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Title: Basics of Thermal Equivalent Circuits

Greetings to all. In this lecture, we will discuss the basic equations to calculate the temperature at various parts of the machine or temperature at various thermal nodes and also we will discuss the basic building blocks to realize the thermal equivalent circuit for any type of electrical machine. Let us consider a machine. This is the machine and with respect to the practical system, the efficiency is not equals to 100 percent and there will be a losses and with respect to the machine, the losses are classified into three types. Copper losses with respect to the I square R loss and core losses, hysteresis and eddy current losses and mechanical losses are nothing but friction and windage losses. These losses are the prime source for the heat generation inside this electrical machine.

So, in order to calculate the temperature at various parts of the machine, we have to identify the different thermal nodes or we have to divide the machine with different nodes. So, these dots represent the thermal nodes. So, we can consider n number of nodes. One can design with 4 thermal nodes, other can decide with 40 thermal nodes to get the higher accuracy and to make the appropriate thermal design, we have to consider the sufficient number of thermal nodes such that we can find the temperature at various parts of the machine.

For example, if we will consider the stator winding node, this red color circle is the stator winding node. At this node, the stator winding copper loss are happening. So, these losses will act as a heat generating source here with respect to the stator winding copper losses. So, same way we have to identify the thermal sources because of the heat generation due to the losses. We will consider these thermal nodes as a thermal sources.

So, with respect to the copper loss at the stator side and rotor side and iron loss at the stator side as well as rotor side and mechanical losses and bearings and friction and windage losses, all those things we will model it as a thermal sources and remaining dots will act as a thermal nodes and then we will design or we will implement the thermal equivalent circuit. So, to implement the thermal equivalent circuit, we have to understand the basic elements. The basic elements to develop the thermal equivalent circuit of any machine are thermal resistance that is R theta and thermal capacitance C t h and the power loss I am representing with current source. So, these power loss are responsible or prime source for the heat generation. This node I am representing with 1 and temperature at this node will be T1 and temperature at node 2 is T 2.

So, with respect to the temperatures level, let us say T 1 is higher than T 2, then heat flow will be in this manner. So, heat will flow always from higher temperature to the lower temperature. The thermal resistance is not a dissipating element. It will offer the impedance for the heat flow as per the material type. So, thermal resistance is equals to r theta is equals to change in temperature or difference in temperature between the 2 nodes divided by power loss.

It will give the thermal resistance. Already, we have derived this equation. This equation is an empirical based equation and units are Kelvin per watt or degree centigrade per watt. Thermal capacitance is similar to the electrical capacitor. I am not considering any analogy.

The capacitance or capacitor is equals to a storing type of element and it stores the heat energy with respect to the 2 different thermal nodes. That is this is node 1 and this is node 2. Temperature at node 1 is T1 and temperature at node 2 is T2. So, with respect to the difference in temperature of these 2 nodes, it will store the heat energy as per the material type and Cth is equals to heat energy with respect to the change in temperature joules per Kelvin. The units are in terms of mass and specific heat.

We can represent mass into specific heat capacity. So, mass will be units are kg and specific heat units are joules per kg into Kelvin or we can write in this manner kg Kelvin and power loss. I am representing with a heat generating source that is current source I have drawn here. So, the thermal modeling or thermal network for any electrical machine or any system, we can model by utilizing these 3 basic elements. Let us consider a simple thermal network and a thermal designing is a 3 dimensional problem like radial, axial as well as tangential.

The thermal design is nothing but a 3 dimensional problem. If we will consider all direction of heat flow, then the thermal modeling will be a complex job. So, first we will consider only 1 dimensional heat flow and we will develop the network based upon this 1 dimensional network. We will realize the thermal network for 3 dimensional aspect. Let us consider a 1 dimensional network and with simple elements, all these 3 elements, so thermal resistance and thermal capacitance and the power loss element.

So, heat is flowing in this direction. This is temperature at node 1 is T1, temperature at node 2 is T2. So, this is the basic 1 dimensional thermal network. If we take the electrical network or C network, this is resistance and capacitance and this is the current source I s. Current flowing through the capacitor is I c, current flowing through the resistor is I r.

So, here the source current is equals to current flowing through the capacitor plus current flowing through the resistor. So, I c is equals to C dV by dt plus I R is nothing but V by R. So, similar to this type of equation in thermal system or thermal circuit analysis also, we can write the power loss equation. P is equals to C th into dt by temperature, change in temperature with respect to the time plus the heat flow with respect to the thermal resistance, that is change in difference in temperature T1 minus T2 divided by R theta. So, this equation represents the transient equation for the thermal network.

