Photonic Integrated Circuit
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Lecture 10
Phase velocity and Group velocity

Hello all, in the last lecture we saw dispersion, group velocity dispersion and group index and also looked at phase velocity as well. So, this is basically how light propagates through the medium. So, let us take a simple example and then understand why we are very particular about this dispersion why it is important to understand we will take optical fiber as an example because we understand it very well and we use it in our daily life. So, let us look at optical fiber and the effect of dispersion in optical fiber.

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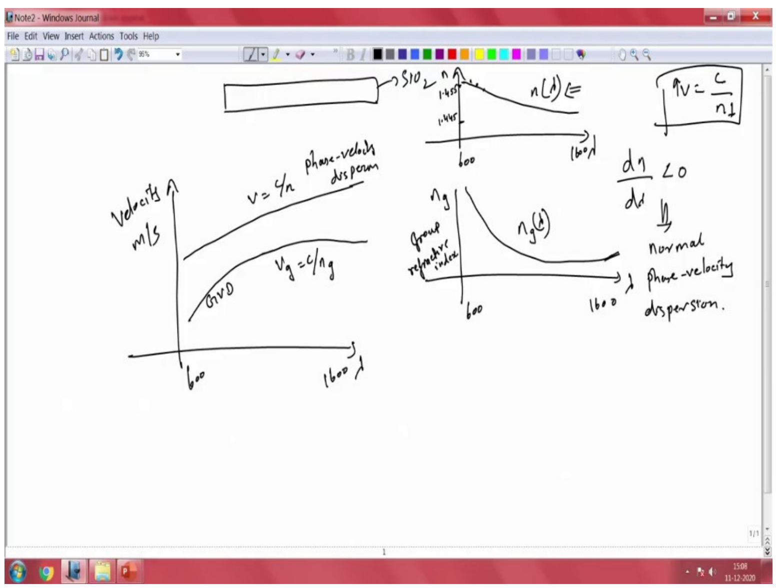

So, if you take a simple optical is made out of silica and we know all material shows material dispersion that means as a function of wavelength the refractive index changes. So, since this is we are working in the transparent region let us say this is 600 nanometers to 1600 nanometers this is a thousand nanometer bandwidth and your refractive index actually slightly reduces.

So, this is n as a function of lambda. So, the change is rather small I mean just it can be anywhere between 1.455 to 1.445, let us say 1.445 in this range. So, this is how the refractive index changes as a function of wavelength. So, let us look at how the group index varies. So, this is for all practical purpose you can this as refractive index at a particular frequency.

So, that is what, that is how you get this. So, now let us look at the group index of this. So, the group index in same range would look something like this. So, this is n_g times lambda and the group index this is group index n_g and this is just n group refractive index. So, in this spectral range your $dn/d\lambda$ is negative. So, $dn/d\lambda < 0$ for any of this wavelengths that we have. So, that means you are in normal phase velocity dispersion let us say, means around this wavelength we just discussed it is going to be this way. So, let us look at the face how the velocities are changing.

So, this is the dispersion that we saw let us look at how our velocities would look like, how fast the waves are going to move. So, this is velocity as a function of wavelength now. So, this is both as a function of wavelength. So, 600 and 1600 here. So, let us look at the phase velocity. So, you can look at this trend. So, the trend is reasonably flat but the trend is moving down. So, there is a reduction in refractive index. So, what is the implication of reduction in refractive index. So, we know that this the speed of light the velocity is given by c/n.

So, when your refractive index reduces your speed will increase. So, the maximum you can get is of course v equals to c that means when n equals to 1. So, that is free space. So, in this case the refractive index is reducing. So, what you expect to see is a slightly increasing trend your velocity is c/n. So, that means your phase velocity dispersion will have such a trend. So, let us look at group velocity here. So, the group velocity depends on how your refractive index profile here for a group index.

So, here you will see something of this kind. So, this is group velocity dispersion and that is $v_g = \frac{c}{n_g}$ and you can see here for particular region of wavelength the speed is nearly equal but then when you look at the phase velocity they are not. So, none of these wavelength will travel at same speed however if you look at the group velocity you will see a region of wavelengths where the difference between the velocities are very small. So, you can see here the slope here is rather small. So, this is the region that we try to exploit.

