Course Name: Design of Electric Motors

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Title: Analysis of Copper Function and Output Function w.r.t the Electric Machine D^3L Product Equation

Greetings to all, in this lecture also, we will continue the same analysis with respect to the D cube l product, how to analyze the sizing equations. First we will see the power equation, what we have derived in the last lecture. So, D square is equal to f of lambda that is f naught of lambda is nothing, but the output function into delta into lambda divided by D naught s square into pi square by 240 root 2 into efficiency cos phi g power factor, copper free factor kcu and winding factor k w and Bg magnetic loading and actual current density and D cube l product and synchronous speed. This is the power equation we have derived in the last lecture.

Now, we will analyze the torque equation. The mechanical power if you will represent in terms of speed and torque, Ns is the synchronous speed that equation is 2 pi NT by 60.

By utilizing these two equations, we can derive the equation for torque developed by the machine. It is equals to f naught of lambda minus del into lambda divided by D naught s square into some constant pi square by 240 into root 2 efficiency power factor and copper free factor, winding factor into magnetic loading and current density and D cube l term into synchronous speed. Analyze this equation, then torque is equals to f naught of lambda minus delta is a constant into lambda divided by outer diameter of the stator square into pi square by pi square will not come here because of this side and both sides pi is there. So, 1 pi will be cancelled each other pi by 8 root 2 into efficiency power factor into copper fill factor winding factor and Bg Jl into D cube l.

So, this is the torque equation, equation number 21. So, from this equation, we can see that the torque developed by an electrical machine is directly related to the volume product that is D cube l e. This product is not only the volume into the outer diameter of the stator. We can represent this D cube l e is nothing but D square l into E D square l into D one more D. This is nothing but the D cube l product.

It is depending upon the volume and it is also related to the magnetic loading and then electric loading with respect to the actual current density that is J l and power factor cos phi g. So, by changing the function and one more thing is output function by maximizing the output function, we can maximize the output power as well as the torque developed by an electrical machine. So, we will see now how to maximize this function because this D square l or D cube l term is already there in the earlier equations and this equation also and flux densities with respect to the air gap and actual current density and power factor. So, power as well as torque depending upon these values. The key thing with respect to the D square l and D cube l product equation is nothing but output function involved in the D cube l sizing equation.

If I will rewrite the sizing equations, P naught is nothing but with respect to the D square l product, I will write first whatever the equations we have discussed. P naught with respect to the output power with respect to the D square l product is nothing but pi square by 120 cos phi g is nothing but power factor and then efficiency, flux density are nothing but magnetic loading at the air gap and electric loading with respect to the air gap, current density and D square l product, winding factor, synchronous speed. This is the final equation with respect to the D square l term. And now what we have derived power in terms of D cube l, that equation is f naught of lambda minus del into lambda divided by D naught s square into pi square divided by 240 root 2 into efficiency cos phi g into k c u, copper fill factor, winding factor, actual magnetic loading and actual current density with respect to the stator and D cube le product into synchronous speed. This is with respect to the D square l term and this is with respect to the D cube l term.

These two sizing equations we can utilize it for designing any machine with respect to the dimensions. If you want to calculate the dimensions of the stator with respect to the inner diameter and length of the core, we can utilize the first equation. If you want the stator geometry completely, we can see here stator lamination where we require the inner diameter, outer diameter and slot area and slot dimensions and flux densities at the different parts of the iron. If you want to consider, then we have to go ahead with the D cube l product equation. That is what we have analyzed as of now.

We can see here. Now, we will see how to maximize this function in the D cube l equation. f of lambda f naught of lambda is the key. So, how to maximize this function with respect to the different flux densities values? If you will maximize this function, automatically we can maximize the output power. That we will see now.

Let us consider the equations, whatever the equations we have derived with respect to the slot geometry, we can see here lambda is nothing but inner diameter of the stator divided by outer diameter of the stator and f of lambda is nothing but copper function. First, we will analyze with respect to the copper function and then output function. We have to maximize these two. These are the roots we have derived and a is nothing but constant b also constant, but a and b depending upon the flux densities at the different parts of the stator core. Here, we can see this is the flux densities at the stator back iron and this is flux densities at the teeth.

This is flux density at the air gap. These three flux densities we have to select precisely with respect to the material. Then, calculate the G c and G t constant and substitute here. Here, I am considering 4 pole machine and slot fill factor. Here, K s we have taken as a stamping factor, steel stacking factor.

I am considering one. There is no air ducts and steel stacking factor is nothing but K s. I am considering K s equals to 1 and B c by B t constant and G t and G c were evaluating first with respect to these two equations. This is G c equation and then this is G t equation. Based upon that, I am calculating what is G t value and what is G c value with respect to the different flux densities where B g, B t, B c are different values.

I am considering based on that. I am considering G c and G t values differently. Initially, 0 and 0 both G c and G t constants are 0 and then G t is nothing but flux density at the air gap to flux densities at the teeth ratio. I am considering 0.

25 like that 0.5, 0.75. To see the output function variation with respect to the G t values and G c values, I consider different values here and based on that, calculate the values of a and b. These are the a and b values constant. Once you know the G c and G t, here what we are doing is we are varying G c value and G t value also we are varying.

