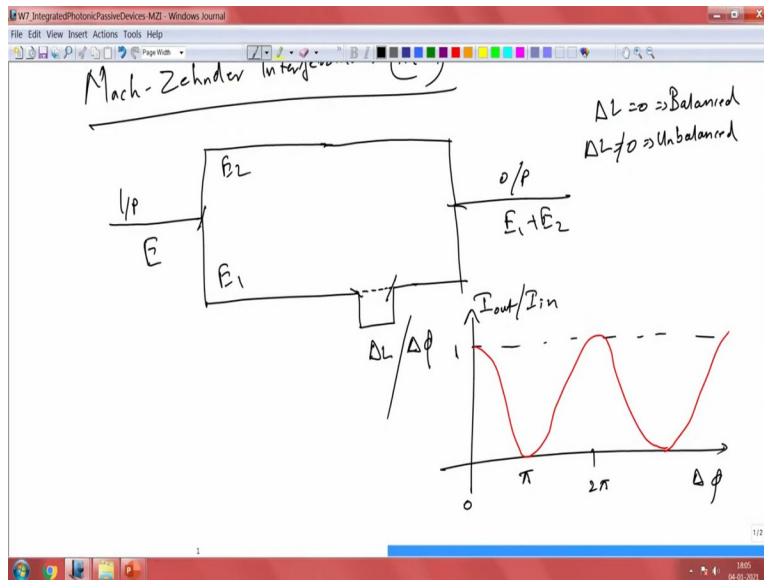


**Photonic Integrated Circuit**  
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**Lecture 31**  
**Mach-Zehnder Interferometer**

Hello everyone, let us continue to look at one of the interesting passive device. Well, we call it passive device, but we also use it in active configuration as well. So, this is called Mach-Zehnder interferometer; so there are multiple interferometers proposed and demonstrated. There is there is a long list of things that you might have studied in your introduction to photonics course. But, here one of the practical architecture that one can use for a lot of interesting applications is Mach-Zehnder interferometer. So, this was named after pair who came up with with this particular configuration.

So, let us look at how this Mach-Zehnder interferometer works. Particularly, in this device it is not a fundamental device. So, we are going to construct it from whatever we just studied in the earlier lectures. There are two (comp) there are two ways to do this. We will, we will look at the two implementation; one is by using directional couplers, very simple directional couplers. And then the next configuration is by using Y-splitters and Y-combiners; so let us look at these two configurations. And then see what all the interesting realization that we can do with this Mach-Zehnder interferometer. So, let us look at that.

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So, we all know interference; so this is a special kind of interferometer. So, Mach-Zehnder Interferometer, so MZI; so in short this is what we use. So, the configuration is rather straightforward; so let me do a simple skeletal diagram there. So, you have a single input and you split this input into two; so we can split this and you allow them to propagate. And you could make an additional length for one, so there is  $\Delta L$ ; and then we put it together back again, and this is the output. So, you have  $E$ , so you have  $E_1$  and  $E_2$ , and now you are going to put  $E_1$  plus  $E_2$ ; so, this is this is basically the configuration. So, when you do not have this particular section, we call this as balanced Mach-Zehnder.

So, when  $\Delta L$  equal to 0, we call this as balanced configuration; and if  $\Delta L$  is not equal to 0, we call that unbalanced. So, what is this particular  $\Delta L$  is going to do? So this is going to have additional phase. So, when the light is propagating through a medium, the length determines what is the phase accumulation that this particular wave has. So, additional distance it travels, it is going to have some additional phase. So, what is the implication of that phase here? Let us let us look at the effect of this  $\Delta \phi$ . So, this is simple interference that you see between these two waves.

So, the ratio between  $I_{out}$  by  $I_{in}$ ; so this is 1 let us say. So, when the phase difference is 0, when the phase difference is 0, you will have maximum; and it is going to go down and up and down and up. So, whenever you have  $\pi$  phase difference, you are going to completely

destructively interfere you will know that. So, when you have  $2\pi$  you are back; and  $3\pi$  you will have same thing again, this is a simple cosine function. So, this is how a simple Mach-Zehnder interferometer works. So, how can we implement this by using integrated optical devices or in a waveguide geometry?

