Foundations of wavelets and multirate digital signal processing.
Professor Vikram M Gadre
Department of Electrical engineering.
Indian Institute of Technology Bombay.
Lecture -7.
Module-1.
Moving from Z domain to frequency domain.

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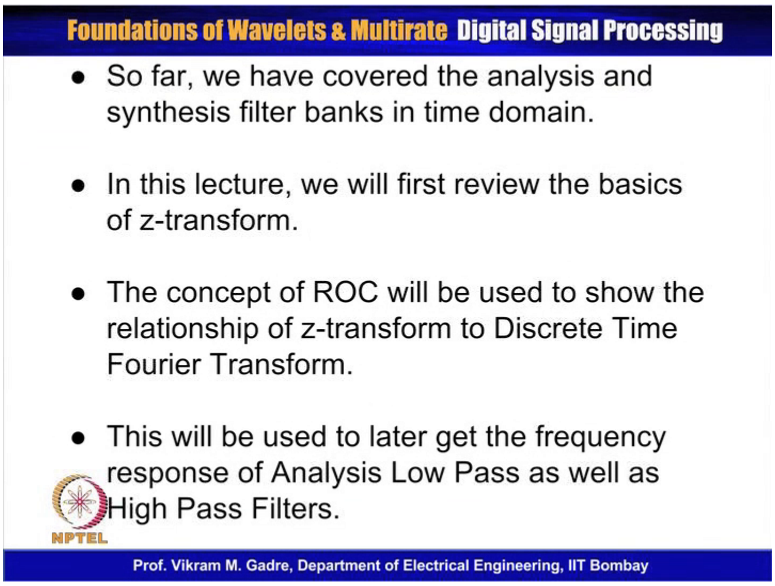

A very warm welcome to the 7^{th} lecture on the subject of wavelets and multi-wave digital signal processing. Recall that we had ended the previous lecture by hinting at the structure of the synthesis filter bank corresponding to the Haar multi-resolution analysis. Today we begin by looking at both the analysis side and the synthesis side once again. I mean the analysis filter bank and the synthesis filter back in the Haar multiresolution analysis. So, I will just put on the 2 filter banks clearly.

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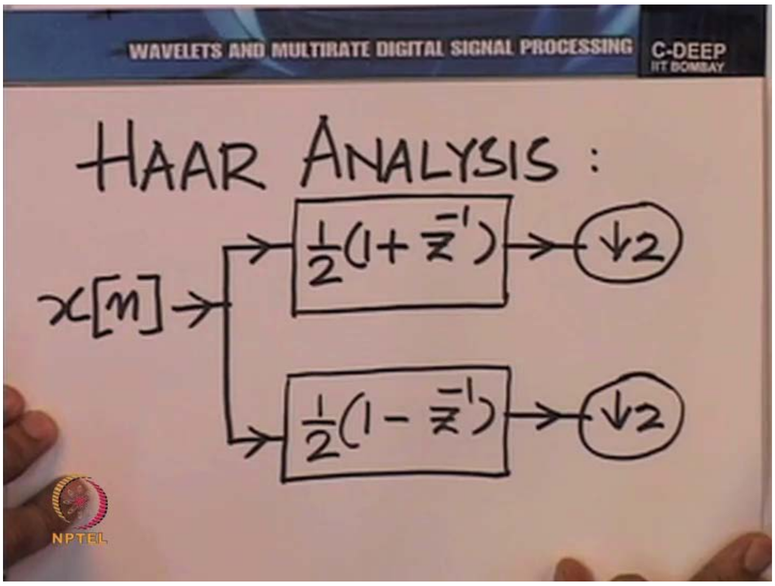

You will recall that on the analysis side we had a structure like this, we had the sequence, let us call it X of N corresponding to this the function in V 1. Essentially it was subjected to the action of 2 filters, filter of the form half 1 + Z inverse and other one of the form half 1 - Z inverse, followed by a downsampling by 2.

