Advanced 3G and 4G Wireless Communication
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Lecture - 15
Correlation of PN Sequences and Jammer Margin

Welcome to another lecture in the course on 3G and 4G wireless mobile communication systems, in the last lecture, we started with analyzing the properties of P N sequences.

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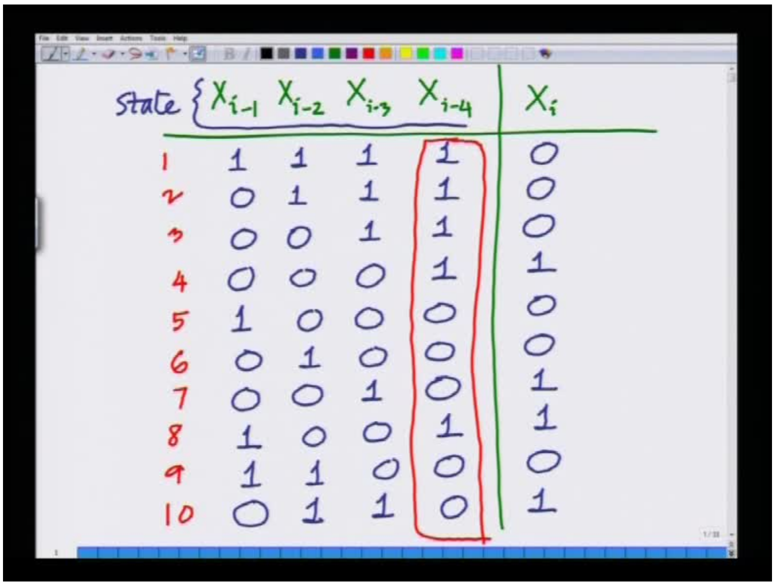

For instance, we had looked elaborately at the generation of a P N sequence using a linear feedback shift register.

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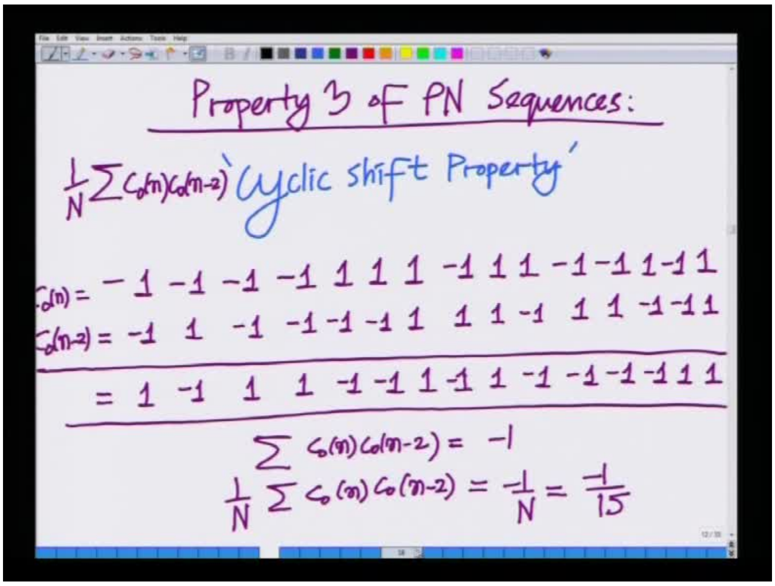

We had then subsequently looked at several properties of the P N sequences for instance here I have the shift property, or the cyclic shift property of the P N sequence.

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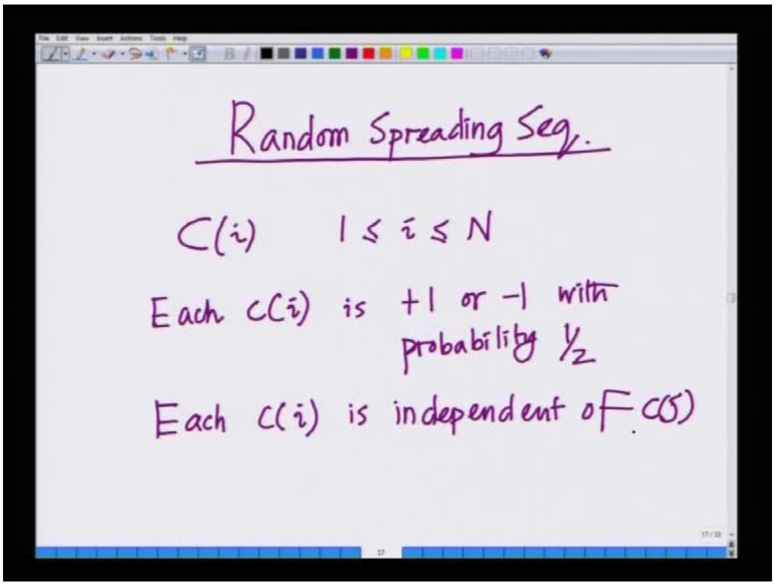

Then we had also looked, we had also started considering random spreading sequences, where each shift is generated randomly as plus or minus 1 with probability a half, and each chip c i is independent of another chip c j.

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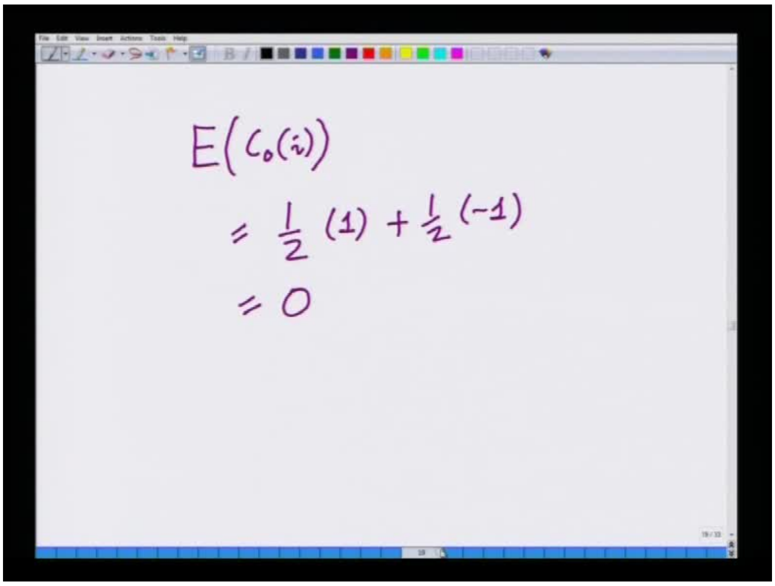

We said that the expected value of each chip is 0.

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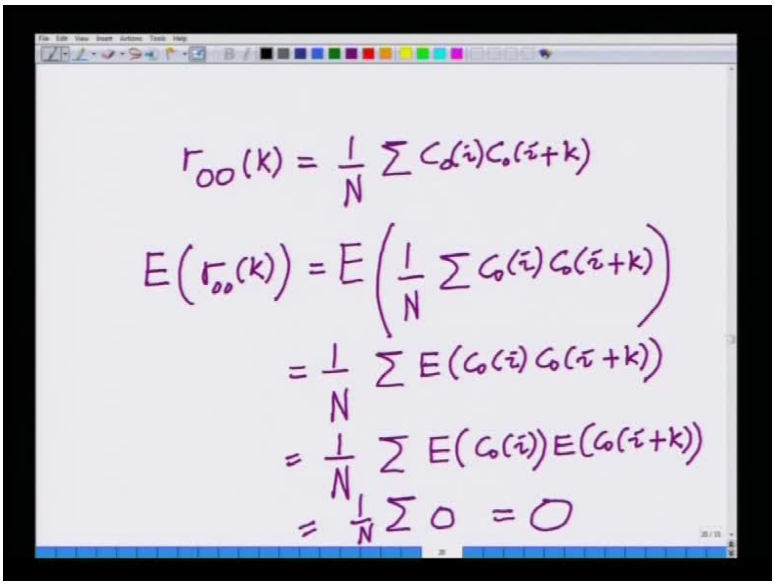

We also said that if you look at the correlation that is the self correlation, that is c 0 i c 0 i plus k summation divided by n that is correlation for a lag of k, that correlation value, expected value of that is 0.

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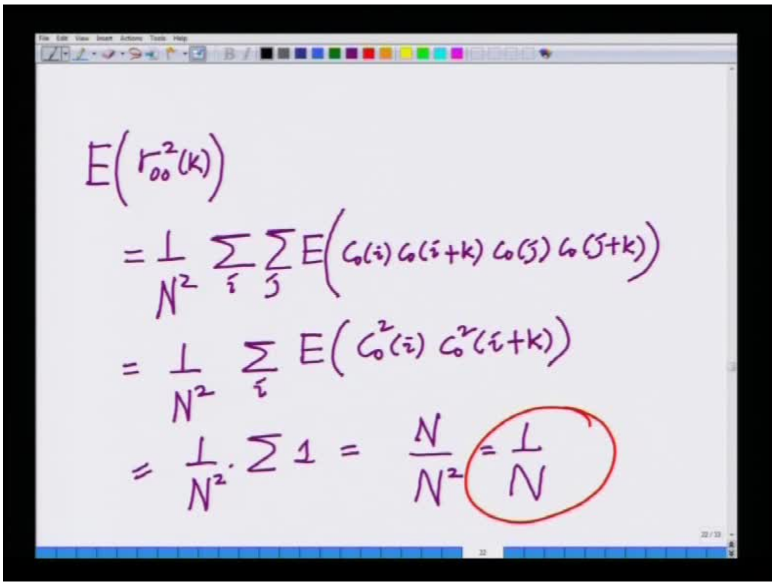

And the variance of that self correlation at lag k is 1 over n, for k not equal 0, that is for any lag that is strictly greater than 0 that is k equals to 1, 2 and so on. And for any lag that is less than strictly less strictly less than 0 that is k equals to minus 1 minus 2 and so on.

