Circuit Analysis for Analog Designers
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Lecture - 03
Power Conservation and Tellegen's Theorem

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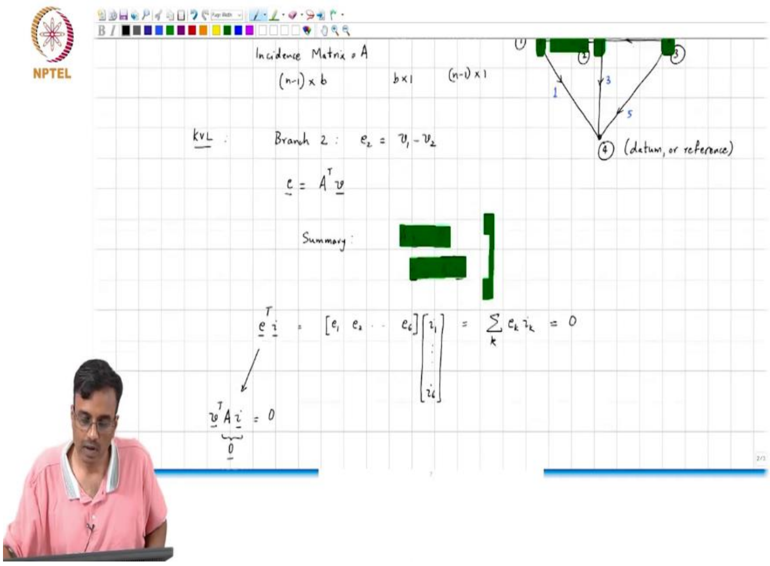

One interesting aspect that I would like to point out is the following. What interpretation can we give to this product $\underline{e}^T\ \underline{i}$. This is nothing but in our particular case be sum over all the branches. What is this? Let me write this out in full form. So, this is nothing but e is a column vector. So, e transpose is a? Row vector.

And how does that look? $[e_1 \quad e_2 \quad \cdots \quad e_6]$

First of all are we is this product legal, what is the size of e transpose? 1×6. i is 6×1. So, the product is legal, so that is simply this multiplied by i1 all the way to 6 right.

$$\underline{e}^T\ \underline{i} = [e_1 \quad e_2 \quad \cdots \quad e_6] \begin{bmatrix} i_1 \\ \vdots \\ i_6 \end{bmatrix}$$

And so this is nothing but sigma over k e k i k.

$$= \sum_k e_k\ i_k$$

What interpretation can we give to this quantity? It is the sum of all instantaneous powers being dissipated in the branches correct ok. And intuitively what do you expect that to be?

If you take all the branches in a network right and compute sigma $e_k\ i_k$ which is basically saying it is telling you each $e_k\ i_k$ is simply the instantaneous power that is being dissipated in the kth branch correct. And the summation is basically adding up all the powers in all the branches at any instant in time. So, what would you expect to see if you form this product sum for all branches $e_k\ i_k$? You expect this to be 0. And why does that make sense?

Law of conservation of power right. Remember power and energy are two different things. Power is an instantaneous quantity. Energy is an integral right. Energy is of course conserved, but this is also telling you that energy is conserved at every instant. So, nature does not for good reason believe in saying you know at this point in time you know you take more energy than you generate you return it to me at a later time right.

Because nature seen too many scams in the past right. So, it knows that you know you should neither be a borrower nor a lender right ok. So, at every time, the you know power is conserved right. So, total power generated and total power dissipated in all the branches is the same ok.

So, let us see if that is being predicted by our kcl, kvl equations right. And so what is e transpose? e transpose is simply v transpose A and of course this is i alright, and we know that this guy is 0.

$$\underline{e}^T\ \underline{i} = \ \underline{v}^T A\ \underline{i} = 0$$

So, this must be the scalar 0 alright. No, it is this A right ok. So, this is simply saying I mean it may not be it is not very surprising that this is true right. It is probably something that you expected anyway, but what is surprising is the following. So, let us say we have one network like this.

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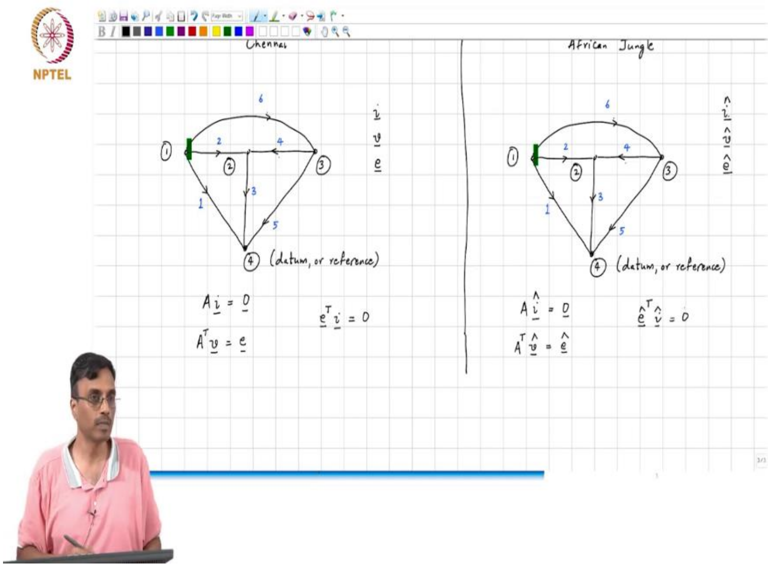

So, this is let us say a network that we have in Chennai and somewhere deep in the jungles of Africa right. You have somebody who is totally unknown to you set up another network with exactly the same graph alright. The only commonality between both these networks is that their skeletons are the same.