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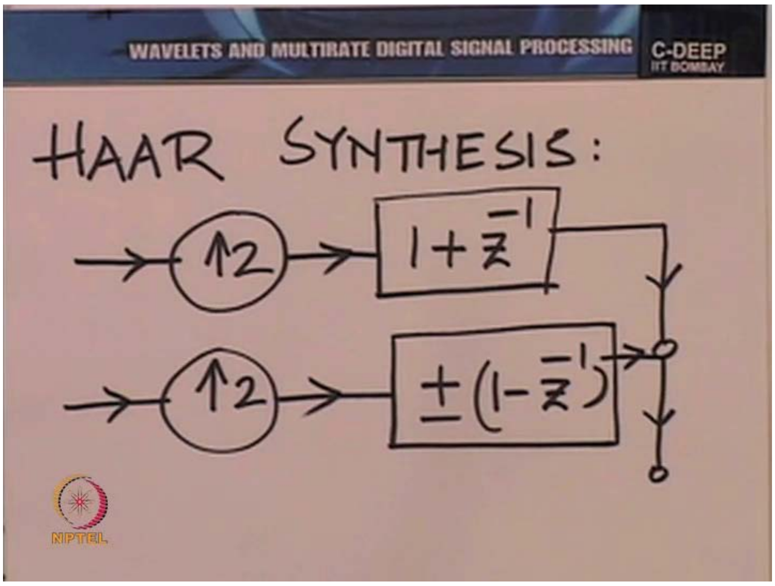

We had also derived the structure of the synthesis filter bank and interestingly we saw the structure was very similar, in fact it looked almost like a mirror image here. Except for the fact that you had an upsampler instead of a downsampler followed by the 2 filters once again.

So, here the filters have the form 1 + Z inverse and 1 - Z inverse. Now, the only catch is I am going to allow for an ambiguity of sign here + - 1 - Z inverse. A small variation from last time. A subtle point but important. Now, there are several things to which we must pay attention. One is if you look at the structure of the analysis in the synthesis filter bank, the filters are almost identical. These are the 2 filters for the analysis side and these are the 2 filters for the synthesis side. Please note I am allowing an ambiguity here.

What would the physical meaning of ambiguity be? It would just determine where I place the sum samples and the difference samples. With the + sign the sum samples will get placed at the even locations and the difference samples at the odd locations. With the - sign, it would be the other way around. So, let us leave that ambiguity for the moment. If we try and resolve the ambiguity right away, we might actually get more confused. So, allowing that little ambiguity for the moment, let us analyse this whole structure in general and then resolve the ambiguity.

Anyway, coming back to the filter bank structure as we had seen, one very beautiful thing that we noticed about the Haar filter bank is that the filters on the analysis side and the synthesis side are almost the same. Except for this sign ambiguity. And if you really look at it, these 2 filters are also very meaningful in another domain of which we shall soon have more information. Now, what we are going to do from here onwards is to look at this general structure, you see we want to understand why the Haar multiresolution analysis though attractive is not adequate.

I have been saying all the while that if we understand the Haar multiresolution analysis and if we understand the filter bank corresponding to the Haar multiresolution analysis, we understand quite a great deal about filter bank and multiresolution analysis. Most of the concepts are captured. Then why should we look for other multiresolution analysis. That is a subtle point which we shall see by going into another domain. So, our objective here is to look at the frequency domain behaviour.

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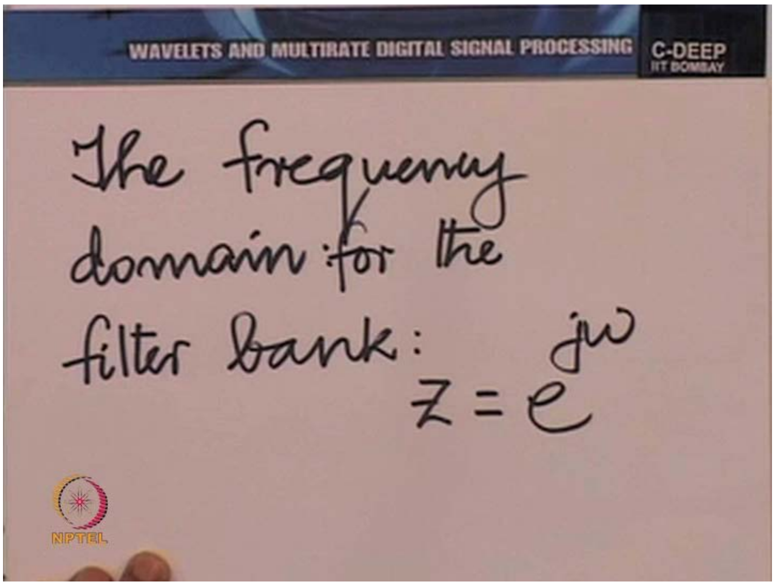