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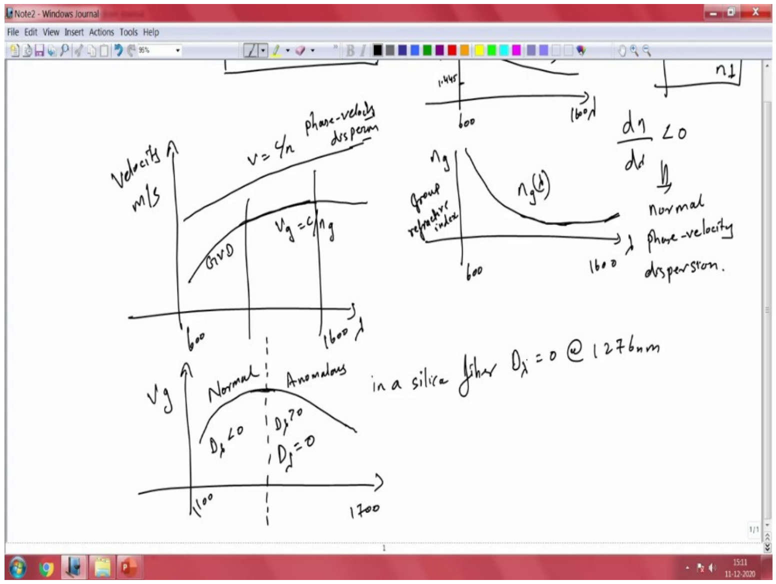

So, let us zoom out and then see what how this actually happens. So, let us zoom in this region. So, when you zoom into this region this is the group velocity specifically that we are interested in let us say now we are zooming in into a 1700 let us say, what you have here is something like this and at a certain point you have 0 dispersion. So, D that is your dispersion coefficient you remember we discussed that is actually 0 at some point it is flat and you have positive side where $D_\lambda < 0$ and your $D_\lambda > 0$ and this region is called normal and this region is called anomalous.

So, you see a region where you could have 0 dispersion that means your light is travelling with the speed or the group of waves that are travelling here without any difference between them and this normally happens in silica fiber. So, this $D_\lambda = 0$ happens at around 1276 nanometer. One can modify this based on fiber parameters but this is something that one can actually change by using various fiber dimensional parameters doping parameters refractive index and so on. So, this is how a group index changes and let us look at the dispersion itself let us look at how the dispersion would look like.

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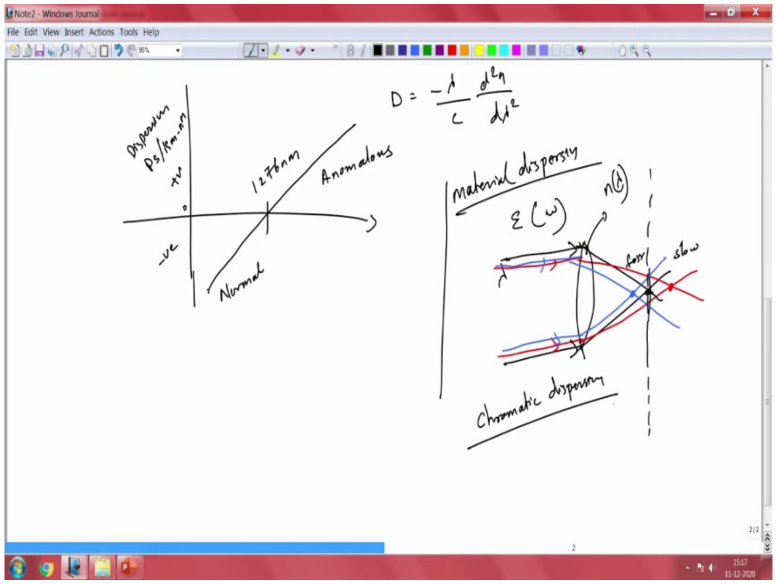