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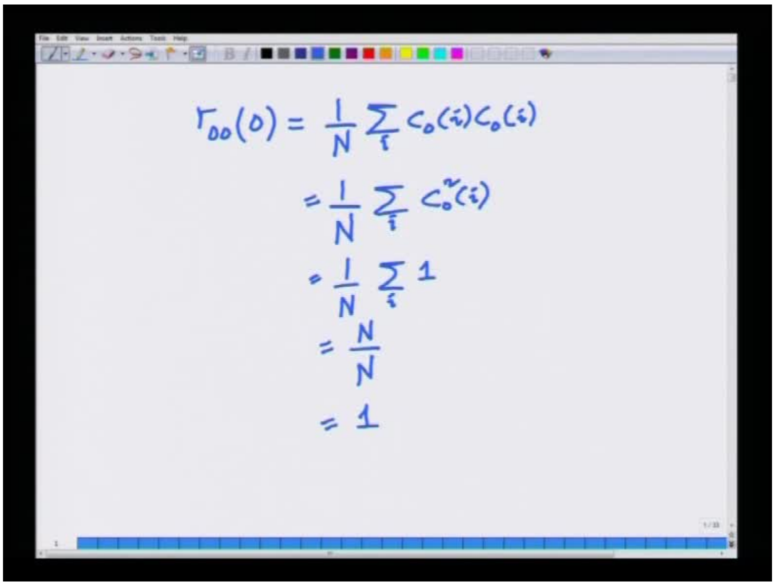

Now, to complete this discussion, let us look at the correlation at lag 0 and that is fairly simple, if I consider the self correlation r 0, 0 at lag 0 I will define it as 1 over N summation over i c 0 of i into c 0 i plus k, but k here is 0. So, this is simply c 0 i in other words this can be written as 1 over N summation over i c 0 square i. Now, remember each c 0 is i is plus or minus 1 with equal with equal probability in any case c 0 square of i is always 1 because c 0 i is plus or minus 1. Hence, this is 1 over N summation of i 1, which is equal to N over N, which is one hence the self correlation at lag 0 is 1 which is which is trivial because it is you are correlating the sequence with itself.

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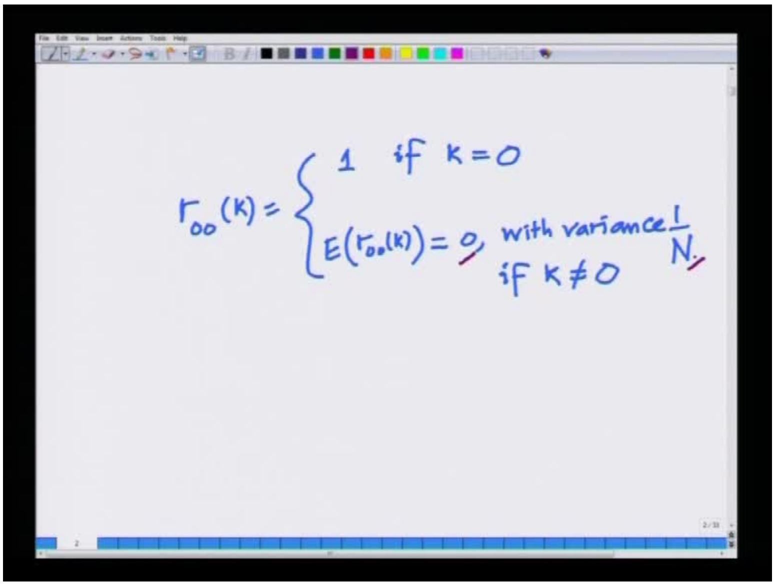

Now, let me summarize the properties of the correlation of the self correlation or the auto correlation r 0, 0 of k as we have seen is equal to 1 if k equals 0 and its expected value this is a random variable remember. So, it is expected value of r 0, 0 k equals 0 with variance 1 over N if k naught equals 0 that is if k equals 0, then it is a deterministic quantity r 0, 0 k is 1 that is r 0, 0 is r 0, 0 of 0 corresponding to lag 0 is 1 if k is not 0 then r 0, 0 k is a random variable with expected value 0, and variance 1 over N or power 1 over N. As we saw earlier expected r 0, 0 square of k equals 1 over N that is its variance is 1 over N.

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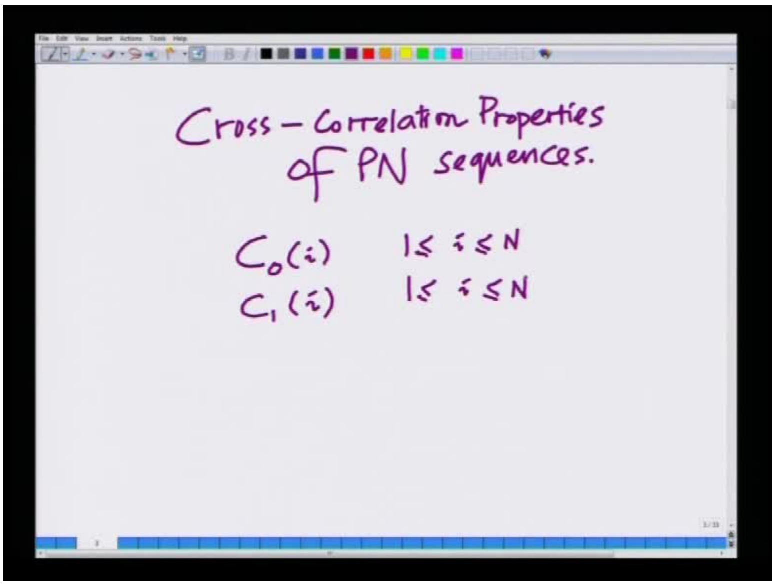

Now, looks look at the cross correlation properties of two random spreading sequences. So, let us look at the cross correlation properties of these P N sequences, or these random sequences, if I look at the cross correlation for that purpose let me consider 2 code sequences c o i and c 1 i such that, 1 less than equal to i less than equal to N and 1 less than equal to i less than equal to N by cross correlation, I mean the correlation between the code sequence 0 and 1 corresponding to a lag k.

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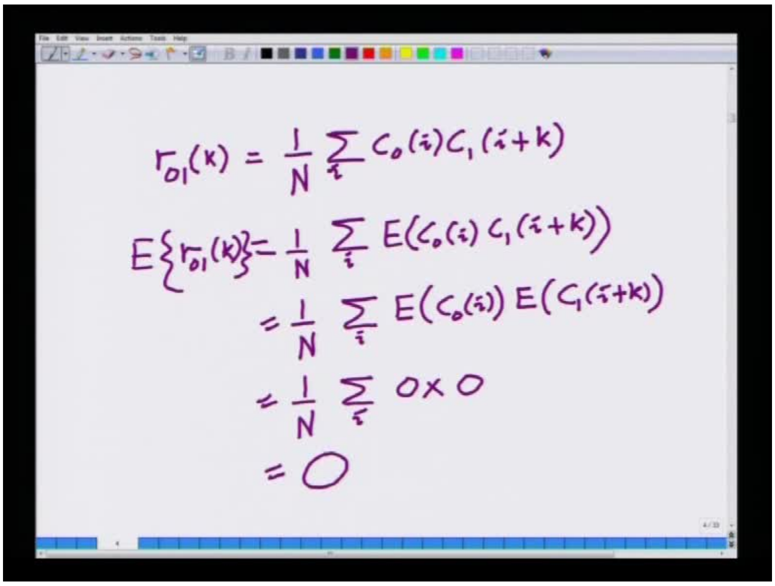

So, similar to r 0, 1 let me define r 0, 1 the correlation between sequence 0 corresponding to user 0 and the correlation between sequence 1 corresponding to user 1 as r 0, 1 of k equals 1 over N summation, summation over i, c o of i into c 1 of i plus k. Now, if I take the expected value of this cross correlation for lag k, if I look at the expected value that is expected value of r 0, 1 of k that is equal to 1 over N summation over i, I am going to take the expectation directly inside. Since, we have done this a couple of times because the summation and expectation are both linear operators hence, we can interchange them. So, this becomes expected c 0 i expected c 1 i plus k and there is a summation over i and you divide over 1 over.

Now, this as we have seen yesterday that the code sequences corresponding to user 0 and user 1 are independent because not the chips are generated independently. Hence, this expectation of c 0 i into c 1 i plus k is nothing but the product of the expectations expected c 0 i expected c 1 i plus k, which is equal to 1 over N summation over i expected c 0 i is 0 expected c 1 i is 0. Hence, this is 0, 0 hence, the expected cross correlation 0, 1 k for any lag.

Now, observe that this is true not only for any non zero lag, but this is also true for k equals 0 because even when k equals 0 unlike in the cross correlation c 0 i into c 1 i are both independent. Hence, this again splits into expected c 0 expected c 1 i which is 0 hence the cross correlation is much simpler for any lag k 0 or non zero the cross expected value of the cross correlation is always 0, and going by the similar analysis as we did for the auto correlation.

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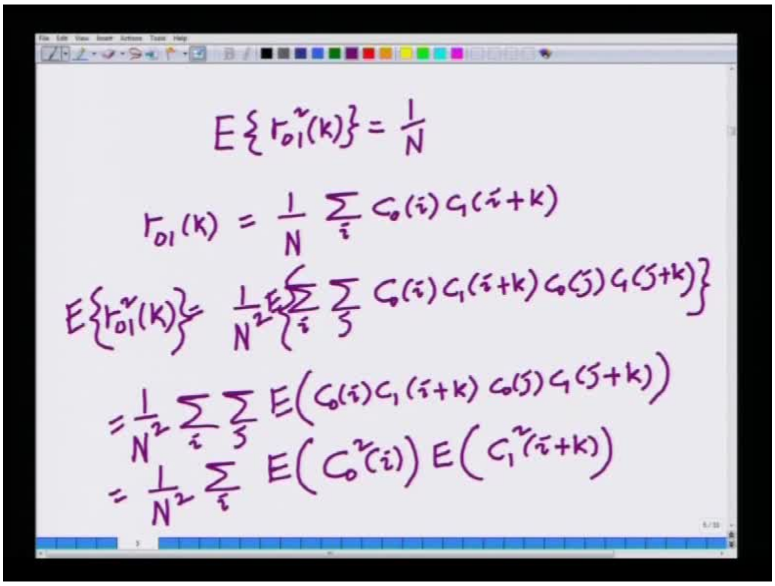

One can easily prove that the expected value of r 0, 1 square of k equals 1 over N, this is not very difficult, let me highlight the salient steps of this derivation r 0, 1 of k equals 1 over N summation i c 0 of i into c 1 of i plus k. Now, the product r 0, 1 square of k is r 0, 1 k times r 0, 1 k which is 1 over N square summation i, summation j c 0 i c 1 i plus k c 0 j c 1 j plus k, I will take the expectation on both sides. And now, I will move the expectation operator inside, and that is easy to see that this reduces to summation over i summation over j expected c 0 i, c 1 i plus k c 0 j c 1 j plus k.