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The top screenshot shows a diagram of a Mach-Zehnder Interferometer (MZI) in a waveguide geometry. It features a central section of length  $L$  where the waveguide is split into two arms. The upper arm has a refractive index  $n$  and the lower arm has a refractive index  $n'$ . The wave number in the upper arm is  $k_0 n$  and in the lower arm is  $k_0 n'$ . The phase difference between the two arms is denoted as  $\Delta\phi$ . The diagram is annotated with "MZI using wave waveguid." and "Phase change". Below the diagram, the phase difference is given by the equation:

$$\Delta\phi = \int_{\text{upper arm}}^2 k_0 n_{\text{eff}}(z) dz - \int_{\text{lower arm}}^2 k_0 n'_{\text{eff}}(z) dz$$

A boxed equation below this states:

$$\Delta\phi = k_0 L \Delta n_{\text{eff}}$$

The bottom screenshot shows a diagram of a waveguide with an input plane (I/P) and an output plane (O/P). The input electric field is  $E$  and the output is  $E_1 + E_2$ . A graph below the diagram plots the normalized output intensity  $I_{\text{out}}/I_{\text{in}}$  against the phase difference  $\Delta\phi$ . The graph shows a cosine-squared function with a period of  $2\pi$ , indicating constructive interference at  $\Delta\phi = 0, 2\pi, \dots$  and destructive interference at  $\Delta\phi = \pi, 3\pi, \dots$ . The diagram is also annotated with "MZI using waveguid." and "I/P" and "O/P".

So, in a waveguide geometry we take a very simple waveguide, we put out a Y-splitter; and then we allow it to propagate through a waveguide section, and then we can combine this. So, this is basically your Mach-Zehnder interferometer using wire waveguide; so this is all single mode in configuration. In this case  $\Delta\phi$  is 0; you can see here there is no difference between these

two. So, all got their own widths, this is all identical width. So, this will always have as a function here; so there is no phase difference. They will always have high signal; so their interference is going to be always be constructive in nature.

So, let us write this  $\Delta\phi$  as a function of waves that are propagating through these two different arms let us say. So, let us assume that they are not out of same material or same system. So, we could we could have two different refractive indices. So, there is a change in the refractive index here; so I can again call this as waveguide 1 and let us say this is 2. So, now this has a different refractive index let us say; this has  $n$  dash and this is  $n$  let us say. So, now there is a difference in this particular geometry, and if that is the case your  $\Delta\phi$  is not any more 0. So, it is going to be non-zero.

And what is that phase difference is going to be? Let us say from branch one, so this is our first branch as a function of length  $L$ . So, this is the length that we have for both, this is also  $l$ . So, now it is  $k$  naught  $n$  effective as a function of  $l$  dl. So, if it is just single number, we can say  $n$  effective  $k$  naught  $n$  effective, minus the second arm. So, this is upper arm, this is lower arm  $k$  naught  $n$  effective dash into dl. So, you could have a section of this with slightly different index and that should be taken care by this; so, this is how your phase will change.

So, phase change this  $\Delta\phi$  is going to follow this particular relation between two arms; the upper arm and then lower arm. So, now you can actually find out what this is by simplifying this. So,  $K$  naught is constant, we are going to use the same wavelength; and the length is also identical here  $l$ . So, then the general case for the  $\Delta\phi$  is nothing but  $K$  naught  $l$  delta  $n$  effective  $l$ .

So, this  $\Delta n$  effective is what will create this phase difference; so this is the phase difference equation that you should keep in mind. So, phase difference is this, how about the power difference? So, whether there will be output power that will be affected by this phase? Yes, of course this I out by I in we drew this one. So, let let let us put that into a proper format here.

