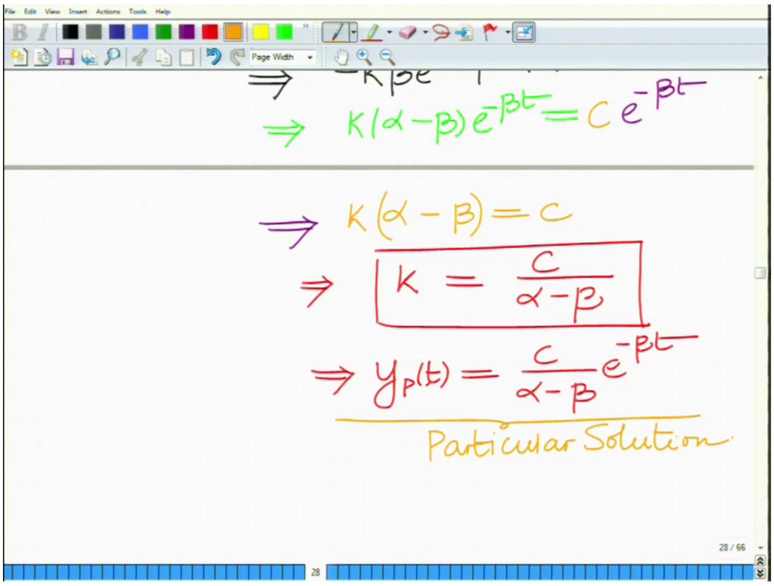

Now, we have d y p t d t plus alpha y p t equals x t this implies d by p t d t is you can see minus K beta e raised to minus beta t plus alpha times K e raised to minus beta t this is equal to x t this implies that K times alpha minus beta e raised to minus beta t equals x t this implies or this is equal to you know x t is given as K e raised to minus beta t.

This is equal to K e raised to minus beta t, this implies this or I am sorry this is C. We are given x t equals C e raise to minus beta t. we have x t the input signal is C e raised to minus beta t correct and therefore, we have alpha minus beta equals C this implies or K times alpha minus beta equals C which in turn implies K equals C over alpha minus beta.

We have the particular solution implies y p t equals C over alpha minus beta e raise to minus beta t. This gives us the particular this gives rise this gives us the particular solution which is C over alpha minus beta times e raise to minus beta t we have founded by assuming the particular solution y p t to have the same form as the input signal that is it is the form some constant K times e raised to minus beta t and this constant K has been determined subsequently determined as C over alpha minus beta.

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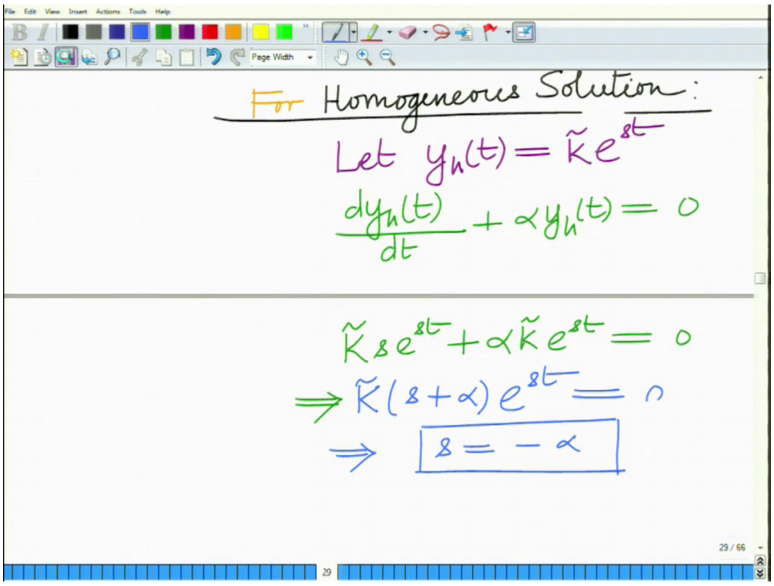

Let us now find the homogenous solution. Now, to find the homogenous solution to find the homogenous solution now let us assume y h t equals sum K tilde e raised to s t now remember the homogenous solution can be found as by equating d y h t d t plus alpha y h t equals 0, now, differentiating y h t, we have K tilde s e power s t plus alpha K tilde a power s t equals 0 which implies K tilde plus alpha e power s t or which implies which implies K tilde s plus alpha e power s t equal to 0 which implies s equals minus alpha.

