Advanced Microwave Guided-Structures and Analysis
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Lecture 46
Analysis of Guided Structures (cont.)

So, welcome to this session, which is essentially the continuation of the discussion of the alternate mode sets, which are going to be subsequently used for the analysis of the partially filled rectangular waveguide. So, the alternate TE to x and TM to x relevant mode sets, which are pertinent to this particular problem at hand. Let us go to the lecture slides.

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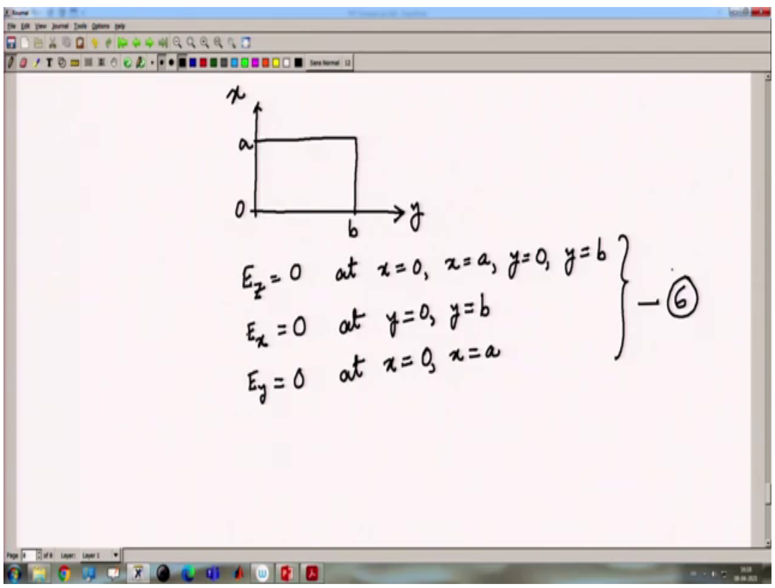

Now, for the rectangular waveguide, which is given here, the boundary conditions on the problem would mean, that the tangential components of the electric field will vanish at the conducting walls that means Ez must be equal to 0 at x equal to 0, x equal to a, y equal to 0, and y equal to b. Similarly, Ex must be 0 at y equal to 0, and y equal to b, and Ey must be 0 at x equal to 0, and x equal to a.

So, this must be satisfied by the mode functions for the rectangular waveguide. So, let us call this equation 6 which define the boundary conditions that needs to be satisfied by the mode functions, the TE to x and TM to x modes. In the rectangular waveguide, or in the homogeneous rectangular waveguide, the rectangular waveguide filled up with a homogeneous material.

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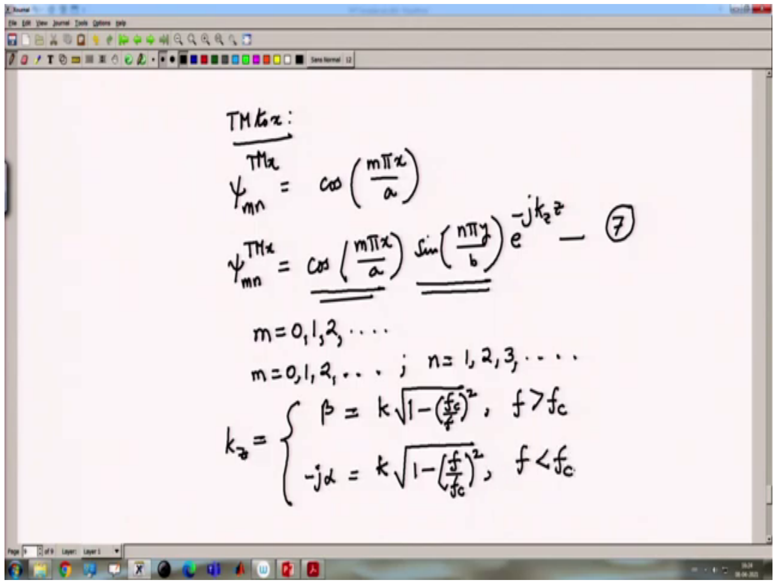