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The top screenshot shows the following equations:

$$P_{out} = \frac{1}{2} \cdot P_{in} \cdot (1 - \cos \Delta\phi)$$

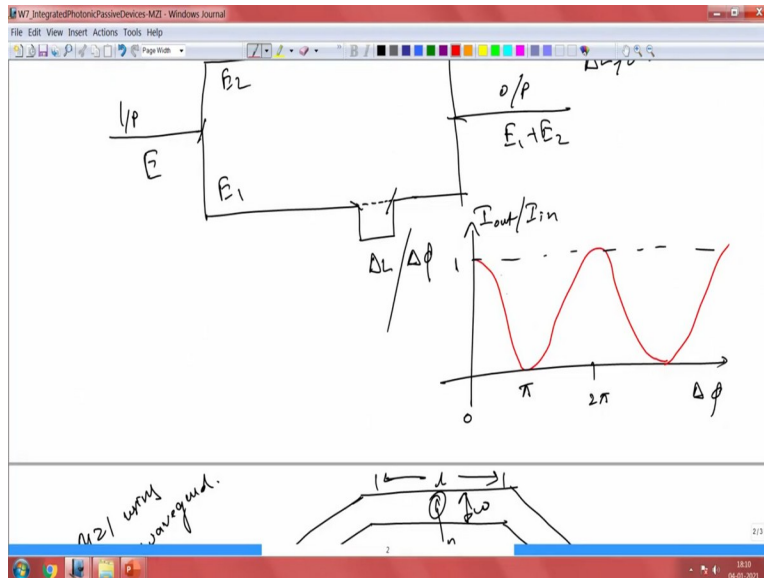
$$P_{out} = P_{in} \cos^2\left(\frac{\Delta\phi}{2}\right)$$

The bottom screenshot shows a diagram of a Mach-Zehnder waveguide (MZI) with a phase shifter. The diagram includes labels for waveguide width  $w$ , length  $L$ , refractive index  $n$ , and phase difference  $\Delta\phi \neq 0$ . A graph above the diagram shows transmission versus phase difference  $\Delta\phi$ , with minima at  $\pi$  and  $2\pi$ . The phase change is defined by the equation:

$$\Delta\phi = \int_1^2 k_0 n_{eff}(x) dx - \int_1^2 k_0 n'_{eff}(x) dx$$

$$\Delta\phi = k_0 L \Delta n_{eff}$$

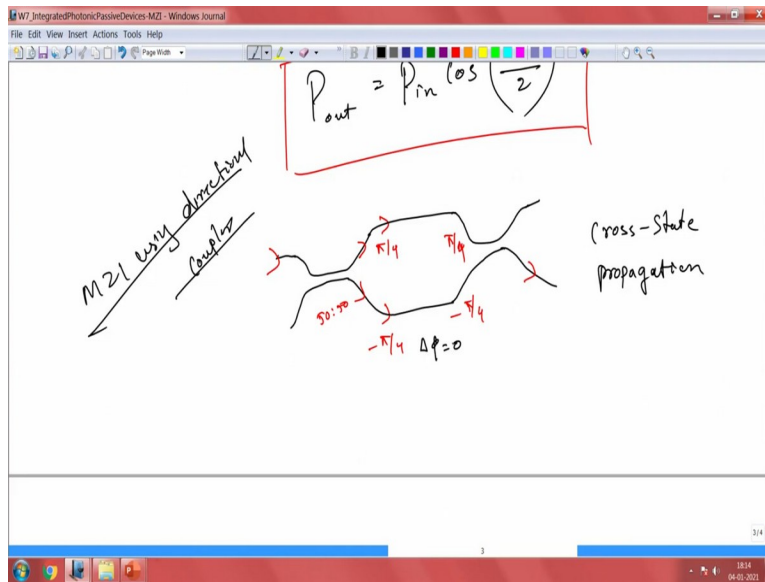
$$P_{out} = P_{in} \cos^2\left(\frac{\Delta\phi}{2}\right)$$



So,  $P$  out that is the power out will be equal to half  $P$  in times  $1 - \cos \Delta\phi$ . Or in other form,  $P \cos^2 \Delta\phi / 2$ ; so this is the identity. So, this is how your output power will change when  $\Delta\phi$  is not 0; so it will reduce the output power. So, this is again an important relation that you would want to remember, keep that in mind. So, now as I mentioned you can also implement this by using directional coupler. So, we have easily implemented this using a Y-splitter; so here you are splitting and here you are combining.