So, this term with respect to the thermal capacitance and this term with respect to the thermal resistance. In a thermal network of any electrical machine, assume there are n number of thermal nodes are there. This is for with this equation is only for with respect to one particular node. For n number of nodes, if you want to represent the generalized equation, Pi is equals to C th I, here I represents the I is an integer or variable, I equals to 1, 2, 3 and so on up to n. It is representing the number of nodes and n number of thermal nodes, we have considered to realize the thermal equivalent circuit.

So, C th I into change in temperature T I divided by dt plus sigma, one more variable I am considering, that is J equals to 1 to n, because the thermal resistance we will define with respect to the change difference in temperature with respect to the two nodes. So, consider this is i th node and this is j th node. Same way, here some other node is there j 1, here also some other node is there j 2. So, with respect to I th node, what is the heat flow with respect to this I th node? That is nothing but j equals to 1 to n t I minus T j divided by R theta thermal resistance with respect to the i and j. We have to define the thermal resistance relation with respect to the all nodes.

With respect to the two nodes, we can write in this fashion, in this term with respect to the different nodes, if you want to represent the thermal resistance term with respect to the power loss, we can use this term, term 3 and this is term 2 with respect to the capacitance and first term with respect to the power losses. If you will represent in terms of matrix form, power loss matrix is equals to capacitance matrix to solve the n number of equations, as n nodes we have means n number of equations we will get right and n unknowns we have to calculate. n unknowns are temperature at n nodes we have to calculate by analyzing this equation C th matrix into change in temperature matrix divided by dt plus 1 by thermal resistance. I am representing with conductance matrix that is G into temperature matrix. This is the matrix form of equation.

This is equation number 3, equation number 1, this is equation number 2. So, here this is a power loss matrix and this is a capacitance matrix. This is thermal conductance matrix and temperature matrix. Consider any electrical machine with n number of thermal nodes in thermal network. Then, equation 3, we will utilize to solve the temperature at various parts of the machine and realizing the thermal network or thermal equations with respect to the all three dimensional heat flow is a difficult task.

So, first we will analyze with respect to the one dimensional heat flow. Under steady stand still condition, there is no supply. Under no supply condition, there is no loss right in the system. That means, in the above equation, there is no power loss matrix directly. We can write C th into temperature matrix change in temperature matrix is equals to minus conductance matrix into temperature.

So, this is the equation with respect to the no supply condition where there is no supply is given to the machine.

There is no losses inside the machine. So, such that there is no heat generation. Then, this equation we can get it. Next, under steady state condition, this term will be 0 right.

The change in temperature term with respect to the capacitor similar to the electrical circuits under steady state condition capacitor will not draw any current right. So, here also we can represent under steady state condition power loss matrix is equals to conductance matrix into temperature matrix. Here, the change in temperature matrix is equals to 0 with respect to the capacitor. Under steady state condition, we will see the heat distribution is happening with respect to the thermal resistances. Now, we will see the matrix with respect to these 4 different types of matrices.

So, power loss matrix is equals to 1 P 1, P 2, P 3. It is n into 1 matrix and so on up to P n. It is a column matrix and a temperature also column matrix T 1 temperature at node 1 temperature at node 2 temperature at node 3 and so on up to T n. This also n into 1 and the thermal resistance matrix or conductance matrix is equals to generally conductance is equals to 1 by thermal resistance right. So, here conductance matrix is equals to sigma j equals to 1 to n conductance with respect to the node 1 to different nodes and then conductance with respect to the node 1 and 2 conductance with respect to the node 1 and 3 and so on up to conductance with respect to the node 1 and n.

So, just see the all terms with respect to this matrix with respect to the above equation. Then, we can realize this matrix. This is n by n matrix. So, same way here conductance with respect to the node 2 and node 1 and summation term j equals to 1 to n conductance with respect to the node 2 and other nodes and same way we can write all other terms here also. So, G n 1 conductance with respect to the node n and 1 and G n 2 and so on sigma j equals to 1 to n conductance with respect to the node n and j.

So, this is a n by n matrix and thermal capacitance matrix C t h is equals to it is also n by n matrix and it is a diagonal matrix C 1, C 2 and C 3 and so on up to C n. This is the thermal capacitance matrix. So, by analyzing this equation, we can find the temperature at various thermal nodes of the machine like this point, this point, wherever we want to analyze the temperature, we can find the temperature. If let us say a temperature at particular node is greater than the operating limit, safe operating limit, consider this is the node at this particular point, the temperature operating temperature is greater than the safe operating limit. In this situation, whatever the thermal system we have implemented that is not suitable.

