Digital Communication
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Lecture - 07
Channels and their Models (Part – 2)

Hello, everyone. In the last class we have started discussing some channel models; we have discussed some physical channels their properties and also some channel models, simplified models with which we can work in we can develop techniques. And, find out what is the maximum what is possible rate at which we can transmit through the channels and so on. So, for doing a good analysis and for finding out techniques for transmitting at the highest rate through the channels it is important to start with some simpler models first. So, that we can get sufficient insight into the techniques and the behavior of the channel. And, then we can complicate the model more and more and see if we can solve for the most general more general channels.

So, in the last class we have discussed some particular channel models, binary symmetric channel to start with that is a simplest model we have discussed; where we transmitter 0 or 1 and we received 0 or 1. And, there is a probability of error which is same for transmitted bit 0 as well as 1. And, then we have discussed binary eraser channels for which the transmitted symbol is either 0 or 1 and the received symbol is 0 1 or eraser; by eraser what we mean is that a bit which cannot be distinguished as 0 or 1. And, then we have discussed binary asymmetric channel which is basically the generalization of binary symmetric channel or probability of error for 0 is different from probability of error for 1.

And, then we have discussed a further generalization of binary symmetric channel that is discrete memory less channel; where the transmitted symbol may be taking value from any finite set and the receipt symbol also may take values from another finite set. And, then the channel is characterized by these 2 finite sets which are called respectively the transmit alphabet and received alphabet and also the probability transition matrix; which is which gives the conditional probability of the received symbol given the transmitted symbol.

Then, we have discussed discrete channel with memory it is basically generalization of discrete memory less channel; we have not discuss this in detail. But this is when the behavior of the channel depends on how it behaved for the pervious symbols. And, then we discussed discrete channel with feedback; where the transmitter is assumed to know what as been received so far at the receiver. So, that the transmitter can possibly use that information for transmitting future values, future symbols.

So, the one of channels most important many important results is that with feedback the capacity of discrete memory less channel does not increase. So, in this class we will discuss some more channel models which are important for practical purpose; to start with we will discuss continuous valued channels which can carry continuous valued signals. So, we will first consider the discrete time version of such a channel all the channels we have discussed so far are discrete time; we will continue to assume discrete time channels.

So, in each use of the channel the value that is transmitted through this channel is continuous value; previously we have assumed that the transmitted symbol takes values from its finite state. So, it can take only finitely many values. So, 0, 1 or 0, 1, 2, 3, 4 or a to z having 26 values. So, now we will consider channels which can transmit through which we can transmit continuous values.

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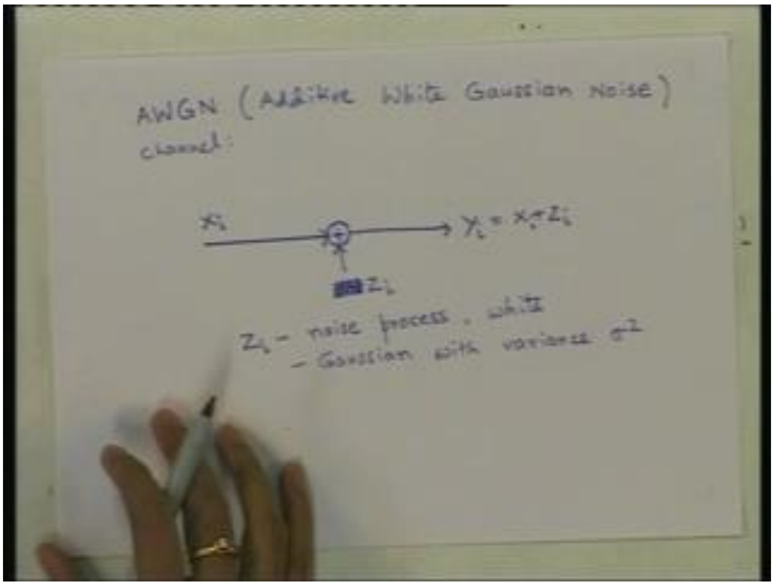

So, the simplest model we will discuss is AWGN channel, additive white Gaussian noise channels. Here, we have the channel through which we can transmit a real value X a received value is Y; and Y is nothing but X plus some noise Z. So, Z is noise and Y is X plus Z sorry this is simply Z; Z is being added; this is noise. So, this is additive because the noise is added to the transmitted signal. Then, it is white because the noise is assumed to be ((Refer Time: 07:00)) noise; that is when we use the channel many times in sequence; the noise samples that is if we consider the first use as if we denote the first transmitted symbol by X 1 and second transmitted symbol by X 2 and so on. And, similarly denote the first noise sample as Z 1 and second as Z 2 and so on. And, similarly the received symbols Y, Y 1, Y 2 and so on. Then, we can denote the I th use of the channel as the following that is X i is transmitted Z i is added with X i. And, we received Yi which is nothing but Xi plus Z i this is a i at the i th symbol interval this is what is happening.