So, if you remember in our Y-splitter discussion, when there is a phase a complete  $\pi$  phase difference between these two; you will not be able to couple the light out, they will all leak out. Look at it what is happening here, the same thing here when  $\phi$  is the phase difference is  $\pi$ ; then there is a complete leakage of power and the power out will go to 0. And that is exactly what is happening here. So when there is a phase difference of  $\pi$ , then there is a destructive interference; and you will not see any light going through the waveguide. So, whatever we learn there, we are applying it here. Now, so let us look at how one can use the directional coupler approach; so, we are going to build a directional coupler based on (12:11).

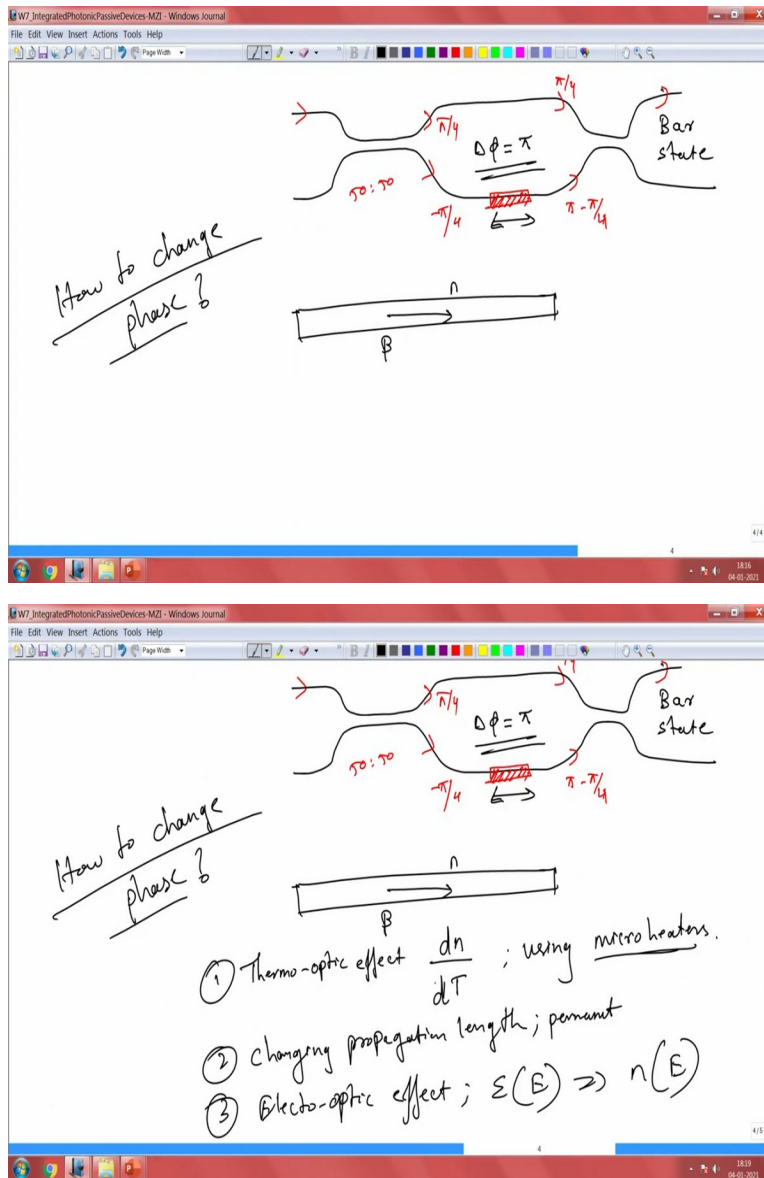
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So, Mach-Zehnder interferometer using directional coupler; so let us look at it what we can do better when it comes to directional couplers. So, the directional couplers you have one coupler here and it can go through; and then other coupler like this. So, you have one coupler here and then it goes out; and then another coupler, so this is one configuration. So, now you are putting your power in this particular arm, so when it is a 50-50 splitter; then you will have modes going into these two. So, they will be propagating through as minus pi by 4 and pi by 4; and when they reach here, again pi by 4 minus pi by 4.