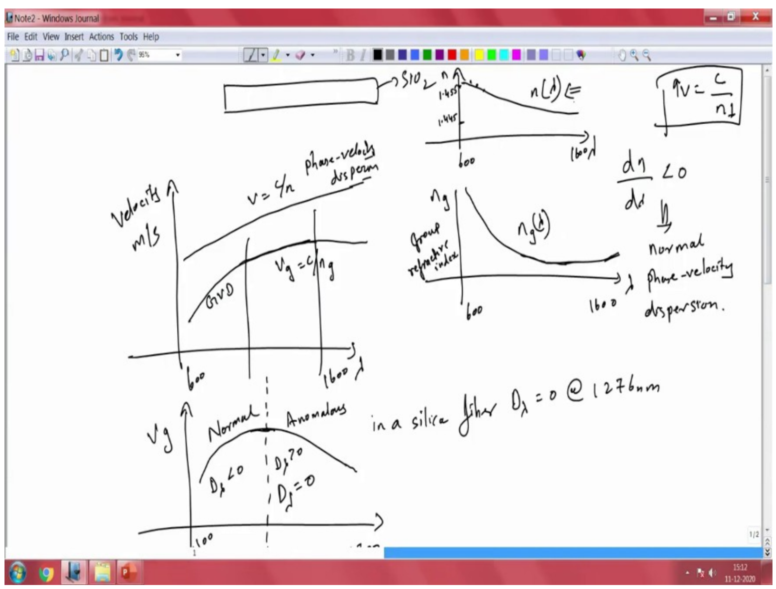

So, this is our group velocity let us look at the dispersion. So, dispersion as a function of picosecond per kilometer nanometer and if this is 0. So, when you go from normal to an anomalous region. So, normal is negative. So, this is negative side and this is positive side. So, you go from negative to positive. So, this is cross point is 1276 nanometer for all practical purpose. So, it can be shifted here and there and this region is anomalous and this region is normal. So, this is how one can find how different waves travel in a medium as a function of the frequency and also function of wave refractive index that you have.

So, this everything starts from the material dispersion. So, it is important to understand the material dispersion and then the group index and from group index we calculate the pulse dispersion. So, the dispersion is an important that is the reason why we call it is important. So, this 0 dispersion point is very interesting. So, 0 dispersion point you make sure that the waves do not lag with respect to each other they travel at the same speed as the neighboring period. So, that is an interesting point.

So, the chromatic dispersion is literally 0 at that point the next thing to understand apart from dispersion is absorption but before that let me also briefly discuss this dispersion of material say. So, it is frequency dependent we know that because, so we know that your epsilon is a function of frequency and this has direct implication on our refractive index and speed but every material has its own dispersion including glass materials and solid materials including glass semiconductors everything.

So, we make lenses out of glass and this material is used or profiled in such a way to focus light. So, you have a light beam coming in and you want to focus the light certain position. So, the problem here is we assume that when you focus it always comes to a certain point but there is a interesting fact that you should understand here that this is only true for a single lambda but then if I have let us say a red photon coming in.

So, I have red light coming in and it will actually focus at a slightly longer position, the focal point to be slightly longer compared to a blue photon that comes through here and that will be focused nearer. So, now this is a real problem because when you have a white light and you want to focus this light onto a focal plane here, let us say I have the focal plane here my lens is going to focus red light away from beyond the image plane and my blue light is going to be focused before the image plane.

So, the effect of this is a blurred image. So, if you are making a multi-color image with this particular lens that has a dispersion. So, your refractive index is a function of lambda and this is what we call chromatic dispersion because of different wavelength of light and associated refractive index you are going to focus it at different places, the same effect happens when you propagate light through a dispersive medium.

So, whether the light is going to arrive fast or slow depends on the wavelength that you have and the chromatic dispersion of this particular medium and this is one of the reason why in communication dispersion compensation through signal processing and various algorithms is heavily used to fight against this chromatic dispersion. It is simply impossible to get rid of this dispersion, you have to live with this, but this is known.

