Course Name: Design of Electric Motors

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Week: 02

Lecture: 09

Title: Magnetic Circuits with Multiple Windings and Permanent Magnets

Greetings to all of you. In the last lecture, we have discussed the magnetic circuits, some example problems. Now, in this lecture, we will discuss the magnetic circuits with multiple winding excitations and permanent magnets. First, we will see the magnetic circuits with AC excitation. I have considered the same magnetic circuit, what we have discussed in the last lecture. Here, we can see AC supply we are giving at the electrical side.

Flow variable is current and effort will be voltage, AC voltage. Both voltage and current are sinusoidal quantities displaced by 90 degrees and current will be lagging. We can see the current and voltage waveforms here. Analysis and approach for analyzing this circuit, everything is same.

Only while calculating the magnetic fields, we have to consider the RMS quantities. B equals to mu N, I write, where we have to calculate the RMS quantity instead of peak or instead of average.

So, B equals to mu N, I into RMS quantity and remaining things will be same. Here, RMS quantity, we can calculate it V by impedance. Based on that, we can calculate.

Next, we will see the magnetic circuits with multiple windings. Let us consider the magnetic circuit having a core permeability mu and length of the core is lc and the windings. One winding is placed at this place or in the window 1 and other winding is placed in the window 2. The effort and flow variables in coils V 1 and i 1 with respect to the first coil and V 2 and i 2 is with respect to the coil 2. Apply a thumb rule and find the magnetic flux or magnetic fields, same way, whatever we have done as of now.

Apply a thumb rule. In both coils, the flux is in this direction with respect to the thumb. Fingers, I am representing with current, B fields with thumb. We can see here, current is entering this side going in and coming out in this fashion. Then, the flux lines are in this direction B fields or flux lines.

Flux lines, we can represent it with flux. Flow variable representation will be d phi by d t. To represent the flux lines only, we can go with flux or d phi by dt, but flow variable is d phi 1 by d t and flux with respect to the coil is this one, phi 2. This is phi 1. The flux linking with respect to the each coil consists of different components.

The phi 1 is the flux which is linking or which is associated with the coil 1, consists of the magnetizing flux and leakage flux component. The leakage flux component is nothing but the flux which is generated with respect to the current in coil 1 and it is not linking with any circuit, any magnetic circuit or any coil. That is nothing but phi l 1 leakage flux. Next, phi m is the magnetizing component. It has two components.

One is with respect to the coil 1 that is phi m 1 and one is with respect to the coil 2. We will take separately coil 1 coil. I am drawing here. This is phi m 1, phi m 2 is coming this side and the leakage flux with respect to the coil is phi l 1. The phi m 1 is the magnetizing flux component.

The flux which is generated with respect to the current 1 and it is linking with respect to the all magnetic circuit that is flowing through the core and linking with the coil 2 also. I will draw with some different color. We can see here the phi m 1 is linking with coil 1 and coil 2 and flowing through the complete core. This is phi m 1. Similarly, phi m 2 is the current which is generated with respect to the coil 2 current and it is linking with all coils and as well as complete core.

This is generated with respect to the current I 2 and phi m 1 is generated by the current in coil 1 is that is phi m 1. So, the flux component which is linking with respect to the coil 1, we can write it phi l 1 plus phi m 1 plus phi m 2. This is the leakage flux only linking with the coil 1. It is not linking with core or coil 2. Phi m 1 is the magnetizing flux generated with respect to the current I 1 and linking with the rest of the circuit core and coil both coils and phi m 2 is generated with respect to the current I 2 and linking with the both coils as well as core.

Phi 2 again we can write it as phi l 2, the leakage flux associated with coil 2. This is phi l 2. Then phi m 1 plus phi m 2. These are just flux representations, not the flow variables. Flow variable is always d phi 1 and d phi 2 by d t.

Now, we will apply the same analysis to visualize or to analyze the circuit. Second step, we have to draw the equivalent circuit, the MMF with respect to the coil 1 and MMF with respect to the coil 2 and the capacitances. I am considering this capacitance or this magnetic length path is ly and this magnetic length path also ly and middle portion magnetic length path is lx and there will be flux lines here also. So, flux lines will be there. Some flux lines will flow here also.

This is ly and this region is lx length of that magnetic path. So, I can draw the equivalent circuit in this fashion. This is capacitance with respect to the lx portion and then the capacitance with respect to the magnetic length path ly. I am showing here this is CMy and CMy this side also. The flow variable here is d phi 1 by d t here d phi 2 by d t.

Now, we have to analyze this circuit with respect to the KVL. MMF should be equal to from the loop 1. This is loop 1 and loop 2. In loop 1, MMF is equal to MMF with respect to the CMy and CMx. So, V CMy is nothing but 1 by C My integral I d t.

I is nothing but d phi 1 by d t into d t plus 1 by CM x integral of d phi 1 by d t minus d phi 2 by d t into d t.

If we will solve this thing, then we can get 1 by CMy into flux phi 1 by CMx into phi 1 minus phi 2 difference in both flux values.