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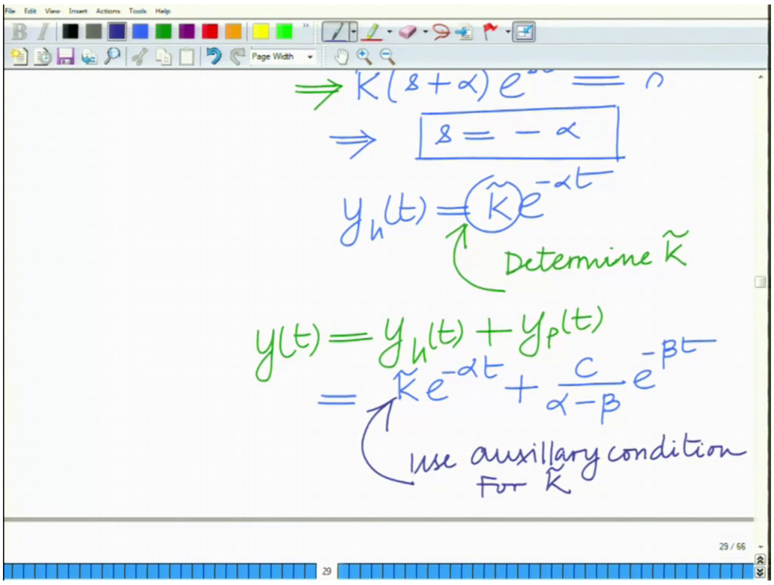

We have derived s equals minus alpha which implies y h t equals K tilde e power minus alpha t, this is the homogeneous solution, but we have to remember still determine K tilde at this point we still have an unknown determined K tilde and that can be done as follows we have y t equals y h t plus y p t homogeneous solution plus particular solution. This is K tilde e power minus alpha t plus C over alpha minus beta C over alpha minus beta times C over alpha minus beta times e raised to minus beta t now how do we determine the value of this unknown constant K tilde and for that remember we use the auxiliary condition all right.

Determine this constant there is a single unknown K tilde we have a single auxiliary condition that is the value of the signal output signal at time t equal to 0. I use the auxiliary condition to determine K tilde. What we can do now is basically use auxiliary condition to determine K tilde.

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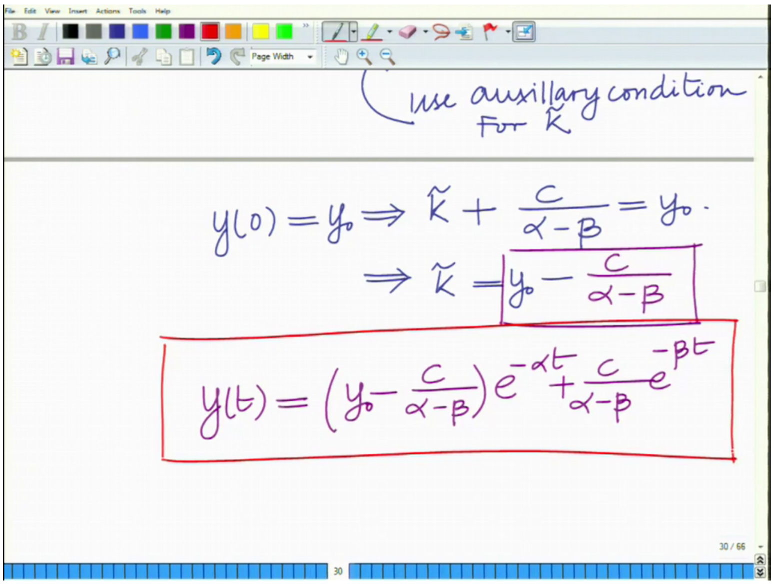

We have y of 0 because y naught implies K tilde plus C over alpha minus beta equals 0 implies K tilde equals minus C over alpha minus this implies I am sorry this is equal to y naught implies K tilde equals y naught minus C over alpha minus beta. Therefore, we have the solution y t equals y naught minus C over alpha minus beta e power minus alpha t plus C over alpha minus beta e raised to minus beta t. This determines the complete output signal.

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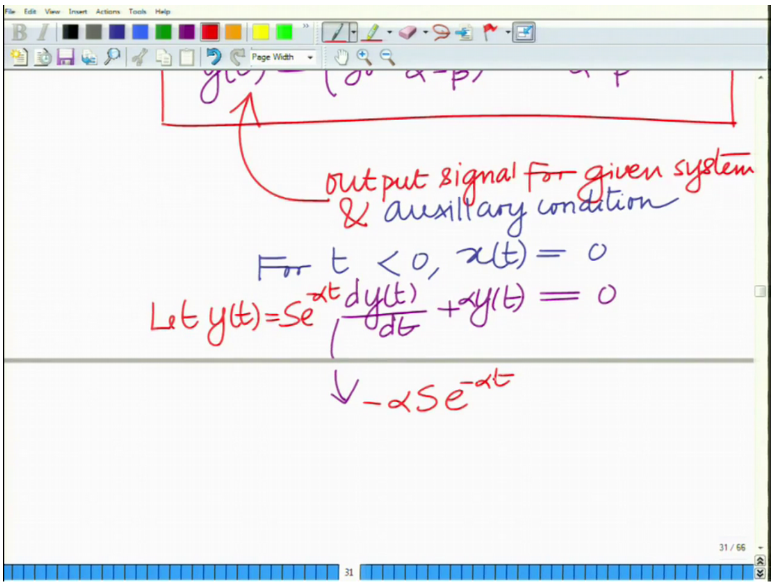

Output signal for given system and also remember the auxiliary conditions are important in this case there is a single auxiliary condition that is a value of the output signal y 0 at time y 0 at time t equal to 0 which is y naught.