Now, we can clearly see if i is not equal to j then all these split into independent terms this is expected c 0 i into c 1 expected c 1 i plus k into expected c 0 j into expected c 1 j plus k because the chips corresponding to different code sequences, and the different chips corresponding to the same sequence are also independent. Hence, this splits into four different expectations except, when i equals j in that case this reduces to summation over i expected c 0 square i into expected c 1 square i plus k, and we know each c 0 i and each c 1 i plus k is either plus or minus 1 hence, each of these quantities c 0 square i c 1 square i plus k are both one hence, this product is 1.

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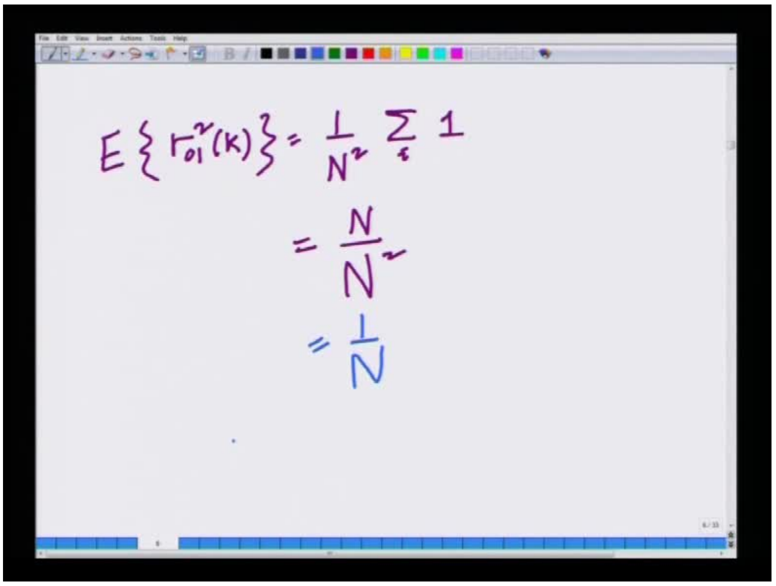

Hence, this reduces to expected r 0, 1 square k equal 1 over N square summation over i of 1, which is N over N square, which is equal to 1 over N. Hence, if I look at the cross correlation at lag k, the relationship is much simple.

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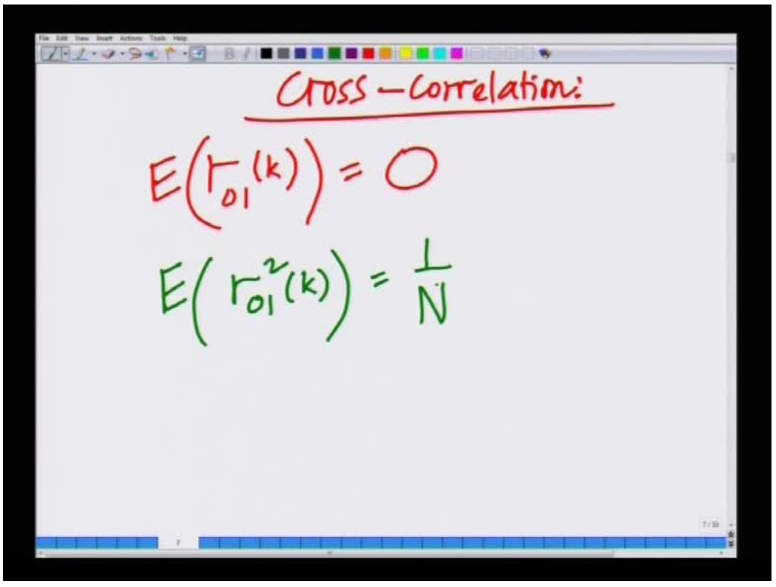

If I look at cross correlation at lag k r 0, 1 k that is the cross correlation at lag k expected r 0, 1 k is 0 and the power or the variance that is expected r 0, 1 square of k equals 1 over N. So, the cross correlation has a much simpler relation, if I look at the cross correlation at lag k the expected value is 0 and the power of the variance in that is 1 over N. So, if you look at it, if you look at these code sequences these long code sequences used by different users they have a very where each chip can be assumed as pro, as randomly generated with randomly generated as plus or minus 1 with probability half or plus 1 as and half for minus 1.

You can see that each of these behaves as a random sequence such that, the correlation expected value of the correlation is 0 and its variance in the correlation, variance of the correlation is 1 over N except for the auto correlation, when the lag is 0 the then the x the correlation becomes that is if you, self correlate a sequence at lag 0 then the expectation, the expected value is deterministic, in fact it is always 1.

So, except for that single case the correlation and the cross correlation are always expected value is always 0, and random and the power or the variance of this correlation is 1 over N, and that plays a big role in distinguishing these information symbols, that are transmitted corresponding to the different users on the in a CDMA system because these codes remember are orthogonal except, I mean remember we started with orthogonal CDMA codes except now, we have moved from deterministic codes to random codes and we are still saying that in the expected sense, the correlation between these codes is 0.

I mean earlier we said the deterministic correlation that is a 0 transpose or c 0 transpose c 1 was 0. Now, we are saying that does not hold in a deterministic sense, but yet in an expected sense that is if, I look at the expected value of code 0 dot product with code one that is still 0, but if I look at the power there is still a small power, small power in the cross correlation which is the variance, which is 1 over N and that slowly tends, that tends to 0 as N tends as the, sequence N becomes as the length N becomes larger and larger that variance tends to 0 alright. So, this plays an important role in multi user CDMA which is the topic, we are going to come to next.

Now, how do this random codes helps in code division for multiple access, that is how do we how do these random codes that we have seen so far help in multiple access of different users on in a CDMA wireless communication systems, how can I multiplex the different information signals compare corresponding to user 0, and one given these random codes c 0 and c 1, let us look at that.

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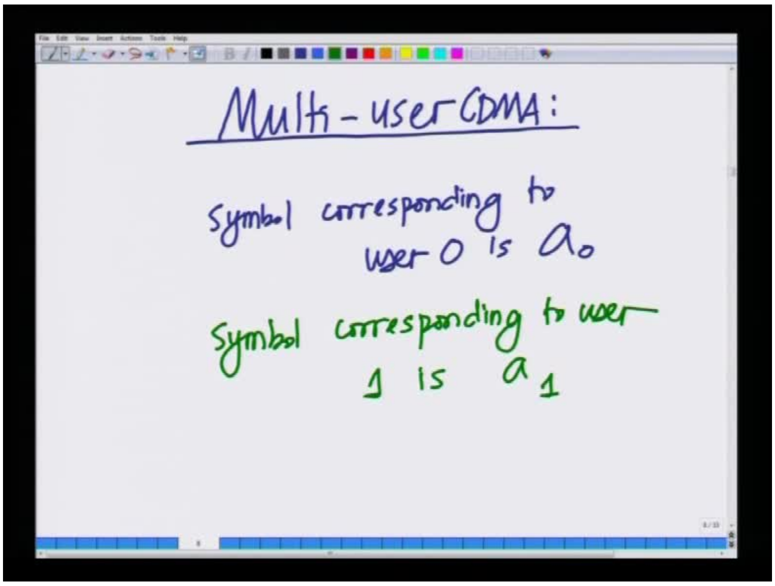

So, let us start with the main topic of this section which is multi user CDMA, if I go into multi user CDMA. First let me assume again as we did before symbol corresponding to user 0 is a 0. Similarly, symbol information symbol corresponding to user 1 is a 1. So, let me write that here symbol to user 1 is a 1 that is a 0 is the symbol corresponding to user 0, a 1 is the symbol corresponding to user 1. Now, we will multiply a 0 by the code sequence corresponding to user 0 and I will multiply a 1 with the code sequence corresponding to user 1 and I will transmit them simultaneously on in the CDMA wireless system.

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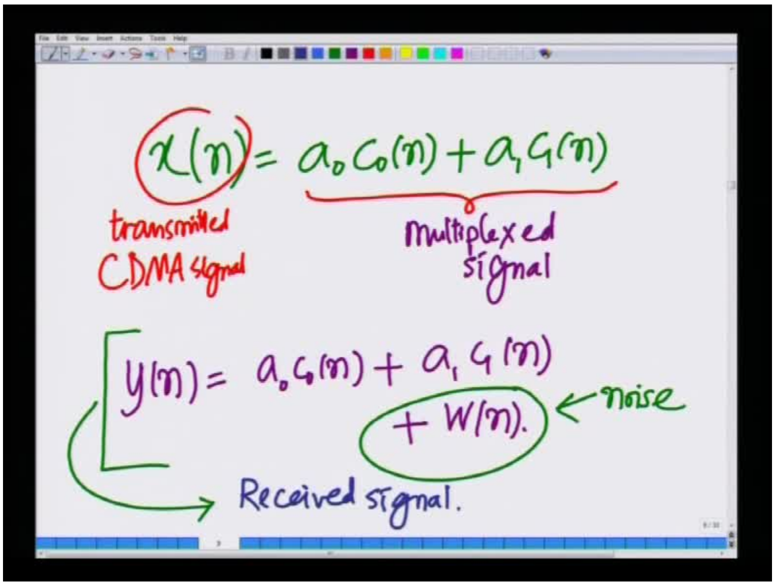

So, let us see what happens when we transmit them each chip y n, which is the transmitted signal or let me in fact call this x n because this is the transmitted signal, this is x n is generated as a 0, c 0 n plus a 1 c 1 n that is the n eth chip of the transmitted CDMA signal. Remember this is the transmitted CDMA signal, which is generated as symbol 0 times chip belonging to c 0 n that is code of user 0 plus a 1, which is symbol corresponding to user 1 times the chip c 1 n corresponding to the code of user 1.

Now, at the receiver what happens is simple I receive this. So, remember this is a multiplexed signal, why is this a multiplexed signal, this is a multiplexed signal why is this a multiplex signal? Because look at this, this signal correspond contains information symbols a 0 and a 1 that is it contains, the symbol corresponding to both user 0 and user 1 at the same time. Hence, it is a multiplexed signal only that the symbols are multiplexed using different codes c 0 n and c 1 n.

At the receiver, you receive this signal I mean this signal is received, in the presence of noise. So, at the receiver we will have exactly x of n except there will be an addition of the noise. So, I will receive a 0 and c 0 n plus a 1 c 1 n plus w of n and this w of n is nothing but the noise, and this is additive byte Gaussian noise alright. Now, what I am so this is the received signal. So, this is the let me write this down again clearly, this is nothing but the received. Now, at the receiver what I will do is I will correlate similar to what we did earlier, I will correlate this signal with the code corresponding to user 0 to extract the signal corresponding to user 0. So, at the receiver let me correlate with 0.