This is the MMF equation with respect to the capacitances. Like this, we have to make the equivalent circuit and we have to realize in this fashion. If we are representing with resistances in terms of reluctance, mathematically, you may get same equation, but it is wrong with respect to the power balance or energy conservation and reluctance is not a dissipating element.

Conventionally, you will see in most of literatures in this fashion, here flux phi 1 and here flux phi 2 like this you will see, but it is wrong. We have to use this type of circuit only. This type of circuit we should not use, but you may end up with same equation, but it is technically and conceptual wise. We should not represent in this fashion and MMF in terms of reluctance, if you will write phi 1 into Ry plus phi 1 minus phi 2 into Rx.

Next KVL with respect to the loop 2, if we will apply from in this loop.

For loop 2, MMF is equal to V CMx plus V CMy. Here, 1 by CMx integral of d phi 1 by dt minus d phi 2 should come d phi 1 by dt plus 1 by CMy integral of d phi 2 by d t into d t.

So, final equation will come MMF with respect to the coil 2 N 2 I 2 is equal to this is N 1 I 1 MMF 1. We can call it this is MMF 2 N 1 N 2 I 2 is equal to 1 by CM x into phi 2 minus phi 1. We will get it 1 by CM y capacitance into phi 2 in terms of reluctance manner can represent phi 1 minus phi 2 into r x plus sorry phi 2 minus phi 1 it is then phi 2 into r y.

Then, next step Faraday's laws we have to apply and we have to find the inductance relation as per the Faraday's law l into I is equal to n into phi. So, phi 1 is equal to i 1 into L 1 that is n 1 into phi 1 flux number of turns with respect to the coil 1 and flux with respect to the coil 1 or flux is linking with the coil 1 is the accurate word that is phi i 1 plus phi m 1 plus phi m 2.

These are the fluxes linking with respect to the coil 1 phi i 1 if you will replace in terms of MMF and reluctance then N1 into I 1 divided by r i 1 reluctance with respect to the leakage flux magnetic path N1 into I 1 divided by r m. This is reluctance with respect to the magnetic flux loop in a core N 2 I 2 divided by r m.

So, we will see the final equation n 1 square into I 1 by r i 1 plus N 1 square into I 1 divided by r m plus N 1 N 2 into I by r reluctance with respect to the core.

The first two terms represent the self inductance terms N square by reluctance is this coming right because we are seeing the flux linkages i 1 into I 1 require we can reframe the equation or rearrange the terms r i 1 plus N 1 square by r m into I 1 plus n 1 n 2 by r m into I 2 this is I 2 i 1 into I 1 flux linkages with respect to coil 1. So, these first two terms represent the self inductance terms. We can call it as i 1 1 that is equals to l l s plus l m s leakage inductance plus magnetizing inductance. This is mutual term mutual inductance. We can write it i 1 1 into I 1 plus i 1 2 into I 2 this is i 1 into I 1 flux linkage with respect to the coil 1.

Similarly, flux linkage with respect to the coil 2 l 2 I 2 is equals to l 2 2 into I 2 plus l 2 1 into I 1. These are the inductance equations with respect to the flux linkages. The performance curves will not change with respect to the other circuits what we have discussed only magnitudes will vary. This is B versus H wave form in a linear zone.

It will be mu will be constant. We will see the linear characteristics and there is no saturation after that we will see the saturation where mu value is almost is equals to 0 and B will not change and only H will vary and inductance versus current wave forms also same. It will vary in this fashion inductance versus current and flux linkage psi versus current. It will vary same as the B H curve in a linear region. It will be constant with a slope l or we can represent in terms of mu also in a non-linear region. The core will saturate and you will see the flux linkage versus current wave form in this fashion where mu equals to 0.

This is the analysis with respect to the multiple winding excitations.

Next we will discuss with respect to the permanent magnets, magnetic circuits with permanent magnets. Let us consider a magnetic circuit with permanent magnet. The permanent magnet length is l m and area is A m. Area of the magnet is a m and there is a core portion as well as air gap portion.

This is the core portion. This core having a permeability of mu will be very high and length of air gap is l g and because of the permanent magnet, you will see the flux lines in this fashion as per our Orsted principles. The permanent magnet flux lines we can see in the core and flow variable here also d phi by d t, but there is no effort variable with respect to the winding, but here effort variable is f m that is a permanent magnet. Source is f m, mmf with respect to the permanent magnet and d phi by d t is the flow variable with respect to the effect f m. Generally, the permanent magnets consist of wider B H loops. We have discussed while discussing the magnetic materials.

These are the B H curves for permanent magnets. It has higher value of retentivity B value and cohesivity H value. We can see here for permanent magnets. The examples for permanent magnets are Alnico, samarium cobalt and neodymium magnets and strontium ferrites etcetera. We can consider it as examples of permanent magnet and the analysis with respect to the permanent magnets is same as the magnetic circuits, what we have discussed as of now.

First step, we have to apply the Orsted principle that already we have seen and flux direction also we have identified. Now, we have to calculate or we have to realize the equivalent circuit of this magnetic circuit. Here, the effort variable is f m like mmf with respect to the permanent magnet and the capacitance with respect to the core in this portion and then this portion also and air gap will be in this region. So, capacitance c m with respect to the core and with respect to the air gap c m g air gap, then another capacitance will come at the core side that is c m.