And how would be get a frequency domain representation of the Haar filter bank? So, what we will do is we will progress as follows. 1st we shall look at the frequency domain for the filter bank. And how would we do that? We would do that by substituting Z equal to e raised to the power J Omega. Let me once again recapitulate a few concepts from discrete time signal processing for the benefit of the class, although one would have done these in a course on discrete time signal processing, it helps to put some of this discussion in perspective.

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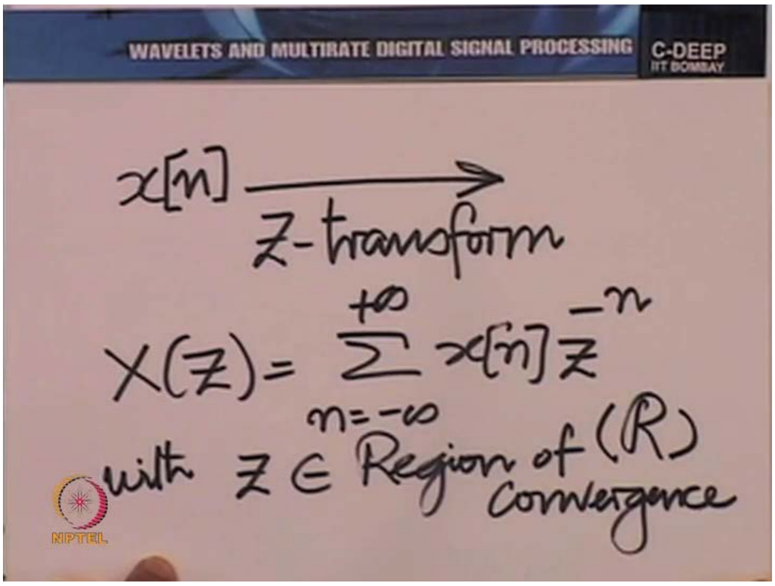

You will recall that we had defined the Z transform last time. We had said that if you have a sequence X of N, its Z transform is capital X of Z defined by summation N running from - to + infinity X of N Z raised to the power of - N with Z belonging to a region of convergence. Now, in particular, if the region, let this region be script R. In particular, if this region includes the unit circle, so you know, let me say a little bit about the regions of convergence of a typical Z transform. Typically R has the following patterns.

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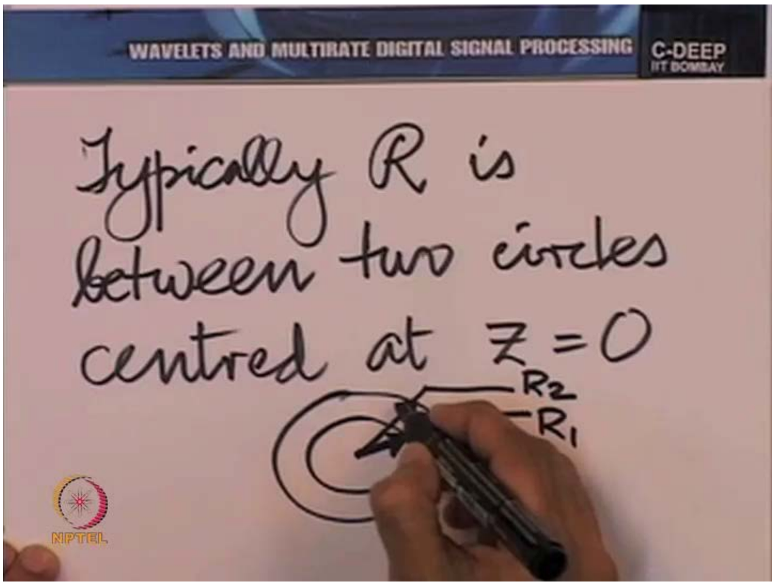