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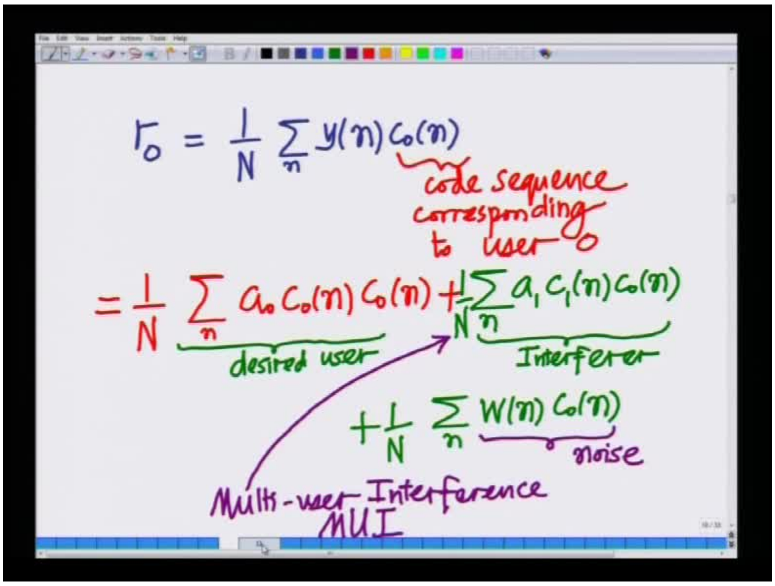

So, let me define r 0 as the correlation 1 over N summation y of y of n with c 0 of n summation over N. That is I am taking y of n correlating it with c 0 of n, which is the code sequence. Remember this is the code sequence, code sequence corresponding to user 0. So, I take 1, 1 over N y of n c of n summation over n, this is the correlating with the code sequence corresponding to user 0.

Now, let us substitute this expression for y of n and see what happens, when we expand this thing expand this received signal that is it get, we get three terms remember y of n has the signal a 0 corresponding to user 0 corresponding to user 1, and it also has the noise components. So, we will have the 3 components here, the first one is y, y n the first one is summation over n a 0 c 0 of n times c 0 of n, this is the term corresponding to user 0. Then we will have the term corresponding to user 1, which is summation 1 over N over n a 1 c 1 of n c 0 of n plus 1 over N summation over n that is the noise, we have the noise term which is w n c 0 of n, this is the corresponding noise term.

Now, let me define some nomenclature at this point we have a 0, which is the desired user view signal, we want to extract that is why we are correlating with c 0 of n. Hence, let me call this as let me call a 0, this is the desired user or the desired component, we are trying to decode the symbol transmitted by user 0 hence, this is the desired user. This user 1, which is not really necessary, but who is interfering with the desired user in the detection is known as the interfering user, or the interferer and this interference in CDMA has a name has a very specific name this is known as multi user interference.

This is known as multi user interference and it is abbreviated as M U I, M stands for multi, U stands for user, I stands for interference. Hence, this is multi user interference, this is a characteristic of CDMA because you are simultaneously multiplexing, the signals information signals of different users they are going to interfere at the decoding process of any user, and that interference is known as multi user interference and this is nothing but the noise component.

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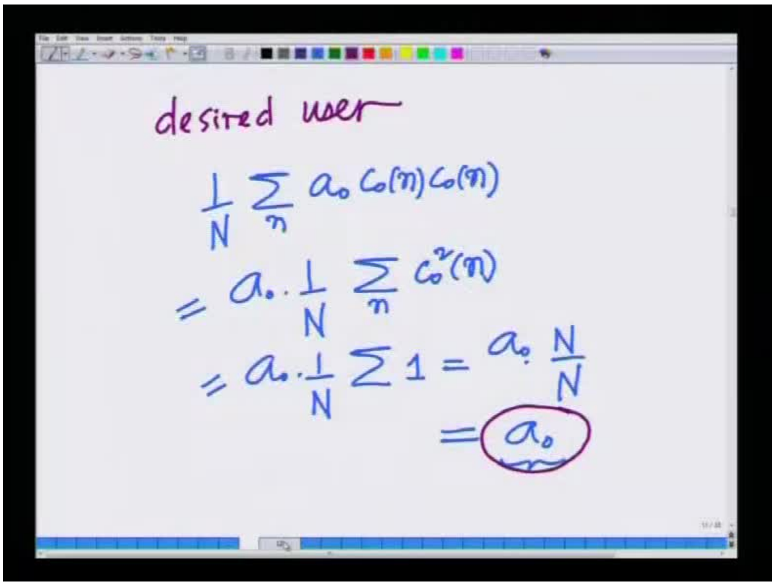

Now, let me write down each analyze each term separately, let me look at the desired user signal, desired user for the desired user, the desired user component that is straight forward that is 1 over N summation over n a 0 c 0 of n times c 0 of n, this is nothing but if I extract a 0 out times 1 over N times summation over n c 0 square of n, and we know that c 0 of n is either plus or minus 1. So, c both cases c 0 square of n is 1 hence, this is a 0 into 1 over N into summation 1 which is a 0 into N over N, this is a 0. So, the result of the correlation of the desired user correlation with c 0 of n, yields nothing but a 0 which is the symbol of user 0 and that is not surprising because what is this?

This is correlation c 0 n with c 0 n that is the auto correlation for a lag of 0, and we said this is the deterministic quantity 1. Hence, once you de spread or de correlate with c 0 of n you get back the symbol a 0 corresponding to user 0. So, this is the symbol 0. So, the result of decoding using a random spreading sequence c 0 of n yields, a 0 which is the symbol correspond to user 0. However, now there is also a multi user interference component look at this, we also have a multi user interference component which is a 1 c 1 n c 0 n. So, let us look at how to simplify this component.

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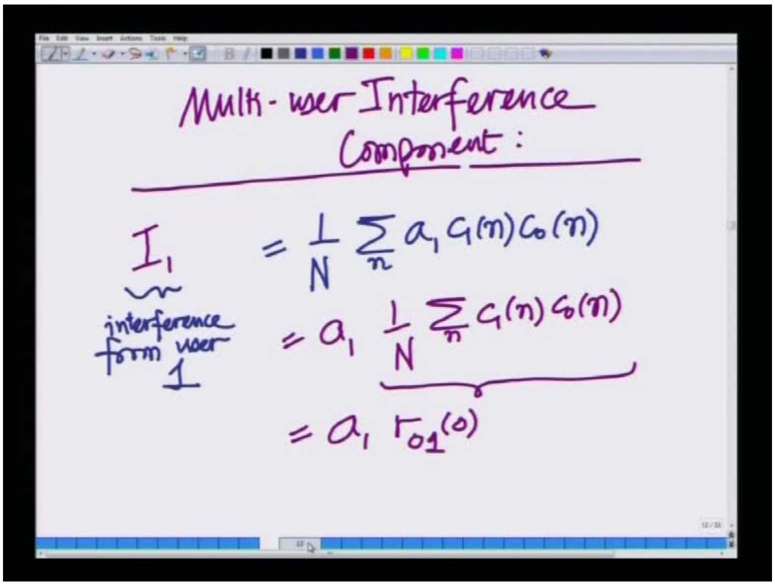

Let us look at the multi user interference component that is the multi user interference. Let us look at the multi user interference component this is given as follows, let me define call this as I 1 this is interference from user 1, this is interference from user 1. So, I 1 is the interference from user 1, this is defined as 1 over N summation over n a 1 times c 1 of n times c 0 of n that is equal to 1 over that is equal to, let me write this in a form that we are familiar with, this is a 1, 1 over N summation n c 1 n c 0 of n look at this, this is nothing but the cross correlation for a lag of 0. Hence, this is a 1 times r 0, 1. So, hence we can simplify this as follows.

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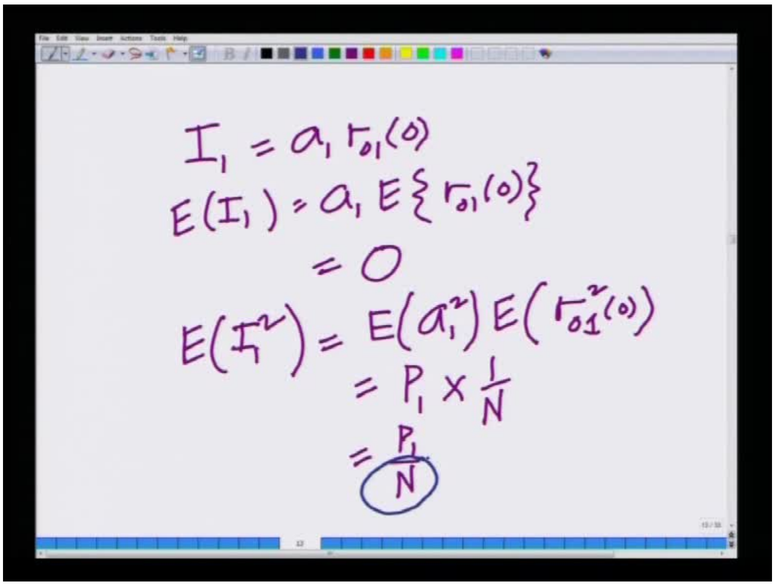

The interference of the multi user interference I 1 is nothing but a 1 times, the cross correlation for a lag of 0 hence, expected I 1 is a 1 times expected r 0, 1 of 0 which is equal to 0 that is what we have seen earlier. And in fact the power in this interfering component that is expected I 1 square, which is the power in this interference component the power in this noise, which is the interference noise because remember it is a distractive influence on the, or a disruptive influence on the signal that we want to detect at the receiver, the power is nothing but expected a 1 square into expected r 0, 1 square of 0.

Now, expected a 1 square is nothing but the power of user 0 because expected a 1 square a 1 is the information symbol transmitted by user 1. Hence, expected a 1 square is the power of user 1 into expected r 0, 1 square that is the power in the cross correlation for lag of 0. We have seen this earlier this is nothing but 1 over N hence, this is p 1 time p 1 over N.

Hence, the multi user interference component is a random variable with expected value 0, it acts as noise with expected value 0 and power p 1 over N, look at this the user p 1 transmits a power, the whole of the power is not acting as interference power p 1 is suppressed by a factor of n which is nothing but the spreading length, or also known as the spreading gain. Hence, a fraction of the power that is transmitted by user 1 is acting as interference at user 0 and that fraction is p 1 over N, and as N tends to infinity that is as a code length n becomes the larger and larger this interference tends to 0.

