Course Name: Introduction to Microwave and Optical Metamaterials
Professor Name: Dr. Debabrata Sikdar
Department Name: Electronics and Electrical Department
Institute Name: Indian Institute of Technology, Guwahati
Week-2
Lecture-8

Lec 8: Electromagnetic Properties of Materials

Hello everyone, welcome to Lecture 8 of the online course on Introduction to Microwave and Optical Metamaterials.

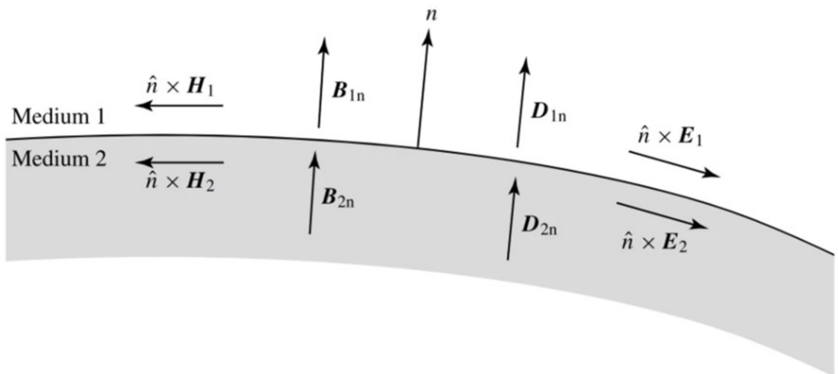
Lecture Outline

- Maxwell's Equations
  - Gauss's law for electric fields
  - Gauss's law for magnetic fields
  - Faraday's law
  - Ampere-Maxwell equation
- The Wave Equation
  - from Maxwell's Equations
  - speed of EM waves

Today's lecture will be on the electromagnetic properties of materials. So here is the outline of the lecture.

Boundary Conditions — when no surface charge

- At the interface of two media of different electromagnetic properties, the electromagnetic field components must satisfy certain boundary conditions.



IIT Guwahati | NPTEL swayam Source: https://www.fiberoptics4sale.com/blogs/wave-optics/optical-fields-and-maxwells-equations

We will look into the boundary conditions in the two cases: when no surface charges are present and when there is the presence of surface charges and their densities. We will then look into the electromagnetic properties of materials. Namely, dielectric permittivity and magnetic permeability, which are epsilon and mu.

And then we will classify the materials based on anisotropy, linearity, magnetization, and conductivity. So, at the interface of two different mediums. which has got different electromagnetic properties. How the electromagnetic field components change is okay; for that, you need to understand the boundary conditions.

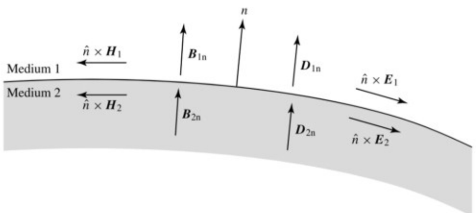
So, here we are studying the boundary conditions with the assumption that there is no surface charge. So, these boundary conditions basically describe the behavior of the electromagnetic fields, namely the electric field, the electric displacement field D, the magnetic field H, and the magnetic flux B, and how all those things are interrelated while we are considering the source-free region. So, in this case, the tangential components of the electric field in medium 1 and medium 2 will be equal. So, that is two different colors showing two different mediums. Here is the interface, so the same thing applies for the magnetic field as well; the tangential components of the magnetic fields are continuous, and the normal components of the displacement fields, like the electric displacement field or the magnetic displacement field, are also continuous.

The normal components denoted by small n show you the direction of the normal, okay. So, this is also continuous. So, this boundary conditions can basically be derived from Maxwell's equation. Right.

Boundary Conditions — when no surface charge

Maxwell's Equations

Divergence equations	Curl equations
$\nabla . D = \rho_f$	$\nabla \times E = -\frac{\partial B}{dt}$
$\nabla . B = 0$	$\nabla \times H = J + \frac{\partial D}{dt}$



- The boundary conditions can be derived from Maxwell's equations.
- From **Curl equations**, the tangential components of the fields at the boundary satisfy.

$$\hat{n} \times \boldsymbol{E}_1 = \hat{n} \times \boldsymbol{E}_2$$
$$\hat{n} \times \boldsymbol{H}_1 = \hat{n} \times \boldsymbol{H}_2$$

- From **Divergence equations**, we have

$$\hat{n}.\boldsymbol{D}_1 = \hat{n}.\boldsymbol{D}_2$$
$$\hat{n}.\boldsymbol{B}_1 = \hat{n}.\boldsymbol{B}_2$$

- The tangential components of E and H must be continuous across an interface, while the normal components of D and B are continuous.

Source: https://www.fiberoptics4sale.com/blogs/wave-optics/optical-fields-and-maxwells-equations

So, here is a quick glimpse of Maxwell's equations, as we know there are two divergence equations and two curl equations.

So, we can start taking the curl equations, okay, and you can see that the tangential components are basically, you know, obtained as the curl of that electric field vector or the magnetic field vector with the normal vector, right? So, it is like the n-cap curl of E; okay, that way you can calculate. So, this is how you can write the tangential components, and the tangential components of the electric and magnetic fields are continuous across the interface. Now, from the divergence equation, you can see the other two components of this displacement field along the direction of the normal, and you can find out that n cap dot D_1 will be equal to n cap dot D_2. Similarly, it will be applicable to the magnetic field or magnetic flux density as well. So, you can write n cap dot B_1 equals n cap dot B_2.

$$\hat{n} \times \boldsymbol{E}_1 = \hat{n} \times \boldsymbol{E}_2$$

$$\hat{n} \times \boldsymbol{H}_1 = \hat{n} \times \boldsymbol{H}_2$$

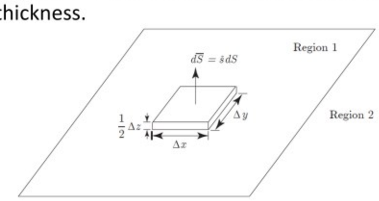
$$\hat{n}.\boldsymbol{D}_1 = \hat{n}.\boldsymbol{D}_2$$

$$\hat{n}.\boldsymbol{B}_1 = \hat{n}.\boldsymbol{B}_2$$

Right. So, the first two tangential components are coming from the curl equations, while the divergence equation gives you the other two boundary conditions, right? So, that is what the takeaway message is: the tangential components of the electric and magnetic fields must be continuous across an interface, whereas the normal components of D and P should be continuous, right? So, that is what we have done. When the boundary conditions were obtained, no surface charge was present. Now, let us consider the case when surface charge and kind density are present, and we will see that this particular concept will have a lot of practical usage. And it is often convenient, in particular mathematically, to define regions where electric and magnetic fields are zero. So, let us assume here that there is a plane boundary surface at z equals 0; this particular plane, as you can see, is basically separating two regions: region 1 and region 2.