Because for a given length of a fiber the dispersion is already well defined and well characterized and based on this you can compensate when you receive the signal. So, that is about material dispersion and its effect on the chromatic dispersion of the medium. So, with that we have come to an end to the whole dispersion discussion. The next thing about the material is about the loss associated in the material. So, we saw how fast light can move through the medium and single frequency, multi-frequency and effect of fiber, silica fiber on the dispersion, so everything we understood now.

Well now the question is when I put an intensity I the input let us say I put some 1 milliwatt of power at the input, will I get 1 milliwatt after I travel maybe one kilometer or 10 kilometer or 1 centimeter for that matter, will I get the same intensity? The question is very valid you want to know whether I have any loss in the system and this strongly depends on again the material property whether this material has gain or whether this material has loss or in other case the material has no gain or no loss let us say it is a completely passive transparent material.

So, that is an idealistic view but it still exists. So, one is a completely transparent material and then you have lossy material and then you have gain material; loss, lossless, gain. So. these are the three types of material that you could have and let us look at what is the implication of material that are lossy because whatever discussion we had so far and whatever basic understanding you have in photonics are primary assuming that it is transparent lossless medium.

But now, let us take a medium that has a loss. What is the origin of the loss? This is again something that we have discussed in the early part of our lecture about when we discussed about dispersion model. So, when you look at a refractive index, so it is a complex refractive index. So, that is what we have n + ik is how we wrote it. So, n is the real part that talks about the density of the material and k which is the damping part. So, the damping part is nothing but the energy absorbing part and that part is related to the susceptibility of the material itself. So, let us look at how we can differentiate between loss and gain in a material.

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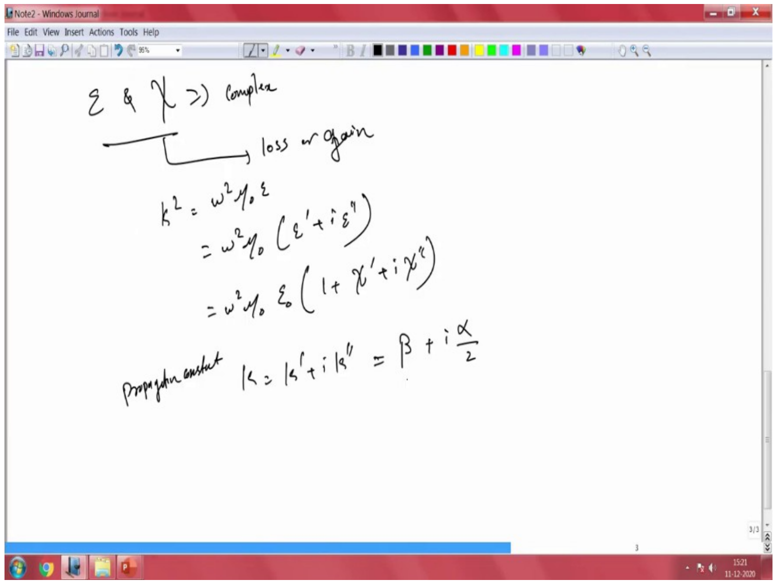