So, we have to make the appropriate design with respect to the thermal system. We can use either forced cooling or some liquid cooling. We can utilize liquid based cooling. Otherwise, in the other side, if the temperature at all nodes with respect to the different nodes is coming less than the safe operating limit, then no need to modify the thermal design. So, this is the basic equation to analyze the temperature at various parts of the machine.

So, if you rewrite this equation number 3, that is d T by d t is equals to, you want to find the temperature at various nodes, right. So, that is d T change in temperature matrix d T with respect to the time is equals to power matrix into 1 by C t h or we can represent with inverse C t h inverse matrix plus conductance matrix into temperature matrix into inverse of C t h matrix. So, this is equation number 4. By utilizing this equation number 4, we can find the temperature at various nodes of the machine. Next, we will discuss the basic building blocks to realize the thermal equivalent circuit of a machine.

So, first we will realize the equivalent circuit with respect to the one dimensional aspect. Then, we will utilize the same network for other two dimensionals also like axial direction. We will find the thermal network, radial direction also we will assume that same kind of network will be there. Then, we will club all those thermal network. Then, we can find the three dimensional thermal network.

At the end of this lecture, we will see the three dimensional thermal network. Let us consider a simple cylindrical structure. In an electrical machine, we can see the most of the structures will be in the form of cylindrical structure, right. So, here heat is flowing in this direction and power loss at the node 1 will be P 1. This is node 1 and this side it is node 2 and heat is flowing in this fashion and temperature at node 1 is T 1 and temperature at node 2 is T 2.

There is no power loss at the midpoint of the cylinder and heat distribution is uniform. Heat flow through this cylinder is uniform and power loss with respect to the node 2 is t 2 and length of this structure is the cylinder is l x. So, at node 1 length is equals to l equals to 0, I will consider at node 2 l is equals to l x, I will consider. So, in order to represent this kind of thermal network with respect to the basic elements, we can utilize the thermal resistance model directly because there is no heat source, right, in this network. We are not considering any thermal source with respect to the losses.

So, directly heat is coming from some other node to this node and from this node to this node it is flowing. So, P 1 and P 2 just I am representing with respect to the losses or heat flow from the other networks or other nodes. So, the equivalent representation of this structure, we can make it in this fashion. The temperature at node 1 is T 1, temperature at node 2 is T 2. This is the thermal resistance if there is no heat source within this structure.

This is the basic building block 1. Same way, if you want to find the temperature at midpoint of this structure T average, then in an ideal case, we can write in this fashion, right. This is the midpoint temperature. So, T 1 plus T 2 by 2 will come, the mean temperature. So, this side it will be T 1 and this side it will be T 2. The thermal resistance for a uniform heat distributed cylindrical structure and resistance also I can make it equally half half.

So, R theta by 2 and R theta by 2, this is basic building block 2 under the assumption that there is no heat generating source. So, these two are the basic building blocks. Next building block is we will consider the heat generating source at the midpoint of the cylinder. At this particular point, I will consider the heat generating source.

For example, stator winding node or stator core. So, here losses are happening that is P L. Then, how to represent this kind of network and assumptions are same. Heat distribution is uniform and it is a one dimensional heat flow axial direction. We are considering one dimensional heat flow and heat source with respect to the losses P L and this heat is injected at the node where the average temperature is T a v. Then, we can represent this kind of network as a T type network.

So, here the average temperature is T a v and the power loss component is incorporated here. This side temperature is T 1 and this side temperature is T 2. The heat is flowing in this direction with respect to the power losses and this is R theta by 2 and this is also R theta by 2 and this one is minus R theta by 6. So, this is just mathematical equations we have to derive with respect to this structure and then, we can get this kind of thermal equivalent circuit. Anyone is interested to derive this thermal network, they can go through the Poisson's equation based one dimensional heat flow method for cylindrical structure.

The proof is given in Lipo text book or you can go through any type of thermal literature, thermal equivalent circuits literature where heat flow is assumed in one dimensional. The Poisson's equation is lambda z is equals to partial derivative with respect to the temperature double derivative in the z direction. Here z direction represents the axial direction that is equals to minus q l. Here q l represents the heat flow density. Heat flow density is equals to power loss with respect to the volume.

This is the heat flow density. So, from this equation, we can start deriving the equivalent thermal network. At the end, we will see the thermal equivalent circuit in this fashion. Otherwise, directly just use this kind of basic building block to realize the thermal equivalent circuit. So, if heat generating source is there at the midpoint of the cylinder, then how to represent the thermal network means this is the structure.