Now, for t less than 0 rights for t less than 0 since the input is 0. The only output response holds for t less than 0 1 can determine the output as follows for t less than 0 we have x t equals 0 which means d y y t d t plus alpha y t equals 0 and the signal that satisfies the solution that satisfies this is again simply the homogenous solution. This implies y t equals, let y t equals sum S e power minus alpha t.

Substituting this in this differential equation we have minus alpha S e power minus alpha t. This is now what satisfies this is basically we already know that what satisfies.

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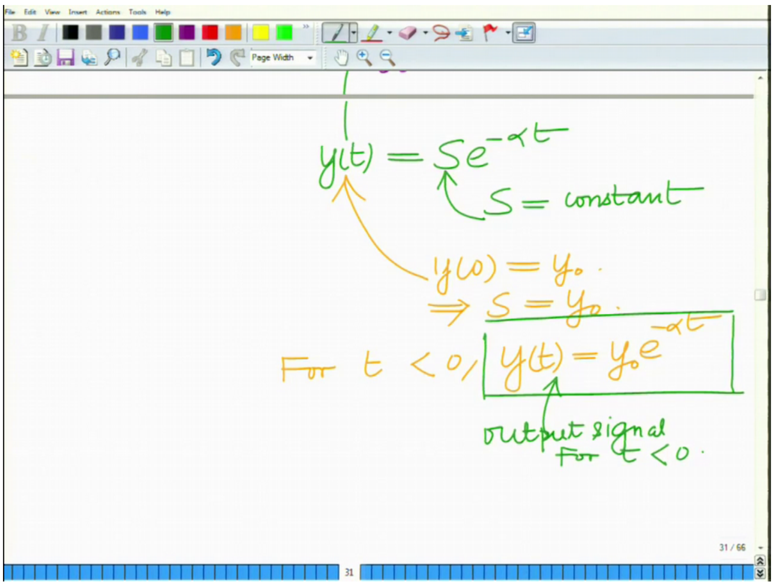

This is the homogeneous solution S e for you know the form S e power minus alpha t where S equals some constant and now that you determine that by using once again the auxiliary condition y of 0 implies s equal to y naught. Implies S into I equals y naught which implies S equals to y naught. For t less than 0, we have the output y t equals y naught e power minus t. This is the output signal; this is the output signal for at time t less than 0 when the input is 0.

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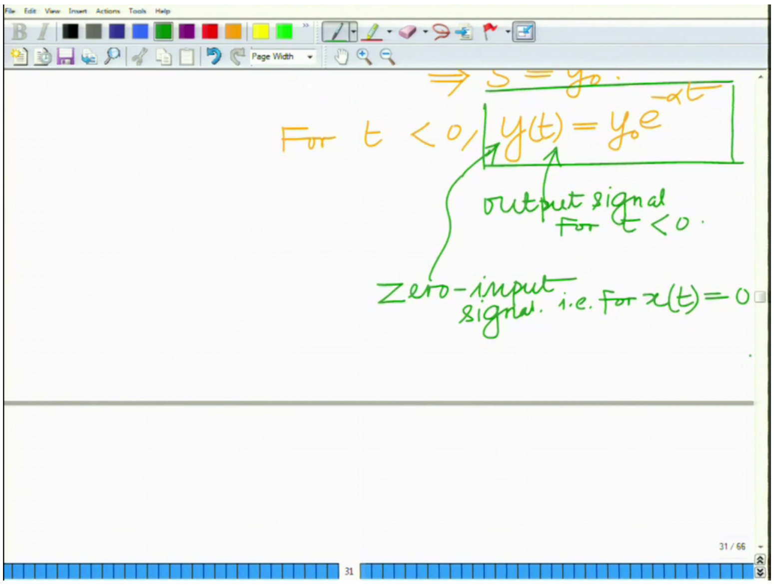

You can think of this also as the 0 input signals this is basically your 0 input signals. You can think of this as basically this is the output of the system when the input x equal to 0.