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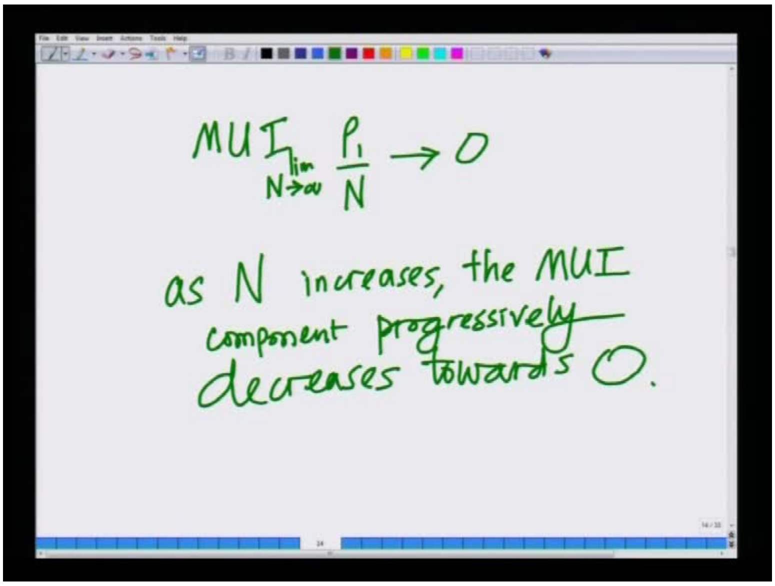

So, let me also note that additional point M U I, the multi user interference p 1 n tends to 0 as n tends to infinity. So, if you look at long spreading it goes as N increases, the M U I multi user interference component progressively, decreases towards 0. As the multi user as the spreading length N increases, the multi user interference component progressively tends towards 0.

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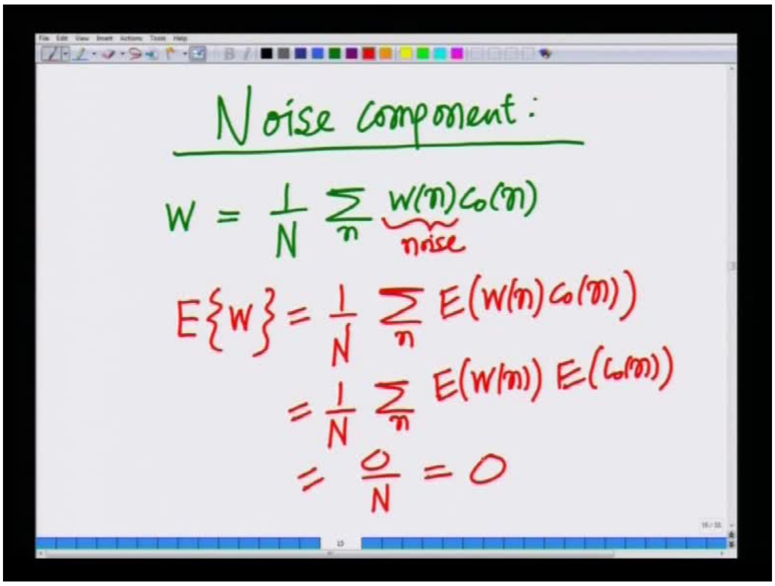

Now there is the third component which is the noise component, and let us look at what happens to the noise component. So, there is a third component, which is the noise component, what happens to the noise, let us look at the noise component, let me call it as w which is equal to 1 over N summation over n w n times c 0 of n. Now, the expected now given that this is noise remember, this arises w n is noise. So, w is nothing but the noise component at the output of the de-correlator.

Remember we are de-correlating with the spreading sequence c 0. So, this is the noise at the output of the de-correlator, we are interested in the power and expected value of this noise. First let us start with the expected value of this noise that is simple, if I look at expected w that is given as 1 over N summation over n expected w n expected c 0 n which is equal to remember w n, the noise and the chip sequence they are obviously, independent because the noise is the noise process at the receiver.

The chip sequence is randomly generated at the transmitter, both of them are independent random processors. So, this is nothing but summation over n expected w n expected c 0 of n and both these expectations that is expected value of the noise and expected value of the chip both are 0 because this noise is also 0 mean the chipping sequence, the random spreading sequence is also 0 mean. Hence, this is nothing but 0 over n which is 0. So, the expected value is trivial that is 0 that is also fairly, straight forward to see that the expected value of the noise at the output is 0 because the expected value of the noise at the input is 0, the system is linear. So, the expected value of the noise at the output is also 0 that is another way to see it, but more importantly. What is the power?

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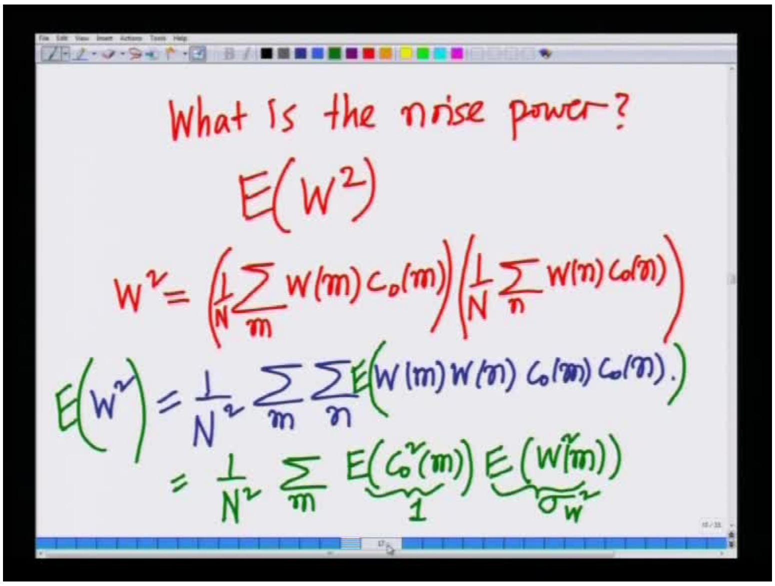

If I were to look at what is expected, what is the noise, what is the noise power that is if I were to ask what is expected w square, what is that value, let us look at that for that we have to look at expected I will write w square as summation over m, one over N summation over m w m c 0 of m, this whole thing product with 1 over N summation over n w of n c 0 of n. I can take, I can combine these two summations as I can combine these 2 summation as w square equals 1 over N square summation m, summation n w of m times w of n times c 0 of m times c 0 of n.

Now, when I take the expected value of w square, if I take the expected value both in the left and the right I will move the expected value direct expectation operator directly inside the summation that is expected w m w m c 0 m c 0 n. Now, similar to the cross correlation between the spreading sequences, you can see that this w m, w n, c 0 m, c 0 n are all independent if m is not equal to n. So, if m is not equal to n, this splits into 4 expectations expected w 0 m into expected w n, expected c 0 m into expected c 0 of n all of which are 0. Hence, this is 0 if m is not equal to n if m is equal to n that is the only term, which survives that is in N square.

I am assuming now, m equals n this now splits into expected c 0 square of m remember m is equal to n. So, c 0 m into c 0 n is nothing but c 0 square of m similarly, w m into w n is nothing but w square of m hence, and w and c 0 are independent. Hence, this splits into two independent expectations, expected c 0 square m into expected w square of m. Now, expected c 0 square of m c 0 square of m c 0 of m is either plus or minus 1.

So, in both cases this is expected, this is one expected w square of m has an interesting interpretation this is nothing but the power in the noise because expected, w square of m it is the average value of w square, which is nothing but the average value of the square of the amplitude of the noise, which is nothing but the power of the noise. Hence, this is sigma w square alright, the noise is w hence this is sigma w square.

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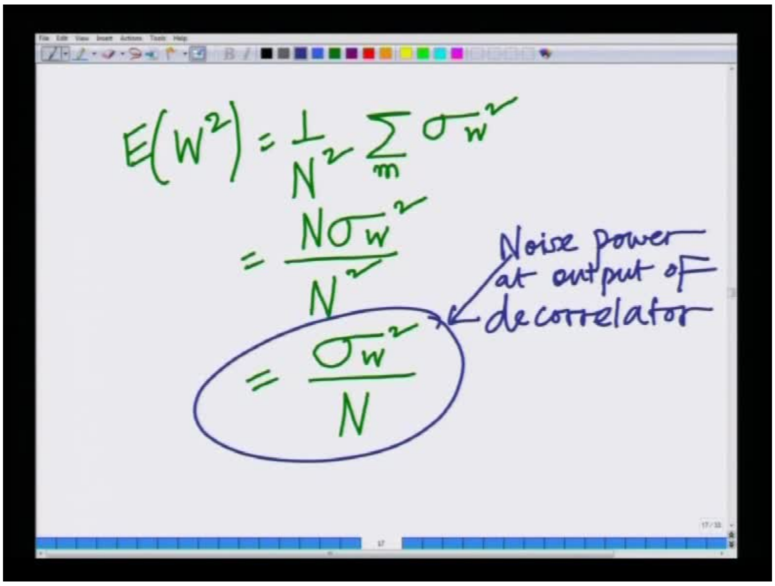

Hence, this expectation reduces to expected value of w square equals 1 over N square, summation sigma summation over n sigma w square, this is nothing but N sigma w square divided by N square, this is nothing but sigma w square divided by N, this is the noise power at output of this is the noise power at the output of the de-correlator.

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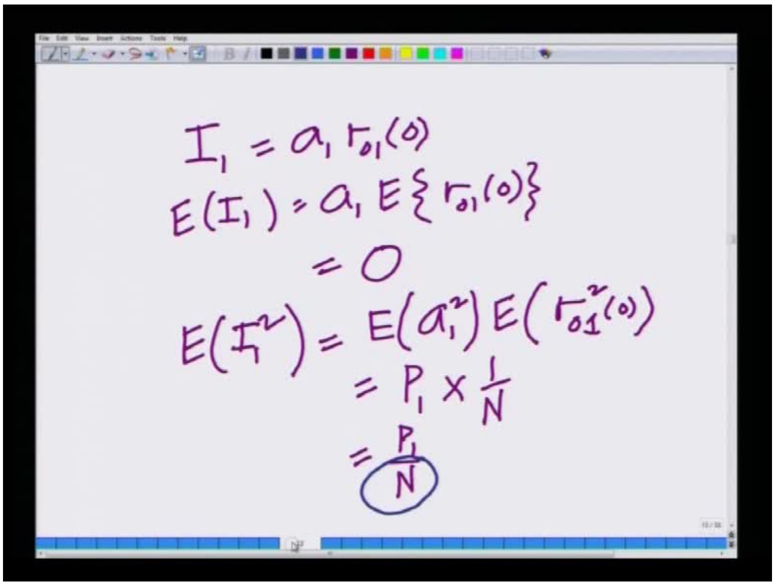

So, we have two components. So, we have looked at these things, we have looked at the multi user interference component, we said that the multi user interference component is p 1 over N.