And, then Z i is a noise process noise random process; and Z i is white which means that the different random variables Z i is for different i s are independent of each other. So, this is white. So, this the fact that it has 0 auto correlation for non zero lags; that is that correlation between Z i and Z i plus T for any T is 0; this implies that in the frequency domain the power spectral density of Z i the process Z i is flat in frequency domain; that means, it is white. And, it is Gaussian. So, Z i is also Gaussian; and let us assume that it has variance sigma square; sigma square is a variance of Z I each Z i. So, these are basically independent and identically distributed with Gaussian with variance sigma square. So, this is discrete time AWGN channel.

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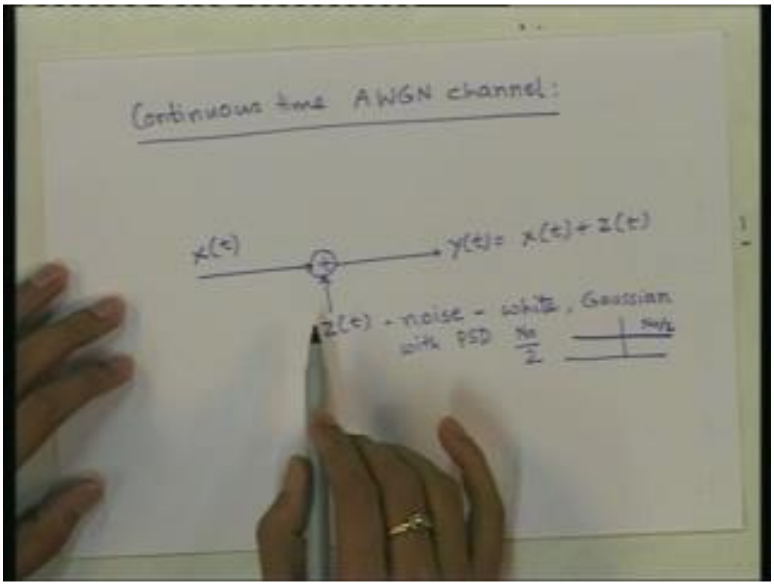

Now, we can similarly, have the continuous time version of this channel; where we will be transmitting a continuous waveform. So, this is continuous time AWGN channel; where we will transmit instead of discrete samples we will transmit continuous waveforms. So, X (t) is a random process we will transmit this is transmitted signal, this is the noise signal and this is the received signal this is X (t) plus Z (t); this is noise just like the discrete time version; this noise is assumed to be white noise that is the autocorrelation function of this random process; the noise random process is the direct delta function it is white and Gaussian with power spectral density N naught by 2. This is usually the N naught is the one sided power spectral density, N naught by 2 is the 2 sided power spectral density. So, the power spectral density of this noise looks like this is flat at N naught by 2. So, this is the continuous time AWGN channel.

Now, in both these cases we have assumed white noise; which means that the noise as the at the present time does not depend on the noise values in the past. So, that itself means that the channel is memory less just like discrete memory less channel we considered; where the where each use of the channel is independent of the others. So, similarly here also the noise values at any instant is independent of all the noise values in the past. That means, the noise is independent, noise is noise does not have any memory. So, that the channel itself does not have any memory; what happens to a signal value transmitted now does not depend on what happened to signal values transmitted before? So, the channels in these channels the channels do not have any memory.

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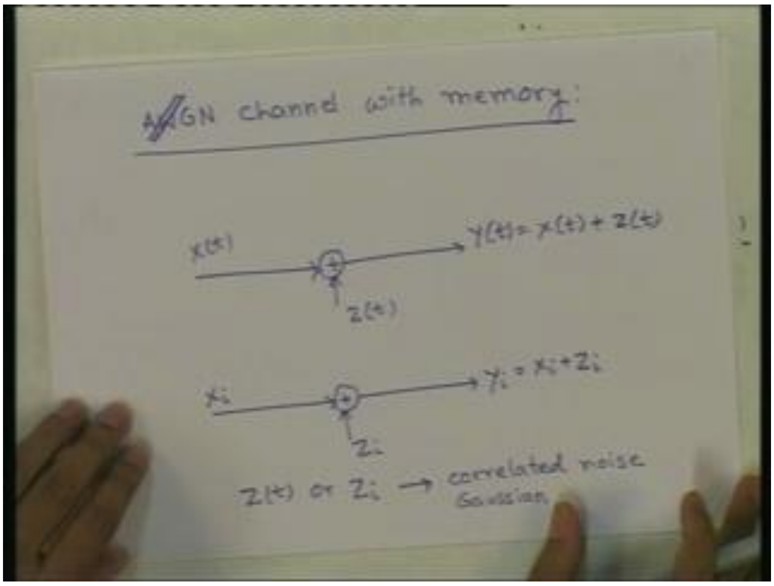