So, this is in one dimensional. If I will consider two dimensional heat flow in radial as well as axial, here axial direction we have considered. If you want to represent the thermal network in two dimensional way, assumptions and analysis everything will be same and I will consider the same network. So, network will be same. Here heat is flowing in both axial direction that is in this fashion and radial direction that is in this fashion. Both axial as well as radial heat is flowing and there is a heat generating source at the midpoint of the cylindrical structure that is Pl and heat flow is uniform with respect to the radial as well as axial direction.

The heat flow rate is Q and the power losses we can see here with respect to the node 1 and node 2, the power losses with respect to the heat flow. Then in axial direction, we will represent the thermal network with respect to the T type network. This is in the axial direction. Same way we can represent the thermal network in radial direction also. This is for axial direction, this is for radial direction and then we will add these two networks to realize the two dimensional thermal network.

The accurate analysis if we will do that is a complex task and the equations also complex and thermal network will be a complex network. So, to realize the simplest thermal network, we will utilize the one dimensional heat flow networks and then we will add these two networks. This is with respect to the axial, this is with respect to the radial and here heat flow with heat generation with respect to the power loss are incorporated at this particular point and temperature at this particular point will be t average and temperature at the node 1 is T 1 and node 2 is T 2. The heat is flowing in this fashion and the power loss I can say P 1 at this particular point heat is flowing with respect to this kind of losses whether at this particular node or some other node and this is temperature T 3 and this is temperature T 4. So, at this radial direction this is T 4 and this is T 3 the temperatures and the thermal resistances in the same as the one dimensional network we can represent here.

This is R x naught by 2 or R x we can represent directly R x and R x by 2 and this is R x by 6 R x is nothing but the thermal resistance with respect to the axial direction R y is nothing but thermal resistance with respect to the radial direction minus R y by 6 here R y by 2 here also R y by 2 heat is flowing in this direction. This is the approximate thermal equivalent circuit model for this kind of system where heat generation with respect to the power loss is happening at the center point of the circuit and heat flow is happening in both radial as well as axial direction that is two dimensional heat flow.

Other approximate thermal model or basic building block with respect to the two dimensional heat flow is in this fashion. So, these two resistances like this resistance and this resistance with respect to the axial and the green color resistances are with respect to the radial. This is R x by 2 and R x by 2 here R y by 2 with respect to the radial and this also R y by 2.

This is temperature T 1 and T 2 this is T 3 and T 4 and the power losses are incorporated at the center point of the network. Here power losses are incorporated and temperature at the midpoint is T a v. So, either this network or this type of network we can utilize to represent the heat flow or thermal network for a two dimensional heat flow. This is basic building block 4. In order to represent the heat flow in all three dimensional aspect, consider the same network here.

Heat generating source is there at the midpoint of the system and heat is flowing in all three dimensional radial axial and tangential directions. Then the thermal network we can realize by utilizing the same one dimensional network. So, this is with respect to the radial, this is with respect to the axial, same network we will utilize it. I will draw here. So, this is the thermal network with respect to the axial, thermal network with respect to the radial is this one, thermal network with respect to the tangential is this one.

So, I am considering same building block with respect to the one dimensional, but three dimensional means three such building blocks we will take and we will connect in this fashion. So, here last component will be incorporated and temperature will be T a v and this is T 1 temperature at node 1 is T 1 temperature at node 2 is T 2 in radial direction. Then axial direction it will be T 3 and T4 in tangential direction it will be T 5 and T 6 heat is flowing in this manner. So, here this direction is axial and radial will be in this direction and tangential direction means upward coming out of the paper or we can see here three dimensional x y and z. All three directions the heat is flowing and the equivalent thermal network we can see here this is basic building block 5.

So, by utilizing these five basic building blocks we can realize the thermal network for any type of machine. Here also the resistance values will not change in a three dimensional network also. So, with respect to the radial axial it will vary. So, the values will be same for radial I represented with R y for axial it is x I mentioned.

So, same way we can consider here also this is for axial R x by 2 r x. So, I am sorry this thing I considered radial direction right. So, r y by 2 r y by 2 minus r y by 6 and this is axial direction r x by 2 r x by 2 and this is minus r x by 6 same way here r z by 2 r z by 2 and this is minus r z by 6. So, the values and basic building blocks will not change only three networks we have to consider and we have to connect and we have to develop the three dimensional thermal network. So, with this I am concluding this lecture. In this lecture we have discussed the basic equations to find the temperature at different parts of the machine and basic building blocks to realize the equivalent circuit of an electrical machine. Equivalent circuit means thermal equivalent circuit. Thank you.