Boundary Conditions — presence of surface charge and current density

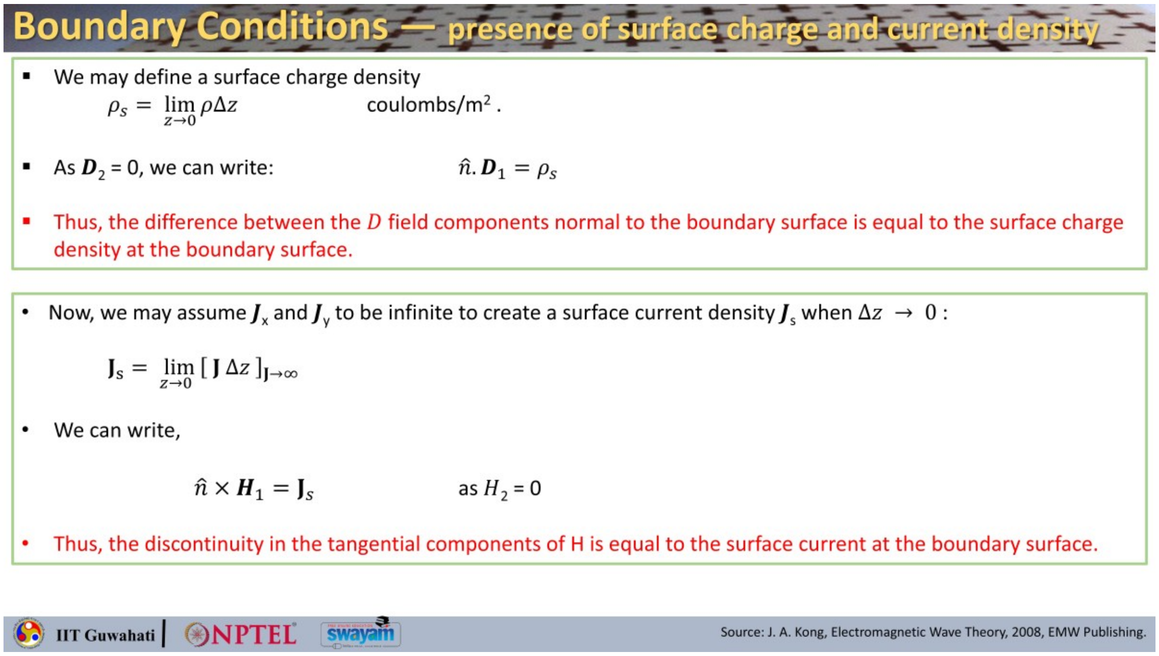
- It is often convenient, in particular mathematically, to define regions where the electric and magnetic fields are zero.
- Assume that there is a plane boundary surface at $z = 0$ separating Regions 1 and 2, we can derive the boundary condition for H by using a small pill-box [as shown in Fig.] and letting Δz go to zero.
- **The media occupying such regions are called perfect conductors**, which are idealizations of media where the fields inside are vanishingly small.
- We assume that all fields in Region 2 are zero, $\boldsymbol{E_2} = \boldsymbol{H_2} = \boldsymbol{B_2} = \boldsymbol{D_2} = \boldsymbol{0}$.
- **Electric charges and currents** are located primarily in a very thin layer on the **surface of perfect conductors**. Thus, on the surface of perfect conductors, we assume $\boldsymbol{\rho}$ is infinite contained in a zero thickness.



IIT Guwahati | NPTEL swayam Source: J. A. Kong, Electromagnetic Wave Theory, 2008, EMW Publishing.

We can derive the boundary condition for the magnetic field H by placing a small pillbox that has a width of delta x. The length of delta y and the height is like, you know, delta z. So, you can see half of delta z is on the region 1 side, okay, and another half will be below that, which will be in region 2, right? The media occupying these regions is called a perfect conductor. which are basically idealizations of media where the field inside will be very small or vanishingly small. Now, let us assume that region 2 has no charge or anything.

So, you can safely assume that E_2, H_2, B_2, and D_2 all equal 0. So, all the fields in Region 2 are basically 0. Now, where are the electric charges and currents located? They are basically located in a very thin layer on the surface of the perfect conductor, and we can assume that the conductivity, ρ, is infinite when it is contained in zero thickness. So let us define, you know, the surface charge density. So you can see that rho s can be written as rho times delta z when z tends to 0, and it has a unit of coulombs per meter squared.



So, that is basically your surface charge density. Okay, and as you already know, D_2 is 0. So, we can safely write that n cap dot D_1 will be equal to this surface charge density, right? So, you can see that the difference between the electric displacement field components normal to the boundary is basically equal to the surface density available at the boundary surface. So, this is basically showing you the difference between the two D field components; D_2 is 0. So, we do not need to write it again, but the difference is basically the surface charge density at the boundary surface.

Now, if you assume that you know J_x and J_y, these are basically current density surface current densities. It is okay to be infinite to create a surface current density Js when you have this thickness going to 0. So, how does it look? So, Js can be defined, as you know, as the limit when z tends to 0, and you have J delta z, and this J is very high. So, this is a formal definition of the surface current density. And then you can write that the tangential component of the magnetic field will be equal to that particular surface current density because on the other side, H_2 is 0.

$$\rho_s = \lim_{z \to 0} \rho \Delta z$$

$$\hat{n}.\ \boldsymbol{D}_1 = \rho_s$$

$$\mathbf{J}_s = \lim_{z \to 0} [\, \mathbf{J}\, \Delta z \,]_{\mathbf{J} \to \infty}$$