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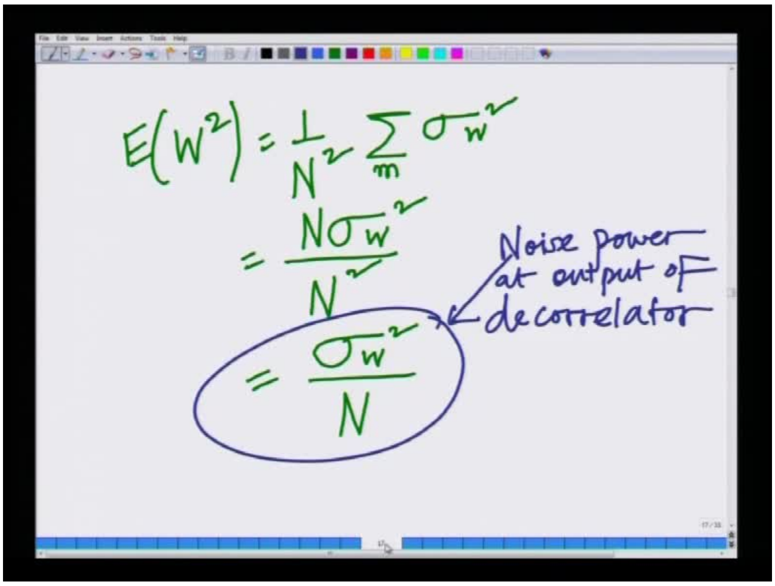

And we have looked at the noise component, we have said the noise component has variance sigma w square over N, look at this this interesting thing that similar to multi user interference even the noise at the output, the noise variance is sigma w square at the input. However, at the output when you dispread or de-correlate with the spreading sequence, the noise power also reduces by a factor of 1 over N and as N tends to infinity that is this you have large codes. The noise power at the output tends to 0 hence, this is an interesting factor, which is de-correlating with the spreading sequence suppresses the interferer and it also suppresses, the noise by a factor of n. Hence, it results in a gain of N in S N R that is the reason it is known as the spreading gain hence, the S N R at the output.

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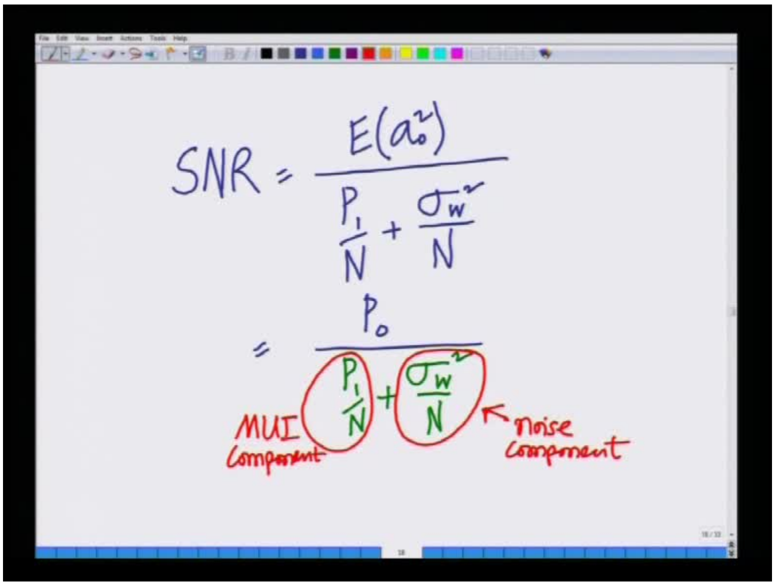

Let us now write the expression for the S N R, the S N R at the output remember the output is nothing but a 0, which is the desired signal in the presence of the interferer which is p 1, which has power p 1 over N and the noise which has power sigma w square over N. Hence, the S N R is nothing but expected that is the power in the desired signal, which is expected a 0 square divided by the power, in the interferer plus the power in the noise, which is sigma w square over N expected a 0 square is nothing but the power in the signal of user 0, that is nothing but p 0. So, that is nothing but p 0 divided by p 1 over n plus sigma w square over N, as we said earlier this is the multi user interference that is the M U I component. And this is the noise component, and look at this both of them are suppressed by a factor of N.

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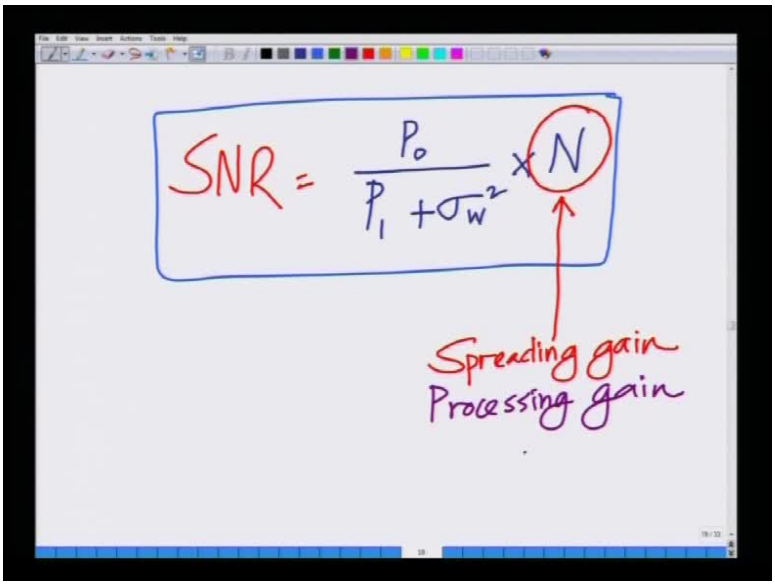

Hence, the net S N R, S N R is equal to at the output of the receiver is equal to p 0 divided by p 1 plus sigma w square into N, this is the expression for the S N R and most important point to note here is that, you have a factor of N that is your S N R, there is a factor of there is a gain of capital N, and as a spreading length N increases your gain S N R gain is increasing, this is known as the spreading gain of the system because it arises from the spreading code.

And this also has another technical name in CDMA, this is known as the processing gain. In a CDMA system there is a gain of capital N, which is the length of the spreading sequence, this is known as either the spreading gain or the processing gain of the CDMA systems. Hence, you can transmit at a very low transmit power because at the output of the receiver, you have a gain of capital N that is why in a CDMA systems typically, you have very low transmit power.

So, the idea is essentially you are transmitting over a wide bandwidth remember CDMA is a spread spectrum system except, you are transmitting at a very low power. So, you are using a wide bandwidth and very low power alright. So, that is the essential S N R of CDMA system. And let us look at the advantages of this code division for multiple access system.

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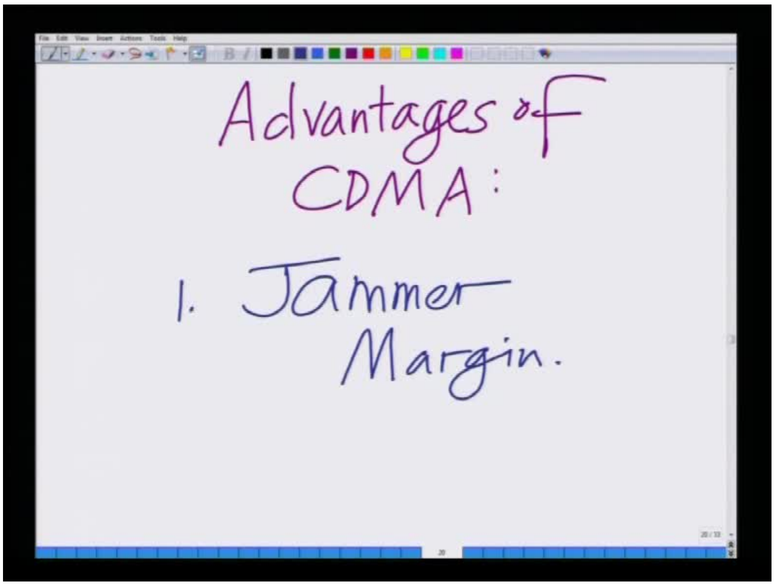

Let us look at the advantages, what are the advantages. So, far we have looked at CDMA that is code division for multiple access, but what are the advantages of this CDMA system over other traditional systems, like T D M A or F D M A based on time division or based on frequency division for multiple access, first the first advantage of this the, first advantage of this CDMA system. The first advantage is what is known as jammer margin. The first advantage is what is known as jammer margin or protection against jammers or protection against jammers.

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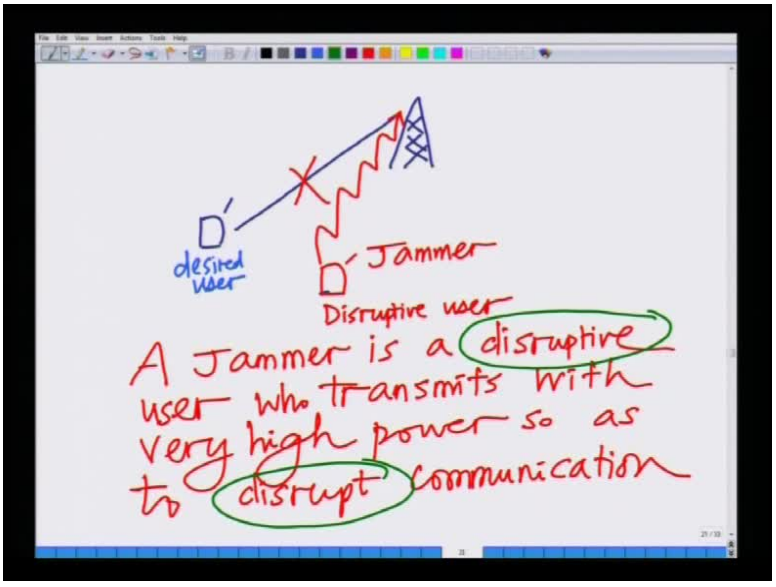

Now, what is a jammer let us let me describe this schematically, let us see I am trying to transmit to a base station, let us say a user desired user is trying to transmit to a mobile station. There might be another disruptive user, let me draw this with a different shade, there must be, there is another disruptive user who is transmitting with a very high power in the same base station to disrupt, the communication of this desired user.