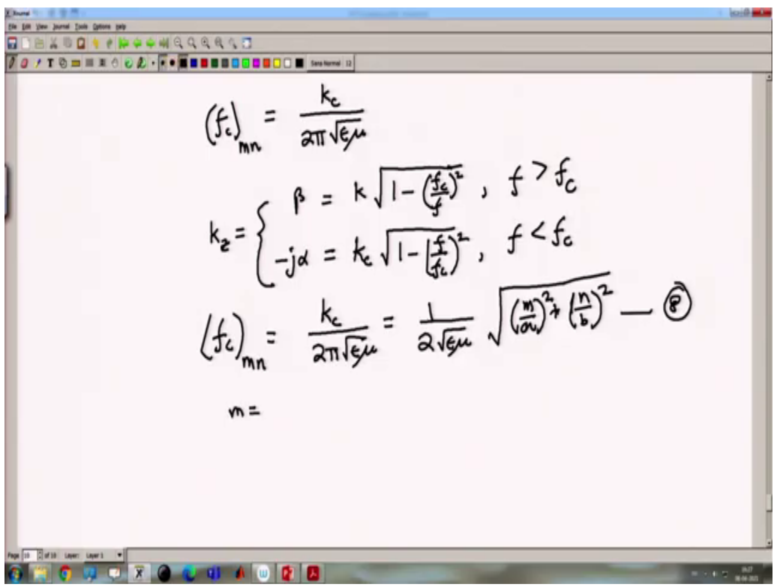

So, therefore in order for this boundary conditions to be satisfied for modes that are TM to x, the psi function for the TM to x mode will be cos(m pi x/a), sin(n pi y/ b), exponential to the power minus j kz z. Because at y equal to 0 and y equal to b, the Ex has to vanish that is dictating sin(n pi y/ b) and at x equal to 0 and x equal to a, the Ey has to be 0. Because at y equal to 0 and y equal to b, Ex has to be 0. So, which is determining this component and because at y equal to 0 and y equal to b, Ex has to be 0, which is determining this component. And at x equal to 0, and x equal to a, the Ey has to be 0, which is determining this component.

So, in this case m can take the values of 0, 1, 2, etcetera. And n can take the values of 1, 2, 3, etcetera. and kz is given by beta, that is equal to $k\sqrt{1-(\frac{f_c}{f})^2}$, for f greater than fc, where fc is the cut off frequency and kz equal to minus j alpha equal $k\sqrt{1-(\frac{f}{f_c})^2}$, for f less than fc for the m nth mode, or the cutoff frequency for the m nth mode is given by $\frac{k_c}{2\pi\sqrt{\varepsilon\mu}}$, kz is given by beta that is $k\sqrt{1-(\frac{f_c}{f})^2}$ for f greater than fc where fc is the cut off frequency. And kz is given by minus j alpha, that is equal to $k\sqrt{1-(\frac{f}{f_c})^2}$ where (fc)mn the cutoff frequency of the m nth mode is given by $\frac{k_c}{2\pi\sqrt{\varepsilon\mu}}$ and that is equal to $\frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2}$. So, let us call this equation 8.

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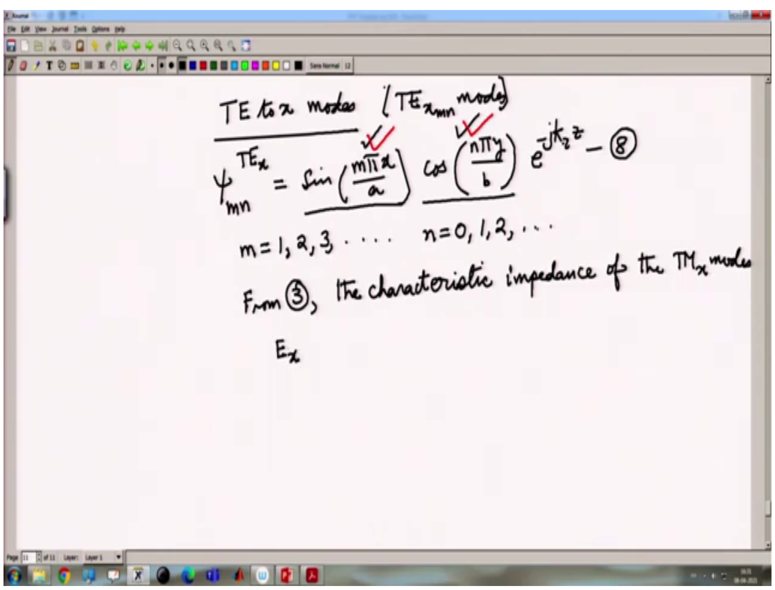