$$\hat{n} \times \boldsymbol{H}_1 = \mathbf{J}_s$$

Boundary Conditions — presence of surface charge and current density

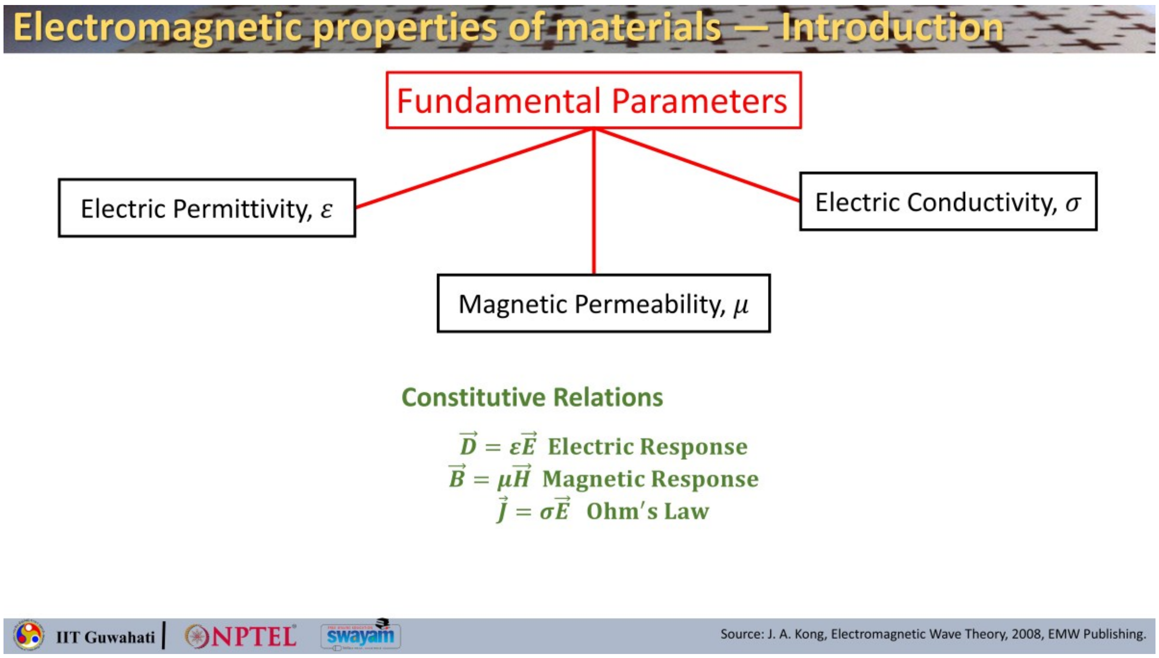
S.No.	Vector Form	Scalar Form	Description
1.	$\hat{e}_n \times (\vec{E}_1 - \vec{E}_2) = 0$	$E_{t1} - E_{t2} = 0$	Tangential electric field ($\vec{E}$) is continuous.
2.	$\hat{e}_n \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$	$H_{t1} - H_{t2} = J_s$	The discontinuity of the tangential H field equals the surface current.
3.	$\hat{e}_n . (\vec{D}_1 - \vec{D}_2) = \rho_s$	$D_{n1} - D_{n2} = \rho_s$	The discontinuity of the normal $\vec{D}$ equals the surface charge density.
4.	$\hat{e}_n . (\vec{B}_1 - \vec{B}_2) = 0$	$B_{n1} - B_{n2} = 0$	The normal component of $\vec{B}$ is continuous.

So, here again you can say that the discontinuity in the tangential component of the magnetic field, or you can say the difference between the tangential components of the magnetic field, is basically equal to the surface current that is available at or flowing through the boundary surface. So, finally, we can put all the equations from the boundary conditions together in the presence of surface charge and current density, and this is how it looks in vector form. So, you can write n cap directly, or you can write, okay, e_n cap, okay? This is also just showing you a different notation because in different books you may find directly n cap, or it may also be written like this: unit vector along that n, which is the surface normal. So, in a few books, you will see that the electric field vector is shown as bold e, okay? I have shown the other notation; they may also come with the vector sign, the arrow on top. So that they are all similar.

So, you can see that this is how it is written. So, it means the tangential component of the electric field is 0, okay. The difference between the two is 0; that means it is continuous, okay. Here also you can see that the discontinuity between the tangential magnetic fields is basically because of the surface current. So, if there is no surface current, there will be no discontinuity in the tangential magnetic field.

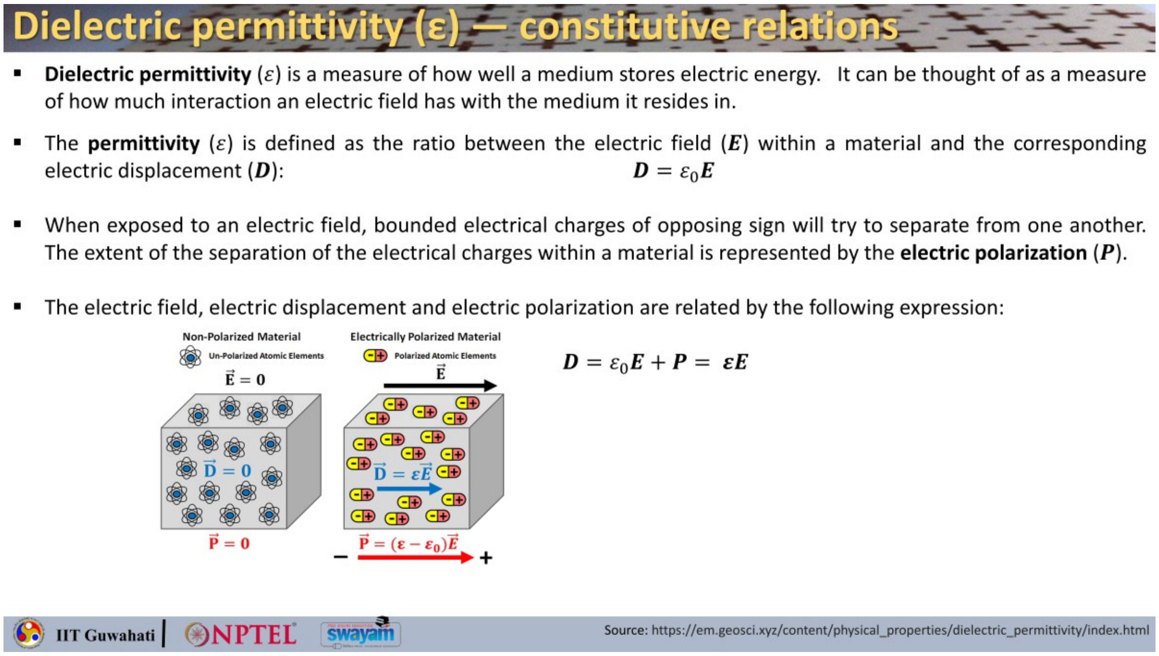
Similarly, the discontinuity in the normal electric flux density or electric flux will be the same as the surface charge density. So, in the scalar form, you can simply write dN1 minus dN2, which will be equal to sigma rho s, okay, not sigma rho s. The normal component of the B field, which is basically the magnetic displacement field or the magnetic flux density, is also continuous. So, the difference is 0. So, an easy way to remember is that when the difference comes out to be 0, they are basically continuous.

Now, because of the presence of surface charge density, the tangential magnetic field will be discontinuous. And also because of the surface charge density, the normal component of the electric field flux density will be discontinuous.



So, now let us look into the electromagnetic properties of materials, and we will see that there are three main properties. One is the electric permittivity, magnetic permeability (mu), and electric conductivity (sigma). So, epsilon, mu, and sigma—these three are very important parameters, and from the electric response, you will see that dielectric permittivity is basically a property that characterizes the degree of electric polarization in a material under the influence of an external electric field.