So, this user is the desired user. However, there is another malicious or there is a disruptive user, this is a disruptive user, who is transmitting with a very high power, so as to cause interference and block the communication. So, what is a jammer? This is a jammer essentially. So, who is a jammer? A jammer is essentially a jammer is a disruptive user a jammer is a disruptive user. A jammer is a disruptive user, who transmits with very high power, so as to disrupt communication.

The key word here is disrupt it is a jammer is essentially, disruptive or a malicious user who transmits with a very high power. So, as to cause interference that is to cause heavy interference to you signal so, as to block your communication or stall your communication. Now, so jammers are a significant problems specially, in secure communication. So, jammers are a significant problem.

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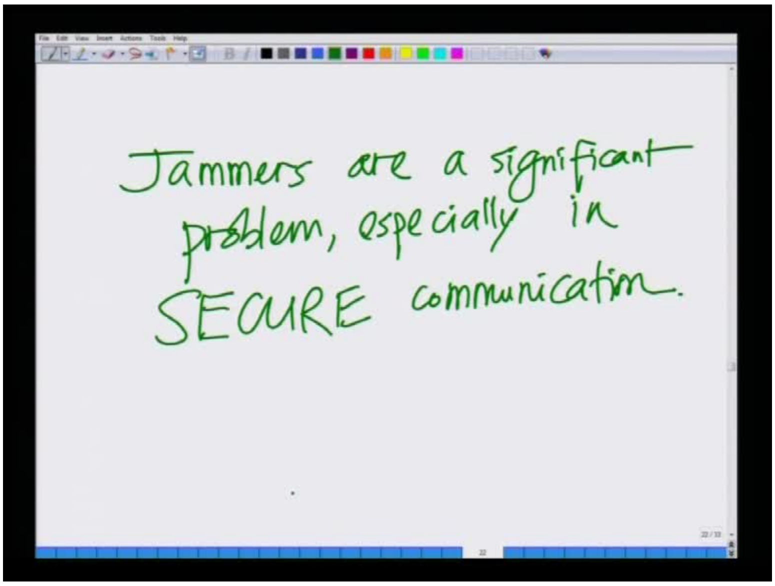

So, jammers are a significant problem especially in secure, the jammers are a significant problem in especially secure communications especially, when you have high security communication link. For instance, if you have military communication links, where information needs to transmitted very securely, jammers can cause a significant problem by disrupting the communication link and in fact, one of the earliest users of CDMA. Therefore, was during the times of World War 2, so that you could provide a secure communication link for the army. So, that it is resilient to jammers. Now, how is this done the idea is very simple. So, CDMA can provide jammer separation or provide a margin against jammer and the idea is very simple, as we have seen here.

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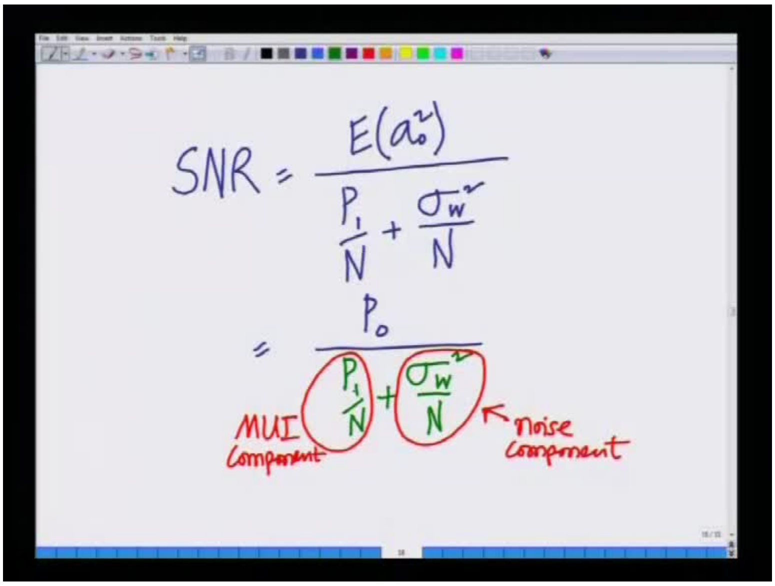

If the interferer transmits with a power p 1, this interference is suppressed by a factor of N. So, you can employ a very large spreading court to suitably suppress, the power of the interferer and the same logic can use to suppress, the power of a jammer. So, let us say a jammer is transmitting with a power P j.

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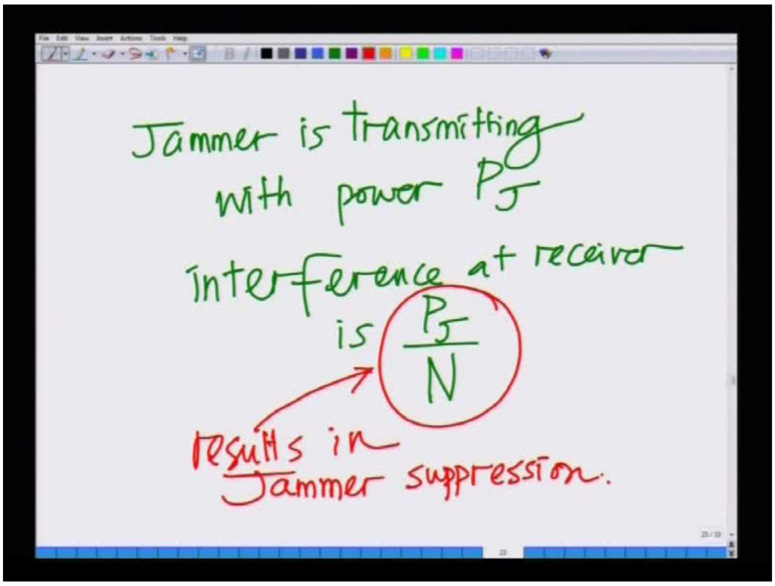

So, let us say jammer is transmitting with power P j then the S N R the interference look the interference power at receiver is P j over N. So, by using CDMA I am able to suppress, I am suppressing the jamming jammer power of the jammer by a factor of N in fact you can employ very large spreading sequences of length 1024, 2048 and so on and so, forth or even in the tens of thousands. So, that this P j over N is suppressed by 20 d b, 30 d b, 40 d b so on and so forth. So, I can employ a suitably long spreading sequence to suppress the jammer power, below a desired level. So, this results in so employing a long spreading sequence results in jammer suppression results in jammer suppression.

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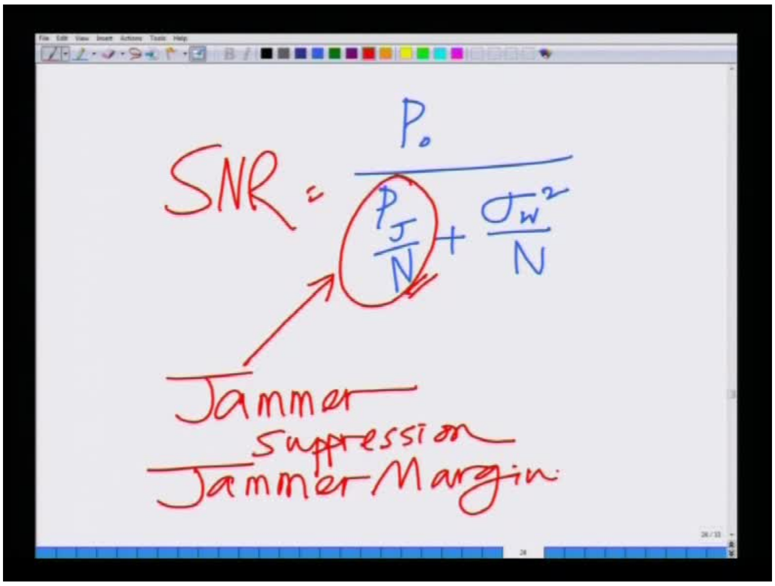

Hence, my CDMA my power at SNR at the output is nothing but as we have seen earlier SNR is p 0 divided by P j over N plus sigma square, w square over N. Hence, by employing a long spreading code I am able to suppress, the power of this jammer. Now, because I am suppressing the power of the jammer, this will not cause significant problem because now, the power of the jammer is reduced by a factor of N.

So, this provides jammer suppression, this provides jammer suppression and this is also known as a jammer margin, a jammer margin is nothing but now you have a more margin in terms of transmit power because the jammer power is being suppressed by a factor of N, if you dint have this margin then you have to transmit at a very high power, so as to overcome the interference caused by the jammer.

Now, the jammer power is being suppressed by using by using the by de-correlating with the code because remember. Now, each user is transmitting along a code, as long as the jammer is unaware of this code and these codes are protected very securely, then the jammer power is suppressed at the output. Which means, now we have more margin in terms of the transmit power. Hence, CDMA is a very key technology in secure communications because it suppresses the malicious users who are known as jammers.

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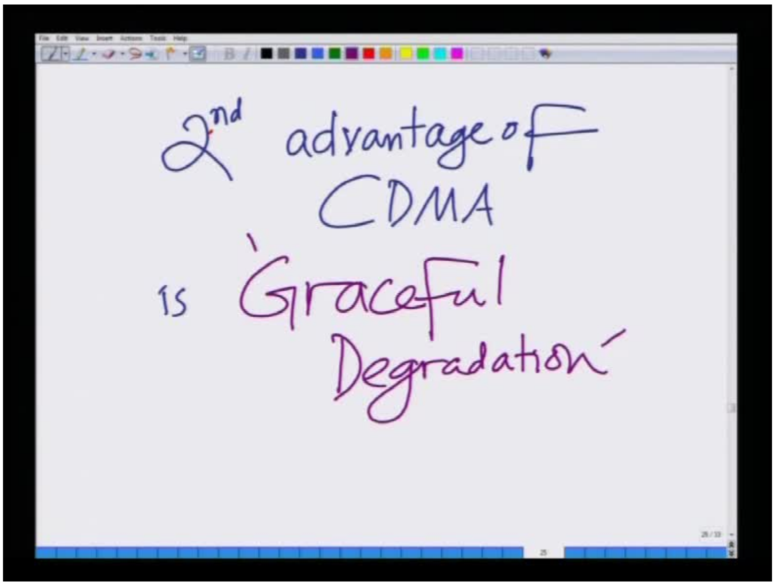

So, let us come to the second advantage of a CDMA system, this advantage is slightly technical. So, we will have to go over it in some detail. So, let us come to the second advantage of CDMA system, the second advantage of a CDMA system. So, let me write second advantage of CDMA second advantage of CDMA is what is known as is graceful degradation, the second advantage of CDMA is also known as graceful degradation. Now, what is graceful degradation for this, we have to build up slight amount of theory.