R is between 2 circles centred at the origin. So, it has an appearance something like this. And both of these are centred at the origin. So, you have R2 and R1. So, you know here, the radius is R1 I mean and here the radius is R2. Now it could be true that R2 might be infinity or R1 might be 0, so we have to allow these possibilities.

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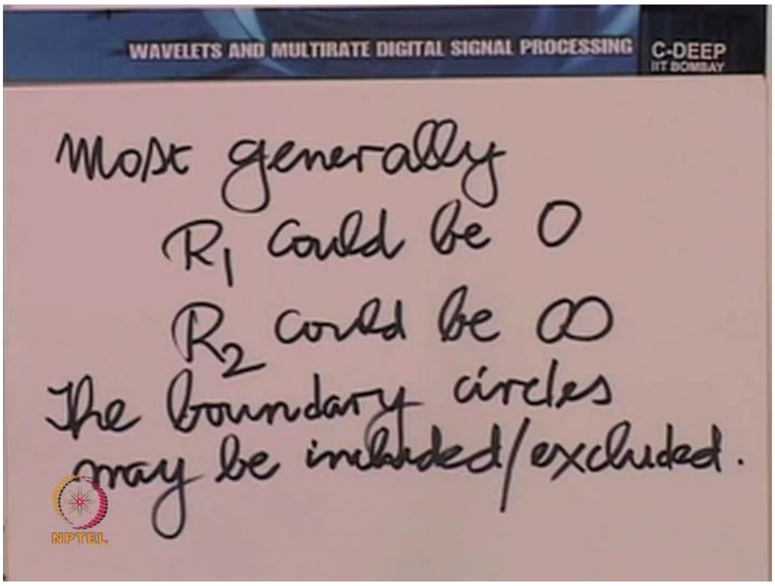

Most generally, R1 could be 0, R2 could be infinity, moreover, the boundaries may or may not be included.

The boundary circles may be included or excluded. In fact, if these values, R1 and R2 are nonzero and finite, most of the times, the boundaries are excluded. It is only when R2 becomes infinity or when R1 becomes 0, then there is the question is the boundary included or excluded? And that is not a trivial issue, whether the boundary is included or excluded makes a difference to the properties of the system. Well, all this is essentially to recall a few points about the Z transform and now coming to the frequency domain.

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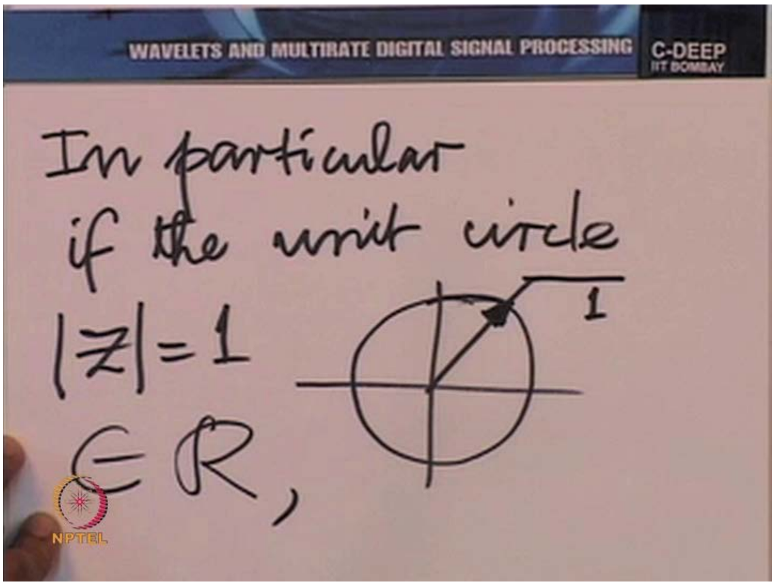

So, in particular, if the unit circle, that is mod Z equal to 1, it is a circle you see, mod Z is a circle of radius 1 in the Z plane. So, if the unit circle is included in R, then we have a frequency response for the system, the system has a frequency response.