Similarly, the magnetic response is given by this equation, which is B equals mu H. So, mu basically tells you the permeability; it means how well a medium can store magnetic energy. Conductivity sigma tells you how much the current density will be in the presence of an electric field. So that's basically Ohm's Law. So these are collectively known as constitutive relations because this is how matter reacts to your electromagnetic field.



So, let us first define electric permittivity. Dielectric permittivity is basically a measure of how well a medium can store electric energy. So, it can be thought of as a measure of how much interaction an electric field is having with the medium it resides in and how to quantify it. So, the permittivity is basically defined as the ratio between the electric field E within a material and the corresponding electric displacement. So, you can write D equals ε_0E, right? So, that is basically in a vacuum, right? So, we will see how we can now do it for the material.

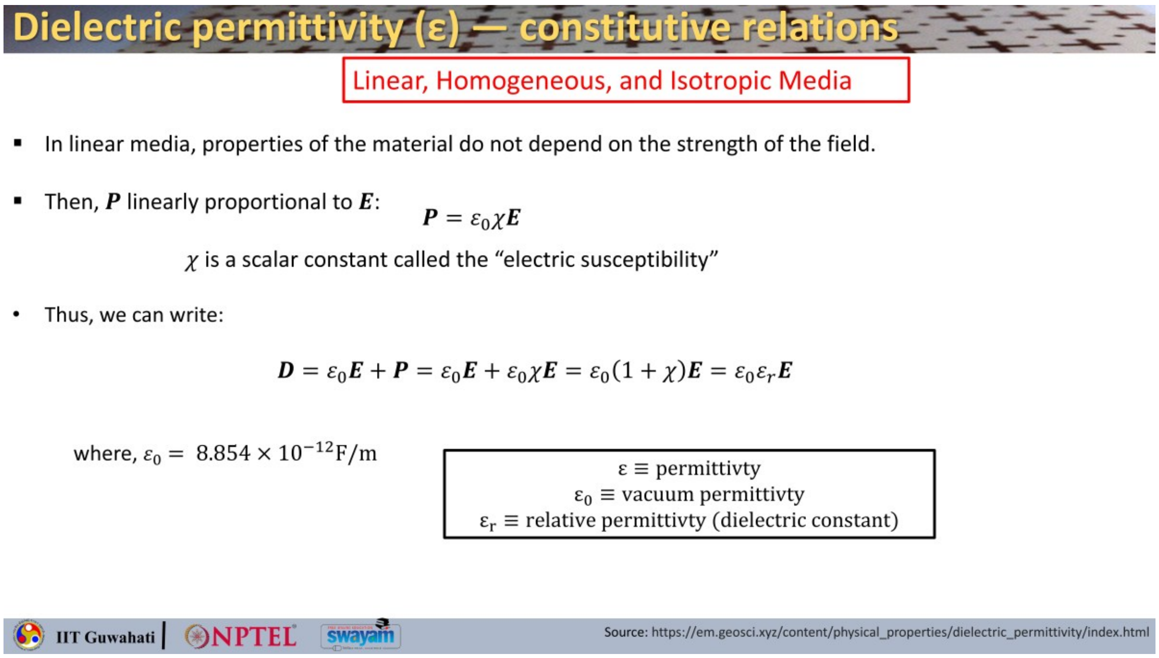
$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$$

So, when exposed to an electric field, bound electrical charges of the opposite sign will try to separate, like this, okay. So, they separate from one another, and the extent of the separation of the electrical charges within a material is represented by the electric polarization P. This is the case of a non-polarized material, non-polarized because there is no external electric field. So, the polarization is 0; overall, you know the electric displacement is also 0. But then, if you have some external electric field, you will see that the polarization is taking place.

So, the displacement can then be written as D equals epsilon E, and because of this, you know polarization can be written as P equals epsilon minus epsilon naught times E. So, how do you get this? It looks straight away from here that the electric field, electric displacement, and polarization are basically related by this. So, normally you have this one okay when this is without the material okay, but then when the material is present, this extra term adds up okay. So, the electric field, electric displacement, and electric progression are related by the following equation. So, you can write D equals epsilon naught E plus P.

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \boldsymbol{\varepsilon E}$$

This is where the response of the material comes into the picture. So, as you see, P is basically epsilon minus epsilon naught times E. When you put it here, you see it only remains to be D equals epsilon naught E. So, what is this epsilon? It is basically the permittivity. So, that tells you the ratio of displacement to the electric field.