Now, the next channel is the obvious generalization of this that is a AWGN channels with memory. So, AWGN channel with memory. So, here we have the same this is the continuous time version X (t), Z (t), Y (t) which is X (t) plus Z (t). And, the discrete time version is same except that t is replaced by a discrete index I; in both these cases previously we have considered Z (t) and or Z i to be white. Now, the generalization is that Z (t) and Z i need not be white. So, their power spectral density need not be flat in the frequency domain. And, as a result there is correlation in the time domain in the noise. So, what happens to the transmitted signal at time instance t depends on what happened to the signal values in the past. Because the noise values at t depends on or is correlated with the noise values in the past. So, the noise values; since the noise in different time instances are related there are correlated; the that distortion that is coming, that is affecting the signal at different points of time are correlated.

So, the channel as memory. So, simply Z (t) or Z i are correlated noise again Gaussian this is till this is not white; so this is not white noise. So, this is not called AWGN channel but additive Gaussian noise channel with memory. Because the noise now is possibly not white; if it is not white then the channel has memory. So, this is not AWGN, but this is additive Gaussian noise channel with memory.

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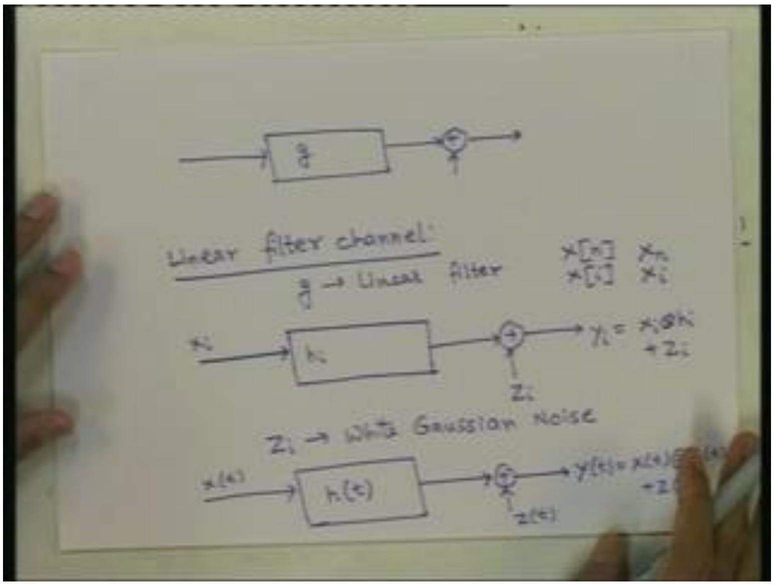

Now, channels with memory are also thought of in a different way in practice; that is previously we have considered channels in general of the form of the following signals goes through some function g. And, then some noise is added; this is a general model of channel we started with; the alphabet may be different, it may be continuous values some continuous set, it may be some finite set. Similarly, the received value may be finite set or a continuous set; the noise itself may be in different form, it may be correlated uncorrelated and so on. But in the channels we have discussed in this class we have not considered any g. That means, it is directly being just added by a noise; in general channels can go through a some function. And, the simplest functions that we considered are the linear filter channels; that is the signals let us first take the discrete time version. So, linear filter channel; in this case g is the linear filter.

So, in the discrete time version of the channel there will be a discrete time filter the this is. So, this is we have a signal X i and there is a linear filter with impulse response; let us say h i; h i may be again a fired filter or IIR filter that is finite impulses response filter or infinite impulse response filter; that is h i may be non-zero for i greater than equal to 0. And, it may have non zero values till infinite infinitely many non-zero values or it may terminate after some time; after some time that coefficients may impulse response may become 0. So, whatever it is there is a discrete time filtered and the output of the filter then is added with the noise. So, Z i then Y i; what is Yi then? Yi is the convolution of X i with h i and then plus Z i the noise; and Z i is assumed to be white Gaussian noise.

And, the continuous time version of this channel is simply X (t) h (t) is a impulse response this is Z (t) and the received signal is Y (t) is X (t) convolution h (t) plus Z (t). So, if you are more comfortable with the discrete time signals being denoted by X [n] instead of X n like this. So, this will then be denoted similar as X [i] instead of X i. So, here we have denoted such a signal as X i instead of putting the i in brackets. So, these should be thought of as discrete time signals going through a discrete time filter. And, a discrete time noise is being added to the output and we receive a discrete time signals. Similarly, here we have instead of everything discrete we have everything continuous as time. So, the continuous time channels are also called waveform channels because they because we transmit waveforms through such channels.