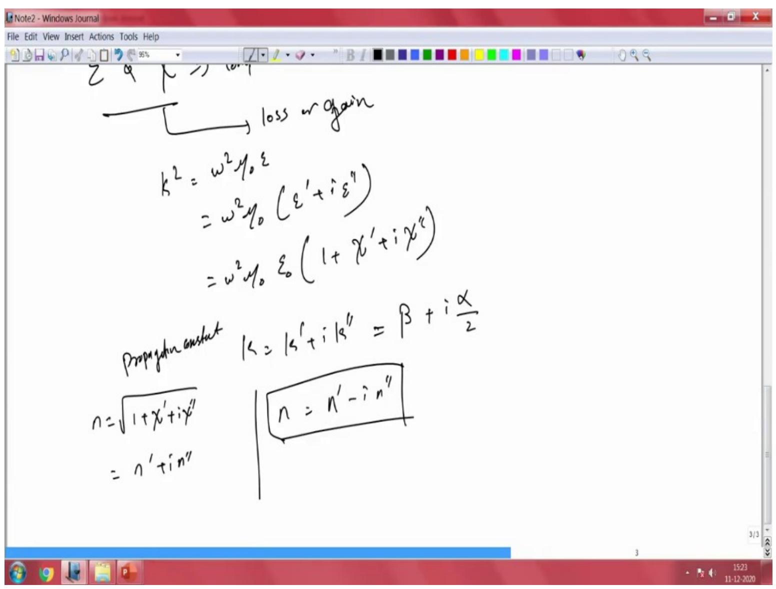

So, for that we need to look at the dielectric constant epsilon we know and your susceptibility. So, both these are complex. So, that is something that we should take into a count and we could have either loss or gain and this depends on certain signage we will see that shortly and what we all know is this from our earlier understanding. So, your epsilon could be written as a real part and imaginary part and even elaborating it further linear and non-linear sorry real and imaginary part that you have.

So, now the propagation constant k will also become complex and that is given by $k=\beta+i\frac{\alpha}{2}$. So, beta is our propagation constant, the real part of the propagation constant and $\alpha/2$ is the absorption part. So, the refractive index can be written as $n=n'+¿$. So, this is basically coming from here. So, you can write, let me just write it down here. So, n is nothing but $n=\sqrt{1+\chi'+\chi^{}¿}¿$ and that is nothing but$n=n'+¿$. So, this is your real part of the refractive index and this is your imaginary part of the refractive index.

And now the impedance that you have in the medium also becomes complex. So, once you bring in the absorption part, so then we need to understand how the propagation of a wave will manifest. So, you have a wave that is propagating through the medium I have now a complex refractive index and how the wave is going to respond to this or what will happen to this wave.

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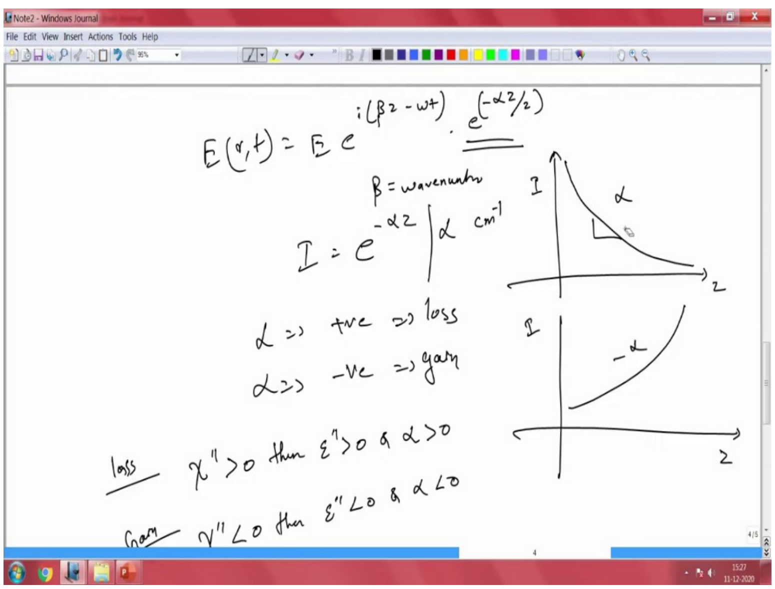

So, let us pick up a very simple optical wave that is propagating along z direction. So, one can write that as $E(r,t)=E e^{i(\beta z-\omega t)}$. So, this is something that we already know. So, beta is nothing but your propagation constant. So, you do not have to get surprised when what happened to k, k has become beta here because it is a real part of the refractive index, in this case times $E(r,t)=E e^{i(\beta z-\omega t)} e^{-\alpha z/2}$. So, this is basically what you get when the light is propagating through the system.

So, beta is nothing but the wave number the propagation constant that you have along z and now you look at the damping factor now here. So, this is the damping factor and your intensity of light is going to damp at this particular rate. So, this is how your intensity is going to reduce as you move along length of your propagation direction.