G c involves in B g and B c and G t involves in B g and B t. By considering different values, how to select these values as per the standards or as per the studies, I will show that table also. Now, we are varying these values and A and B, we can calculate it here. These are the A B values. So, with respect to these A B values, here I have considered the same A B values with respect to different G and different G t value and different G c values.

First A value, then B, then lambda value. Lambda value I am varying 0 to 1. We have done the analysis. The feasible values for lambda is 0.

4 to 1. That is what we have concluded in the last class. So, lambda value I am varying from 0 to 1. How it will vary with respect to the output function? We will see. With a step of 0.02, I am varying the lambda value and here f of lambda I am calculating that values are presented here and f lambda also is presented here.

f naught f lambda is nothing but output function. This is the output function and this is the copper function. Similarly, for different values of a and b, this is set 2 values and these are the set 3 values and these are the set 4 values and set 5 values. 5 set of points I have considered here. We can observe here a1 and b1 like 5 columns, 5 rows are there.

This is set 1, 2, 3, 4 and 5. 5 sets I have considered different a and b values. The same values I am substituting here and I am estimating the output function and copper function. If I plot the waveforms f of lambda as well as f of f naught of lambda with respect to the lambda. These are the curves for lambda versus f naught of lambda.

Here it is lambda, y axis is f naught of lambda. How the f naught of lambda function is varying with respect to different values, we can observe here. This is with respect to the set 1. We can identify which curve is representing which one. So, this one is nothing but set 1.

Then this is set 2 and set 3, set 4 and set 5 with respect to the 5 operating points of a and b. I have considered the f naught of lambda curves. I have drawn here f naught of lambda curve. From here, we have to choose the lambda value precisely to find the f naught of lambda value.

We have to maximize this function. Let us take this curve I am taking. In this curve, f naught of lambda maximum at this particular point where the lambda value is nothing but 0.35. This is f of f naught of lambda.

With respect to the 0.35, D I s and D naught s difference will be very small like lambda value is very small means the space for rotor is coming down because of this reason, we have concluded the lambda value should be 0.4 to 1 or up to 0.75 to make the feasibility with respect to the rotor design also. We can select 0.

5 at this particular point. In this range, we can select any point and see the f naught of value, f naught of lambda value. For example, lambda equals to 0.5. Then f naught of lambda value is at this particular point.

Then f naught of lambda is equals to 0.12. We have to select the lambda value as well as f naught of lambda value to solve the power equation. These are the curves with respect to the f naught of lambda versus lambda that is output function equations or curves with respect to the lambda. So, the operating curve we can see the optimal operating points with respect to the maximum values is nothing but the red color line. If it is matching with respect to this range, then we can analyze the decubal product and we can realize the machine main dimensions.

Similarly, other side we can see the f of lambda versus lambda function, copper function with respect to the lambda curves. If f of lambda function is equals to 1, what will happen at this particular point? If I will consider f of lambda equals to 1 means a lambda square minus 2 b lambda plus 1 is equals to 1. That means a equals to 0 and b equals to 0 or we can say lambda equals to 0. That is also fine to attain this f of lambda equals to 1.

At this situation, what it will happen means f of lambda equals to 1 means lambda equals to 0.

Then from the slot area equation, I will go back to the note pad. Slot area equation a s is equals to what? f of lambda in terms of a s. f of lambda is equals to a s into 4 q s divided by pi d naught s square minus delta divided by d naught s square. Here, if we neglect this term and if we will substitute f of lambda equals to 1, then a s is equals to pi d naught s square by 4 into 1 by Q s. If I will bring Qs also this side, q s into a s is equals to pi d naught s square by 4.

Here, Q s into A s represents the total stator area. The total stator area is equals to pi D naught s square by 4. Actual area, if we will see here, the stator lamination, single lamination, the total area if we will calculate pi D naught s square by 4 circle area. And with respect to the slots, each slot having the area of As and number of slots are Q s, then Q s into a s equals to pi D naught square pi D naught s square by 4, which will eventually representing that there is no space for iron at the stator side. Complete stator is having the copper only, all slots complete stator is occupied with copper windings only.

This is the stator complete thing is occupied by copper. Similarly, if I will consider f of lambda is equals to 0 for different values of a, b and lambda. For example, a equals to minus 1 and b equals to 0 and lambda equals to 1. This is one possible solution, a equals to 0 and b equals to minus 1 and lambda equals to 1. And similarly, if lambda equals to 1 and b equals to 2 and a equals to 1, for this for this possible solutions f of lambda is equals to 0.

f of lambda equals to 0 means from the slot slot area equation 4 q s divided by pi D naught s square is nothing but 0 from the above equation. From this equation, if we will substitute f of lambda is equals to 0, a s equals to slot area is equals to 0. That means, here there is no slots here, slot area is not there. That means, we cannot place the windings. There is no space for windings means completely the stator is occupied by iron, iron core no space for copper windings.