And when they combine they will come out this side; so this is what we call the cross-state, cross-state propagation. So, this is how a simple directional coupler, where you have 50-50 coupling and you do not have any change in the phase at all; so, here delta phi equals to 0. So, let us take a scenario where your delta phi is pi; so that is our other scenario.

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So, in that scenario let us look at how we can do this; so again I am going to launch it from the top. In this case delta phi is equal to pi let say and how am I doing this? I am doing some changes here. I am I will briefly come to that how I can change it in a minute; but then I can change this particular section. I can do anything I want, so I can change the length, or I can change the property of waveguide there. So, now I am putting the same light through here and I am getting a 50-50 splitting. And I can get the light here and here; so now the phase is pi by 4 minus pi by 4 simple as that.



But, now I have an additional  $\pi$  here, so that  $\pi$  minus  $\pi$  by 4 is what I am going to get here; and here it is nothing but  $\pi$  by 4. Now, in this scenario the light is going to come out at this end; and what we call the bar state; so, this is the bar state. So, by using directional couplers you can see here, now I can switch between the two different outputs here. I can either go on the top or I can go to the bottom waveguide here. So, how did I do that? I just change the phase that I have here; so the ways to change the phase now. So, this is this gives you some phase  $\pi$ , but how to change phase is the question to ask here?

So, when you take a very simple waveguide; it has a certain refractive index  $n$ ; and you will propagate through this with certain  $\beta$ . So, now what are the possible ways to change this? One way to do this is by using thermo-optic effect. So that means your refractive index is a function of temperature; when the temperature changes, the refractive index would change. So, you can locally heat this and that would result in change in the refractive index. And that change in the refractive index would result in change in propagation constant; and you will have some phase difference here.

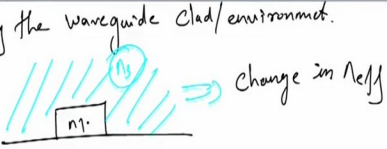
So, you can achieve this thermo-optic effect  $dn$  over  $dT$ ; you can locally heat this by using micro heaters. So, in the following lectures, I think later on we have a section talking about application and also different platforms. So, there I am going to show you real case studies; I am going to show you how we can use these heaters, micro heaters. And show you how the response of this particular Mach-Zehnder changes; we are going to see practical examples there. But, in this case we are just looking at the concept and theory behind this. So, the concept here is using thermo-optic effect that you can apply.

The second way to do that is by change the length, changing propagation length. So, this is this is permanent this is permanent; while in a micro heater case, this is tunable. You can apply the amount of current flowing through this micro heater, you can do that; but, in this case it is permanent. And the other way that you can do this is by using for example electro optic effect.

So, here we know that  $\epsilon$  is a function of electric field; so that means your refractive index could be changed as a function of electric field. So, by applying necessary electric field, you should be able to change the refractive index. So, when refractive index changes, everything changes; your propagation constant changes and your effective refractive index also changes resulting in change in your phase.

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④ Changing the waveguide clad/environment.