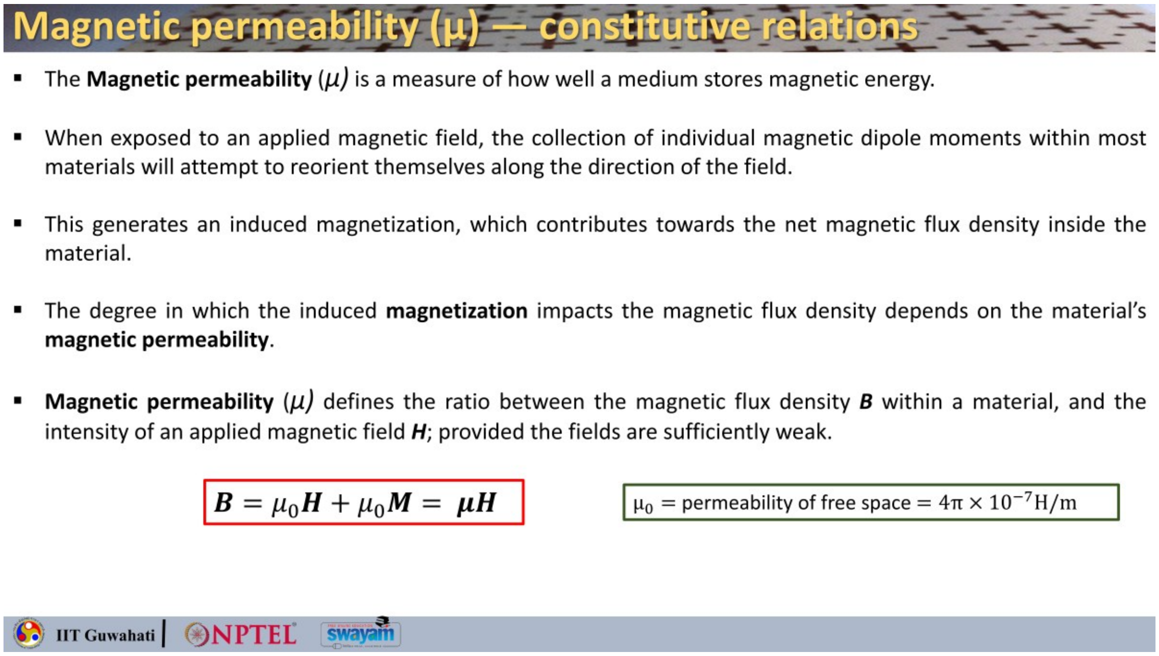


Now, let us look into these constitutive relations in different types of media. So, let us consider the most idealized one that is a linear, homogeneous, and isotropic medium. So, what are these terms: linear, homogeneous, and isotropic? Linear medium means that the properties of the material do not depend on the strength of the field, okay? So, you can see that the polarization is basically linearly proportional to the electric field. So, P equals $\varepsilon_0\chi$E, okay? That is also how you can write. that. So, chi is called the electric susceptibility. So, how can you express your displacement field from here? So, D can be written as epsilon naught E plus P. So, P can be replaced with this particular equation: epsilon naught chi E, where chi, as I mentioned, is electric susceptibility. So, you can take epsilon in common, and you get 1 plus chi. So, this 1 plus chi is nothing but your epsilon r, the relative permittivity that we all know, okay.

So, D can be written as epsilon naught epsilon r E, and these two epsilons multiplying give you that epsilon that you saw in the previous slide. So, it is very handy to know what epsilon naught is. So, always remember that epsilon gives you the permittivity, epsilon naught is the vacuum permittivity, and epsilon r is nothing but the relative permittivity with respect to vacuum.

$$\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E}$$

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_0 \boldsymbol{E} + \varepsilon_0 \chi \boldsymbol{E} = \varepsilon_0 (1 + \chi) \boldsymbol{E} = \varepsilon_0 \varepsilon_r \boldsymbol{E}$$



Similarly, let us look into magnetic permeability, which gives you another constitutive relationship. It tells you how well a medium can store magnetic energy. So here, obviously, when you are exposing an applied magnetic field to your medium, the collection of individual magnetic dipole moments within most materials will attempt to reorient themselves according to the direction of the field. So, this particular act basically generates induced magnetism, which contributes to the net magnetic flux density inside the material. And the degree to which the induced magnetization would impact the magnetic flux density depends on this material's magnetic permeability. So, let us see how it is done or expressed mathematically. So, a more formal definition would be, say, magnetic permeability defines the ratio between the magnetic flux density B within a material and the intensity of the applied magnetic field H.

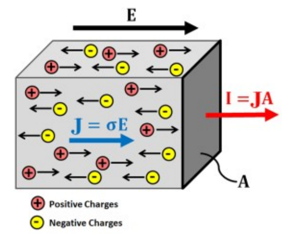
Provided the fields are sufficiently weak. So, what does it mean when you have B equals μH plus μ_0M? So, what is M? M is basically magnetization, and in most materials, you will see that this is not happening. So, you take non-magnetic materials, but then if there is magnetization happening, this overall thing can be expressed in terms of H and then μ. So, this μ is basically the permeability, which is again μ_0 and μ_r. So, you can think of, you know, vacuum permeability times the relative permeability giving you the actual permeability of the material. So, this is the permeability of free space; I am sure you all know this.

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M} = \boldsymbol{\mu H}$$

Conductivity (σ) — constitutive relations

- The **conductivity** describes the degree to which a material conducts electricity.
- When an electric field is applied to a material, free charges within the material experience an electrical (**Coulomb**) force.
- This force causes the free charges to move through the material along the direction of the applied field (***i.e. electrical current***).
- *The ease at which electrical charges move through a material under the influence of an electric field depends on the **material's electrical conductivity**.*
- Electrical conductivity (σ) can be defined as the ratio between the current density (**J**) within a material and the electric field (**E**). This relationship is known as **Ohm's law** and is given by:

$$J = \sigma \boldsymbol{E},$$
$$\sigma = \ 1/\rho \quad [\Omega.\mathrm{m}]$$



And then the last one in this constitutive relationship, an important parameter is conductivity, which gives you the degree to which the material conducts electricity. So when an electric field is applied to the material, free charges within the material will experience an electrical, or you can say, Coulomb force. And this force will cause the free charges to move through the material along the direction of the applied field, which is electrical current. So, you see, the field is this way. So your positive charges are moving along the direction.

And that will be the direction of the kind flow. Electrons will basically move in the opposite direction, and the reverse of their movement will give you that kind of flow, right? So, the kind is basically in this area. So, J is the kind of density. If this cross-sectional area is A, when you multiply J and A, you get the kind, right? So, the ease with which, or we can say how easily, the electrical charges can move through the material under the influence of an electric field basically depends on the material's electrical conductivity, right? So electrical conductivity, sigma, can be defined as a ratio of the current density within a material to the applied electric field.

So that's Ohm's law: J equals sigma E. So sigma is basically J divided by E, right? So, you can also correlate this conductivity with resistivity. So, that is 1 over rho, okay. So, in many cases, you know that the electrical properties of the materials are also characterized by their electrical resistivity. So, that is why we are showing both conductivity and resistivity here; they are just reciprocals of each other, okay.

$$J = \sigma \boldsymbol{E},$$

$$\sigma = \ 1/\rho \quad [\Omega.\mathrm{m}]$$

Electromagnetic properties of Materials — Important parameters

- **Velocity of the EM waves:**

Compare:

$$\nabla^2 \boldsymbol{E} = \mu_0 \varepsilon_0 \varepsilon_{\mathrm{r}} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} \text{ and } \nabla^2 \boldsymbol{E} = \frac{1}{v^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\Rightarrow v^2 = \frac{1}{\mu_0 \varepsilon_0} \frac{1}{\varepsilon_{\mathrm{r}}} = \frac{c_0^2}{\varepsilon_{\mathrm{r}}}$$

In Free Space (Vacuum):

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299{,}795{,}638 \text{ m/s}$$

- **Refractive index of a material:**

Refractive index is a material property that describes how the material affects the speed of light travelling through it

$$n = \frac{c}{v} = \sqrt{\varepsilon_r} = \sqrt{1 + \chi}$$

IIT Guwahati | NPTEL swayam

Source: https://em.geosci.xyz/content/physical_properties/electrical_conductivity/index.html

So, that is easy to correlate. Now that we understand the basic electromagnetic properties and their constitutive relations, we will move on to how to find the velocity of the EM waves. So, velocity can be found by comparing these equations, okay? We have seen these wave equations, okay. So del square E was given as mu naught epsilon naught epsilon r times dot square E by dot E square, and then you can compare this also with this form, okay. So, where these three variables are basically represented as 1 over v squared.