So, continuous time AWGN channel, continuous time linear filter channel, continuous time additive Gaussian noise with memory they are called an waveform channels instead of called them continuous time channels; we can also called them waveform channels. Because we transmit continuous time signals that is waveforms through the channels. Now, we have discussed this linear filter channel and here also we see that what happens to the signal now; depends on what happened before that is what was transmitted before. In other words what we receive now depends on not only what we transmit now but also on what we transmitted before.

And, as a result this also, these channels also have some kind of memory that is they remember what happened in the past. So, there is also some relation between this linear filter channel and the additive Gaussian noise channel with memory that we have discussed just before this. Because both of them; obviously have some kind of memory. And, one this form of channel can be converted with some operation at the receiver to a channel of the previous type. And, a channel of the previous type can also be converted into a channel of this type with some manipulation at the receiver.

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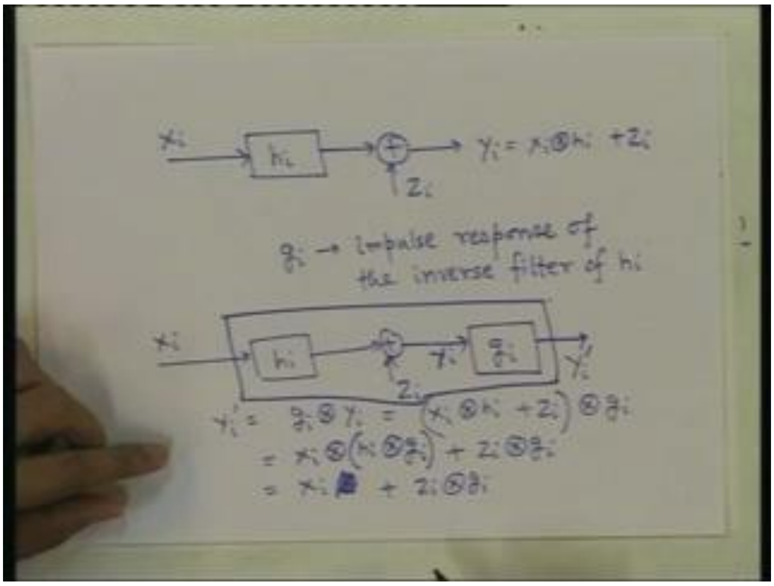

We will see that now. Suppose, we have a linear filter channel where we have X i there is a liner filter channel liner filter with a h i impulses forms. And, then a noise is added Z i we have Y i which is X i convolution h i plus Z i. Now, suppose that we can implement some inverse filter of this; we will not go into the intricacies in the design of the inverse filter the it may not be stable. And, all such intricate issues we will have go through but let us say we have an inverse filter of this channel filter. Suppose, that g i is the impulse response of the inverse filter, this is the impulse response of the inverse filter of h i. Then, what we can do is at the receiver we can simply pass the signal through that filter g i, this is Yi is actually received. And, then we pass it through g i then we have another signal let us called it Y i prime. Then, the what is the relation between Y i prime and this X i?

Yi prime this discrete time signal is g i convolution Y i which is nothing but X i convolution h i plus Z i. So, this convolution g i. So, this convolution this or this convolution this there are same things. So, now when we convolve g i with this what do you get X i convolution h i convolution g i. Now, we can do this convolution first and then this it does not matter convolution is commutative operation then Z i convolution g i. Now, this is nothing but what is the convolution of h i and g i; g i is the inverse impulse response, impulse response of the inverse filter of h i. That means, the convolution of h i and g i is just the delta function. So, when we convert with X i it becomes X i itself. So, we have simply X i plus Z i convolution g i. So, it is like as if the signal is just added with some noise; if we considered this as the whole this whole thing as the channel; this whole thing as a channel.

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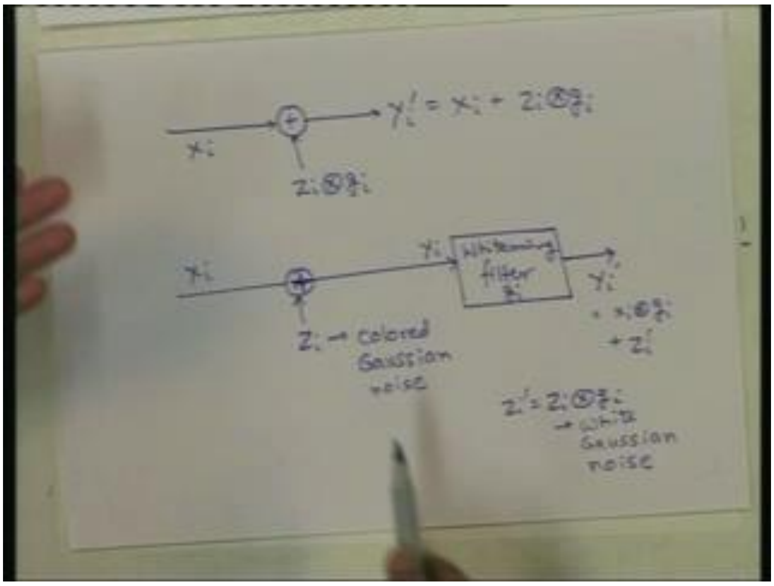