So, here flow variable again d phi by d t. The permanent magnet flux is constant with respect to that. We can say that we can consider here flux. It depends upon how much magnetic fields we have to store at the air gap. It depends upon that flow will be there.

d phi by d t will be there. Once it will fully charge, if there is no d phi by d t. Step 3, we will apply the Ampere's law. Integral h dot d l is equals to n into I. Here, there is no n into I term with respect to the winding. It is equals to 0 and permanent magnet l is h m into l m with respect to the core h c into l c and with respect to the air gap h g into l g.

This is the MMF equation as per the Ampere's law. From here h g, the magnetic field intensity with respect to the air gap is equals to minus of h c l c plus h m l m divided by l g. This is equation number 1.

If we will consider the core has the permeability value is very high and there is no MMF required to magnetize the core or very less magnitude of MMF is required to magnetize the core. So, we can neglect this term and h g is equals to minus h m into l m by l g. Why we are neglecting this term means, mu value is very high and the magnetization required for the core is with respect to the MMF is negligible or very small because of that reason I neglected this thing.

This is equation number 2.

Now, the flux relations we will see. The flux in the core as well as flux associated with the magnet flux in the air gap everything is same. So, phi c is equals to phi m is equals to phi g. So, phi m is with respect to the magnet B m magnetic flux density into area of the magnet phi g is nothing but flux density with respect to the air gap and area with respect to the air gap. From here, we can get the B m is equals to B g by A g divided by A m.

This is equation number 3 and we can write the relations B m and B g values with respect to the mu and H. This is mu into H m and this is mu g into H g. This thing we have to find from the BH curve of permanent magnet material B g is equals to mu g into H g. If we will substitute that term in the equation number 3, then we will see mu g into H g into A g divided by A m.

Here h g value we have to get it from equation 2. Substitute here equation number 2, then flux density with respect to the magnet is equals to mu g into minus H m into l m divided by length of air gap into a g by A m.

If we will rearrange the terms mu g or mu naught into l m into A g by A m and l g term also there into H m.

This is the equation representing the B m and H m flux density and flux intensity with respect to the magnet. If we will draw a line with respect to the BH curve of the permanent magnet, only second quadrant I am considering. It will be in this fashion for permanent magnet wider BH loop.

If we will draw a load line with respect to the equation number 4, we will see in this fashion. This is the line equation with respect to the fourth equation. The cross sectional point is nothing but operating point. We have to design the magnet with respect to this operating point.

This point will give the maximum energy product. This minus represents the operation or operating point with respect to the second quadrant. Next, we will see how much volume is required to establish the flux density B g in the air gap. How much volume of the magnet is required to establish the required flux density at the air gap? Volume of the magnet is equal to area of the magnet into length of the magnet. Area of the magnet, we have to get it from the B flux relation B m A m is equal to B g into A g. From there, A m is equal to flux density at the air gap divided by flux density of the magnet into A g.

l m, we have to get it from the ampere's law. That is equation number 2 H m l m by, if requires, we require l m. So, H g l g by H m, we can write it. Then, rearrange these terms and substitute B g value is equal to mu g into H g. Then finally, we will see B g square into A g l g divided by mu naught into B m into H m, the magnetic flux density with respect to the magnet and flux intensity with respect to the magnet. This is flux density square divided by mu naught into B m H m into A g into l g product.

We can represent it as a volume of the air gap that is V g. This side, V m is there. This equation tells that in order to establish the magnetic flux B g at the air gap, we require a magnet of volume V m and the B m and H m value should be maximum. Next, we will see how to solve the magnetic circuits with respect to the permanent magnets and windings inside the magnetic circuit. Let us consider this is a magnet and here we have electric side coil to produce the required mmf V and number of turns N and flow variable with respect to the coil is d phi by dt and flux if we will talk that is phi.

This permanent magnet length is l m and area is a m. The approach to solve the problems associated with magnetic circuits, single excited and permanent magnets or permanent magnets with air gap approach will be same with respect to the different parts or different lengths of magnetic paths. We have to identify the regions where the flux density's values may change, but flux is same in the core. Flux density's will change as per the cross sectional areas of that particular region. I am marking the locations with respect to different parts of the core.

This red color one and then we have to mark it. This is 1, 2, 3, 4, 5, this is 6 and magnet portion 1 is 7. Draw the equivalent circuit and solve the problem similar to the network theory concept. Here, mmf with respect to the coil is n 1 into I 1 or N into I simply and the capacitance with respect to the region 1 is C 1 and capacitance with respect to the region 2 is C 2, then C 3. So, we represent all the reluctance paths in terms of capacitances where the energy is stored C 6, C 5 and the permanent magnet mmf.

We can represent it here Fm. There is no number of terms F m and this side flow variable is d phi 1 by dt. This side we have to consider d phi by dt with respect to the magnet and analyze the circuit. With this, I am concluding this lecture. In this lecture, we have discussed the magnetic circuits with multiple windings, magnetic circuits with permanent magnets and the respective equivalent circuits and analysis. Thank you.