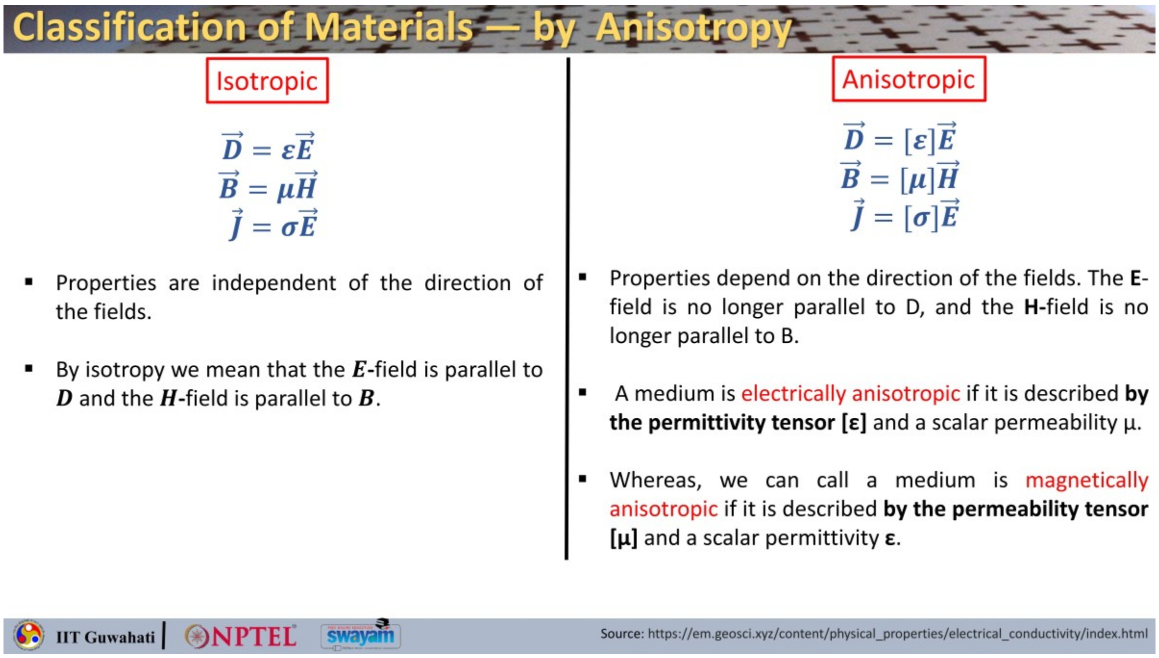
$$\nabla^2 \boldsymbol{E} = \mu_0 \varepsilon_0 \varepsilon_{\mathrm{r}} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} \text{ and } \nabla^2 \boldsymbol{E} = \frac{1}{v^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\Rightarrow v^2 = \frac{1}{\mu_0 \varepsilon_0} \frac{1}{\varepsilon_{\mathrm{r}}} = \frac{c_0^2}{\varepsilon_{\mathrm{r}}}$$

So, that means 1 over v squared equals this, okay. So, you can just break the fraction and see that 1 over mu naught epsilon naught is nothing but c naught squared. Okay, c naught is the speed of light in a vacuum, and 1 by epsilon r remains there. So, this is the typical relationship between velocity, the speed of light, and permittivity, right? So, if you take, you can write V equals C naught divided by the square root of epsilon r; that is also fine, and the square root of epsilon r, if epsilon r is a real quantity, will give you the refractive index, and that is how, in the optical field, people work with refractive index in most cases, right? So these are the typical values that we all know. So, as I mentioned, the refractive index is a very important term that defines the material property that tells you the speed of light in that particular medium. So, the refractive index can be defined as c over v, where c is the speed of light in a vacuum divided by the speed of light in that particular medium, which comes out to be the square root of epsilon r, which is also related to the electrical susceptibility of the material by the square root of 1 plus chi.

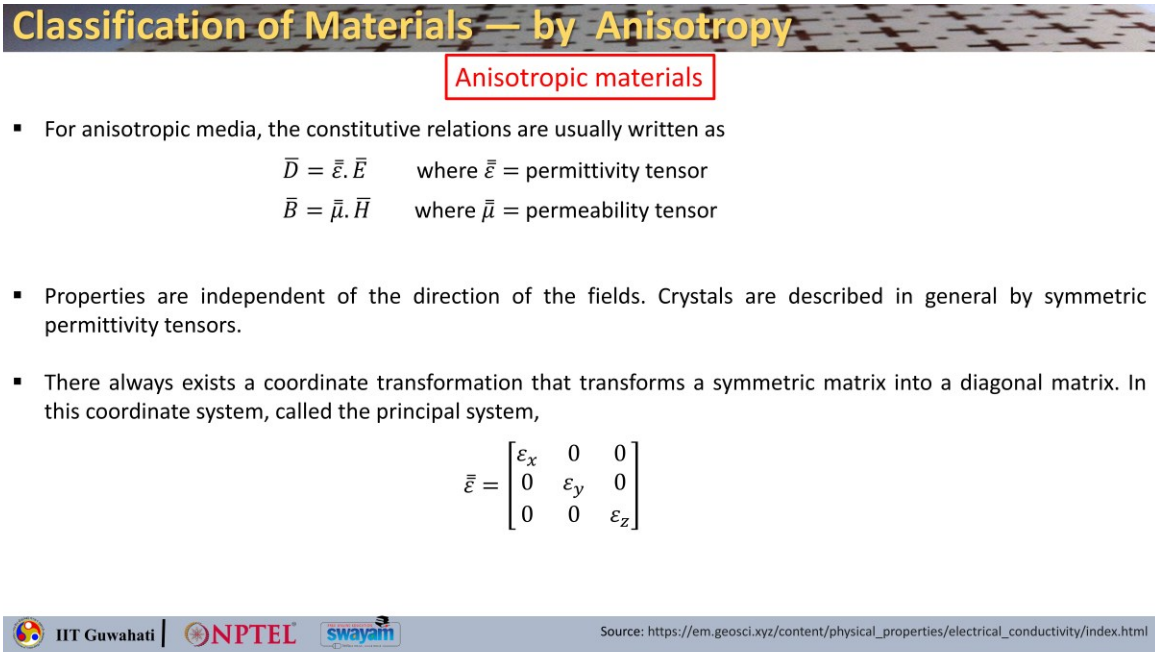
$$n = \frac{c}{v} = \sqrt{\varepsilon_r} = \sqrt{1 + \chi}$$

So you're just writing ε_r as 1 plus χ, okay.



So now we'll see how the materials can be classified based on different electromagnetic properties. So we'll start the classification with anisotropy. So the first thing you should know is what the isotropic ones are. So we have seen that in isotropic materials, the properties are basically independent of the directions of the field, right? So you can write D equals epsilon E, B equals mu H, and J equals sigma E, so all the ones with the arrows are basically the vectors. Okay, so here, because it is isotropic, you understand that here E and D are basically parallel; similarly, H and B are also parallel.