Then, what is happening in effect is that we are transmitting X i it is going through as it is not through any filter. And, it is added with some noise which is a convolution of Z i and g i and then we are receiving Y i prime. So, this is X i plus Z i convolutions g i. Now, what is this is nothing but the addicting Gaussian noise channel except that this noise is now not white. But as we discussed before the noise is a colored noise it is no more a white noise. So, it is similar to it is the same channel as we discussed as additive Gaussian noise with additive Gaussian noise channel with memory; where the noise is not white but it is colored noise.

So, by doing a simple operation of just putting an inverse filter at the receiver; we have converted the channel linear filter channel into a channel where which is additive Gaussian noise channel with colored noise. And, the signal itself is otherwise unchanged. And, just like this we can also convert any additive Gaussian noise channel with colored noise into a linear filter channel also.

So, what we have to do is the put a so called whitening filter at the receiver which will make the noise white but then which will which will so what will happen is if we put take X i. And, suppose originally we have we have a colored noise Z i colored noise Gaussian noise. Then, if we pass it through a whitening filter how to design this filter we will not discuss now but it can be designed. So, whitening filter we pass it through whitening filter the this; if this is Y i this Y i prime will be X i convolution this g i now; gi plus Z i prime which is Z i convolution g i and which is white Gaussian noise.

So, we can convert a additive Gaussian noise channel with additive white colored Gaussian noise channel, additive colored Gaussian noise channel into a linear filter channel also by putting a whitening filter at the receiver. So, these 2 channels linear filter channel and additive colored Gaussian noise channel or additive Gaussian noise channel with memory there are actually equivalent. So, both of them to equivalently because they both are equivalent any of them is called the additive Gaussian noise channel with memory; this 2 different ways the either linear filter or colored Gaussian noise these 2 are just 2 different ways of thinking of additive Gaussian noise channel with memory. Now, we will discuss a very special channel model which is which I think we would be probably the most popular channel now to you that is wireless channel.

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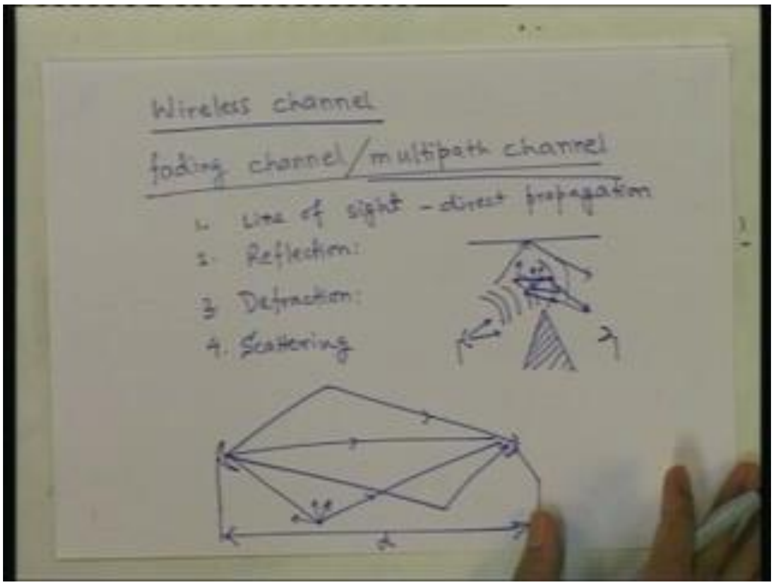

It is the channel model that we will discuss which is very different from all the channels we have discussed so far in at least 1 aspect. And, that is the channel changes with time, the behavior of the channel changes with time. So, first of all let us discuss the different phenomenon that the wirelessly transmitted signal goes through; then we will discuss the suitable model for wireless channel.

So, such wireless channels are called fading channels referred as fading channel or multipath fading channel or simply multipath channel for the reasons we will discuss now. So, what are the different ways the signal travels through space; wireless channel is basically channels where there is no air, where the transmitter and the receiver are not connected by any wire. So, it may be through simply space or it may be there may be some air or some other media medium in between or there may not be anything it is may be just vacuum. So, one is one mode of propagation is simply line up sight, simple line up sight propagation this is the direct propagation.

Then, there are other phenomenon's that other phenomenon that happens in the propagation. For example, reflection were the signal is reflected from smooth surface, smooth surfaces around smooth large surfaces and then defraction; what is defraction? The signal where we transmit some signal from antennae it is seen that even if there is some obstacle here that we are transmitting some signal here. And, if we put 1 antenna here then we will still be receiving some signal though there is an in between obstruction.