So, if this is I then it is an exponential decay along z with slope alpha. So, this is how what will happen when light propagates through a lossy medium and the sign of alpha here I kept it positive, so I am writing it as a decaying here. What happens when it is negative? So, now alpha is positive you get loss. So, alpha is negative you get gain. So, there when you have alpha negative your intensity would increase and this is minus alpha as you move along.

So, when you have a gain medium, when you are going through a gain medium the intensity of light is going to increase exponentially as you move along the propagation direction but then when you have a lossy medium your refractive, your light is going to lose the energy as you move along. So, just to give you a little bit more example, so for the lossy side you have $\chi'' > 0$.

So, what this will do? Then epsilon double dash is greater than 0 and alpha is also greater than 0 and this is for loss and for gain chi double dash is less than 0 then epsilon double dash is less than 0 and alpha is less than 0. So, the gain is represented or also loss is represented per unit length. So, alpha in this case is represented per unit length it can be per centimeter. So, gain can be also per centimeter. So, this is how you can calculate how much loss one could get out of this lossy or lossy system or a gain system that you have the material could have both.

We can easily characterize represent the material properties nicely here. So, the other thing to know to notice here is when you have a certain absorption, you can actually use these alpha positive or negative to find out the length scale required to absorb a certain amount of light or what is the length required to generate a certain amount of light. Let us say you want to create a detector and if the material has a certain susceptibility here.

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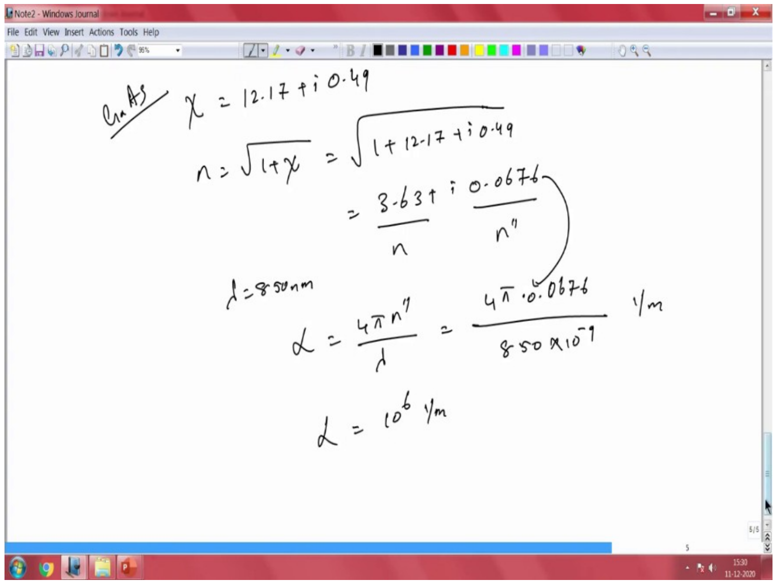