Okay. But if it is anisotropic, that means the properties are basically dependent on the direction of the fields; that means in this particular case you cannot say that the electric field and the electric displacement are basically parallel to each other. Similarly, they are also not, because the material property is now direction-dependent. So, a medium is electrically anisotropic if it is described by the permittivity tensor, as you can see here, and a scalar permeability mu. On the other hand, you can understand that if a medium is called magnetically anisotropic, that means it has a tensor for permeability, which means the permeability is different along different directions, but the electrical properties are the same. So, you can just go with scalar permittivity, okay? So that way, you can do the classification.



So, this is another notation. So, for anisotropic materials, the constitutive relationships will be written as D equals epsilon double prime. So, that tells you the tensor, okay. So, this notation tells you that these are basically vector fields, okay. So, I am just showing you the different notations in different slides to tell you that all these notations are possible in textbooks; different authors follow different versions of notations.

And you can write B equals μH. So, μ is again a tensor in this case. So, here we understand that the properties are basically direction-dependent, right? Now, when the properties are independent of the direction of the fields, crystals are basically described in general by symmetric permittivity tensors. There always exists a coordinate transformation that can transform a symmetric matrix into a diagonal matrix. This coordinate system will then be called the principal system. So, what you can see is that this particular tensor can be diagonalized like this, okay.

So, you can just have epsilon x, epsilon y, and epsilon z to make life simple, okay? The three coordinate axes are referred to as the principal axes of the crystal. So, here you can see that the principal axis of the crystal epsilon can be written as this. So, if you take a cubic crystal where x, y, and z are all equal, that means E, epsilon x, epsilon y, and epsilon z will be equal, and that becomes isotropic, okay. However, if you go for other types of crystals like tetragonal, hexagonal, or rhombohedral, two out of three parameters will be equal. So, either of these two will be equal; the third one will be different.

$$\bar{D} = \bar{\bar{\varepsilon}}.\bar{E}$$

$$\bar{B} = \bar{\bar{\mu}}.\bar{H}$$

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

Classification of Materials — by Anisotropy

Anisotropic materials

- Principal axes of the crystal: $\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$
- For cubic crystals, $x = y = z$ and they are *isotropic*.
- However, in *tetragonal, hexagonal, and rhombohedral crystals*, two of the three parameters are equal. **Such crystals are uniaxial**.
- For a uniaxial crystal, the **permittivity tensor** can be written as: $\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$
- Here, the z axis is the optic axis. The crystal is:
  - positive uniaxial if $\varepsilon_z > \varepsilon$;
  - negative uniaxial if $\varepsilon_z < \varepsilon$.
- **Bi-axial:** In *orthorhombic, monoclinic, and triclinic crystals*, all three crystallographic axes are unequal.
- We have $\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$, and the medium is **biaxial.**

So, that kind of crystal is called a uniaxial crystal. So, in a uniaxial crystal, the permittivity tensor will look like this. So, that says x epsilon x and epsilon y are equal, and you can just represent them as epsilon; the third one is epsilon z, which is different. Now depending on whether epsilon z is larger than epsilon or smaller. So, if it is larger, you call it positive uniaxial; if it is smaller, you call it negative uniaxial. So, these are different types of classifications, and there are materials where all three crystallographic axes are different.

So, you call them biaxial, okay, and the examples are orthorhombic, monoclinic, and triclinic crystals. Okay. So, this is where you have to remember that when epsilon x is not equal to epsilon y and epsilon y is not equal to epsilon z, that is when your medium is biaxial.

Classification of Materials — by Linearity

Linear

- Here, properties of the material do not depend on the strength of the field.
- **Electric polarization** ($\boldsymbol{P}$) is linearly proportional to $\boldsymbol{E}$:

$$\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E}$$

χ is "electric susceptibility"

- Thus:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_0 \boldsymbol{E} + \varepsilon_0 \chi \boldsymbol{E} = \varepsilon_0 (1+\chi) \boldsymbol{E} = \varepsilon_0 \varepsilon_r \boldsymbol{E}$$

$\varepsilon \equiv$ permittivty
$\varepsilon_0 \equiv$ vacuum permittivty
$\varepsilon_r \equiv$ relative permittivty (dielectric constant)

Nonlinear

- Properties depend on the intensity of the field.
- In nonlinear medium, the electromagnetic response can often be described by expressing the polarization $\boldsymbol{P}$ as a power series in the field strength $\boldsymbol{E}$ as

$$\tilde{P}(t) = \varepsilon_0[\chi^{(1)}\tilde{E}(t) + \chi^{(2)}\tilde{E}^2(t) + \chi^{(3)}\tilde{E}^3(t) + \cdots$$
$$\equiv \tilde{P}^{(1)}(t) + \tilde{P}^{(2)}(t) + \tilde{P}^{(3)}(t) + \cdots$$

- The quantities $\chi^{(2)}$ and $\chi^{(3)}$ are known as the second- and third-order nonlinear optical susceptibilities, respectively.

So, next we will look into the classification of materials by linearity. So, as I told you earlier, for the linear case, the electric polarization is basically linearly proportional to the electric field.

So, you can see P equals epsilon naught chi E. Chi is basically the electric susceptibility, and you will see that the displacement is written as epsilon at E plus P, which boils down to epsilon naught epsilon r into E, right? And if you consider non-linear, in that case, the properties basically depend on the intensity of the field, okay. In the other case, in the linear case, they do not depend on the intensity or strength of the field. So, you can see that in a non-linear medium, the electromagnetic response can be defined by expressing the polarization as a power series of the field strength E. So, how does it look? So, epsilon naught chi one plus E(t), and then you will have chi 2 or the chi-square term, which is proportional and depends on E square, which is the intensity; there will also be a chi-3 term coming into the picture, which will depend on the cube of the electric field, and so on.