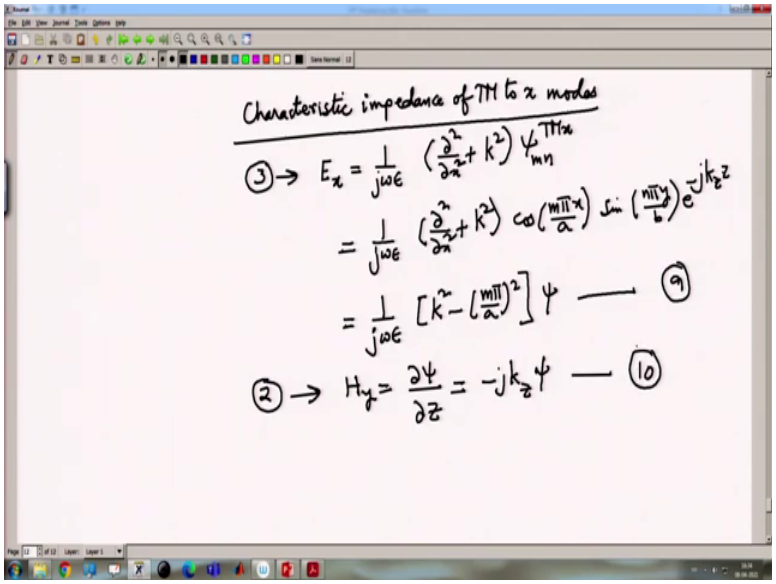

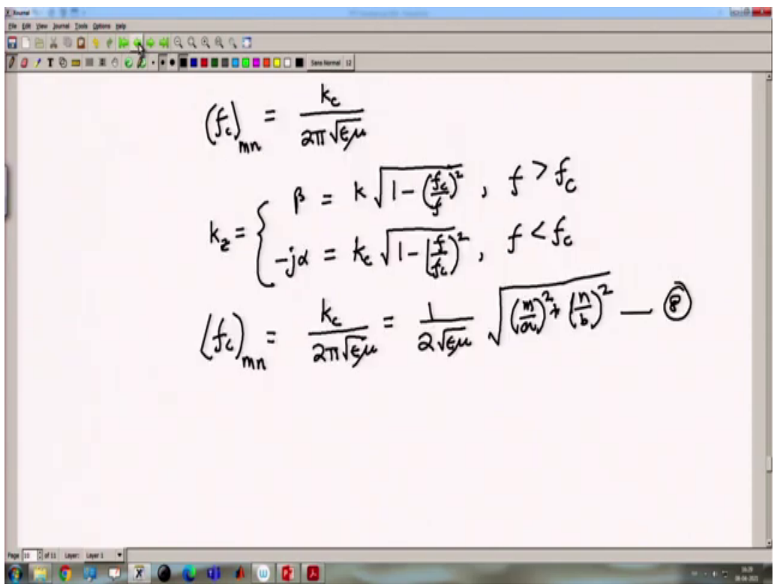

Now, for the TE to x modes, or we can also say TE to x mn modes, the corresponding psi function for the TE to x mn mode is given by sin(m pi x/ a), cos(n pi y/ b), e to the power minus j kz z. Let us call this equation 8, where m is given by 1, 2, 3, etcetera and n is given by 0, 1, 2 etcetera and kz is as given before.