$n_2$   
→ useful for sensing applications

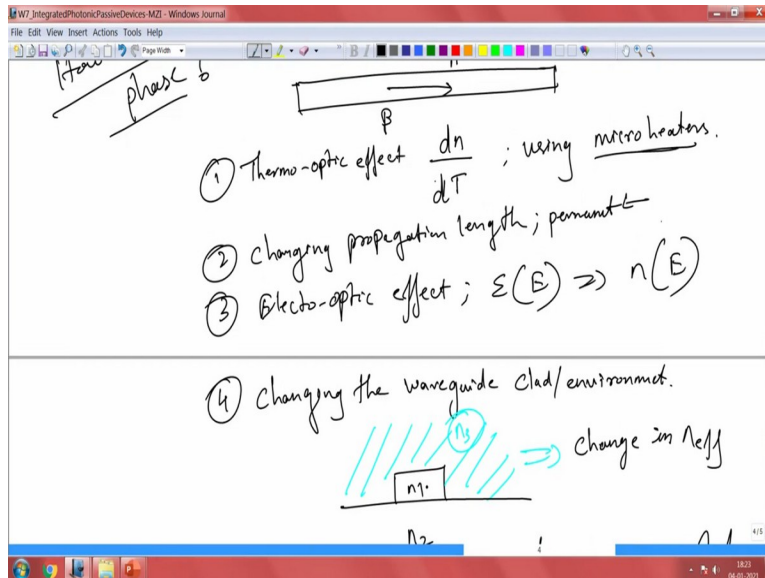
⑤ All-optical / nonlinear process ⇒ high intensity

$$n = n_0 + ik$$

$n_0 + \Delta n I$  → light intensity

Phase change

$$\Delta\phi = \int_{\text{upper arm}}^z k_0 n_{\text{eff}}(z) dz - \int_{\text{lower arm}}^z k_0 n_{\text{eff}}(z) dz$$
$$\Delta\phi = k_0 L \Delta n_{\text{eff}}$$
$$P_{\text{out}} = \frac{1}{2} \cdot P_{\text{in}} \cdot (1 - \cos \Delta\phi)$$
$$P = P_{\text{in}} \cos^2\left(\frac{\Delta\phi}{2}\right)$$



So, the another way to do this is by expose changing the environment or waveguide clad or environment. So, here I will take a cross section; so this is the waveguide with  $n_1$  and you have  $n_2$ . So, you have a certain beta propagating through this medium; so there is a certain beta associated with this. But, then if I put a (ref) a different refractive index on top, like there is  $n_3$  on top; then the beta is going to change now. And by changing this cladding index from let say  $n_3$  to  $n_4$ . You are going to change the effective refractive index. So, this will have an implication in change in  $n$  effective.

Because if you remember effective refractive index capture not only the cross the geometric property, but also the environment; that the waveguide is sitting. So, this is the property we use for sensing, useful for sensing application. So, we change the environment; when we change the environment, the  $n$  effective changes. So, when you change the  $n$  effective, your phase also changes. We had this  $n$  effective here you see here; so when we change the  $n$  effective, you will change the phase here as well. So, that is another way of doing this phase difference; and finally you can do all optical.

This is reasonably hard that will invoke non-linear process; so, this uses non-linear process. So, wherein you use light to change the refractive index of light; so you have real part and the imaginary part of light sorry,  $n$  naught plus  $iK$  in this case. But, then even this  $n$  naught this could be a function of the intensity. So, this could be a function of light intensity; and based on this light intensity, you could change the refractive index. So, this is part of utilizing non-linear properties in the waveguide to do this.

This is not this this can be very small for some material, and this can be large for other material. They based on the material property; but the bottom line is you can do this, but you need higher power. So, you need high intensity of light for this. So, the usual way that we do is through thermo optic or electro optic; and you can also do by changing the environment, which we normally use it for sensing application. But, permanently changing your length, this is something that we use to realize filters; so fixed spectral filters. So, this is again examples that we will see in the case studies. Once we go into the application discussion later on in the course.

So, so far this is this is all I would like to show it to you guys on the use of Mach-Zehnder interferometers. And also understanding how you can build this Mach-Zehnder interferometer by simply using a splitter and a combiner; attached with the simple waveguide. And you can create this phase change by applying change in length; or applying heat or applying field in some cases, applying high intensity light itself.

So, by doing this kind of phase tuning, you can realize very interesting spectral properties, spectral characteristics. And you can also use this as a switch that we saw by using directional couplers, at the input and output end by changing the phase in the arm, we can either put it the bar or it could put it at the cross. So, one can exploit this functionality for many application that you desire. So, with that thank you for listening.