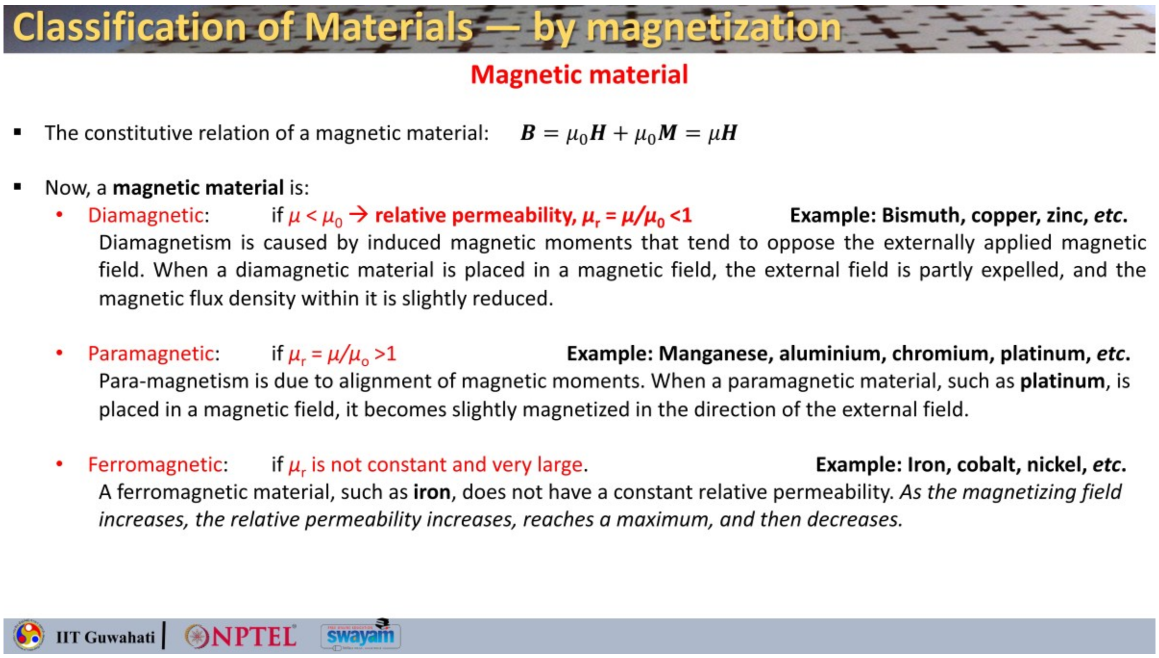
So, that way you can express the polarization as a power series. So, all these quantities, chi square and chi cube, are basically the second and the third order nonlinear optical susceptibilities, right? We can also classify materials by their magnetization. There are magnetic materials, and we have seen that the constitutive relation for magnetic materials looks like this: B equals mu naught h plus mu naught m. So, you can write them as, you know, mu naught mu h, okay.

$$\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E}$$

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_0 \boldsymbol{E} + \varepsilon_0 \chi \boldsymbol{E} = \varepsilon_0 (1+\chi) \boldsymbol{E} = \varepsilon_0 \varepsilon_r \boldsymbol{E}$$

$$\tilde{P}(t) = \varepsilon_0[\chi^{(1)}\tilde{E}(t) + \chi^{(2)}\tilde{E}^2(t) + \chi^{(3)}\tilde{E}^3(t) + \cdots$$

$$\equiv \tilde{P}^{(1)}(t) + \tilde{P}^{(2)}(t) + \tilde{P}^{(3)}(t) + \cdots$$



Now, a magnetic material can be called diamagnetic if your mu is less than mu naught; that means you are getting a relative permeability, which is calculated as mu r, as a ratio of mu over mu naught, and you are getting this as less than 1. So, if you take the example of bismuth, copper, zinc, etcetera, they are basically this kind of material.

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M} = \mu \boldsymbol{H}$$

So, diamagnetism is basically caused by induced magnetic moments that tend to oppose the externally applied magnetic field. When a diamagnetic material is placed in a magnetic field, the external field is partially repelled, or you could say expelled. So, that way, the magnetic flux density basically reduces, okay? You have a paramagnetic substance, which is a completely opposite case. So, you have a relative permeability greater than 1. Examples of such materials are manganese, aluminum, chromium, platinum, etc.

So, paramagnetism is due to the alignment of magnetic moments. So, when you place a paramagnetic medium such as platinum in a magnetic field, it becomes slightly magnetized in the direction of the external field. And then you have ferromagnetic materials. So if your μr is not constant and is very large, something like iron, nickel, cobalt, etc. So ferromagnetic materials such as iron do not have constant relative permeability. As the magnetizing field increases, their relative permeability also increases until it reaches a maximum, after which it decreases. So, these are three different types of materials based on magnetization.

Classification of Materials — by Conductivity

- On the basis of the relative values of electrical conductivity (σ) or resistivity ($\rho = 1/\sigma$), the solids are broadly classified as:
  - **Metals:** They possess very low resistivity (or high conductivity or $\sigma >> 1$).
  - **Semiconductors:** They have conductivity intermediate to metals and insulators.
  - **Insulators:** They have high resistivity (or low conductivity or $\sigma << 1$).

Similarly, you can classify materials based on conductivity. I am sure all of you know this, right? Based on the relative values of electrical conductivity or resistivity, you can broadly classify solids as metals. They possess very low resistivity, which means conductivity is very high, sigma being much greater than 1. You have semiconductors that have conductivity intermediate between metals and insulators, and insulators have very low conductivity. Here is a chart that provides a complete comparison of the orders of values with respect to conductors, semiconductors, and insulators, which is resistivity.

Classification of Materials — by Conductivity

Properties	Conductor	Semiconductor	Insulator
Resistivity	$10^{-6} - 10^{-8}$ Ω/m	$10^{-4} - 0.5$ Ω/m	$10^{7} - 10^{16}$ Ω/m
Conductivity	$10^{6} - 10^{8}$ mho/m	$10^{4} - 0.5$ mho/m	$10^{-7} - 10^{-16}$ mho/m
Temp. Coeff. Of Resistance (α)	Positive	Negative	Negative
Current	Due to free electrons	Due to electrons and holes	No current
Forbidden Energy Gap	$\cong 0$ eV	$\cong 0 - 1$ eV	≥ 6 eV
Examples	Pt, Al, Cu, Ag	Ge, Si, C, GaAs, $GasF_2$	Wood, Plastic, Diamond, Mica

IIT Guwahati | NPTEL swayam Source: https://www.esaral.com/classification-of-solids-in-terms-of-the-forbidden-energy-gap/

You can see that insulators have highly resistive conductivity, ranging from 10 to the power of 6 to 8 mo per meter. Inverse of ohm, okay? Then you have a positive temperature coefficient for the conductor and the other two are negative, okay? That means with an increase in temperature, the resistance will increase, okay. Then you have current, which is coming from free electrons in semiconductors; they are from both electrons and holes and can insulate with no currents. These are the band gaps, as you can see, conductors are 0, okay. Semiconductors can typically be between 0 and 1 electron volt, while insulators are much below that.

So, it is like more than 6 eV; these are the typical examples. So, altogether here we wanted to discuss the different kinds of classification of materials that will be interacting with the electromagnetic field. So, this is not something very new, but these are just fundamentals to brush up on your old knowledge about different electromagnetic properties and their classifications.

So, with that, we conclude here, and we will continue in the next lecture. If you have any doubts regarding this, contact me at this particular email address. Thank you.