So, from this phenomenon there is there the there are theories develop to justify reception of signal even at the in the shadowed region. And, 1 theory is that when wave propagates every point in the wave front. So, see these are the wave fronts then every point in the wave front behaves like point sources of further wave. So, from every point here signal is transmitted in all directions. So, it is natural that we will also receive signal in the shadow of the obstruction. So, this phenomenon is called defraction; this is the phenomenon by which signal travels even on the other side of the obstruction and then scattering; it is seen that the actual received signal at the receiver antenna is often stronger than what can be predicated by refraction and deflection, defraction models together.

So, even if we take care of if even if we see all the refraction reflect reflecting surfaces and defraction that could be happening; we cannot still account for all the signal strength that is received at the received antenna. So, the another phenomenon that contributes to the propagation is scattering. So, this happens from all kinds of small objects or surfaces with roughness. So, rough surfaces are small objects. For example, there may be a lamp post or a tree or chair table these are small objects from which signal is scattered in all directions; as these are not reflectors, these are not large enough or may be sometimes these are not smooth enough to reflect the signal in 1 particular direction. But these signals when they fall into these objects they are scattered in all directions. So, that is called scattering. And, the received signal at the receiver is most of the times the result of all these different modes of propagation.

So, in summary when we transmit a signal from an antenna; and we receive the signal at the receiver antenna the signal in between is propagating in different mechanisms the direct propagation, reflection, defraction and scattering. And, the received signal is the resultant of all the signals that are received through all these mechanisms. And, as one can now see that signal propagates from the transmit antenna to the received antenna by traveling through multiple paths; it may be getting reflected from somewhere, it may propagate directly along one path, it may get scattered from somewhere and reach the receiver antenna along one path. So, it may get scattered from here and actually go in all directions but one of the rays can go reach the receive antenna. So, and there are usually infinitely many such paths through which the signal is propagating to the receive antenna. So, this is called multipath channel that is why wireless channel is called multipath channel; when the signal is traveling through multiple paths to the receive antenna. And, it is quite interesting to think what will be the signal power that we will receive; if the receiver antenna and the transmitter antenna are separated by distance d.

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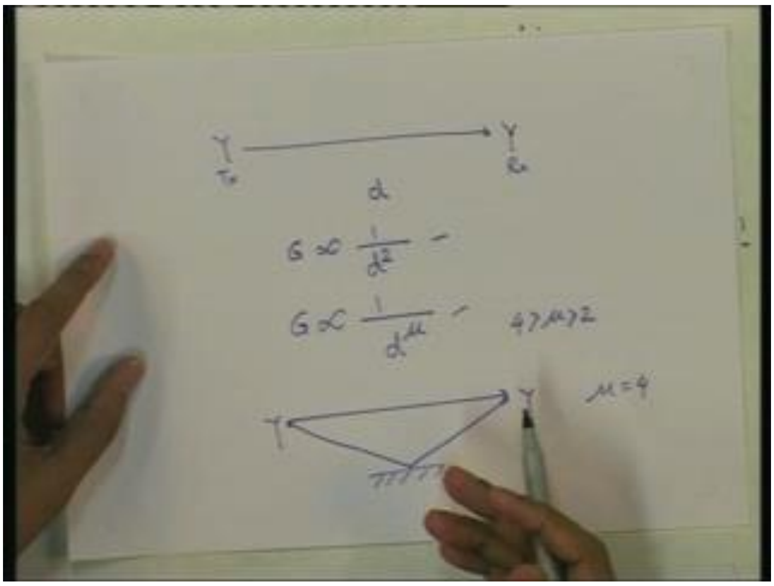

It is known that if we have a if we have only direct propagation say if we have one antenna here, one antenna here; this is the transmit antenna and this the receive antenna this is separate by distance d. Then, if there is only one path the line of sight direct propagation path; then the path attenuation the gain path gain is proportional to G is proportional to 1 by d square; actually it is proportional to d by lambda square. So, it also depends on the frequency of the signal. So, if we take only distance it depends on the distance in this way. And, however if there are other mechanism through which the signal is propagating then this need not be the way the signal power here depends on the distance; it may in general depend in the following d power mu; where mu is some constant usually greater that mu greater than 2 and it is also usually less than 4.

So, depending on what kind of channel it is what kind of environment it is the attenuation may be may depend on the distance between the transmitter and receive antennas in this fashion for different mu values. So, for example if the there is a only one reflecting surface which is called ground reflection model; where the signal passes there is one direct path and there is one reflected from the ground. Then, mu is 4. So, the channel again is proportional to 1 by d power 4. So, the different so it depends on the environment usually. Now, this is about t average gain of the channel this need not be the instantaneous gain; meaning by any point of time the attenuation the signal will go through need not be proportional to this at any point of time; it is usually changing with time. Because there may be obstruction at 1 point of time, there may be may be not be obstruction at another point of time and the channel itself may change. Because the this kind of antennas are moving relative to each other and so on.