So, again we see that the boundary conditions have to be satisfied by the choice of the appropriate psi function along the x, and y direction, namely the Ex has to be 0 at y equal to 0 and y equal to b, and Ey has to be 0 at x equal to 0 and x equal to a, which will determine these two field distributions, or these two potential function distributions.

So, that when we calculate Ex from here, it has to vanish at y equal to 0 and y equal to b and when we calculate Ey from here, it has to be 0 at x equal to 0 and x equal to a. So, that will determine the choice of these two potential functions. This is similar to the rectangular waveguide, which we studied earlier. So, now from 3, if we look at equation 3, the characteristic impedance of the TM to x modes can be calculated.

So, we can say, that for the computation of the characteristic impedance of the TM to x modes, we can use equation 3, to find out Ex as 1 by j omega epsilon, del square del x square plus k square, psi mn, TM x and that is given by 1 by j omega epsilon del square del x square plus k square. Just substitute.

So, cos(m pi x/ a) sin(n pi y/ b) exponential to the power minus j kz z. So, that is 1 by j omega epsilon k square minus m pi by a whole square psi, because of the del, del x square operation, we have minus m pi by a whole square, when we perform the derivative del, del x square on this function. So, it becomes k square minus m pi by a whole square psi. So, that is equation 9. Then from equation 2, we can find Hy as del psi, del z, that is minus j kz psi. Let us call this equation 10.

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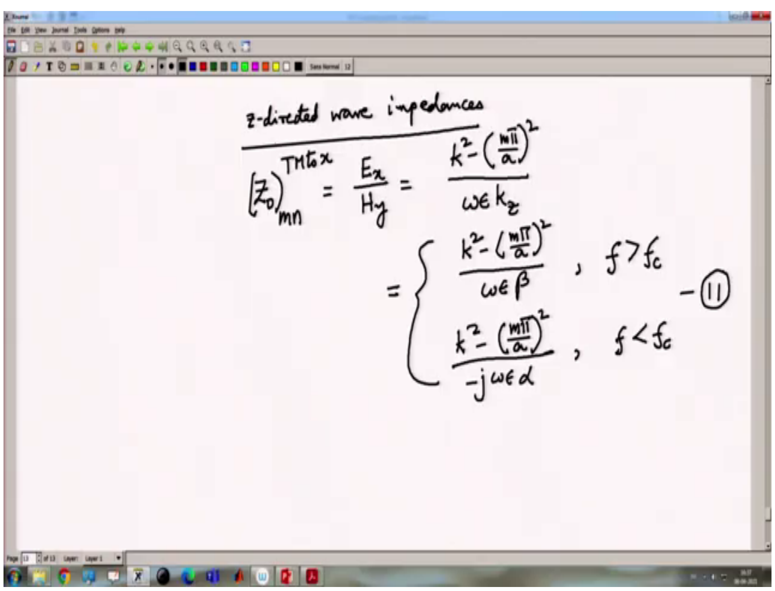

So, therefore, the z directed wave impedances are (Z0)mn for the TM to x mode as Ex by Hy, that is k square minus m pi by a whole square divided by omega epsilon kz and that is equal to k square minus m pi by a whole square by omega epsilon beta for f greater than fc, which is the cutoff frequency fc being the cut off frequency and that is equal to k square minus m pi by a whole square by minus j omega epsilon alpha for f less than f c. So, this is equation 11.