One could be linear, one could be non-linear, one could be time invariant, one, the other could be time invariant, time variant, it does not matter. All that it all that we say is that these two networks are topologically identical right. Now, of course, kcl and kvl must be universally valid. So, A, so what comment can we make about the incidence matrices of both these graphs? Of both these networks is the same. So, A times i is equal to 0 right.

$$A\,\underline{i} = \underline{0}$$

And what is kvl here.

$$A^T \underline{v} = \underline{e}$$

Alright, and this is of course, another network. So, apart from the incidence matrix being the same, there is no other relationship between these two networks. And this is basically let us call these branch currents $\underline{\hat{i}}$. And therefore, $A\,\underline{\hat{i}} = \underline{0}$. And what comment can we make about this kvl here? So, the branch volt current vector and the branch voltage vector and the node voltage vector here are $\underline{i}$, $\underline{v}$ and $\underline{e}$; here they are $\underline{\hat{\imath}}$, $\underline{\hat{v}}$, and $\underline{\hat{e}}$. So, what comment can we make about?

A transpose times V hat equal to e hat alright ok.

$$A^T \underline{\hat{v}} = \underline{\hat{e}}$$

And we know obviously, that e transpose times i equal to 0 alright. $\underline{e}^T \underline{i} = 0$ And of course, e hat transpose times i transpose times i hat must also be equal to 0.

$$\underline{\hat{e}}^T \underline{\hat{v}} = 0$$

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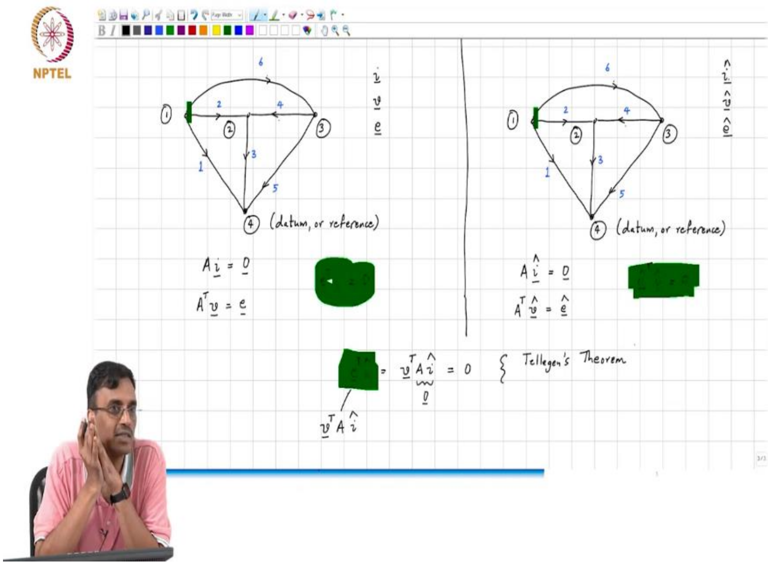

But let me show you an interesting thing and that is let us try and form the product $\underline{e}^T\ \hat{\imath}$ alright. And what physical interpretation if any can we give to this quantity, what are the dimensions of $\underline{e}^T\ \hat{\imath}$?

It is a scalar and it is also got dimensions of power. The only difference between what we did here and what we are doing now is that to compute within quotes the branch power, we are taking the voltage in one branch. And the current in. In a network which is you know thousands of miles away we do not know what the network is apart from the fact that we just know its graphs right. Now, let us see what happens when we do that right, and that is e transpose is what it is v transpose times. A. And what is i-th well this times i hat ok. So, what is this, what is this equal to?

$$\underline{e}^T \hat{\imath} = \underline{v}^T A \hat{\imath} = 0$$

And this is. Oh, well, this is the 0 vector. So, what comment can you make about this quantity? It is 0 right ok. So, this throws up the somewhat surprising the fact that these two are 0 is no surprise at all right. They have something that you physically expect ok. But what is very surprising is the fact that you take the did you find this within quotes some power like quantity where you multiply the voltages in one branch right and the current you take from a network which is completely unknown to you the only thing that you know is that it is.

It has the same graph, surprisingly it seems as if that is also equal to 0 right. We will take a look at the intuition behind why this is true. It is not as surprising as you might at first think right at first you might think that you know how does the network know what current is flowing in the branch. In some network that it does not even know correct and magically all these products add to 0 correct.

So, to show you why this makes it is not as surprising as it one might seem. And you know you might also think that well you know with this all this matrix stuff there is no intuition right I mean yeah it just turns out the math comes out to be you know if I do the math or evidently the answer is 0. So, it must be the truth right. But is there intuition behind this result; and this result is a very important result and this is what is called Tellegen's theorem right.

And quite surprisingly it was discovered somewhat late given that people have been working with circuits for a really long time, this theorem came out sometime in the late 1950s I think ok. You would have thought that somebody would have thought of this within quotes obvious observation a long time ago, but it is actually quite recent relatively speaking.