So, we have to select the lambda value and f of lambda value optimally to achieve the or to make the proper design with respect to the windings as well as iron. That is what we can see here. The curves of f of lambda copper function with respect to the lambda with respect to the different values of G t, G t we are varying here. With respect to the different values of G t, we can see the curves here. So, we have to select the operating points at the middle somewhere.

We have to select the operating points in this region such that we can accommodate the copper as well as iron. So, these curves are varying with respect to the number of poles. If I will change the number of poles, how the output function as well as copper function will vary, we will see. Why those two will vary with respect to the poles means here G c is depending upon the number of poles.

G c equation is depending on the number of poles. Because of that reason, if I will change it to 6 pole here, earlier 4 pole was there. Now, I have increased it to 6 pole. We can see here, here 4 pole machine, I did the analysis f of lambda as well as f naught of lambda curves. We can observe here. If we will increase the number of poles, these curves will change in this manner where f of lambda value as well as f naught of lambda values will vary.

For example, if I will take the operating point for the third case with respect to the 0.42, here it is to near to the 2 f naught of lambda value for 4 pole machine. For third case, it is not at all equals to the 0.2. The same thing we can observe at 0.4. Here, some gap is there. Next, if we will increase the number of poles further, for example, I am substituting the 10 poles here. 10 poles I substituted here with respect to this number of poles here. G c values have been changed because the G c value is depending upon the number of poles. Based on this equation here, G c value has been changed with respect to the number of poles.

Based on that thing, we can observe here the third curve that is yellow one, the maximum possible output function value f naught of lambda is increasing now. We can observe that one. Earlier it was at the black point, this dotted black color point. Now, it increased to this point. Similarly, if I change the number of poles to 20, I am changing the number of poles, then G c will be changed.

Then a and b values also will change. The moment a and b changes, then f of lambda value and f naught of lambda value will change. Here, P equals to 20 is substituted. So, if we increase the number of poles, automatically this point will keep on increases. That means, the maximum output function is varying with respect to the number of poles.

That variation we can see here. If I will increase the number of poles, if you are increasing the number of poles in this direction, we can see the f of lambda equation as well as lambda values. This curve blue curve representing the lambda values that is d I s divided by D naught s curve. This red color curve is representing the output function f naught of lambda. So, by changing the number of poles, we are changing the f naught of lambda output function value and lambda is nothing but D I s by D naught s value. So, this function will eventually changes the output power equation as well as output torque equation.

In order to maximize the power output power, we have to maximize the f of lambda function. So, by considering the different flux density values in different portions or different parts of the iron, we have to maximize this f of lambda. What are the feasible values for B g, B t and B c with respect to the different parts of iron? We will see now.

So, here we can see the maximum possible flux densities with respect to the air gap for induction machine, for salient pole synchronous machine, for non-salient pole synchronous machine and DC machines is presented here. Flux density is at the air gap, flux density is at the stator yoke that means, this one stator yoke is nothing but this one back iron.

The stator tooth means here, this one the gap between two slots and air gap is nothing but here, this one this is nothing but air gap. If I will place the rotor here, the gap between the stator and rotor is nothing but the air gap. We can see here, whatever the gap is there between the rotor and stator in that particular gap, the maximum possible flux density will be this one 0.

72, 0.9 for induction machine, 0.85 to 1.05 for synchronous machine, 0.6 to 1.1 for DC machines. Similarly, at the rotor side, if we will take the rotor lamination, here we can see rotor yoke means this is the one back iron with respect to the rotor structure and poles, commutating poles and other things, the allowable flux density limits with respect to the DC machine also presented here. These flux density values with respect to the different parts of iron depends upon the material. What type of materials we are utilizing? Based on that thing, the maximum allowable value of the flux density with respect to the air gap, with respect to the teeth and with respect to the core, we have to select it and there should not be any saturation.

The saturation flux density should be higher than whatever the values we are considering for the design. Similarly, if we take the electric loading for different machines, this is the A is nothing but actual current density and J is nothing but current density with respect to the stator winding like here A is nothing but electric loading. That is what we have discussed with respect to the d square l product equation and J is nothing but actual surface current density that is ampere per mm square. We can see here the electric loading kilo ampere per meter or ampere per mm, 30 to 65 kilo ampere per meter and current density is nothing but 328 ampere per mm square for stator winding and for copper type of rotor winding in an induction machine 328 ampere per mm square and for aluminum rotor winding, 326.

5 ampere per mm square. If we take the higher values of current densities, we have to make the proper thermal management system. Similarly, for synchronous machines, we can see here different current electric loading values and current density values. So, if you want to select the higher value of the current density, we have to design the proper cooling mechanism or thermal managing system. We have to design it. This is how we can select the current density values and electric loading values with respect to the d square l product equation and d cube l product equations.

So, this A is nothing but electric loading where you will utilize it for d square l sizing equations and J is nothing but current density that is J l. We will utilize it for d cube l sizing equations. With this, I am concluding this lecture. In this lecture, we have discussed the analysis with respect to the output function that is F0 of lambda and F of lambda. We have discussed the copper function and output function, how to maximize it these two things and how to select the flux densities at the different parts of iron.

We have discussed in the next lecture. We will discuss the actual design with respect to the different machines. Thank you.