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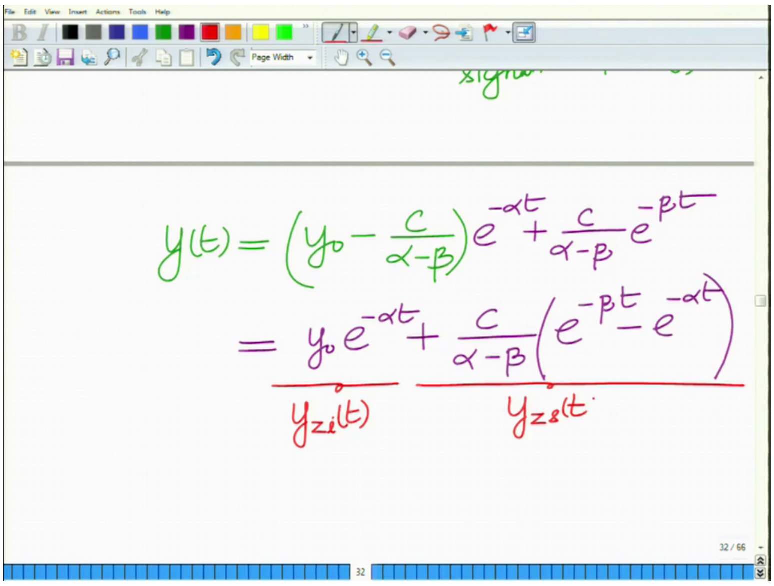

Now, if you look at the output for t greater than 0 that can be expressed as y naught e power minus alpha t plus well, let me write it down I have that y naught minus C alpha minus b C over alpha minus beta e raised to minus alpha t plus 0 alpha minus beta e raised to minus alpha beta minus e raised to minus beta t for t greater than 0 which can be written as y naught e raised to minus alpha t plus C over alpha minus beta C over alpha minus beta times C raised to minus beta t minus e raised to minus alpha t and you can now see that this is basically the y naught e power minus alpha t is the 0 input signal that is output to when the input is 0 and this is basically known as the 0 state signal.

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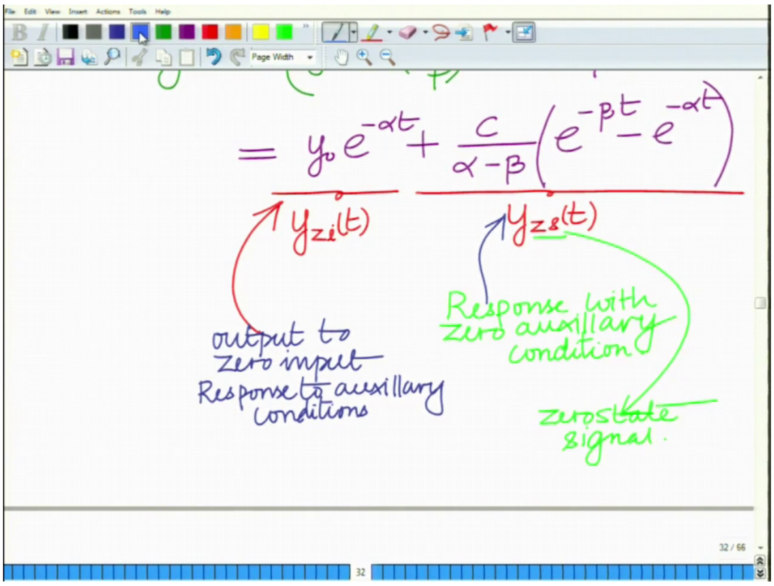

I can express the solution as y zi t which is input which is output to 0 input and this is basically 0 state signal which is the z which is basically the response. This is the output to 0 inputs or basically response to auxiliary conditions and these y 0 states is basically the response with response with 0 auxiliary conditions.

Basically you can see you can express the solution y t also as y zi t which is basically the output to a 0, input that is output signal when the input is 0 plus y zs t right where zs denotes 0 state. The 0 state signal 0 state output is basically output to 0 auxiliary condition and you can see here that if y 0 is 0 the auxiliary condition is 0 y 0 that is the output at time 0 that is y naught is 0 then y z is 0 y zi t is 0, it reduces to the 0 state signal that is the output to 0 auxiliary condition.

Similarly, if the input is 0 then we already seen the output is basically y zi t which is basically the output simply to the thus simply the output to the 0 input that is response simply to auxiliary condition y at time equal to 0 is y naught. Basically that completes this thing. I can express this as the 0 input zi t which is the 0 input signal plus y zs t which is the 0 state.

This zs stands for 0 state signals, 0 state this is basically your 0 state signals. We will stop this module here. We have looked at a couple of interesting example problems all right, pertaining to basically the Eigen function of a LTI system, the differential equation representation of a given system and also the solution or the output signal to a system represented by represented by a differential equation given the input signal and also the auxiliary controls all right. We will stop here and continue with other aspects in subsequent modules.

Thank you very much.