So, even though these things tell you about the average behavior of the channel based on the distance; they do not tell us what is the instantaneous behavior of the channel; how the channel changes with time. And, that is the fact that channel changes with time due to the change of environment is known as the fading. So, the channel gain varies with time. And, because of so there are 2 things; 1 is the variance the variance of the channel with time and another is the channel is going through multiple paths. And, because the channel is going through multiple path it is like the received signal.

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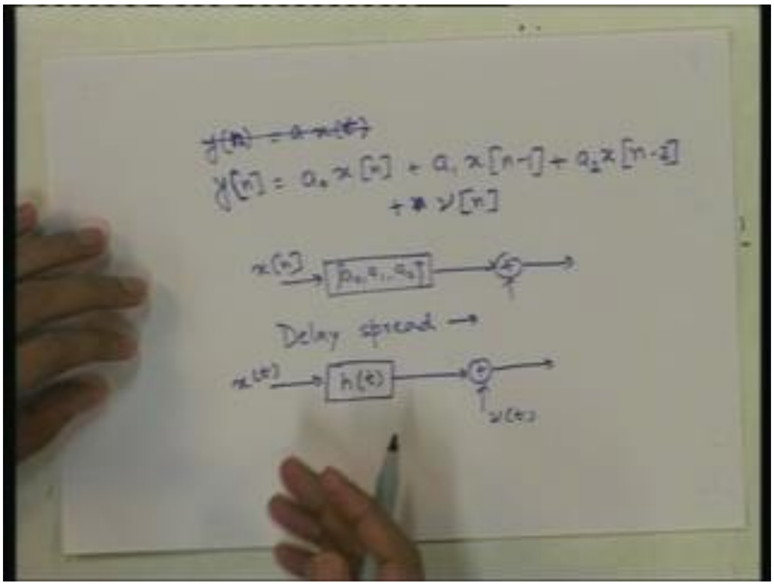

For example, if we consider a continuous time version y (t) will be say some attenuation times x (t) let us call it say take the discrete time version for simplicity say y [n] is a naught x [n] plus a 1 x [n] minus 1; there may be another path through which the signal is reaching the receiver with attenuation a 1 and delay 1.

And, then we may have so many other paths a 2 attenuation times delayed by 2 and so on. So, let us just take 3 paths with 3 different delays; 1 is no delay, another is delay 1, another is delay 2 with 3 different attenuation levels a naught, a 1 and a 2; if this is the and then of course we will have noise, the received signal will also have some noise. So, let us say n naught mu n. Then, it is like this signal is going through a filter. So, this is basically a filter with impulses response coefficients a naught, a 1, a 2; this is the filter impulses response x [n] is going through then noise. So, multiple paths in the channel gives rise to linear filter channel layer and this filter coefficient themselves might be changing with time; there may be this path, may be there at some point of time, there may not be this channel at some other time point of time. Because some reflector might have been there before and it may not be there now.

So, a 1 may be 0 at some point time. So, in general the all these channel coefficients are changing with time. So, there is a multiple multipath channel and the filter itself is changing with time. Now, there are different parameters of the wireless channel; which specify how fast the channel is changing with time or how much multipath is there and such other things? For example, the length of this impulse response. So, in this case the length is 3; 0 to 2 this length is called the delay spread; in the continuous time case usually these samples these delay 1 will mean some microsecond and millisecond. So, this is the delay spread; it will have some for the continuous time case, it will have some continuous h (t).

So, this is the delay spread is the range of values of t for which h (t) is non zero. And, then there is Doppler spread which is basically the Doppler comes because the when the environment is changing or specially it is easy to conceptualize as the receiver or transmitter moving; when we have 1 transmitter antenna and when is 1 receive antenna let us say signal is passing to this from different through different paths.

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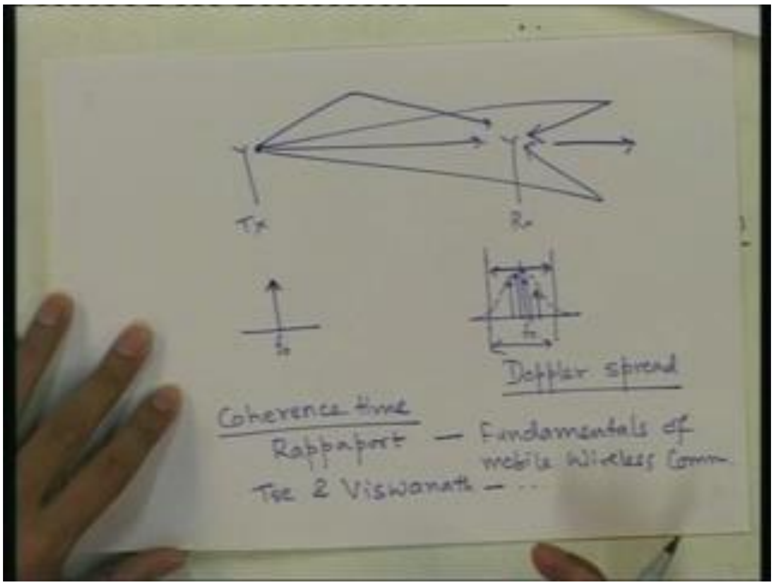