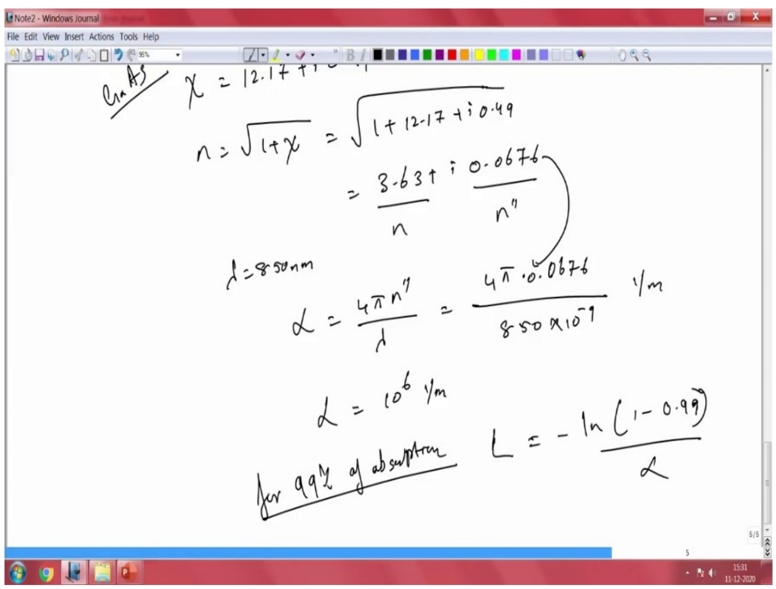

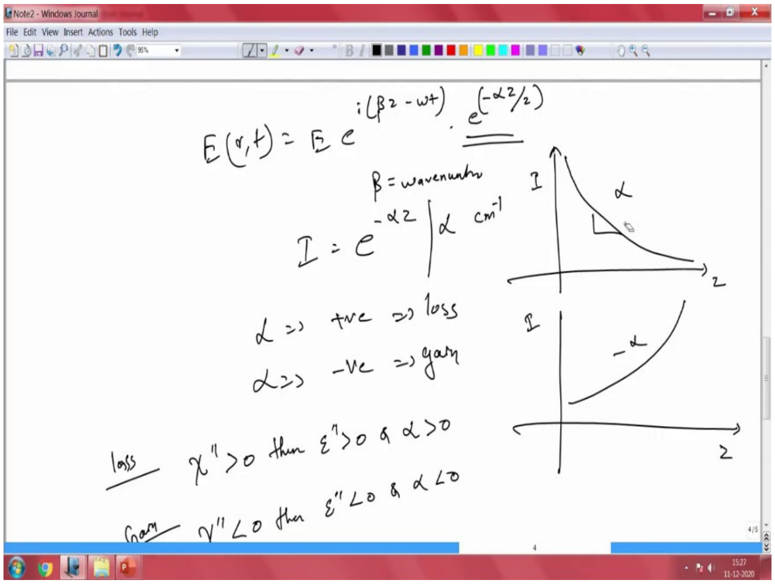

Let us say, I take gallium arsenide as a compound semiconductor and this is my complex refractive index let us say $\chi = 12.17 + i0.49$. So, now immediately this is your susceptibility and now we need to convert this into a refractive index. So, $n = \sqrt{1+\chi}$ and then it is represented by $n = \sqrt{1 + 12.79 + i0.49}$ simple operation if you do you will get. So, this is my n and this is n double that. So, positive and so the real part and the imaginary part.

So, n and k I am not using k here I am trying to use n double dash because there are lot of k's here. So, you have propagation constant wave number also k. so, I am using n double dash here. So, now let us assume that this is what you have and your wavelength of interest let us say is 850 nanometers and I want to calculate how much light I can absorb. So, in order to do that let us look at alpha now. So, I need to calculate the absorption coefficient now. So, the absorption coefficient now is given by alpha over 2. So, that is what we saw here you remember alpha over 2 that is kz.

So, let us say. So, $\alpha = \frac{4\pi n''}{\lambda}$ and this is coming from $\alpha = \frac{4\pi 0.0676}{850 x 10^{-9}}$. So, this is per meter and if you do the calculation approximately this will be $\alpha = 10^6 \frac{1}{m}$ So, based on this alpha one can calculate how much length is required in order to absorb the light. So, it is, let us say if I want to absorb 99 percentage of light, for 99 percentage of absorption.

So, for 99 percentage of absorption what is the length required. So, that means L should be equal to $L = \frac{-\ln(1-0.99)}{\alpha}$. So, this is what I need. So, when you do this you will get something like 4, 4 and a half micron or so. So, this is how one can use the knowledge of the complex refractive index n order to calculate how much of light one could absorb inside the medium and the same thing could be used for amplification as well, all you have to do is change the sign.

So, if you change the sign you will get absorption, generation of photons now, so you will see amplification. So, with that we have a complete understanding now of a dispersive medium and we have also seen what will happen to the propagating wave when you have loss in the system and also when you have gain in the system. So, once we understand this it gives us a good platform to build any circuit or device based on the material understanding.

So, far we looked at isotropic nature of light it means in all the directions you see the same material property and that is a very good approximation for an isotropic or amorphous type of material. But then things can be very different when you go to anisotropic medium and that will be the topic for our next discussion. Thank you very much for listening.