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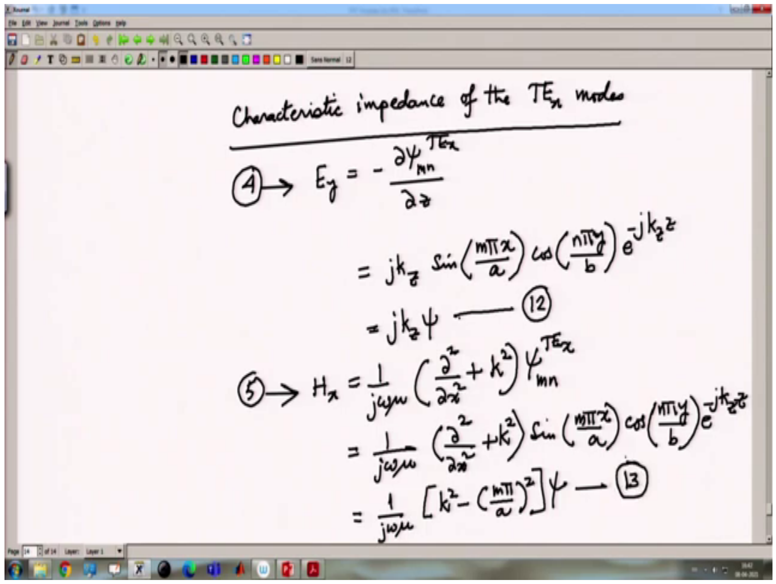

Then for the characteristic impedances of the TE to x modes we obtained from equation 4, Ey equal to minus del psi mn TE to x del z that is equal to j kz, sin(m pi x/ a) cos(n pi y/b) exponential to the power minus j kz z that is equal to j kz psi. This is equation 12.

Similarly, from equation 5, we obtain Hx as 1 by j omega mu, del square del x square plus k square psi mn TEx, that is 1 by j omega mu del square del x square plus k square, sin(m pi x/ a) cos(n pi y/b), exponential to the power minus j kz z. And that becomes equal to 1 by j omega mu k square minus m pi by a whole square times psi. We call that equation 13.

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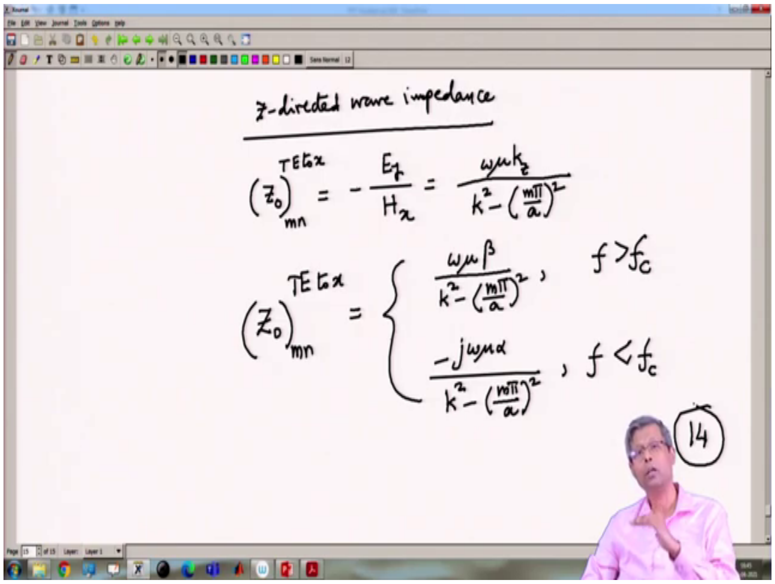

And therefore, the z directed wave impedance becomes z0 mn TE to x as minus Ey by Hx, that is omega mu kz by k square minus m pi by a whole square. So, this can be written as z0 mn TE to x as omega mu beta divided by k square minus m pi by a whole square, for f greater than the cut off frequency fc, or the frequency greater than the cut off frequency fc and that is equal to minus j omega mu alpha by k square minus m pi by a whole square for the frequency less than the cut off frequency fc. So, let us call this equation 14.

So, that in the preliminary framework, we need for the mode functions of the TE to x and the T M to x modes which we are going to now use to analyze and find the dispersion characteristics of the partially filled rectangular waveguide. So, this is where these kinds of mode functions, or alternate mode functions are going to be used to satisfy the new boundary condition, or the additional boundary condition, which is present at the dielectric air interface inside the partially filled rectangular waveguide. So, this will be continued in the next session. Thank you.