There is a there are these 4 paths and this antenna receiver antenna is moving in this direction. Then, these paths, these path lengths are increasing. So, the signal received through these paths the frequency of them will decrease. So, this suppose we are transmitting a sinusoidal signal from here the received signal received from these 2 paths will have a smaller frequency than the transmitted frequency. And, because these path lengths are decreasing as the receiver moves in this direction; the signal received through these paths will have a smaller frequency then the transmitted frequency.

So, even though in the frequency domain we transmit 1 sinusoid at some frequency f naught at the receiver we will receiver some signal at smaller frequency, some signal at higher frequency, some signal here, some signal here. So, there be a signal received around f naught. But if we transmit a single frequency we will not receive a single frequency but those frequency will be spread out in the frequency domain. And, that how much it is spread is this is called the Doppler spread; we know that that this is Doppler; this shift of frequency is called the Doppler shift. But in this case there will not be a Doppler shift but there will be a Doppler spread of the signal. Because some paths the frequency along some paths is decreasing and the frequency along some other paths is increasing.

And, as a result the transmitted signal on a single frequency will be spread out in a band of frequency and so this band is called the Doppler spread. And, also Doppler spread as you can see how much the spread is will also tell you how fast the receiver or transmitter are moving with respect to each other; if they move faster there will be a larger spread if they move slower then there will be a smaller spread.

So, the Doppler spread has relation with what is known as coherence time which denotes what is the time interval after which the channel response does not is independent of the previous time. So, that is how fast the channel is changing. And, similarly there is coherence bandwidth which depends on the Doppler on the delay spread. And, we will now discuss this is more detail; if you are interested you can look at any wireless communication book either by Rappaport fundamentals of mobile wireless communication; there is a book by David Tse and Viswanath; both the books are available in Indian edition now not very costly. So, this is also some fundamentals of wireless communication. So, please look at these 2 books if you are interested more into wireless communication and wireless channels in general. And, we will just summarize by saying that wireless channel model simply one can say that we have linear filter which depends on time.

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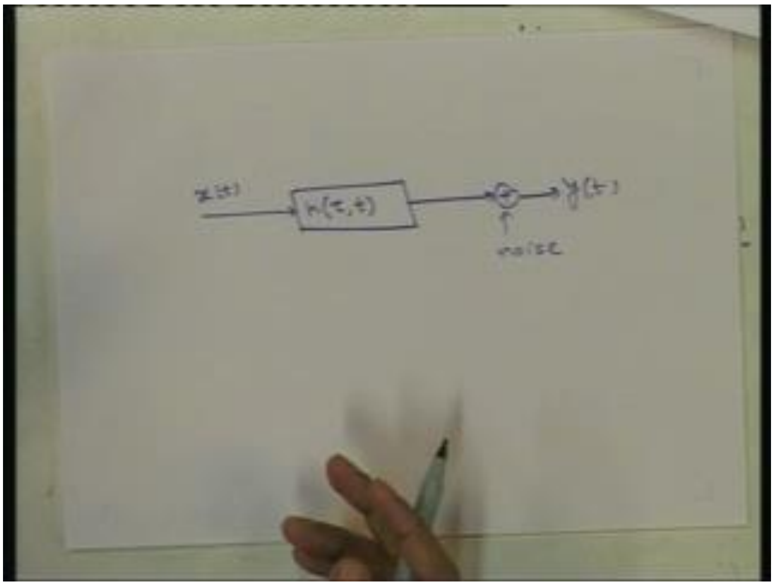

So, h usually it is denoted by 2 time this is x (t) and this is then there is noise. So, h tau is the impulse response at time t and as time progresses the channel itself impulse response itself is changing with time. So, there are 2 time one is the axis in the impulse response, the other is the to denote the time index; with which the impulse response is changing. So, in this class we have discussed linear filter channel. First of all we have discussed continuous valued channel unlike discrete value channel that we have discussed in the last class.

Then, we have discussed continuous time or waveform channel then linear filter discrete time as well as continuous time channel. And, then we have discussed wireless fading channel in little more detail if seeing what happens in a real physical wireless channel. And, also the channel model for such channel; we will possibly use some of the channel models that we have discussed in this these 2 classes. And, even if you did not do not discuss some of the channels it is quite important to know that such channels exist and they are important in general.

Thank you.