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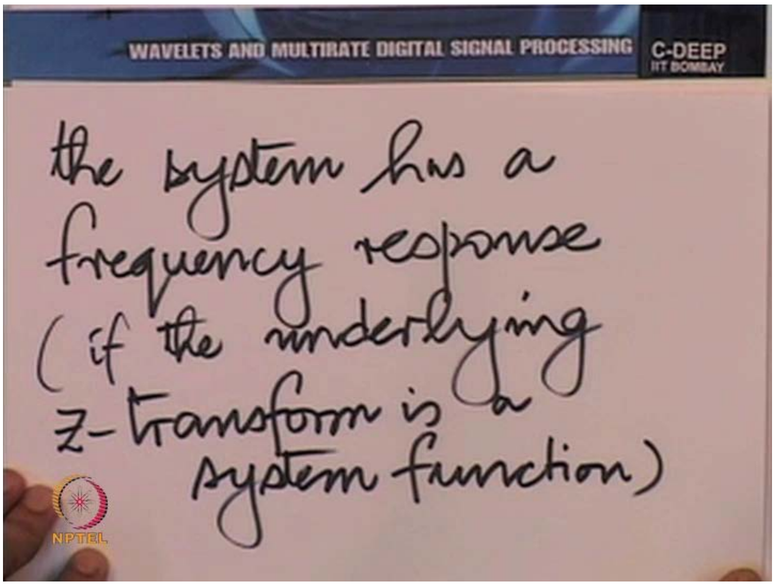

That is, you see, we talk about the system having a frequency response if the underlying Z transform is a system function. If the underlying Z transform is a system function. And of course, otherwise, in case we are talking about a sequence, then we say the sequence has a discrete time Fourier transform.

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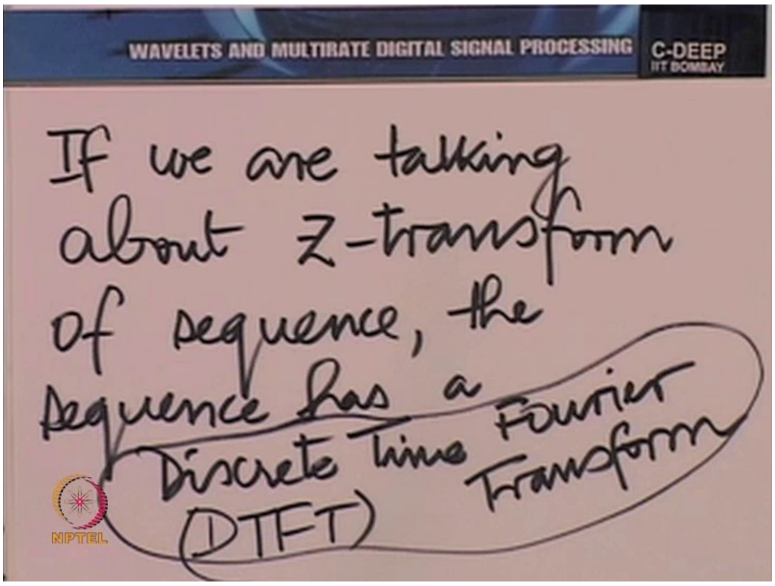

So, if we are talking about the Z transform of a sequence, we say the sequence has a discrete time Fourier transform. This is an important concept, DT FT for short. Either way, what we are saying is in case the unit circle is included in the region of convergence, then we have an even more interesting interpretation of the system function of the sequence in question. And in fact if you look at all these Z transforms encountered in the Haar filter bank, they all satisfy this.