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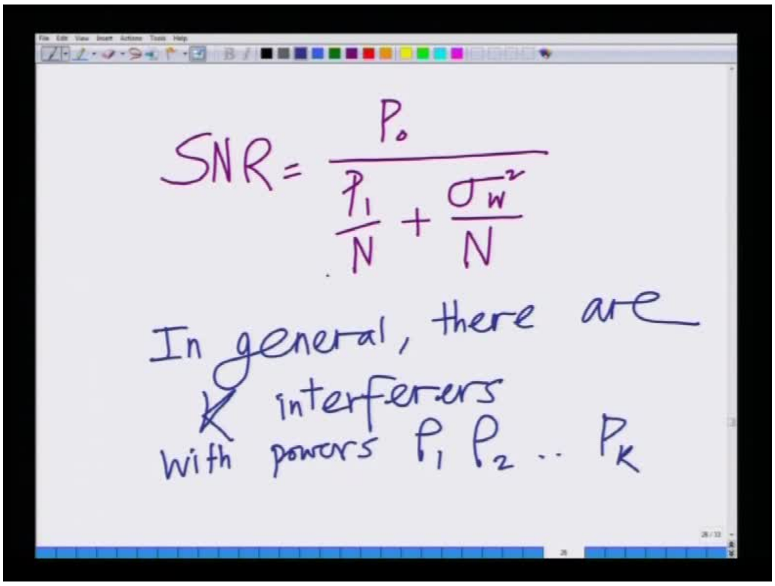

We have seen that the SNR in the presence of two users SNR is nothing but p 0 divided by p 1 over N plus sigma w square over N, this is when I am multiplexing the signals of 2 users that is the desired user is p 0, and the interfering user is p 1 is 1. I can also have multiple users that is I can have interference, that is I can have not only 1 interference, but I can have k interference. So, I can have k in general, there are k interferers in general there are capital k interferers with powers p 1, p 2 up to p k. Now, in the presence of k interference I can simply modify this expression, there will be a factor of p i over N for each of the k users, Hence, the general expression for SNR in the presence of k interference.

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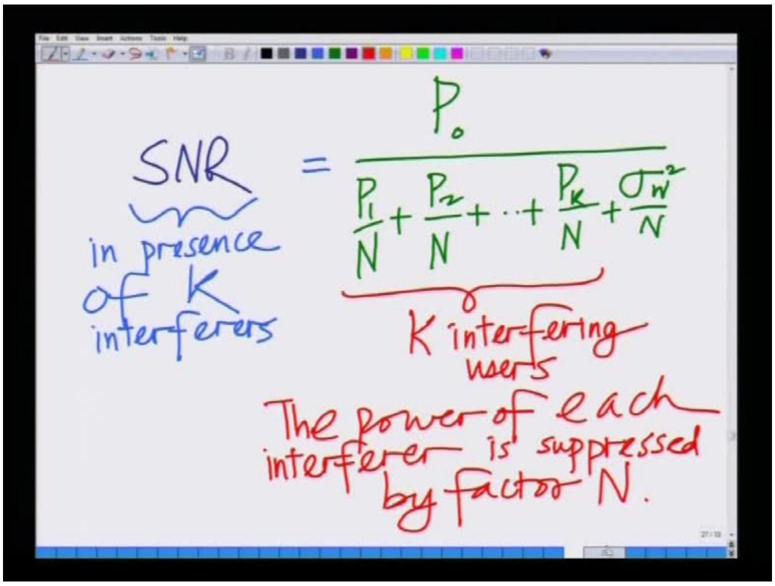

Hence, SNR where this is the SNR in the presence of k interferers, SNR in the presence of k interferers is nothing but is nothing but p 0 divided by the power of each interferer reduced by a factor of. So, these are the k interfering users these are the k interfering users, each the power of each interfering user is suppressed by a factor of N. The power of each interferer is suppressed, or attenuated by N is suppressed by a factor of the power of each interfering user is suppressed by a factor of N.

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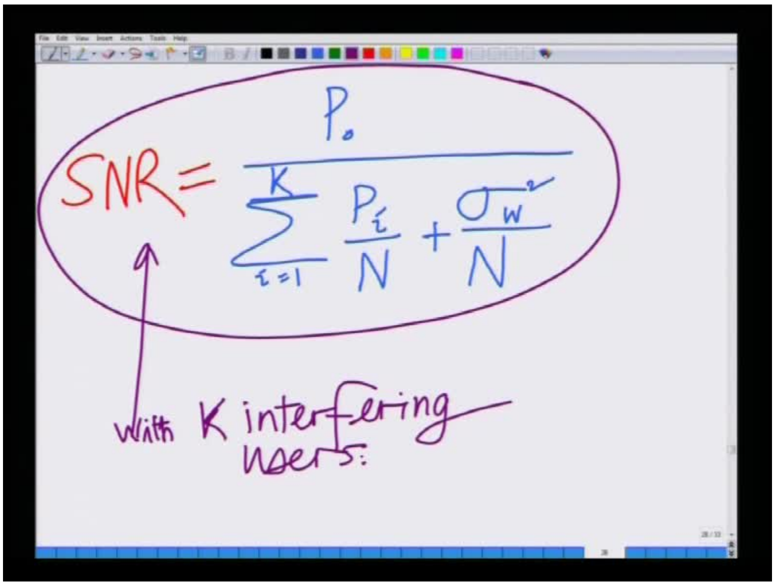

Hence, I can write this succinctly as SNR in the presence of k interfering users is nothing but SNR equals p 0 divided by summation of I equals 1 to capital K, p i over N plus sigma w square over N. This is the power SNR in the presence of so, this is the SNR in the presence of with k or capital K interfering users. Now, let us say there is a new interferer there is a k plus 1 eth interferer, who is the added to the system.

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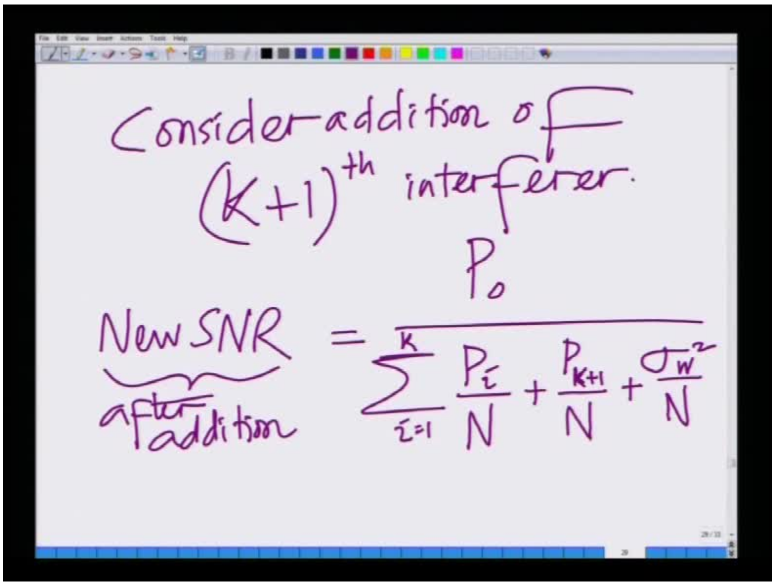

Consider addition of a of K plus 1 eth interferer, what happens when you add a K plus one th interferer, that interferer we interfere with desired user 0. However, his power will also be suppressed by a factor of N. Hence, the new S N R or the new S N R this is after addition, after addition of K plus 1 eth user is nothing but p 0 divided by summation i p i over n i equals 1 to k, there is also a K plus 1 eth user, whose power also adds as interference power p k plus 1 over N plus sigma w square over N. So, the newly added user that is the K plus 1 eth user causes an interference of only P k plus 1 over N at user 0. Now, this user K plus 1 will also cause an interference of use of P k plus 1 over n at user 1, he will cause an interference of P k plus 1 over N at user 2 so on and so forth, he will cause an interference of P k plus 1 over N at user K.

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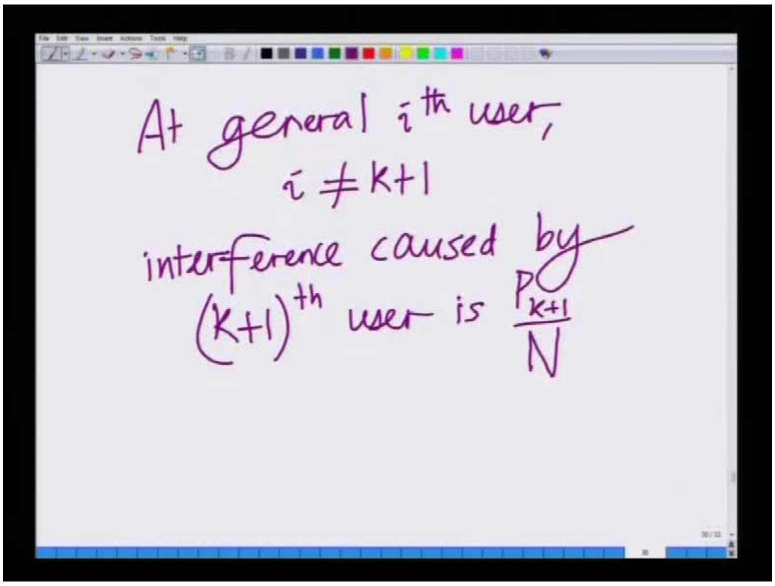

So, at the general i eth user, at the general i eth user, i not equals K plus 1 that is at any i eth user, i not equals K plus 1, interference caused by K plus 1 eth user is P k plus 1 over N. Hence, the interference caused at any i eth use, at any i eth user, i equals 0 1, 0 1 up to capital K, i not equals that is i not equals K plus 1 the interference caused at each user is power P k plus 1 over N.

Hence, his interference power is being equally absorbed by all the users in the system that is he is not causing more interference at one user, and less interference at another user rather his interference power is being equally divided amongst all existing users, this leads to what is known as graceful degradation, because the interference power is absorbed by each user equally, the performance of each user decreases a little bit symmetrically, rather causing too much interference at 1 user, and bringing the power of that user down completely and leaving the other users unaffected, this is affecting each user by a small amount, because this is distributing the interference. Hence, this leads to graceful degradation compare to other existing systems before and this is a very important property. So, because of the time I have to end this lecture at this point, we will start the next lecture again at this point and complete this point and look at other advantages of CDMA.

Thank you very much.